Climate Change Science
Prof. J. Srinivasan
Department of Environmental Science
Indian Institute of Science, Bangalore

Lecture – 35
Stochastic resonance

In the last lecture, we were discussing the theory to explain why ice ages occur every 100,000 years during the last million years. We found that if you propose a simple linear model—that the temperature variation in the polar region is directly related to the incoming radiation in the polar region in summer—then we have a problem, which is that the largest amplitude of variation of radiation from the Sun occurs in the 20,000-year cycle, while the observed amplitude of the temperature variation in the polar region is in the 100,000-year cycle. So, we have to conclude that the linear system model cannot be right.

If the linear system model were right, the forcing amplitude of solar radiation is maximum at 20,000 years; then the temperature should fluctuate with a 20,000-year cycle. It is not fluctuating. It is fluctuating more closely to a 100,000-year cycle. So, the only way to resolve the issue is to propose a non-linear model.

SALTZMAN'S MODEL

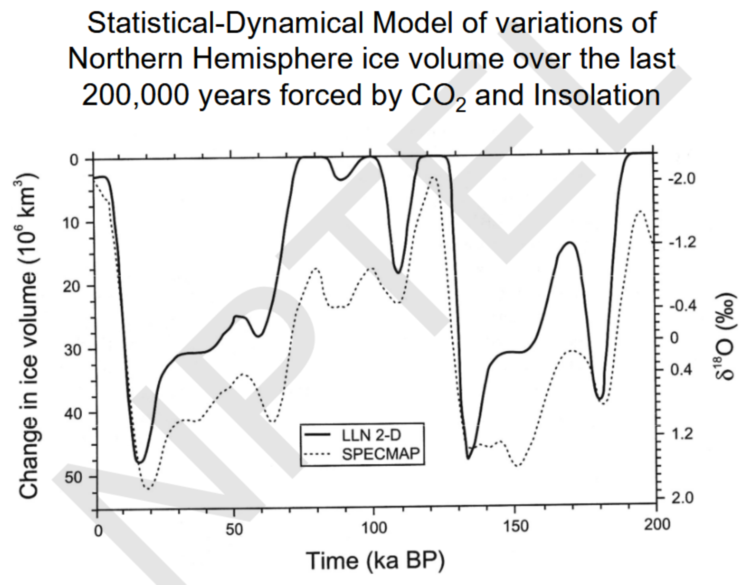
$$\frac{dX}{dt} = -\alpha_1 Y - \alpha_2 Z - \alpha_3 Y^2$$
$$\frac{dY}{dt} = -\beta_0 X + \beta_1 Y + \beta_2 Z - (X^2 + 0.004Y^2)Y + F_Y$$
$$\frac{dZ}{dt} = X - \gamma_2 Z$$

where in this particular case X, Y and Z are the ice mass, deep ocean temperature and atmospheric carbon dioxide.

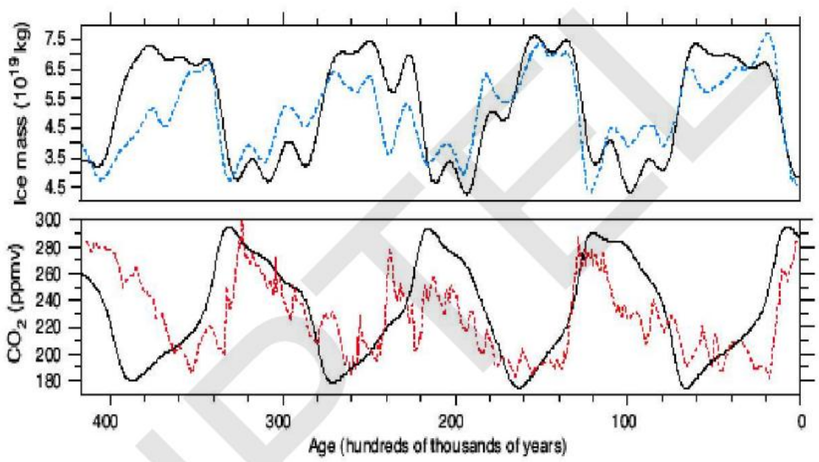
where X is ice mass,
Y is ocean temperature
Z is CO_2

I briefly discussed this non-linear model of Saltzman in which he said the rate of change of ice (which is in X here), the rate of change of deep ocean temperature (which is close to global mean temperature), and the rate of change of carbon dioxide are interconnected, interrelated, and each of these equations also had a non-linear term. So, by putting the non-linear terms in this equation and having three coupled ordinary differential equations, Saltzman solved this problem.

He got the following result for the last 200,000 years, in which the model simulation in black agreed fairly closely with the ice core data—not perfectly, but quite closely. The peaks are matching.



Now, he did one more simulation. This is only for 200,000 years. He followed this up with another simulation for 400,000 years, where black is the simulation and red is the observation.



Solution of the dynamical system climate model of Saltzman and Maasch (1988) for the past 400 thousand years subject to the earth orbital radiative forcing. The model prediction for ice (top panel) and carbon dioxide (bottom panel) are shown. For comparison, the dashed blue curve in the top panel is the SPECMAP ∂18O estimate of ice variations and the dashed red curve in the bottom panel is the Vostok core estimate of CO_2 variation.

Here, blue is the ice mass and red is CO_2. So, this model is able to predict that both CO_2 and the amount of ice on Earth will fluctuate with the 100,000-year cycle by incorporating non-linear terms in those three equations. But the only problem with that model is that those terms introduced by Saltzman are essentially ad hoc. They do not follow directly from either energy balance equations, mass balance equations, or some other conservation law. So, these are postulates that, for example, temperature depends quadratically on temperature, and the ice mass depends quadratically on temperature, and the deep ocean temperature depends quadratically on ice mass. These are hypotheses which are not easy to prove. So, although Saltzman managed to get fairly nice results, people still had doubts about whether this hypothesis was valid.

