Radiation Heat Transfer
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Lecture - 8
Radiation In enclosures

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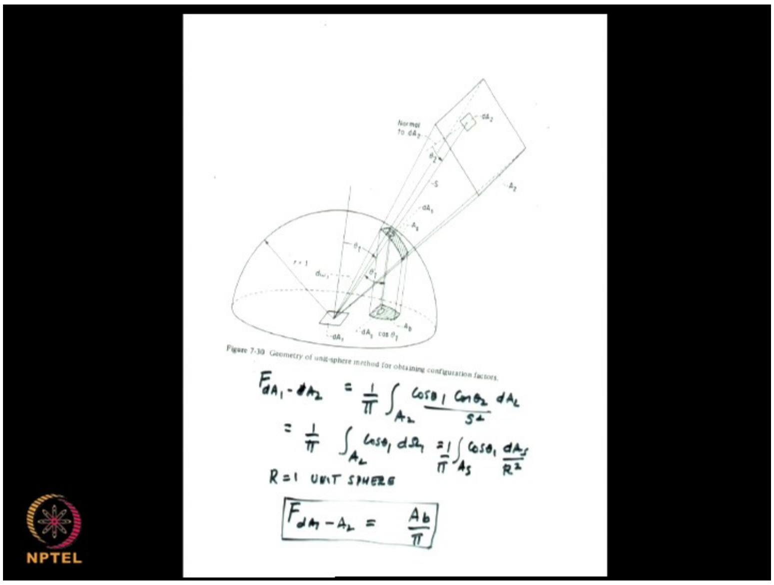

In the last lecture, we talked about an empirical method for finding shape factors; this is used in the industry when we have complex shapes and you may not easily able to find standard expression for that. Here is an example; where we have the object d A 1 here and we want to find d A 1 to d A 2, and so we put a translucent sphere on top; so that the image of d A 2 will fall on this unit sphere. Then we take a camera and take a picture of this area A 2 and we will get this area A B as a picture.

Now, we want to show that $F_{dA1\text{-}dA2}$ is nothing but area of that image by pi. So first we look at the expression da1-dA2; which is $1/\pi \int_{A2} \cos\theta_1 \cos\theta_2 \, dA_2 / S^2$ where θ_1 is the angle between the line joining these two areas, and the normal of d A 1. When we come on this sphere, we know that the line joining the center of this hemisphere to the object; d A S is normal to that line. So θ_2 here is 90 degree. The $\cos\theta_1 \, d\Omega_1$ are substituted by d S and d A 1 on a sphere. When we integrate this we will get $A_s \cos\theta_1$. This is nothing but A b. Finally, we will get $Ab / \pi R^2$; if we take R as unity then we get Ab/π. So this is the simple way to calculate shape factors empirically and it is useful in particular situations, where the objects have compact shape and we may not be in a position to do either the double integration or obtain the shape factor from any simple tables available in books. With this, we have covered all aspects of both estimating the radiative property of surfaces, that is emissivity; absorptivity; and reflectivity. Also we have covered how to calculate the fraction of radiation, leaving one surface which arrives on another surface. We now have got all the tools necessary to solve real problems. The real problem we are dealing with is, in radiative heat transfer between surfaces.

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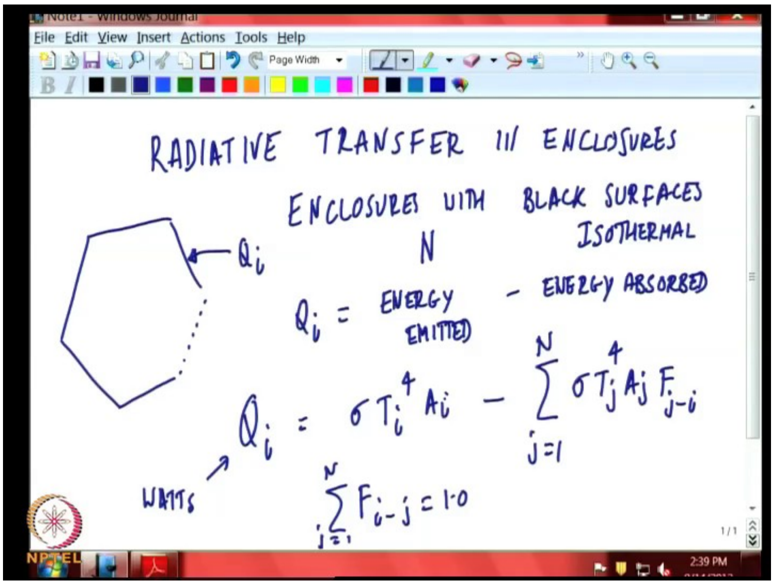

We will now look at radiative transfer in enclosures. Enclosure is nothing but a series of surfaces, which enclose a given area. Not all surfaces in enclosure need be actual surfaces, some of them can be imaginary surfaces defined for convenience. So here enclosures consisting of 1, 2, 3, 4, 5 surfaces and an opening which we can close by a dotted line, that is a virtual surface. We have 6 surfaces in the enclosure. The concept enclosures is useful because ultimately the radiation that leave any one surface has to reach some other surface within the enclosures. It is a closed system and it makes analysis of heat transfer very easy. As the first example we take up an enclosure consisting of only black surfaces. There is no reflection involved in the simplest case and this makes the analysis extremely easy.

