Climate Change Science

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Lecture 21
Meridional Variation

In the previous lecture, it was demonstrated that incorporating a layer representing the stratosphere into the atmospheric energy balance model helps explain why rising carbon dioxide (CO_2) levels lead to warming in the troposphere but cooling in the stratosphere. This behavior was analyzed using energy balance equations at the Earth's surface, within the troposphere, and in the stratosphere. Initially, it was shown that in the absence of solar absorption, the stratosphere would behave similarly to the troposphere: increasing CO_2 would raise its emissivity, thereby warming the stratosphere. However, when solar absorption by ozone is taken into account, and considering the low emissivity of the stratosphere, a consequence of its thinness, an increase in CO_2-induced emissivity results instead in cooling of the stratosphere. Thus, the contrasting thermal responses of the troposphere and stratosphere to increased CO_2 can be understood through energy balance principles and radiative processes within the atmosphere.

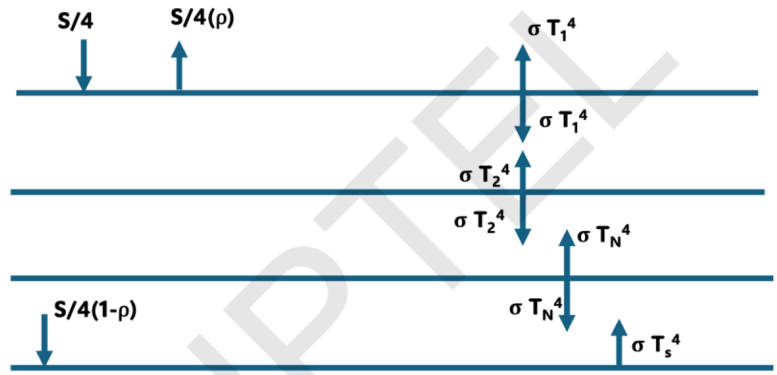
In the troposphere, temperature decreases with height because most of the solar radiation is absorbed at the Earth's surface. The atmosphere is then heated from below via radiation, evaporation, and turbulent heat fluxes. This bottom-up energy transfer leads to the observed temperature lapse rate - a decrease in temperature with altitude.

In contrast, the stratosphere is heated from above. Here, ozone plays a crucial role by directly absorbing incoming solar ultraviolet radiation, which causes the temperature to increase with altitude. This distinct vertical heating pattern sets the stratosphere apart from the troposphere.

These differences in heating mechanisms help explain why an increase in atmospheric CO_2 causes warming in the troposphere but cooling in the stratosphere. In the troposphere, CO_2 traps outgoing longwave radiation, increasing temperature. In the stratosphere, however, enhanced emission due to higher CO_2 in a thin, low-emissivity layer combined with less incoming solar radiation results in net cooling.

The idea can be further extended using multi-layer models of the atmosphere. While one- and two-layer models suffice for explaining Earth's climate system, they are inadequate for planets with much thicker atmospheres. Earth’s atmosphere has a surface pressure of about one atmosphere, making it relatively simple to model.

However, Venus, for example, has a surface pressure approximately 100 times greater than Earth's. Modeling such planets requires considering many atmospheric layers to accurately capture the vertical structure and radiative processes. Hence, more complex models are essential to study climates of planets with denser atmospheres.



N layer model of the earth's atmosphere. All the layers are transparent to solar radiation but opaque to thermal radiation emitted by the surface

To illustrate the concept of radiative transfer in a multi-layer atmosphere, consider an atmosphere composed of 'N' layers. While N can be any number, for simplicity in classroom analysis, we make a few idealized assumptions. First, all atmospheric layers are assumed to be completely transparent to solar radiation, meaning that solar radiation, after partial reflection, passes through all layers and is absorbed only at the surface.

On the other hand, we assume these layers are completely opaque to Earth's infrared (IR) radiation, behaving like blackbodies. As blackbodies, each layer absorbs all incoming IR radiation from below and emits IR radiation equally in both upward and downward directions. Since every layer acts as a blackbody, no emissivity values are needed in this model, which simplifies the radiative calculations.

