ENVIRONMENTAL GEOSCIENCES

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Lecture-36

Law of Groundwater Movement - Darcy's Law and Applications (Part - 2)

Welcome to the SWAYAM NPTEL course on Environmental Geosciences. We are discussing the module seven. In module seven, the important coverage will be of law of groundwater movement, Darcy's law and applications, groundwater fluctuations, pollution of groundwater resources. We have already discussed in lecture one related to the law of groundwater movement. This is also the continuation of the previous lecture.

In this lecture the empirical formulas for groundwater velocity determination will be discussed first then the darcy law definition, validity of darcy law, assumption of darcy law and relation between darcy velocity and interstitial velocity. So now first of all we will see the empirical formulas for groundwater velocity determination. Before Darcy came into picture, certain empirical formula based upon the experimental results were the only way to find out the velocity of groundwater flow. The formulae which were commonly used are, first is the Slichter's formula.

$$V_a = K' \times I \times (D_{10}^2 / \mu)$$

Here in this formula, you can see, where V_a is the velocity of groundwater flow in meter per day. K' is nothing, it is a constant. I, it is the slope of the hydraulic gradient line. D_{10} is the effective size of the particles in the aquifer when it is in mm the hypothetical size which is larger than 10 % of the particles in the sample that is only 10 % of the particles will pass through this size and μ is the viscosity of water depending on temperature so this is one of the formulas Slichter's formula second is the Hazen's formula. Here, Hazen's formula, you can see, again the velocity of ground water flow,

$$V_a = \frac{K' \times I \times D_{10}^2}{60} \times (1.8T+42)$$

So, here we are just seeing the addition of the temperature. So, V_a is the velocity of the ground water flow in meter per day. T is the temperature in degree C. I is the slope of hydraulic gradient line. Here the note is that the values of K' and K" in MKS or SI system are approximately 400 and 1000 respectively. So this is the Hazen's formula. Now based on these two formulas, just we will solve few numericals. We will see how this formula is working. Find out the velocity of the groundwater flow. With the following data, using Slichter's and Hazen's constant as 400 and 800 respectively. Viscosity coefficient of water at groundwater temperature of 10°C is 1. Effective size of the particles in the aquifer is 0.1 mm. Hydraulic gradient is 1 in 80. So these are given. These all are given. Now we have to use the Slichter's and Hazen formula which we have read and we have to solve the numerical. Using Slichter's formula, we are knowing that

$$V_a = K' \times I \times (D_{10}^2 / \mu)$$

So, by this Slichter's formula, we are given whatever the given material, that is V_a, velocity of groundwater flow in meter per day, K' is a constant. I is the slope of the hydraulic gradient line. D_{10} is the effective size of the particles in the aquifer in mm and π is the viscosity of water depending on temperature. So, V_a, just if you will put the value whatever is given here you can see that four hundred we are putting for K'.

$$Va = 400 \times (1/80) \times (0.1^2/1)$$
$$Va = 400 \times (0.01/80) = 0.05 \text{m/day}$$

So, this is about the Slichter's formula. Now, next is the Hazen's formula. We have read the Hazen's formula, this one. So here, V_a is the velocity of groundwater flow in meter per day, T is the temperature in degree C, and I is the slope of the hydraulic gradient line.

Again, after putting the value from the questions, we are getting the V_a value near to the 0.1 meter per day. Now, Darcy's law. Now Darcy's law we will discuss. Darcy's law defines the rate of water flow through porous media assuming a laminar flow. It states that the rate of flow per unit cross sectional area is equal to the product of the hydraulic conductivity of the material and the hydraulic gradient.

So two important parameters are here. It is the product of hydraulic conductivity, this one and the hydraulic gradient. Henry Darcy, a French waterworks in Dijon, revealed a proportionality between the flow rate in clean sand and applied hydraulic gradient. Therefore, Darcy's law demonstrated experimentally that for laminar flow conditions in saturated soil, the rate of flow or the discharge per unit time is proportional to the hydraulic gradient. So this is about the Darcy's law.

The rate of flow or the discharge per unit time is proportional to the hydraulic gradient.

$$Q \propto iA;$$
$$Q = kiA;$$
$$\frac{Q}{A} = v = ki$$

where k is the constant. This k is nothing but this is the coefficient of permeability. Now where Q is the discharge, K is the coefficient of permeability expressed in centimeter per second, meter per day or feet per day. A is the total cross-sectional area of soil mass perpendicular to the direction of flow.