First we look at this case and develop our methodology, then we go on to more complex cases. Let there be N number of surfaces in the enclosure and for each surface , let us say is the i^{th} surface. The heat is arrives at Q_i and we assume all surfaces as isothermal and uniform. This is not a very restrictive assumption because in case the surface is not isothermal. We can divided it into 10 sub surfaces, where each of them can be assumed to be isothermal. We can divide any real surface in which, there is a temperature gradient into large number of small surfaces; each one of them can be assumed to be isothermal. This problem is easy to do and today with the availability of computer and the surfaces can be as large as 1000 or 5000, there is no problem; one can easily solve these equations.

Hence with the i^{th} surface of the enclosure the energy added is energy emitted minus energy absorbed. The energy emitted by the i^{th} surface is nothing but $\sigma T_i^4 A_i$; that is the total radiation emitted by the surface in Watts, and the radiation absorbed. We look at the j^{th} surface; which emits this much radiation and ask what fraction of this radiation reaches surface i, that will be F_{ji}, which is a fraction of the radiation emitted by the j^{th} surface which reaches the i^{th} surface. We sum over all j surfaces in the enclosure, all the N surfaces of the enclosures. We get an expression, which relates the heat added to the enclosure. If the amount of radiation emitted by the surface A_i is larger than the amount which is absorbed; we have to add heat to surface to be keep it isothermal and in steady state and that heat is Q_i.

If the energy emitted is more than energy absorbed, we have to add heat , that is Q_i is positive. On the other hand, if the energy emitted is less than energy absorbed and you remove heat, then Q_i is negative. Now we also know as we go on to simplify this equation; we know that, $_{i=1}\sum^{n} F_{i-j} = 1$. We know that sum of all shape factors in an enclosure is equal to 1. We can multiply this by 1.

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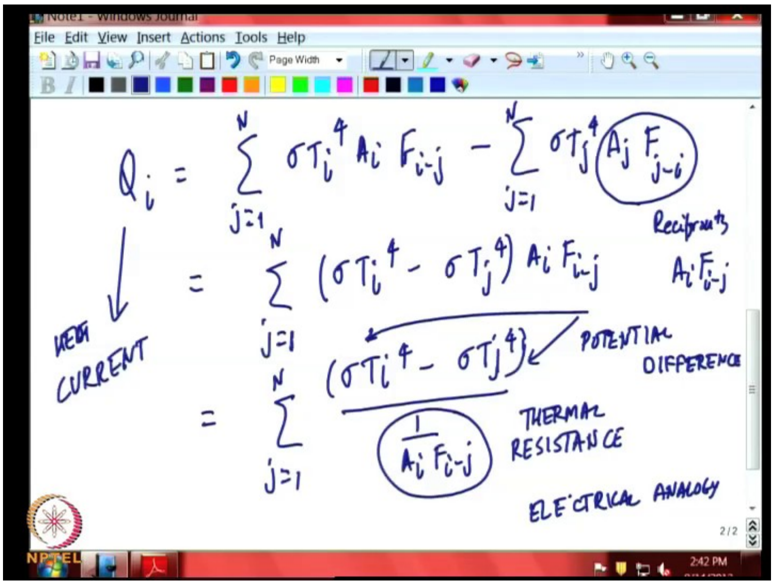

We are going to write it as $_{j=1}\sum^{n}\ \sigma T_i^4\ A_i\ F_{i\text{-}j}\ -\ _{j=1}\sum^{n}\ \sigma T_j^4\ A_j\ F_{j\text{-}i}$. By reciprocity A_jF_{ji} is A_iF_{ij}. We can take A_iF_{ij} common and rewrite this as $(\sigma T_i^4 - \sigma T_j^4) / A_iF_{ij}$. Now we have a very simple expression and this can be written further in a more elegant form, which enables us to interpret it physically. We want to now interpret this result, which we have obtained regarding radiative transfer between different surfaces as the potential difference between the node i and the node j divide by a resistance. We look upon this as a thermal resistance to radiation and this term really is the potential difference. We have an electrical analogy, for radiation. Let us call it as the heat current. This heat current is equal to difference in potential between the two nodes i and j divided by the resistance between the two surfaces.

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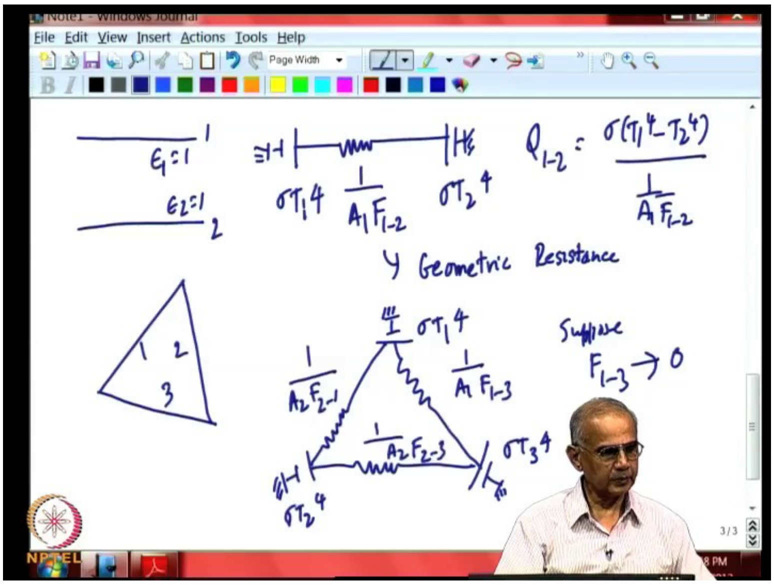

Let us now illustrate this method with a simple example. Suppose we have two parallel black infinitely long plates with surface 1 and 2. By electrical analogy, we will say surface 1 as a potential σT_1^4 and the surface 2 is σT_2^4. Q_{1-2} is nothing but the difference of the potentials of the two surfaces divided by the resistance $1/AF_{1-2}$.