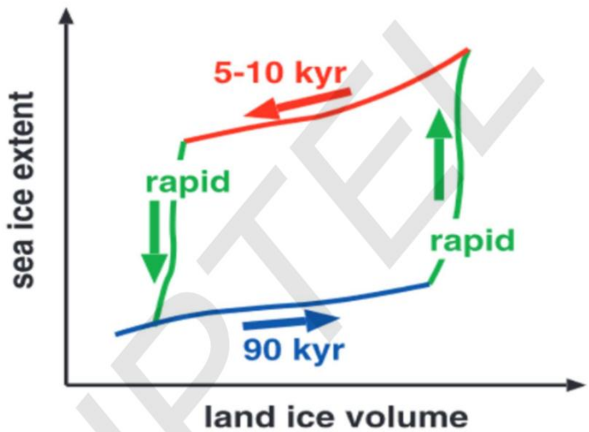
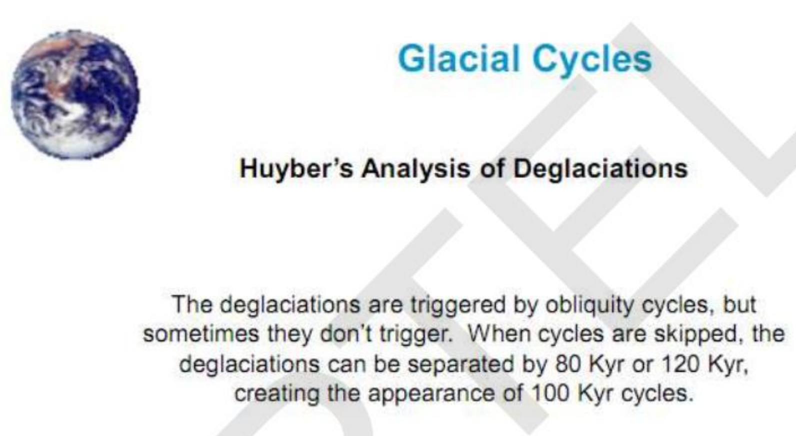


Figure 1. A schematic figure of the hysteresis between land ice and sea ice during glacial cycles, predicted using a simple box model by *Gildor and Tziperman* [2000] and examined here using a model that is continuous in the meridional direction.

The other problem people had was that there is ice over land and there is ice over ocean—that is, sea ice. What happens is that the land ice builds up very slowly over a 90,000-year period and then rapidly. What you see here is that at that time, the sea ice is not changing much. The sea ice increases rapidly only near the end of this period. In the same way, when the land ice starts decreasing, initially, sea ice changes are small, then sea ice changes rapidly.

So, the two different time scales of the sea ice and the land ice would have some impact on the 100,000-year cycle. This was the hypothesis of Gildor and Tziperman, which has some similarities to the hypothesis given by Saltzman. Now, one more hypothesis is by Huybers, who says that yes, ice ages should have come every 20,000 years, but sometimes the occurrence of ice ages was skipped.



He has not given a reason why. He is appealing to a trigger. For example, you saw that there are certain threshold values of solar radiation—only when it goes below that, ice starts forming. So, his argument is that in the 400,000-year period we saw, there were periods where, although the solar radiation went down, it did not go sufficiently down in the 20,000-year cycle. So, that cycle is skipped. So, in his argument, to get a 100,000-year cycle, you have to skip three 20,000-year cycles, and the fourth one—it should come. So, this also is a bit far-fetched, but this brings in the question of some non-linearity in the problem.

$$c\frac{d\tilde{T}}{dt} = Q(1-\tilde{\alpha}(\tilde{T})) - A - B\tilde{T}$$

Lyapunov Potential V
Vmin stable while Vmax is unstable

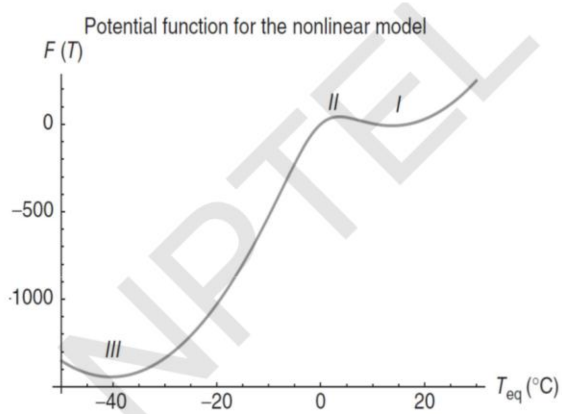
$$\frac{d\tilde{T}}{dt} = -\frac{dV}{d\tilde{T}}.$$

Clearly,

$$V = -\frac{1}{c}\int [Q(1-\tilde{\alpha}(\tilde{T})) - A - B\tilde{T}]\,d\tilde{T}$$

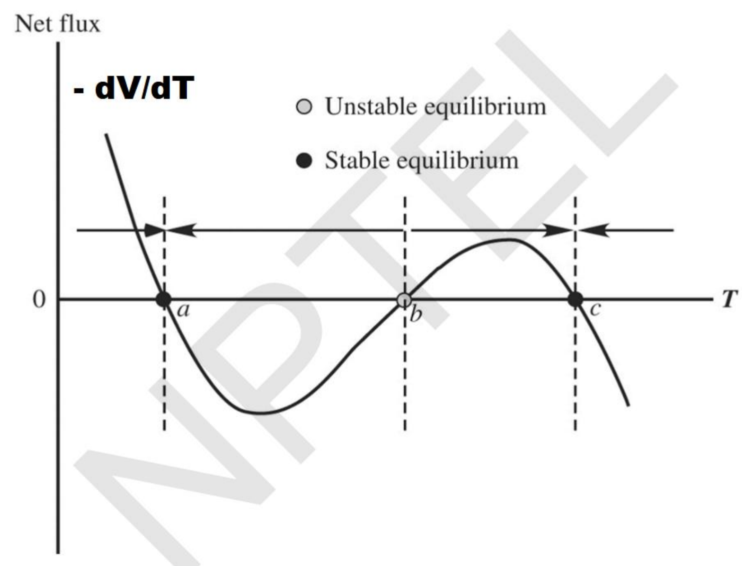
The non-linearity we want to explain better is better done through our energy balance equation, in which we have the rate of change of temperature of the Earth, heat capacity here, incoming radiation, albedo—so we have the absorbed solar radiation—and we have the emitted radiation, which is linearized, as we have done earlier. So, the outgoing radiation, the absorbed radiation, and the difference should drive the temperature.