So Q is equal to A into V. V is the velocity of flow, that is Darcy velocity. And I is nothing, it is a hydraulic gradient, that is $(H_1 - H_2)/L$, where H_1 is the pressure head of water at top of the sample. H_2 is the pressure head of water at the bottom of the sample. And L is the length of the sample. So here you can see the H_1 value, then H_2 value, and this is the length of the sample.

Now Darcy law gives an equation that describes the flow of a fluid through a porous medium. The law was formulated by Henry Darcy based on the result of experiments on the flow of water through the beds of sand forming the basis of hydrogeology.

V is equal to K into i is analogous to Ohm's law in electrostatics, the empirical formula of proportionality between V and I, written as V is equal to R into I, and this is called as Ohm's law. Linearly relating to the volume flow rate of the fluid to the hydraulic head difference, which is often just proportional to the pressure difference via the hydraulic conductivity. So this is V is equal to K into i is very much analogous to the Ohm's law.

One application of Darcy law is in the analysis of water flow through an aquifer. Aquifer we have read, the rocky formations which are holding the water. Darcy law along with the equation of conservation of mass simplifies to the groundwater flow equations which is one of the basic relationship of hydrogeology. Darcy law describes the relationship among the instantaneous rate of discharge through porous medium and pressure drop at a distance. Using the specific sign convention, Darcy law is generally expressed as

$$Q = -KA\frac{dh}{dl}$$

Where Q is the rate of water flow, K is the hydraulic conductivity. A is the column cross sections area and dh/dl indicates a hydraulic gradient. Here we can see in the Darcy's law diagram also. A is the column cross sectional area. Q is the rate of water flow and h is the your this dh/dl is equal to is nothing but it is the hydraulic gradient.

So in this way we can find out the rate of water flow in the porous medium. Darcy law is a simple mathematical statement which neatly summarizes several familiar properties that groundwater flowing in aquifer exhibits. First, if there is no pressure gradient over a distance, no flow occurs. So these are the hydrostatic conditions. But if there is a pressure gradient, flow will occur from high pressure towards low pressure, opposite the direction of increasing gradient.

Hence the negative sign is given in the Darcy law. The greater the pressure gradient through the same formation material, the greater the discharge rate. And the discharge rate of fluid will often be different through different formation materials or even through the same material in a different direction. Even if the same pressure gradient exists in both cases. Now validity of Darcy Law. Darcy Law is valid for laminar flow through sediments.

In fine-grained sediments, the dimensions of interstices are small and thus flow is laminar. Coarse-grained sediments also behave similarly but in very coarse-grained sediments, the flow may be turbulent. Hence, Darcy Law is not always valid in such types of sediments. Darcy law is only valid for slow viscous flow. However, most groundwater flow cases fall in this category.

Typically, any flow with a Reynolds number less than one is clearly laminar and it would be valid to apply Darcy's law. There are some assumptions of Darcy law. The following assumptions are made in Darcy law. That is the soil is saturated, first assumption. Second assumption is the flow through the soil remains laminar in nature.

The flow is continuous and steady and the total cross sectional area of soil is considered. Relation between Darcy velocity and interstitial velocity. Darcy velocity is apparent velocity or fictitious velocity or Darcy flux that is discharge per unit area. This value of velocity often referred to as the apparent velocity is not the velocity which the water travelling through the pores is experiencing. The velocity v is referred to as the Darcy velocity because it assumes that flow occurs through the entire cross section of the material without regard to solids and pores.

Actually, water can flow through pores only and the pore spaces vary continuously with location within the medium. To define the actual flow velocity or interstitial velocity, one must consider the microstructure of the rock material. For naturally occurring geologic materials, the microstructure cannot be specified three-dimensionally. Hence, actual velocities can only be quantified statistically. In reality, the flow is limited to the pores.

Here you can see the void spaces. These are the pores. Only so that the average interstitial velocity or actual velocity or seepage velocity Vs through pore space can be determined by applying continuity equation.

$$Q = A_s \; v_s = A \; v$$

hence

$$v_s = v \frac{A}{As} = \frac{v}{\eta}$$

where A is the total area of soil specimen, A_s is equal to area of pores only and n is the effective porosity. The velocity is divided by porosity to account for the fact that only a fraction of the total aquifer volume is available for flow.

The pore velocity is the velocity of a conservative tracer dye experiences if carried by water through the aquifer. Now, Reynolds number. Reynolds number is a dimensionless quantity that is used to determine the type of flow pattern as laminar or turbulent while flowing through a pipe. Reynolds number is defined by the ratio of inertial forces to that of the viscous forces. It is given by the equation,

$$R_e = \text{Inertia force/ Viscous force}$$

$$R_e = \frac{\rho v_s d_m}{\mu}$$

where ρ is the density of water, v_s is the seepage velocity, d_m is the grain size of the aquifer material and mu is the dynamic viscosity of the water.