We have two surfaces and the flow of heat or current. The current in the analogy is due to the potential difference, which is σT_2^4 in this case divided by the resistance. We notice that, this occurs primarily due to the geometry of the problem. This we will call it as geometrical resistance because this resistance arises on account of the relative configuration of the two surfaces, which is a consequence of geometry of the problem and this is an interesting example and but for only two surfaces.

Suppose, we have three surface enclosures; 1, 2 and 3 as shown in the above figure. We will have three potentials with a resistance between two each potentials which are $1/A_1 F_{1-3}$, $1/A_2 F_{2-3}$ and $1/A_2 F_{2-1}$ which forms a triangle. By electrical circuit theory, we should be able to solve this delta network using standard theorems in electrical engineering. But we must remember that this is only electrical analogy for the convenience of one's understanding of the problem.

Today in reality as soon as we go to more than two surfaces we are going to use a computer. We will write down the expression for the various surfaces and we will get three equations, for the three unknowns T_1, T_2 and T_3 and we will invert this matrix very quickly, using computer. So the final method solution is a matrix algebra, but in order to have a physical understanding of the various factors, controlling the heat transfer between surfaces it is good to draw electrical network.

For example, suppose this is very small, tending to 0. Then this circuit will open up, and the current will flow from 1 2 3 only through 2. No current can flow through the other arm. We are going to use electrical analogy mainly to understanding, what is happening in a given radiative transfer problem. The actual numerical solution is based on solving a set of simultaneous equation, using standard tools available today on the computer. So, this was for enclosures consisting of black surfaces.

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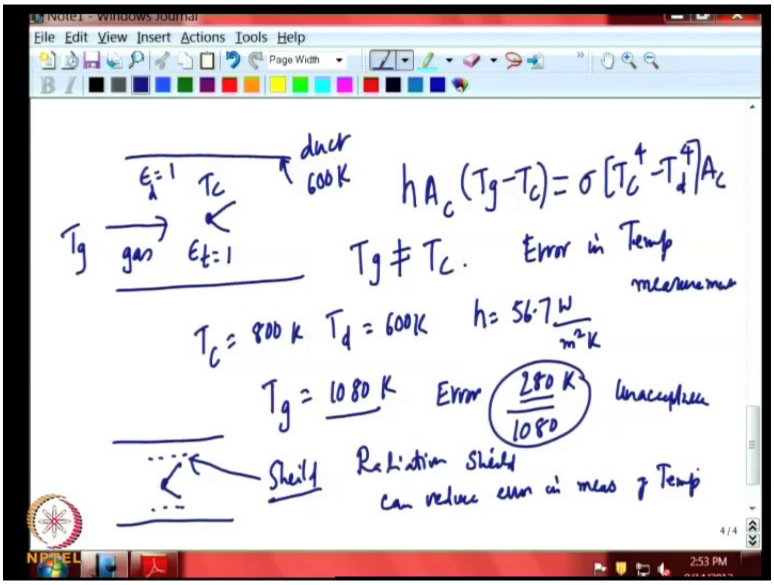

Now we move on to something more ambitious and look for radiative transfer between non-black surfaces. Before that we give one example. Imagine a duct and let us say, we are trying to measure the gas flowing in the duct. Using a thermocouple let us assume that, the walls are at 600 K and the duct is black, and the thermocouple is also assumed to be black. We want to know, what is the error in the measurement of temperature by thermocouple because the thermocouple is supposed to measure the temperature of the gas flowing in this stream. But the temperature recorded by thermocouple T_c will not be equal to T_g. This is because heat transfer from the gas to thermocouple will be by convection and that can written as $hA_c(T_g - T_c)$ and this has to balance, the heat loss by the thermocouple to the duct.

On account of this balance, T_g will not be equal T_c and our interest is to understand what is the error in the temperature measurement. Our aim is to measure the temperature of the gas, but the thermocouple will not get the temperature of the gas to increase because the temperature of the thermocouple will be in between the temperature of the gas and temperature of the duct. If we take a typical example suppose, the thermocouple is reading 800 K, and the duct we assumed is at 600 K, and let us assume that the heat transfer coefficient in this case for convenience is around 50 Watts per square meter per Kelvin.

Typical for gas flow at reasonably high velocities, we will get the temperature of the gas as 1080 Kelvin. So the error of measurements of gas temperature is quite large. It is the difference between the thermocouple temperatures and gas temperature 280 degree Kelvin, divided by the gas temperature. This error is pretty large so an error of 280 Kelvin is unacceptable. Hence what one can do to reduce this error, is to shield the thermocouple. The thermocouple barrier shield, which ensures that the thermocouple does not directly see the wall of the cold wall of the duct there by reducing this error of 280 Kelvin to much more reasonable number. Now this we will see in the subsequence example, how radiation shield can reduce error in measurement of temperature.