Now consider the energy balance at the top of the atmosphere. The absorbed solar radiation is calculated as the incoming solar radiation minus what is reflected. This absorbed solar energy must equal the infrared radiation emitted by the topmost atmospheric layer, which is expressed as $\sigma {T_1}^4$, where σ is the Stefan–Boltzmann constant and T_1 is the temperature of the top layer. Because the atmosphere is fully opaque, no IR radiation emitted by the Earth's surface escapes directly to space; only the top layer can radiate to space.

$$\frac{S}{4}(1-\rho) = \sigma {T_1}^4$$

The topmost layer emits in upward direction to space and in downward direction to the second layer. It can only absorb the radiation emitted by the second layer below it in the upward direction. Hence, the energy balance at the topmost layer yields

$$\sigma {T_2}^4 = 2\sigma {T_1}^4 \qquad \sigma {T_2}^4 = 2\left(\frac{S}{4}(1-\rho)\right)$$

This means that the second layer is warmer than the top layer.

Proceeding to the second layer (just below the topmost one), it absorbs radiation from the third layer below and the topmost layer above, and emits IR radiation in both directions. The energy balance for this layer incorporates the downward emission from the top layer and the upward emission from the third layer. Solving this balance, one finds that

$$\sigma {T_3}^4 + \sigma {T_1}^4 = 2\sigma {T_2}^4 \qquad \sigma {T_3}^4 = 3\left(\frac{S}{4}(1-\rho)\right)$$

This pattern continues as we descend through the atmosphere and a clear trend emerges: for N layers, the N^{th} layer emits radiation equal to $N \times \sigma {T_1}^4$

$$\sigma {T_N}^4 = N\left(\frac{S}{4}(1-\rho)\right)$$

At the Earth’s surface, energy balance involves two sources of incoming energy: (1) solar radiation transmitted through the transparent atmosphere, and (2) the downward IR radiation emitted by the bottom-most atmospheric layer. The surface itself emits radiation as a blackbody, $\sigma {T_s}^4$. Combining the incoming solar and downward longwave radiation, and equating to the surface emission, one obtains the result:

$$\sigma {T_s}^4 = (N+1)\left(\frac{S}{4}(1-\rho)\right)$$

This result is quite significant as it shows that for the same amount of solar radiation absorbed by the Earth–atmosphere system, the surface temperature increases with the number of opaque layers. Specifically, the surface temperature is proportional to (N+1) times the absorbed solar radiation. As the number of opaque atmospheric layers increases, more infrared radiation is trapped, leading to a corresponding rise in surface temperature.

This concept helps us understand the case of Venus, which has an atmosphere much thicker than Earth's. Due to this thickness, more radiation is retained, resulting in a significantly higher surface temperature. To model Venus realistically, we require many atmospheric layers, typically around 70, to match the surface temperature measured by probes that descended through its atmosphere. This illustrates the intensity of the greenhouse effect on Venus, which is far more extreme than that on Earth.

However, it’s important to note that this simple model cannot determine the exact number of layers needed for a given planetary atmosphere. The choice of 70 layers for Venus is based on observational data and more advanced modeling, not on this blackbody-layer approach alone. This model gives a useful conceptual understanding but lacks the detail needed to set parameters like the number of layers or their physical properties.

Therefore, while this simplified analysis provides a qualitative insight into why Venus is so hot - owing to its thick, nearly opaque atmosphere - it falls short of explaining the quantitative aspects. A more sophisticated radiative-convective model is needed to fully capture the complexity of Venus's atmosphere, including variable emissivities, pressure, composition, and other planetary characteristics. We will explore this further when we specifically study the greenhouse effect on Venus, which is one of the strongest known in the solar system.

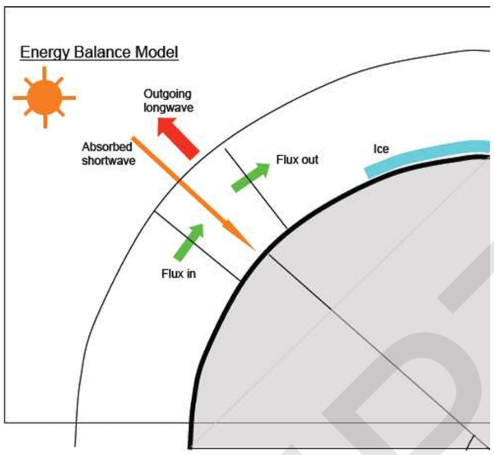
This discussion becomes particularly fascinating when we consider the evolutionary histories of Earth and Venus. According to astronomers, when the solar system was formed around 4.5 billion years ago, Venus and Earth began with similar dimensions, compositions, and atmospheric characteristics. At that early stage, both planets had atmospheres dominated by carbon dioxide, possibly around 70–80% CO_2.