Now, On the basis of the Darcy's law, we will solve some problem. The first problem is, calculate the Reynolds number if seepage velocity of water is 0.017 centimeter per second, grain size of aquifer is 1 mm, and the dynamic viscosity is 0.008 x 10^{-4} Newton second per meter square. So we will put the value in this equation where V_s is the seepage velocity. It is given 0.017 centimeter per second. We will convert in 2 meter per second.

Then ρ, density of water 1000 kg per meter cube. d_m is the grain size of the material, that is 1 mm. It will come to 1/1000 meter. And μ is the dynamic viscosity of the water, which is 0.008×10^{-4} Newton second per meter square. Now putting these all values in the above formula, we are getting R value near to 212.5. So Reynolds number is coming to two hundred twelve point five.

Now next problem. Three wells A, B and C are located in triangle direction with a space of 3000 meter among them. The water levels in the wells are 35, 20 and 5 mbgl. The effective porosity of the aquifer material is 0.12 and the hydraulic conductivity is 20 meter per day. So now the problem is telling to determine hydraulic gradient, then the velocity of groundwater flow and the seepage velocity. So, three different wells are here, A, B and C, and the water level of the different wells are also mentioned here, that is 35 mbgl for the well A, 20 mbgl for the well B and 5 mbgl for the well C. So, based on the above data, we have to find out these things, hydraulic gradient, velocity of groundwater flow and seepage velocity.

So we will see space between the walls are given 3000 meter water level in a, b and c are also given that is 35, 20 and 5 mbgl. Difference in water level between two wells is generally 15 meter. Effective porosity of the aquifer material is 0.12. It is given in the problem. Hydraulic conductivity of the aquifer material is 20 meter per day. It is also given in the problem. Now first we will measure the hydraulic gradient. So hydraulic gradient is the slope of the water table which is caused by change in hydraulic head over the change in distance between the two monitoring wells which is equal to i is equal to dh/dl and i is equal to $(h_2 - h_1)/l$. So here you can see $(h_2 - h_1)$ is the difference of water levels between upstream and downstream points or between wells. dh is the difference in water levels between two wells. dl or l is the distance between two wells.

So therefore, if we will put the value, it will come, fifteen, i is coming to 15/3000, that is i is equal to 0.005. Second is the velocity of groundwater flow. So we have seen in the previous slide that the space between the wells is given, that is 3000 meter. Water level in well A, B and C is also given, that is 35, 20 and 5 mbgl. Difference in water levels between two wells is fifteen meter, we have measured there.

Effective porosity of the aquifer material is given 0.12 and hydraulic conductivity of aquifer material is 20 meter per day. So just we have to find out the velocity of the groundwater flow that is small v. It is the flow per unit cross sectional area of the porous medium which is a quantity of hydraulic conductivity and hydraulic gradient through porous material. This is generally expressed in meter per day. So v=k.i. We have seen this formula earlier. where k is the hydraulic conductivity in meter per day, i is the hydraulic gradient, which is 0.005.

Now, just we will put the value here, we are getting the v value, that is 20 x 0.005, then v value is coming to 0.1 meter per day. So, this is about the velocity of the groundwater flow. Next is the seepage velocity. It is actual velocity of the water that can flow through the pores only. This is expressed in meter per day.

This one is the formula of the seepage velocity, v=k.I/n. In the given problem, we have seen the space between well is 3000 meter, water levels in well A, B and C is 35, 20 and 5 mbgl. Difference in water levels between two well is coming to 15 meter. Effective porosity of the aquifer material was given 0.12 and hydraulic conductivity of aquifer material is also given 20 meter per day. So, just from this formula, we know that the k is the hydraulic conductivity in meter per day, i is the hydraulic gradient, that is 0.05 as calculated in the first part of the problem, and n is the effective velocity. So, just if we will put the value in this formula, we are getting the V is equal to 0.83 meter per day.

Now, next problem is, to determine the coefficient of permeability from the following data. Length of sand sample is given, that is 25 centimeter. Area of cross-section of the sample is also given, 30 centimeter square. Head of the water edge is given 40 centimeter and discharge value 200 centimeter cube in 110 seconds, that is time is given, 110 seconds. Now, L is 25 centimeter. A is given 30 centimeter square, then H 40 centimeter and Q is 200 centimeter cube in 110 second.