As a major fact in meteorology , where routinely the temperature is measured outdoors, it shows the error is a fact, that sunlight may directly fall on the thermocouple or on the thermometer. We tend to either shield thermo meter or provide a special enclosure, which is a shield inside which the thermometer is kept. All these are attempts to show that the thermo couple or thermometer, actually measures the gas temperature or air temperature and not the temperature due to the heating by the sun or due to the fact that this thermometer is losing heat to a cold sink.

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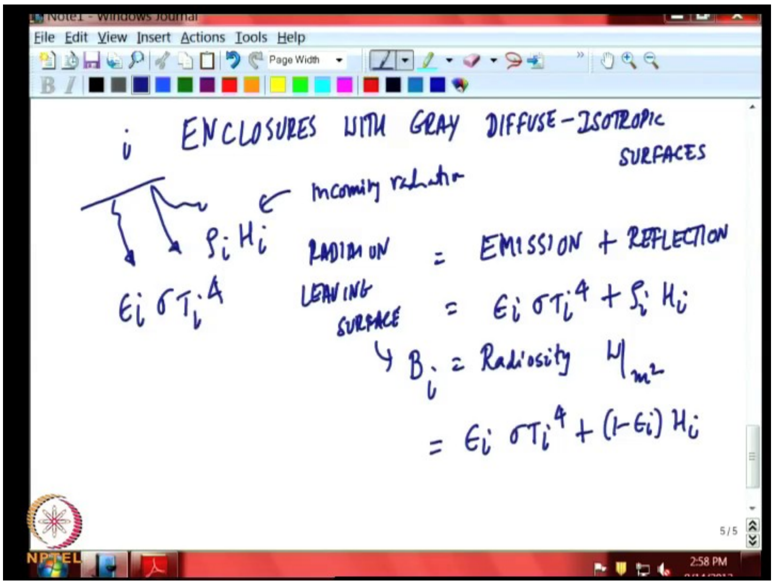

We will take look at enclosure with gray diffuse isotropic surfaces. The problem now is little more realistic, not just enclosures with black surfaces; which is rather rare. Now we have surfaces, where all the surface enclosures are diffuse isotropic. That means they reflect and emit radiation in a diffuse isotropic manner. The angular effects are very simple, but we will make the approximation of the gray surface, which we had earlier argued is not very common, which is a starting point for our analysis.

We will start with the gray surface, but later on we will also look at non-gray. Now in a gray surface enclosure, suppose the surface i not only is the surface, emitting the radiation but which is also reflecting radiation. The reflected radiation is reflectivity of the surface ρ_i H_i, which is incoming radiation. The total energy leaving the surface is a sum of emission and reflection, radiation leaving surface is equal to emission plus reflection.

As we count the photons coming out of a surface, either they are photons emitted by surface or they are photons, which are reflected from that surface. This is $\epsilon_i \sigma T_i^4 + \rho_i H_i$. Now this quantity will come frequently in gray enclosures because in context of black enclosures, we will have only emission, but no reflection. In a gray enclosure, we must worry about both emission and reflection. Since this is going to come very often we will give a name call radiosity.

Radiosity is nothing but radiation leaving the surface that is the sum of emission and reflection per unit area. The radiosity leaving the surface unit area (Watts per meter square) consist of two components, the emission and the reflection. Since we are dealing with gray diffuse isotropic surfaces, we can write this also as $1 - \epsilon_i H_i$ using Kirchhoff's law.

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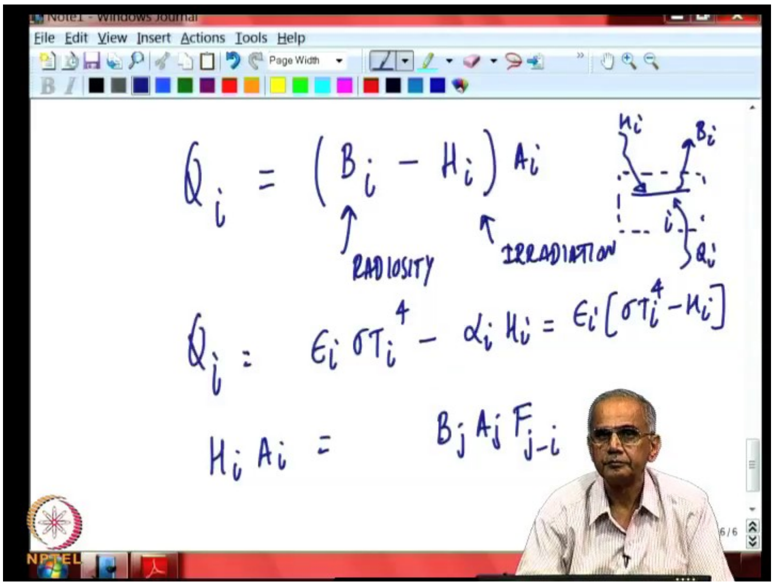

This is a basic building block, of the radiosity is as follows; first staying on the ith surface, the heat leaving the surface is nothing but energy leaving the ith surface. Energy inside on the ith surface multiplied by A_i. If we take a surface i and put a little control volume there, we can see that radiation H_i is coming and radiosity B_i is leaving. The difference between B_i and H_i is a measure of how much heat we must add to maintain this surface at a constant temperature T_i. This is a simple equation, which can be also written in another way. Heat to be added is also equal to black energy emitted, another energy absorbed, both these are equivalent; one is treated by control volume, we actually do not worry about the process of absorption, deflection and emission.