Over time, however, the two planets followed very different evolutionary paths. On Earth, carbon dioxide levels declined substantially, giving way to an oxygen-rich atmosphere. Today, CO_2 levels on Earth are just about 400 parts per million, a dramatic reduction from the early atmospheric composition. In contrast, Venus did not lose its carbon dioxide; in fact, it even gained more CO_2 from its surface. This accumulation of greenhouse gases led to extreme heating, making Venus an uninhabitable hothouse planet.

The contrast between these two outcomes, Venus becoming scorching and uninhabitable, while Earth remained temperate and liveable, raises crucial questions. Given their similar starting points, we must ask why they diverged so drastically. Understanding this divergence is not only a matter of scientific curiosity but also essential for understanding the future of Earth. The concern is that if we continue to burn fossil fuels excessively, releasing massive amounts of CO_2, we might steer Earth toward a Venus-like climate.

This possibility underscores the importance of studying Venus and constructing accurate models of its atmosphere. While the simplified N-layer blackbody model helps to build intuition, especially in illustrating that Venus cannot be accurately modeled using only one or two atmospheric layers, it is not sufficient to quantitatively capture Venus's extreme climate. When we model Earth with N = 1 or 2, we get reasonable results, but the same approach fails for Venus due to its dense atmosphere and high albedo.

To model Venus correctly, we need to build a much more sophisticated atmospheric model, incorporating a large number of layers (around 70) and accounting for complex radiative transfer, albedo effects, and energy absorption at different depths. Such a model will be explored later in the course, providing deeper insight into why Venus became so hot, and helping us understand what steps must be taken to avoid a similar fate for Earth.



Energy balance model with meridional variation of temperature $T(\Phi)$

$$\{S/4\}\, s(\Phi)\{1-\rho\} = (A + B\,T) - D(T - T_m)$$

Where $s(\Phi)$ accounts for the variation of solar radiation with latitude. $(A+BT)$ represents outgoing longwave radiation and transport of heat from equator to poles is represented by the last term T_m is the global mean temperature. The tropical regions with temperatures higher than the global mean lose heat through meridional transport.

Now, before moving on to other topics, we extend our analysis of the zero-dimensional energy balance model to a one-dimensional model that accounts for variation with latitude. This extension is important because the distribution of ice on Earth is not uniform. It is predominantly concentrated in the polar regions. Over Earth's long climatic history, the extent of polar ice has changed significantly. At certain times, Earth experienced periods like the Snowball Earth, when ice covered the entire planet. At other times, the planet was nearly completely ice-free.

To understand such phenomena, it is necessary to model the latitudinal variation of temperature and identify the ice line - the latitude beyond which ice is stable. Understanding where the ice ends and ice-free regions begin is essential for studying Earth's climate stability and feedback mechanisms.

To model this latitudinal variation, two key factors must be considered. First, solar radiation does not fall equally on all parts of the Earth. Due to the tilt of the Earth's axis, the polar regions receive little to no sunlight for several months each year. For instance, polar night can last up to six months, during which there is no incoming solar radiation.

This naturally makes the poles colder than the equator. However, to quantify how much colder, a more detailed energy balance analysis is needed.

Second, when there is a temperature gradient from the equator to the poles, heat transfer occurs. This is a direct consequence of the second law of thermodynamics, which states that heat flows from regions of high temperature to regions of low temperature. Therefore, to accurately model Earth's temperature distribution, we must account for the transport of heat from equatorial regions toward the poles, which tends to moderate the temperature difference between these regions.

So, we now need to extend the simple zero-dimensional Liou's model we used earlier to account for latitudinal variations by considering the variation of solar radiation with latitude, and the transfer of heat from the equator to the poles. To begin this extension, a simplified energy balance equation is written as follows:

$$\frac{S}{4}s(\varphi)\{1-\rho\} = A + BT - D(T - T_m)$$

In this equation, $S/4$ represents the average solar radiation incident per unit area on Earth, considering that the Earth is a sphere and only half is illuminated at a time. The variable $s(\varphi)$ captures how solar radiation varies with latitude (φ). This factor $s(\varphi)$is derived from astronomical considerations, such as the angle of solar incidence and duration of daylight, which vary with latitude.