So Q is equal to volume by time that is 200/110 centimeter cube per second that is 20/11 which is coming to 1.82 centimeter cube per second, i is the hydraulic gradient we are knowing, so i = h/l, which is h value is given, head of the water 40, and 25 is also given length of the sand sample, which is coming to 1.60. By using the Darcy law equation, we can see Q = k.I.A, so k will come to Q/i.A, and if we will put the value, we are getting the k value near to 0.3788 millimeter per second.

Now next problem we will see the quantity of water passing through a sample 6 centimeter height and 50 centimeter square cross sectional area in 10 minutes is 480 centimeter cube under an effective constant head of 48 centimeter. Now the question is what will be the coefficient of permeability of the soil?

So in the problem it is given length of the specimen is given 6 centimeter. Cross sectional area is given 50 centimeter square. Quantity of water is also given 48 centimeter cube. Time of flow is given 10 minutes. So it is coming to 600 second.

Constant head is 48 centimeter. Since the Q is the flow rate of the water flowing through the area A. So it is by formula we are knowing Q = k.i.A, i will convert into H/L.A. So, k = Q.L/A.H. So, if we will put the value in this formula, K value is coming to 0.0002 centimeter per second.

Now next problem is the coefficient of permeability of soil sample of 6 centimeter height and 50 centimeter square cross sectional area under an effective constant head of 48 centimeter is 2×10^{-5} meter per second. So here coefficient of permeability value is given.

If the void ratio of the soil is 0.57, what will be the Darcy velocity and seepage velocity of the soil sample? So these two velocities we have to determine, Darcy velocity and seepage velocity. So first is the length of the sample, that is 6 centimeter. Given cross-sectional area is 50 centimeter square. Given constant head is 48 centimeter.

Coefficient of permeability is also given, 2×10^{-5} meter per second. And void ratio of the soil is mentioned here 0.57. Now we have to find out the Darcy velocity as well as seepage velocity. For Darcy velocity we are knowing v = k.i = k.h/l. If we just put the value of k that is coefficient of permeability and and the constant head by the length of the sample, it is coming to 16×10^{-5} meter per second. Porosity, we are also knowing, it is e/(1+e), that is, it is related to the void ratio, so 0.57/(1+0.57), which is coming to 0.36. Now, for seepage velocity, we are knowing this equation $V_s = V/n$. So, just we will put the value because we are knowing V is the your 16×10^{-5}, which is velocity is coming and n is 0.36. The velocity is also mentioned here. So, it is coming to 44.4. So, it is coming to 44.44×10^{-5} meter per second. So, in this way, both Darcy velocity and seepage velocity can be calculated if the coefficient of permeability and the void ratio will be given in the problem. Now, next problem is based on a cylindrical mould of 50 centimeter square cross-sectional area contains 18 centimeter length of the sand sample. When water flows through the soil under constant head at a rate of 48 centimeter cube per minute, the loss of head between two points that is 8 centimeter apart is found to be 12.8 centimeter. What will be the coefficient of permeability of the soil?

We have to find out. Now cross sectional area of specimen given is equal to 50 centimeter square. Length of the soil sample is also given 18 centimeter. Rate of flow is also given 48 centimeter cube per minute. So we will change into the second.

Length of sample L = 8 centimeter. Then loss of head is also given 12.8 centimeter. Hydraulic gradient we can determine with the help of h/l that is 1.6. Now by using this equation that is Q = k.i.A, we can find out the coefficient of permeability k = Q/i.A. Q value is 0.8, i is equal to 1.6 and A is the 50. Just it is coming to 0.01 centimeter per second. Now just summarizing the lecture 2, we have discussed in the lecture 2, the first point we have discussed that is the empirical formulas for ground root of velocity determination.

In this, we have discussed the Slichter's formula as well as the Hazen's formula. Secondly, we have seen the Darcy's law, few assumptions we have seen, the assumptions where the soil is saturated, the flow through the soil is laminar, the flow is continuous and steady, the total cross-sectional area of the soil is considered. Darcy's law we have seen, Darcy's law defines the rate of water flow through porous media, assuming a laminar flow. It states that the rate of flow per unit cross-sectional area is equal to the product of the hydraulic conductivity of the material and the hydraulic gradient. And validity of Darcy law is by Reynold's number here should be less than one.

Darcy law is only valid for the slow viscous flow. Darcy law is valid for laminar flow. In very coarse grain sediments, the flow may be turbulent. Thank you very much to all.