Now, the best way to understand this equation, when changes in albedo could be non-linear, is to rewrite the equation based on what is called the Lyapunov potential. You write dT/dt and take it to the side, and write the right-hand side as –dV/dT, where T is temperature. So essentially, V is nothing but the integral of this equation—just rewriting it, nothing very unusual. Now, when you rewrite it this way, then you plot V as a function of T—this integral—then you will have this kind of shape.

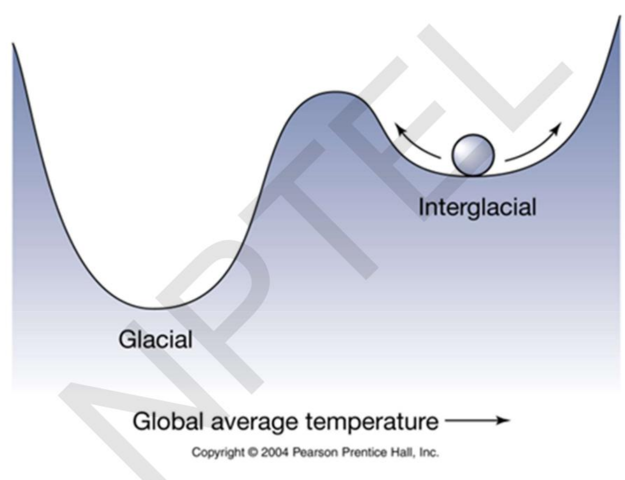


The Lyapunov potential has a shape in which it has two minima—here and here. One corresponds to snowball, and the other corresponds to the present temperature. The idea is that the system switches between one state and another state depending on the perturbation coming there—noise or whatever it is—that pushes this over this hump and takes it down here.

Similarly, to go from the ice age to the present climate, we need a large push above this hump to come here. So, the idea is that the trigger to push the system from one stable state to another requires some external trigger, and that could come from various sources.

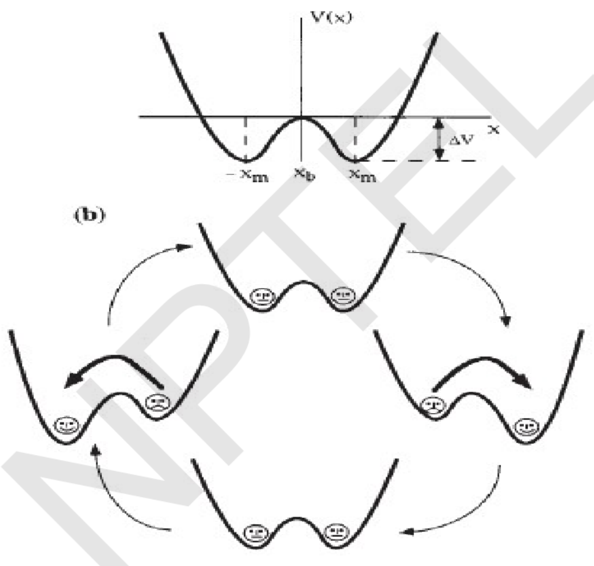


I repeat what we discussed when we talked about global mean temperature. In the graph shown above, ‘b’ was the last ice age, ‘c’ is the present climate, and ‘a’ was the snowball region. We saw that this last ice age state was unstable. The slightest perturbation switched it either to the present climate or to the snowball. So, this result is plotted in terms of dV/dT to explain the role of potential.



Essentially, we are saying that to shift the Earth's climate from one state to another in a non-linear problem requires something to push it over that hump which separates the two states. Once something pushes it up there, it will move to that other state. What pushes it up could be related to either land glaciers, sea ice, or some weather-related noise. For example, you can move from interglacial to glacial if there is a major volcanic eruption that cools the Earth so much that it shifts this from here to here. Some shock is needed to push it up.

Now, the best way to understand this is through looking at the snapshot of an animation done by Chris Boulton of the University of Exeter in the United Kingdom.



Here is a Lyapunov potential showing two stable states, and the Earth is in one stable state here and remains there. There are perturbations to this state, shown by this time series here, and you see the potential itself is changing on account of changes in temperature and ice cover. Suddenly, the potential becomes suitable for it to come this way. So, you can see that this animation represents very nicely how the Earth remains in one state for a long time because it is extremely stable.