We merely count the photons coming in and photons going out and the difference in medium has to balance by the heat order. On the other hand, if we look at the surface and ask how much energy is absorbed, this energy coming into surface absorbed is α_i. Using Kirchhoff's law we can add this as, that is also an equivalent statement of the same expression. One is through the concept of radiosity and other is through the concept of absorption minus emission. Both are equivalent and here we will be using this interchangeably to get information about any given problem.

The next question we would like to ask is what is H i. The energy arriving at the surface i, $H_i A_i$, has to be equal to energy leaving surface j, $B_j A_j$ times the fractional radiation leaving j and arriving at i. So the logic is same as what we did in black enclosure except that, instead of σT^4, the black body emission, we are using the concept of radiosity. Hence radiosity is a convenient way to handle problems by both emission and reflection. It combines both total energy leaving surface j into area of j and this is a fraction arising at surface of i. Now we sum over all surfaces j 1 to n, which is your incoming radiation at surface i.

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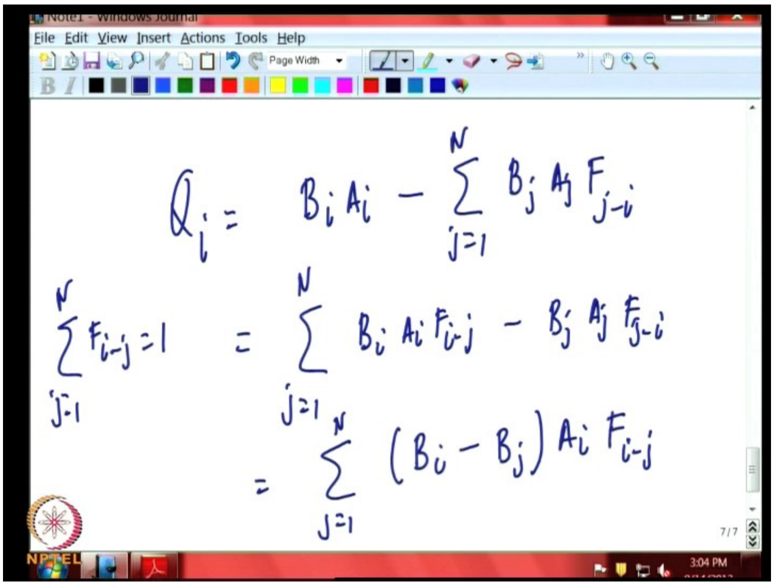

Now let us combine all that and see how it looks like. In surface i, where it is leaving surface i is $B_i A_i - H_i A_i$. We are going to rewrite as $_{j=1}\sum^{n}$ energy leaving surface j is $H_j A_j F_{j-i}$. Like we did earlier for the case of black surfaces, we can multiply numerator and denominator by 1. We have $_{j=1}\sum^{n} B_i A_i F_{i-j} - B_j A_j F_{j-i}$. By using reciprocity $A_i F_{i-j}$ is $A_j F_{j-i}$. So we get a very clean result once more.

We must recognize exactly same as the expression, where obtained earlier for enclosure with black surfaces, but now instead of σT^4 our potential difference is the radiosity. That is the only difference that is appearing there so once we recognize that, it will be quite easy to proceed further.

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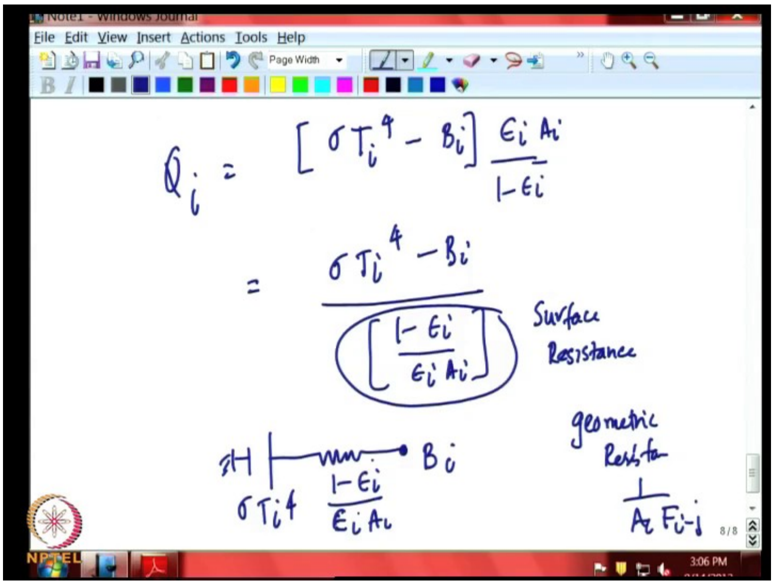

We can also combine the two expressions we derived for the heat transfer and we can write the Q_i by eliminating the irradiation; we can write as $[\sigma T_i^4 - B_i]\ \epsilon_i A_i / 1 - \epsilon_i$. Now this comes by combing the 2 equations we wrote for Q_i a few minutes back and eliminating H_i from the equation, the irradiation. We get the expression as $1 - \epsilon_i / \epsilon_i A_i$. This is useful because it says that if we take a surface at the certain potential it shows the resistance between that surface and the radiosity point B_i and that is due to the surface not being a black surface and hence this quantity is call the surface resistance.