Next, (1 - ρ) denotes the absorbed fraction of solar radiation, where ρ is the albedo, or the fraction of radiation that is reflected. So, the left-hand side of the equation represents the solar energy absorbed by a given latitude band.

On the right-hand side, the outgoing longwave radiation (OLR) is approximated by a linear function $A + BT$, where T is the temperature at a given latitude. This is a simplifying assumption made to allow for analytical solutions, rather than resorting to complex numerical methods. This approximation is justified because satellite measurements of OLR and corresponding surface temperatures reveal an approximately linear relationship between the two, especially for the purpose of broad climate modeling.

The final term in the equation represents the heat transport between latitudes. The model assumes that if a latitude is warmer than the global mean, it loses heat to cooler regions, and if it is colder than the global mean, it gains heat from warmer areas. This is a simple and intuitive representation of heat transfer.

In a more rigorous treatment, this inter-latitude heat transfer would be modeled using a diffusion equation, akin to Fourier’s law of heat conduction. However, solving such a diffusion model would require numerical methods, making the analysis significantly more complex. For the current discussion, we will proceed with this simple analytical model to examine how temperature varies with latitude, which provides valuable insights while remaining tractable within a classroom setting.

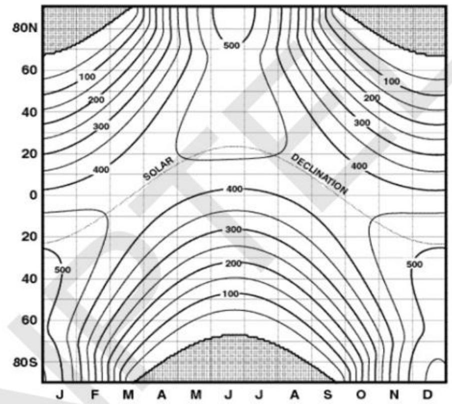
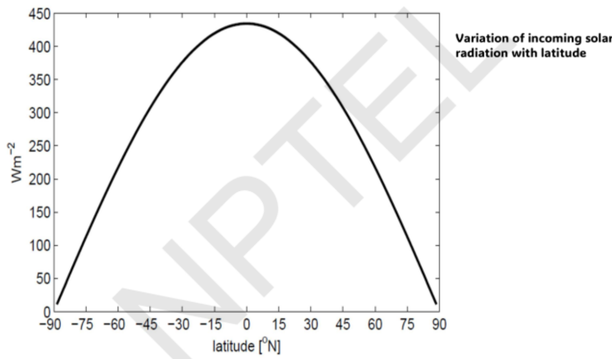
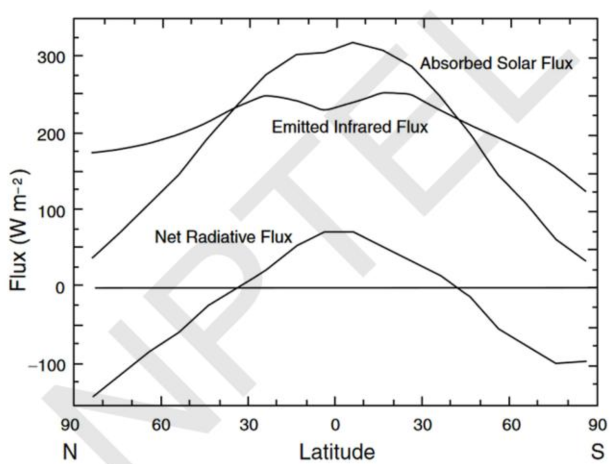


Figure 1.2: The seasonal variation of the zonally (or diurnally) averaged insolation at the top the atmosphere. The units are W m^{-2}.



In this extended model, we must incorporate how solar radiation varies with both latitude and season. However, since the focus here is on the annual mean temperature, we will average over all months and represent solar radiation (S) as a function of latitude only. As mentioned earlier, we account for this variation in meridional direction by the term $s(\varphi)$. The representation of the annual mean solar insolation variation across different latitudes is shown in the above figure. The result used here is a symmetrized version of the actual distribution. While the real variation is not perfectly symmetric because the Earth's orbit is an ellipse, not a perfect circle. We ignore this slight asymmetry for the sake of simplicity in this model.



Because the incoming solar radiation varies with latitude, the absorbed solar flux is highest at the equator and lowest at the poles. However, this variation is not entirely smooth. The reason for this irregularity lies in the variation of Earth's albedo with latitude. Albedo is influenced by several factors: the type of surface (land or ocean), vegetation (forest or desert), and the presence and extent of clouds. All these elements affect the fraction of solar radiation that is reflected versus absorbed.