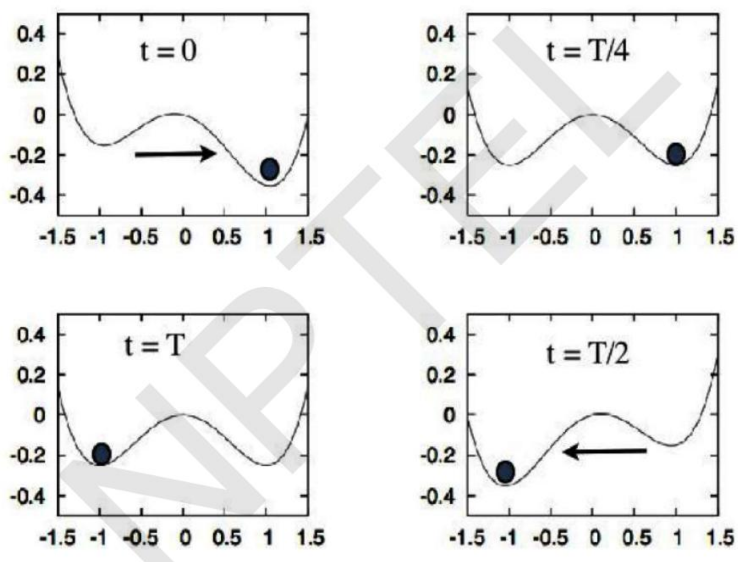
But as the potential changes in response to changes in Earth’s temperature, sea ice, and land ice cover, there comes a point at which the ball is able to jump over the hump. The hump is very weak here, and so the ball jumps over it and suddenly comes to the new state. You can see the sudden jump. This is the abrupt change in climate we are concerned about, which may happen in the future.

I want you to look at it a couple of times to be convinced that Earth can go from one state to another depending on the shape of the Lyapunov potential as well as the trigger—like a volcanic eruption or other noise in the system—which shifts this ball when the Lyapunov potential has the right shape. It will suddenly shift to this new state.

So, this is a nice work showing how even random noise can shift a system from one state to another. This is called stochastic forcing. That is, the forcing is not steady—it is fairly random—and under certain conditions, it can shift the Earth from one state to another if the Lyapunov potential has the right shape. So, the idea is that the Ice Age conditions change the Lyapunov potential, and then some random event shifts it.

If you understand this approach, then one can justify the argument about skipping 20,000-year periods. Imagine that in one 20,000-year cycle, the conditions were such that the ball could not shift to this state. It happened two or three times, and then on the fourth time, it shifted to the other side. So, the idea is that the shape of the Lyapunov potential was not the same during various times when the 20,000-year variation of solar radiation should have triggered either the collapse of the ice age or its establishment.

Let us look at it again. As the Lyapunov potential changes shape, there comes a point at which the Earth, in this state, can shift to the other. So, it depends on both the shape of the Lyapunov potential and the perturbation the system receives at that time.



At time t equals 0, one-fourth, half, and T—you can see that at this point, the shape of the potential is such that a little kick will shift it to the other state. So, going from one state to another—it shifts at a later point because the Lyapunov potential shape has changed slightly compared to before. The valley may have gone up. Earlier, the valley was steep, and it couldn't climb; now, the valley is shallow, so it climbs more easily. That's the logic here.

Let us look at another example of changes in Lyapunov potential (refer to the figure below). On one side, the system is in a deep well and will not come out easily. But as the well changes shape, at this point, it is suitable to change to the other state. This is based on a simple algebraic model for the Lyapunov potential.

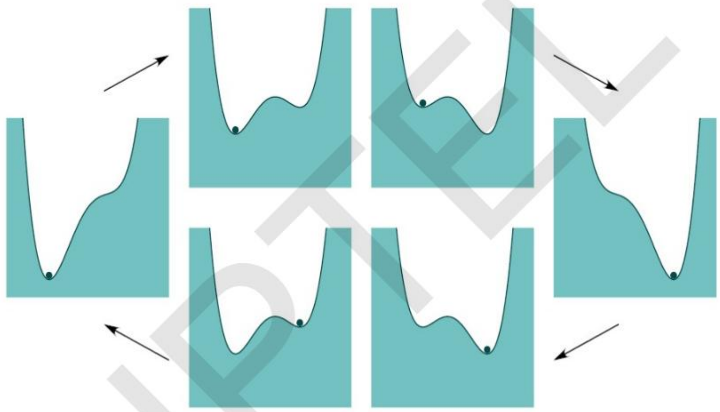
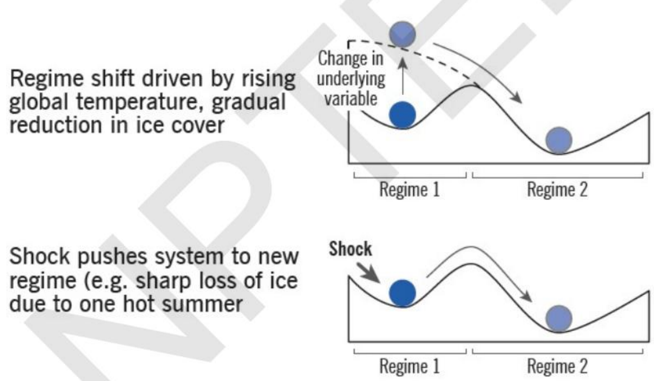


FIGURE 4. The potential $V(x,t) = \frac{1}{4}x^4 - \frac{1}{2}x^2 - \lambda(t)x$, with $\lambda(t) = K\cos(2\pi t)$, when K exceeds the threshold λ_c. In the deterministic case, with $\varepsilon \ll 1$, the overdamped particle jumps to a new well whenever $|\lambda(t)|$ becomes larger than λ_c, leading to hysteresis. Larger values of ε increase the size of hysteresis cycles, but additive noise of sufficient intensity decreases the size of typical cycles, because it advances transitions to the deeper well.

These are good examples, but I will say the best way to understand this would be to actually solve a problem. Here is an example where a regime shift occurs when some shock—such as a sharp loss of ice due to one hot summer (a random event)—shifts the system to a new state.