So this must be in contrast with the concept of geometrical resistance, which we derived a few minutes ago; which was $1 / A_i F_{i-j}$. This is purely a geometric factor in this relation. This resistance is fundamentally dependent on surface properties. When emissivity of surface i is 1, this term drops out completely and does not exist. It exists only when the surface is not a black body, than does a resistance. So if the surface is a black body then the radiosity point is same as black body emission point are equal. .

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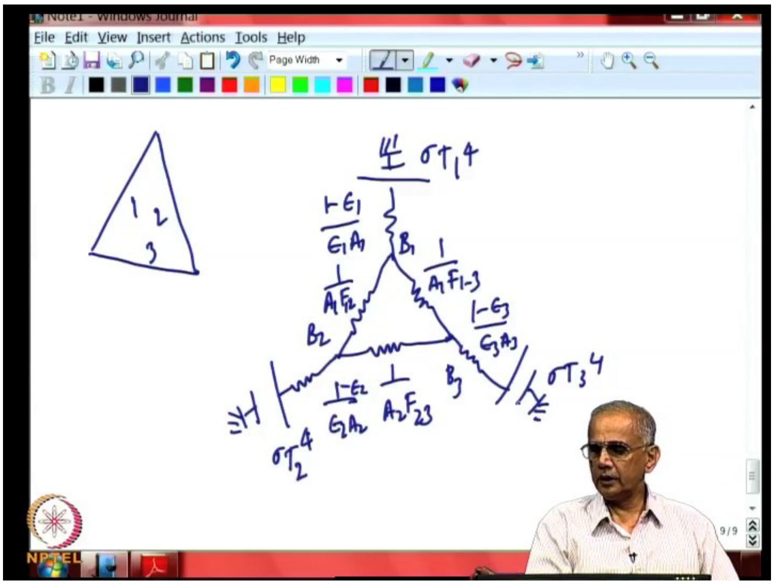

Now let us write a expression, now for a three surface enclosures so that each surface has its own resistance namely $1 / A_1 F_{1-3}$, $1 / A_2 F_{2-3}$ and $1 / A_1 F_{1-2}$.

So we have that complete resistance diagram. Now for an enclosure consisting of three gray diffuse isotropic surfaces and they consist of three potentials here as we have three surface resistance systems because the surface is not a black body and three geometric resistance term; on account of the relative configuration of the three surfaces.

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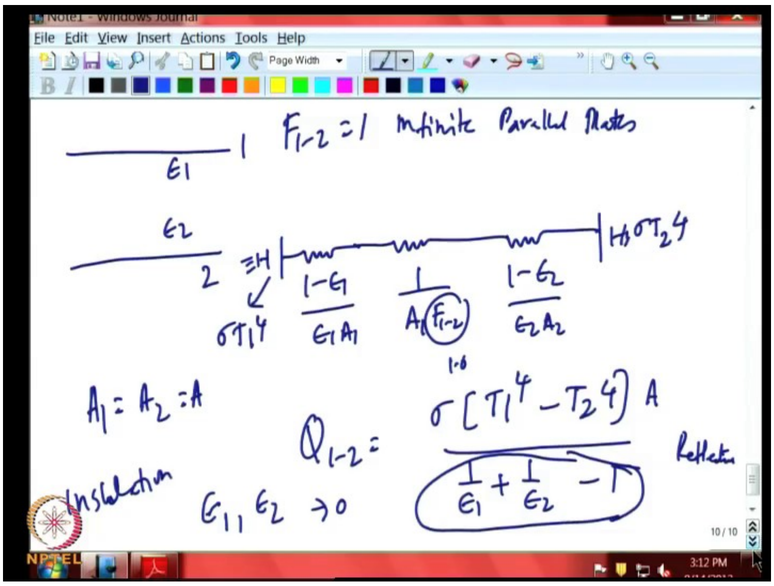

Let us take a very simple case first. Let us take two infinite parallel plate's, emissivity ϵ_1 and ϵ_2. We assume that we know the surface. Therefore F_{1-2} is 1 because of infinite parallel plates. We can write the electrical analogy for this case as, $1 - \epsilon_1 / \epsilon_1 A_1$. This is σT_1^4. Therefore, we have $Q_{1-2} = \sigma[T_1^4 - T_2^4] A / [1/\epsilon_1 + 1/\epsilon_2] -1$. This is the expression for radiative heat transfer into infinite parallel plates and it clearly shows that the major geometrical resistance is right here. We can see that if we want to reduce heat transfer between two parallel plates, then ϵ_1 and ϵ_2 become very small.

The denominator here becomes very large and it reduces the heat transfer between the two surfaces. This is used in practice in insulation application. When we insulate a particular object, we take a large number of reflective surfaces and we line them up.