For clarity and reference, the actual measured values of absorbed solar flux and emitted longwave (infrared) flux are provided. These are derived from observational data, including satellite measurements. A key feature shown in these data is that around 30° North and 30° South, the two curves representing absorbed and emitted radiation intersect. This intersection indicates that between these latitudes, essentially the tropics, the Earth absorbs more solar energy than it emits as longwave radiation.

In contrast, at latitudes beyond 30° North and South, the Earth emits more radiation than it absorbs. This establishes a net energy gain in the tropics and a net energy loss in the mid-latitudes and polar regions. To maintain energy balance and achieve a steady state, this imbalance must be corrected through heat transfer.

This heat transport from the tropics to the poles is represented in the model by the term $D(T - T_m)$ in the energy balance equation. Here, D is a diffusion-like coefficient, T is the local temperature, and T_m is the global mean temperature. This term models the idea that warmer regions (like the tropics) lose heat to cooler regions (like the poles). Without such a mechanism, the polar regions would continue cooling indefinitely due to their net radiative loss, violating the principle of energy conservation in a steady-state climate.

The variation of absorbed solar radiation with latitude, which was illustrated earlier, can be approximated using a simplified analytical expression as follows:

$$y = \sin(\varphi)$$

$$s(\varphi) = s(y) = 1.241 - 0.723y^2$$

According to this representation, solar radiation incident at the equator is approximately 1.241 times the global mean, whereas at the poles it is only 0.518 times the global mean. This highlights a significant gradient: the polar regions receive less than half the solar radiation compared to the equatorial region.

To proceed with the modeling, we introduce the concept of an ice line, defined by a critical latitude y^*, beyond which the surface is assumed to be ice-covered. For latitudes greater than y^* (closer to the poles), the albedo is taken as 0.62, a typical value for ice and snow. For latitudes less than y^* (closer to the equator), the albedo is assumed to be 0.32, representative of land regions. These values are chosen for simplicity and can be adjusted, although the abrupt change at $y = y^*$ is not physically realistic. In nature, such transitions are gradual. However, the simplification enables easier analytical treatment of the problem.

With these assumptions, an equation governing the temperature variation with latitude is expressed as follows:

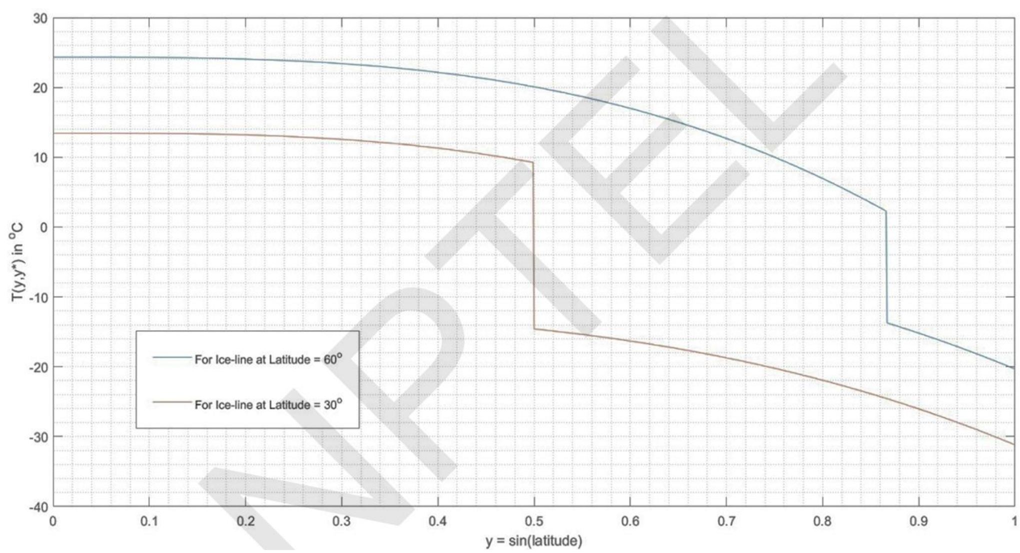
$$T(y) = \frac{1}{B + D}\left[\frac{S}{4}(1.241 - 0.723y^2)\big(1 - \rho(y)\big) - A + DT_m\right]$$

A key unknown in this formulation is the global mean temperature, T_m. This is determined from the global energy balance equation, which states that the globally averaged absorbed solar radiation equals the globally averaged emitted longwave radiation.

$$T_m = \frac{1}{B}\left[\frac{S}{4}(1 - \rho_m) - A\right]$$

To compute this, one needs to evaluate the mean albedo, ρ_m , which itself is dependent on the position of the ice line. This is achieved by integrating the albedo function over latitude, weighted by the surface area at each latitude band.