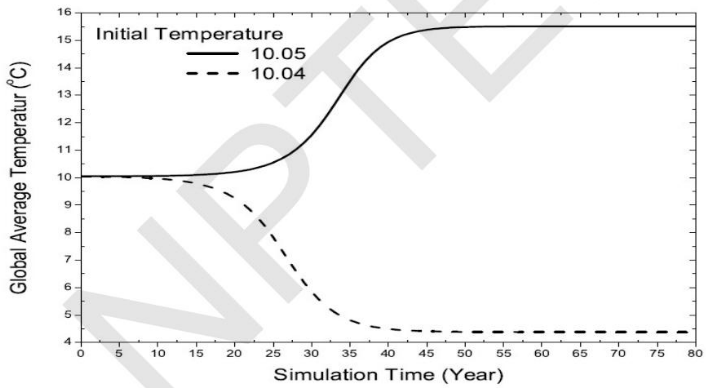
Figure 4.8b **A regime shift in the climate patterns of the Northern Hemisphere?**



This time, the conditions are more favorable than the previous time. That is the point.

Now, the best way for you to understand this problem, in the context of this course, is to solve a simple problem. Here is a problem where the rate of change of Earth's temperature is written in terms of incoming radiation.

$$C\frac{dT}{dt} = \frac{S}{4}\{0.607 + 0.056 \times \tanh[0.294 \times (T - 10.106)]\} - \varepsilon\sigma_0(T + 273.15)^4$$



T is the global mean temperature, and the albedo is written as a tanh (hyperbolic tangent) function—we did this earlier. This is an example. Then, the outgoing radiation is expressed via the Stefan–Boltzmann law and emissivity. Now, all of you should try to solve this problem on the computer, and you will find that depending on the initial temperature we give—say, 10.05°C—it shifts to the warm state, 15°C, which is the present climate. But if you give a slightly lower value, just 0.01°C less, it will shift to the glacial state around 4.5°C.

So, this equation is very sensitive to the initial condition. Whether the system will shift from the ice age condition to the snowball state or to the present climate depends on the temperature during that ice age. You can see a small change of 0.01°C shifts the solution one way or the other. This shows the impact of stochastic forcing. If the system is sitting very close to an unstable equilibrium, a slight push will shift it.

So, you can solve this equation, and today, all of you have access to software to solve this nonlinear equation by marching in time. It is very simple, and you will see your answer is very sensitive to the initial temperature. This is one example.

Now, another example uses the same equation (please refer to the figure shown below), taken further with an imposed variation in solar radiation—a cyclic variation, representing the 20,000, 40,000, or 100,000-year cycle of variation. And again, (1 – albedo) has that tanh function. Then, there is the outgoing longwave radiation (OLR), and a white noise term. This white noise is a random input to the system—it has no preferred frequency. If the noise occurs at all frequencies, it is called white noise.

$$C\frac{dT}{dt} = \frac{S[1 + A\cos\omega(t)]}{4}[1 - \alpha_m(T)] - \varepsilon\sigma_0(T + 273.15)^4 + a\overline{\Gamma}(t)$$

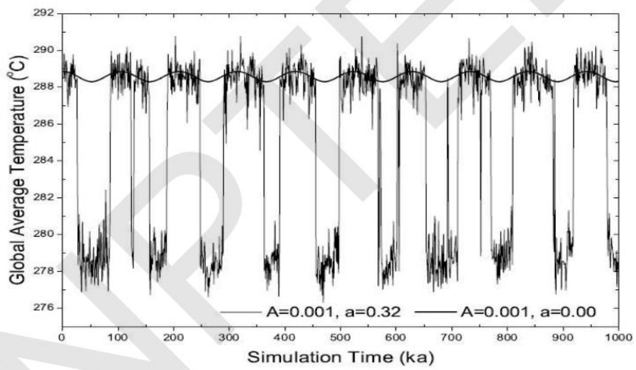


Fig. 6. The solution trajectory of Equation (5) when $A = 0.001$ and $a = 0.32$ (thin line) or 0.00 (strong line).

So, if you solve this equation, it can be shown that either you will get a solution which is periodic according to this forcing cos ωt. Or it will jump between the snowball state and the present climate at random. This depends upon the value of this noise. If there is no noise, it sits in. It follows this simple law. If there is a slight amount of noise, here it is 0.32, then it starts fluctuating between the two states. So, this shows the role of random noise, random perturbation on the solution of a non-linear equation. This equation is non-linear, both because of the outgoing longwave radiation and the albedo function. Now, in this paper, the author has given various values of A. A is the amplitude of the solar radiation, and small a is the noise amplitude. You can say if there is no noise, there is no stochastic resonance. If the noise is only 0.22, as soon as it goes to 0.26, it jumps. It starts having stochastic resonance. If the noise is too high, there is no stochastic resonance. If there is no forcing solar amplitude, then there is also no stochastic resonance.

Case	a	A	Stochastic Resonance Happened?
1	0	0.001	No
2	0.22	0.001	No
3	0.26	0.001	Yes
4	0.28	0.001	Yes
5	0.32	0.001	Yes
6	0.39	0.001	No
7	0.32	0	No

So, you can see that it is a very interesting example. You can see that he gets stochastic resonance if and only if there is a small amplitude of variation of the incoming solar radiation. And the noise is in a certain range. If the noise is too large or too small, it does not happen. So, there is a narrow range of noise amplitude during which this stochastic resonance takes place. If the noise is too high, there's no resonance either. And if there is no forcing in the solar amplitude, there is also no resonance. So, you can see this is a very interesting example. Stochastic resonance occurs only if there's a small amplitude variation in the incoming solar radiation and the noise is within a certain range. Too large or too small, and it doesn't happen. This is a very nice illustration of the role of noise in a nonlinear system. Remember, this won't happen in a linear system. If this alpha were constant and the system were linear, nothing would happen—it would just follow the forcing. But because of nonlinearity from albedo and OLR, even a small amount of white noise is enough to trigger oscillations.

Now, another issue I want to briefly mention—and I'll talk about it in more detail later—is the importance of continental location. The presence of landmass at high altitude is a prerequisite for developing large ice sheets. If there's only ocean in the polar regions, ice cannot form easily, because the ocean has large thermal inertia and a large mixing layer—it resists forming ice.