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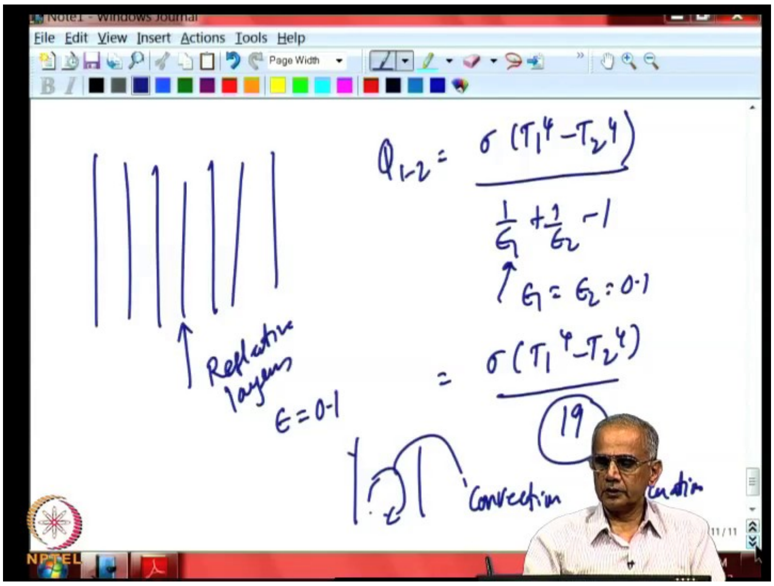

We have large number of reflective layers, these are nothing but plastic, thin plastic sheets coated with aluminum and so emissivity tends to be typically of the order of 0.1 or less. If we just take two parallel plates, we can see that Q_{1-2} is σ. So let us say $\epsilon_1 = \epsilon_2 = 0.1$. This will be 10+10=20. If we reduce the heat transfer by a factor, and it was only black surfaces we will get the expression numerator, by having reflective gray diffuse surfaces with emissivity of the order of 0.1.

We are able to reduce the heat transfer by factor of 19 times so this is now very common application both in spacecraft as well as in other application on the ground, where we cover a surface to be insulated with the large thin parallel sheets of plastic coated with metal, like aluminum and this is very inexpensive and very effective insulator we can have. We must remember, that this works extremely well in space; where there is no other mode of heat transfer except radiation, but on the ground there will always also be convection between the plates.

We have to cut on radiation, but we cannot cut down convection. Hence the convection will go up but we can reduce convection by evacuation. That is by reducing the amount of matter between the two surfaces, reduce the pressure so that the number of molecules between the two surfaces is small, and then we cut down the convert heat transfer between the two plates. In space there is vacuum so there is no other mode other than radiation, so it works very well as per on the ground. Also we can use effectively provided we confine some other ways to suppress convection.

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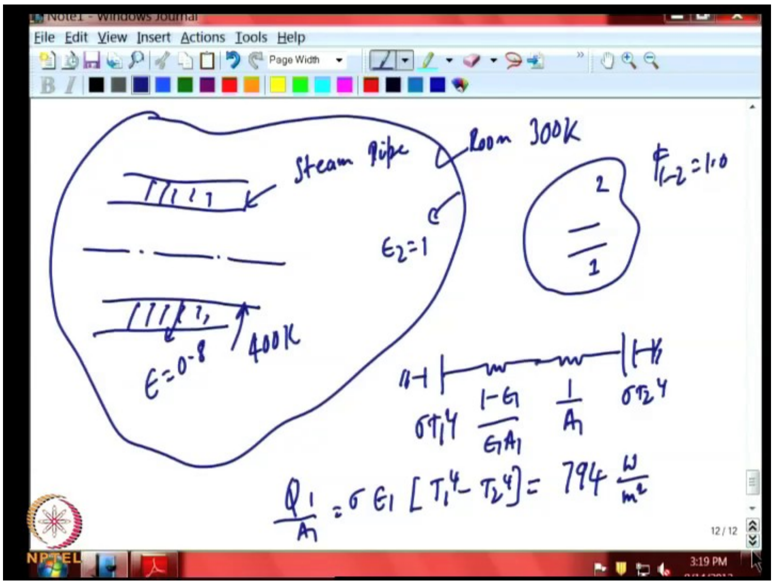

Now let us take an example, to illustrate this issue further. Suppose, we put between two plates a shield. For example let us now take a simple steam pipe. Let us say the temperature in the steam pipe is 400 Kelvin, about 120 degree centigrade and its emissivity is 0.8. We would like to know what the heat laws by radiation in the room surrounded and the room walls are 300 K. Now this problem can be solved by, treating the pipe as a surface and surrounded as shown above. We can do a simple resistance model. Let us take two surfaces as 1 and 2 so we have σT_1^4 $(1-\epsilon_1 / \epsilon_1 A_1)$ $(1/A_1)$ F_{1-2} where F_{1-2} is one.

Let us assume room to be a black body, so that room emissivity is 1. We will get now σT^4. Therefore, heat transfer rate per unit area of the pipe is nothing but ϵ_1 $\sigma[T_1^4 - T_2^4]$. The minus 1 plus 1 cancel out and only $1/\epsilon$ is left behind. We can estimate this because we know emissivity is 0.8, T_1 is 400 K; T_2 is 300 K. We will get number like 794 watts by meter square so that is the kind of heat loss.

We notice that, this heat loss is very It is more than the heat loss by free convection from this steam. Hence it really is useful now to add some insulation on the steam tube to reduce heat loss not by condition, but also by radiation. This is one example of typical application in industry, where a pipe containing steam going long distances; will cause large heat losses which are inevitable.