For example, if we assume y^* =0.5, which corresponds to an ice line at approximately 30° latitude, and compute the mean albedo accordingly, we obtain a global mean temperature of approximately -6.32°C.



When the temperature is plotted as a function of latitude using the derived equation for two different ice line positions at 30° and 60° latitude, a sharp discontinuity in temperature is observed precisely at the ice line. This abrupt change is an artifact of the step-function assumption for albedo, where it jumps suddenly from 0.32 (typical of ice-free regions) to 0.62 (typical of ice-covered regions). The discontinuity in temperature arises directly from this artificial discontinuity in albedo.

Had a smoother albedo variation with latitude been used, for instance a gradual transition zone around the ice line, the resulting temperature profile would also be smooth. However, introducing such a realistic albedo function would complicate the mathematics, likely necessitating a numerical solution rather than an analytical one. The simplified step-function approach was deliberately chosen to enable an analytical solution solvable with basic tools, such as a calculator, making it accessible without the need for computer programming.

Nevertheless, this model serves as a valuable demonstration of how albedo assumptions strongly influence the latitudinal temperature profile, particularly the presence or absence of sharp transitions. Students interested in exploring this further are encouraged to implement a numerical approach with a smoother albedo profile using basic programming techniques. This would yield a more realistic and continuous representation of how temperature varies with latitude.

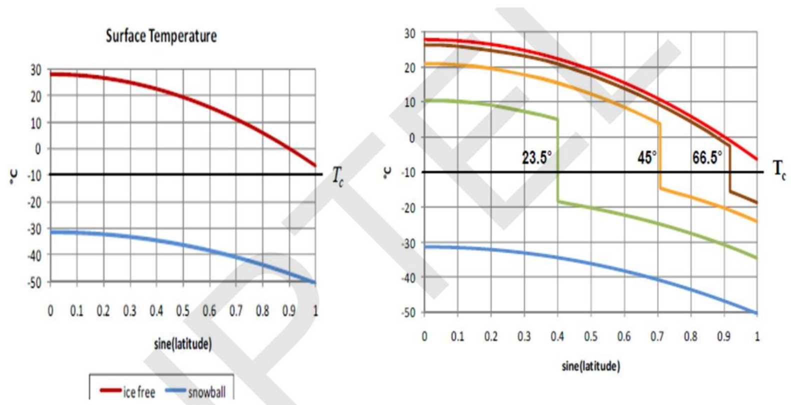


Figure from James Walsh, Oberlin College

This simplified energy balance model has also been illustrated in the above figure by Professor James Walsh of Oberlin College, showcasing its ability to reproduce a range of climatic scenarios. These include a completely ice-free Earth, where the ice line is at 90° latitude, and a fully ice-covered Earth, or "Snowball Earth", where the surface temperature ranges between -30°C and -50°C. The model can also simulate intermediate cases, with ice lines at various latitudes such as 23.5°, 45°, or 66.5°, each yielding distinct temperature profiles. Although the model makes several simplifications, its elegance lies in its ability to replicate diverse global climate states using only basic assumptions and calculations.

Despite its simplicity, the model offers important insights into how the location of the ice line affects the global temperature distribution. However, real-world temperature variation with latitude is continuous, not abrupt. Such discontinuities in this model are artifacts of using a step-function for albedo. To capture more realistic behavior, one could use a smoother albedo function that gradually transitions between ice-covered and ice-free regions. Alternatively, a diffusion term can be added to the equation to model meridional heat transport more accurately. This term would be of the form $diffusion\ coefficient \times \frac{d^2T}{dy^2}$, where y is the sine of latitude. Including this term, however, makes the equation analytically unsolvable and necessitates a numerical solution using appropriate software.

Nevertheless, for conceptual understanding and hands-on calculations, this simple analytical model provides a solid foundation for studying how energy balance, albedo, and latitude interact to control Earth's climate and ice distribution. It serves as an accessible entry point into more sophisticated climate modeling.