The role of the location of continents

The position of the continents with respect to the poles is probably one of the most important factors controlling the long-term fluctuations in global climate.
The presence of large land masses at high latitudes appears to be a prerequisite for the development of extensive ice sheets, because the large accumulations of ice associated with ice sheets cannot form over the ocean.
At the start of the present ice age, not only were North America and Europe located in the higher latitudes near the north pole, but Antarctica was located at the south pole.
This continental configuration led to extensive glaciation of both North America and Eurasia, as well as full continental ice sheets covering Antarctica.
During the ice age that occurred about 300 million years ago, the southern portion of the supercontinent Pangea was at the south pole. The result was extensive glaciation of what is now Africa, South America, India, Antarctica, Australia, and the Arabian peninsula.

On land, forming ice is easy. So, the configuration of continents matters.

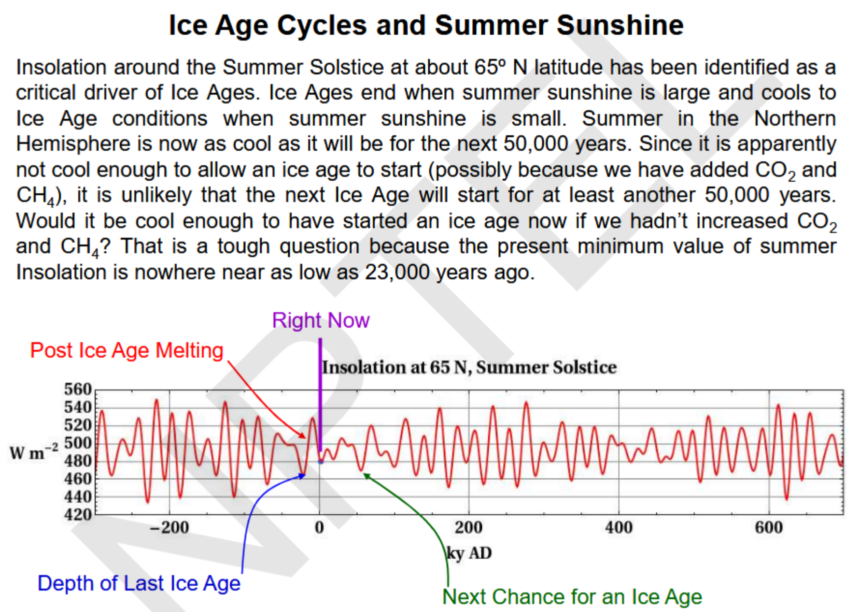
This issue is important because if you look at data from the last 5 million years, the continents were not located as they are today. Ice ages had a 41,000-year cycle 5 million years ago, but in the last million years, they show a 100,000-year cycle. One reason could be the shift in continental configuration. We'll return to this when we discuss Snowball Earth.

At 65°N,
obliquity accounts for about a third of the insolation variability, and precession two thirds.
The direct effect of eccentricity is small, but eccentricity regulates the intensity of the precessional variability.

The Earth's orbit will be nearly circular in the coming 50,000 years, so that precession effects will be smaller than usual

Now, the other issue to remember is that at 65° North, near the polar region, the obliquity or the tilt accounts for about one-third of the radiation variability. Precession accounts for two-thirds, and the impact of precession depends on eccentricity. The direct effect of eccentricity is small, but it regulates precession. Right now, the Earth's orbit is almost circular, so the effect of precession is not large. But a long time ago, when eccentricity was higher, precession played a very important role. That is why you see that in the period between 1 million and 5 million years ago, we had a 41,000-year cycle. So, the role of obliquity also depends on how eccentric the orbit is. The more eccentric the orbit, the more dominant the effect of precession over obliquity.

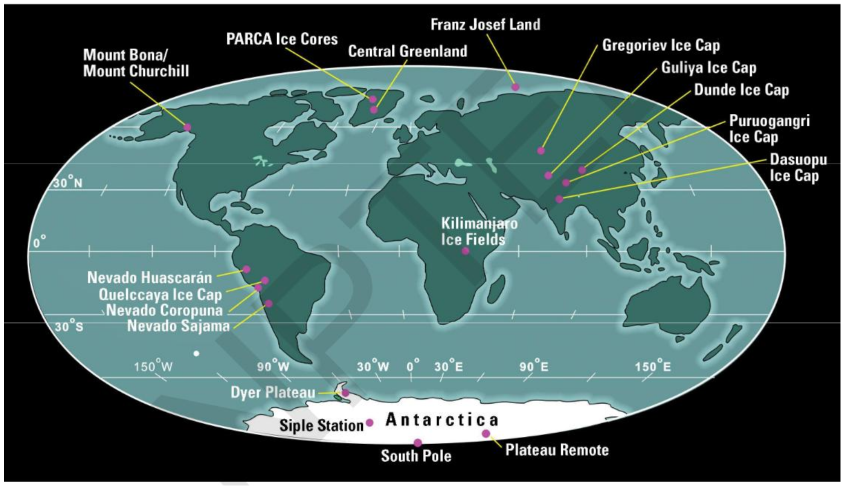
So, people have examined when the next Ice Age will occur. There's a lot of debate. If human beings had not interfered with the global climate by increasing carbon dioxide, the expectation was that the next Ice Age would come around 50,000 years from now. But there is an argument that, due to the increase in carbon dioxide from 280 parts per million to now 420 parts per million, the chances of the next Ice Age occurring 50,000 years from now are very, very low.