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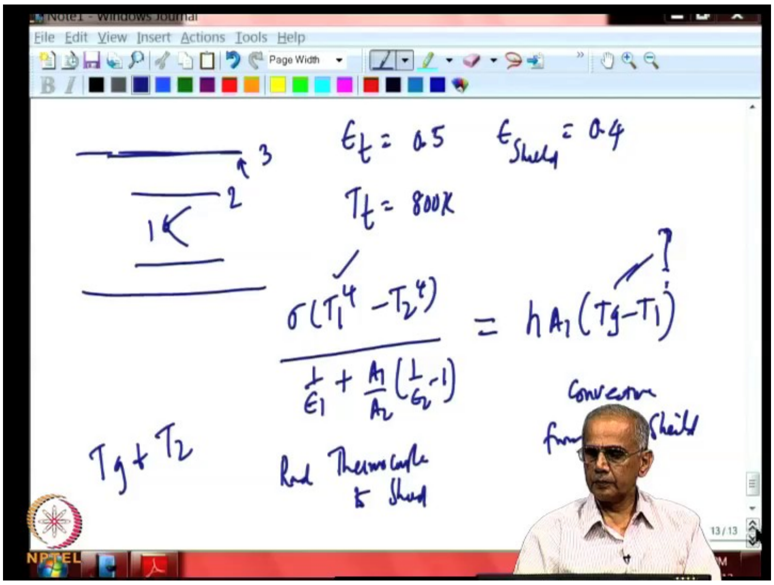

Now let us go to the example of the thermocouple shield, we thought of a wall of duct with the thermocouple there and we put a shield here. Let us say emissivity thermocouple is now 0.5, the shield thermocouple has an emissivity of 0.4; for convenience. Let us say thermocouple is measured at the temperature of 800 K we would like to know the actual gas temperature. The shield is surface 2, a duct wall is surface 3 and thermocouple is surface 1. We can calculate the heat transfer from the thermocouple to the shield which can be written as $\sigma[T_1^4 - T_2^4] / [1/\epsilon_1 + A_1/A_2(1/\epsilon_2 - 1)]$. We get this by writing down the electrical energy between 1 electrical circuit and 2, the heat transfer rate plus the radiant transfer from the thermocouple to the shield has to be equal to that from the gas to the shield and to thermocouple.

This is a balance between rate transfers from thermocouple to shield with convective transfer from gas to shield. We know the thermocouple temperature, as 800 K. We want to know the gas temperature; given the measured temperature and we do not know the shield temperature. There are two unknowns T_g and T_2.

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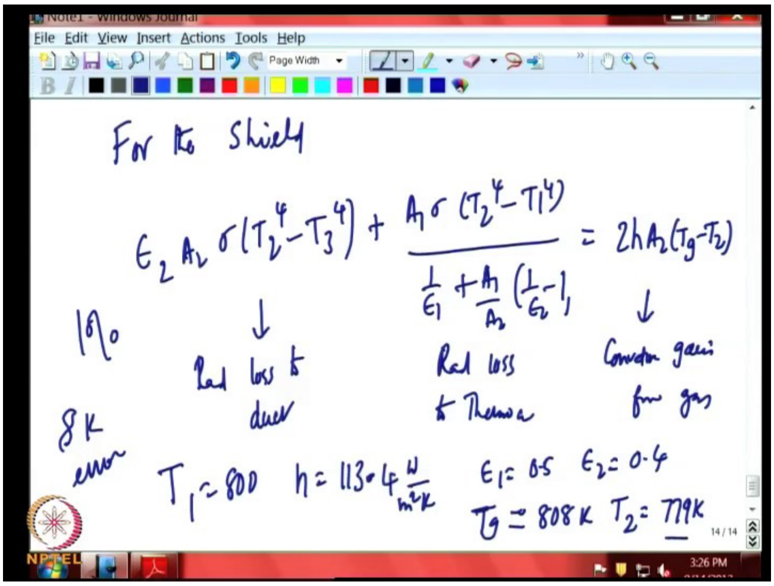

We write one more energy balance, for the shield. First thing is radiate transfer between the shield and the duct. Then we need to derive between the shield and the thermocouple and this has to be equal to heat lost by shield, by convection . This is radiative loss to thermocouple and convective gain from gas. Now we have 2 equations and 2 unknowns. We can solve it numerically. We find that, if T of the thermocouple is 800, heat transfer coefficient by gas is 130.4 Watts per meter square Kelvin.

This is a typical value for high speed flow in a duct, then we already assumed epsilon of thermocouple is 0.5; epsilon of the shield is 0.4. We will find that the temperature of the gas is around 808 Kelvin, and that of the shield is 779 Kelvin. Now this illustrates the important role of the shield. When the shield is not there, we had an error of the order of 280 Kelvin.

When we put the shield, the thermocouple was not directly seeing the wall of the duct and it was exchanging radiation only with the shield the shield already had a high temperature; so radiative loss of the a thermocouple to shield becomes small, so the temperature of the shield gets closer to the gas temperature. Thus we can see there is only around 1 percent the error, so 80 Kelvin error, which is equivalent to 1 percent. So we reduce the error from almost 25 percent to 1 percent by just adding one shield.

The using of shield is a very popular measure in the measurement temperature because it drastically reduces the error introduced by radiative transfer. In the above example we showed clearly the importance of shield and reducing the error and this was done by using simple shape factors and radiative transfer enclosures. We will continue along this line and look for more complicated examples in the next lecture and we will see how to extend this idea to a much more complex problem, as well as more realistic problems, including those which are non gray.

Thank you.