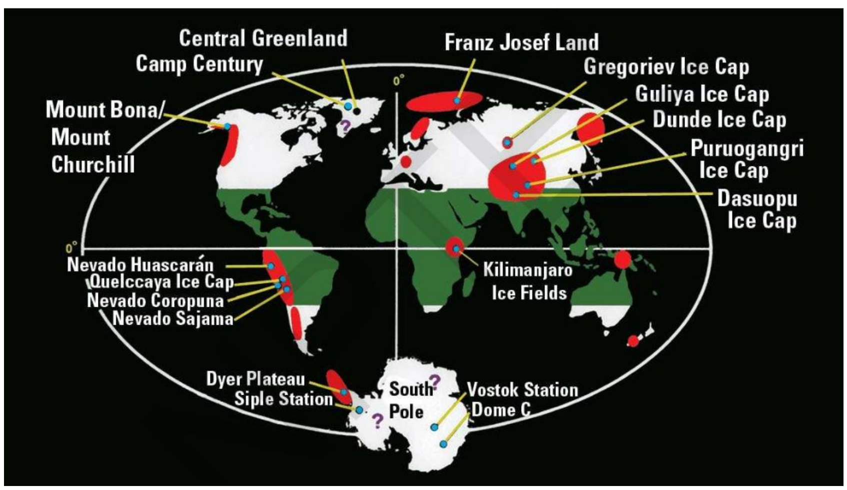
This closes the discussion on Ice Age cycles. To briefly summarize: we saw that the amount of radiation falling in the polar region during summer plays a critical role. That is the Milankovitch hypothesis. But that hypothesis cannot fully explain why we observe a 100,000-year Ice Age periodicity in the last million years. To explain that, we need to invoke nonlinear phenomena. One of them could be stochastic resonance, as we saw in the animation. Or it could be due to land ice depressing the continent, causing a delay in the Ice Age. So, there are various hypotheses—none of them fully accepted. But we all agree that the trigger for an Ice Age is incoming solar radiation. When the Ice Age actually ends or starts depends on some nonlinear phenomena that we do not yet fully understand.

Now, the next topic we are going to take up is understanding what happened during the last Ice Age, which is very important for understanding the role of carbon dioxide. If you recall, in the one-million-year dataset we have from ice cores, the carbon dioxide and the temperature in Antarctica varied almost together. So, from that dataset, we could not show whether it was the temperature increase that caused CO_2 to rise, or the CO_2 increase that caused temperature to rise. That was not resolved.

For that, we need a much higher resolution dataset, and that was obtained recently from ice cores in mountain glaciers—Himalayas, South America, Greenland, Kilimanjaro, and Antarctica. So, lots of data have been gathered over the last 40 years.

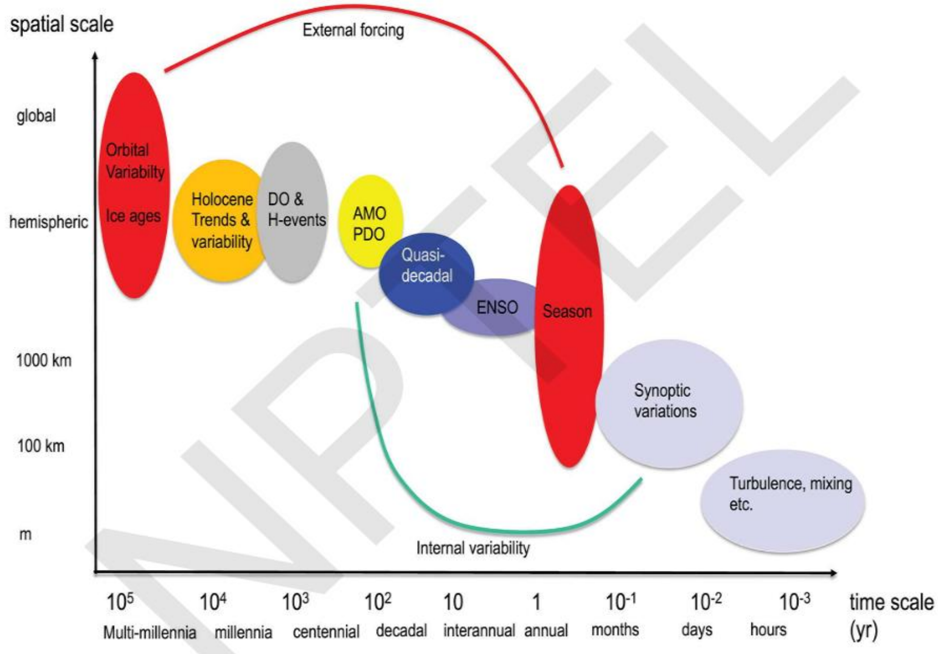


In addition to all this ice core data, we also have ocean core data, which spans much longer periods.

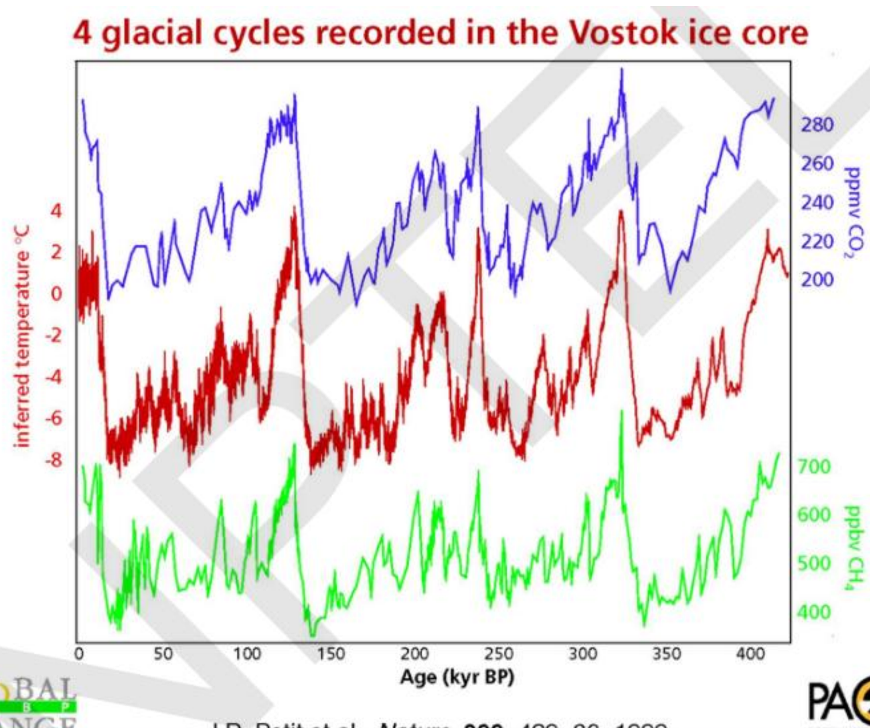


Ice core data goes back, at most, 800,000 years, but ocean cores can go back millions of years because you can drill as deep as needed into the ocean to study all the deposits. Now I want to remind you that we are talking about changes in Ice Ages due to orbital variation, which involves timescales of 10,000 to 100,000 years. Most of us are only familiar with external forcing from the seasonal cycle—winter and summer—caused by Earth's axial tilt. But many other things are happening on Earth that are not caused by external forcing.

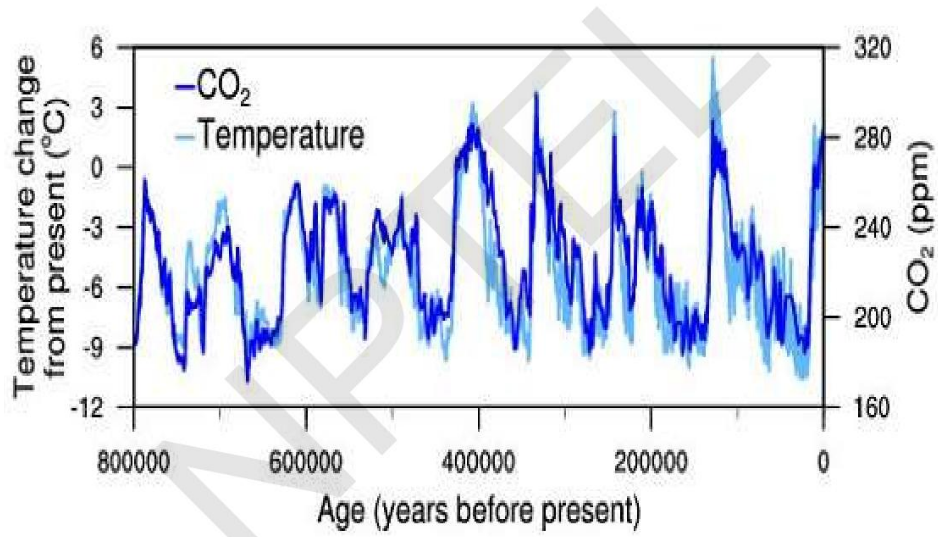
We've seen Dansgaard–Oeschger (DO) events, Heinrich events, decadal variations of the Atlantic Meridional Oscillation, the Pacific Decadal Oscillation, and El Niño phenomena in the Pacific. All these are internally forced—internal variability.



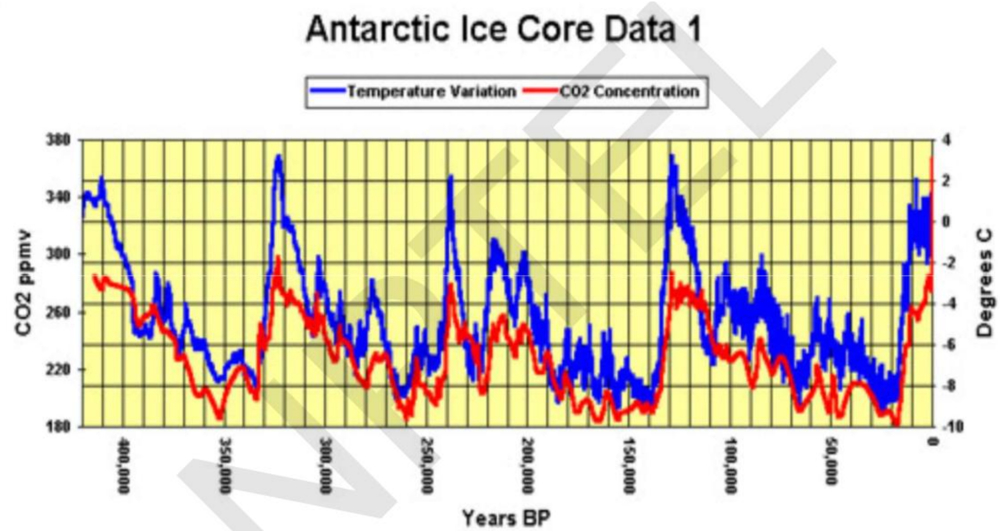
So now we are going to look at timescales on the order of thousands of years to understand why sudden changes take place and what role carbon dioxide plays. Let me remind you again that, from the Vostok ice core, all we could say was those changes in carbon dioxide and methane closely mirrored changes in temperature. We could not determine which one leads or lags.



J.R. Petit et al., *Nature,* **399**, 429–36, 1999.



To show that more clearly, this figure superimposes CO_2 and temperature over the last 800,000 years. You can see that we cannot really differentiate between them.



One more graph—this also shows, on a slightly different scale, red for carbon dioxide and blue for temperature. You can see that the peaks of CO_2 coincide with the peaks of temperature. From this figure, we cannot conclude whether CO_2 causes the temperature increase, or whether the temperature increase causes the rise in CO_2. That we cannot resolve with the ice core data alone.

So, we needed much more accurate data. In the last 10–20 years, people have obtained very high-quality, high-resolution data for the last 20,000 years. From that, we hope to resolve the issue.

The variations in Carbon dioxide and methane are closely associated with changes in temperature in Antarctica during the past 700,000 years.

Did change in temperature occur on account of the increase in greenhouse gases or vice-versa?

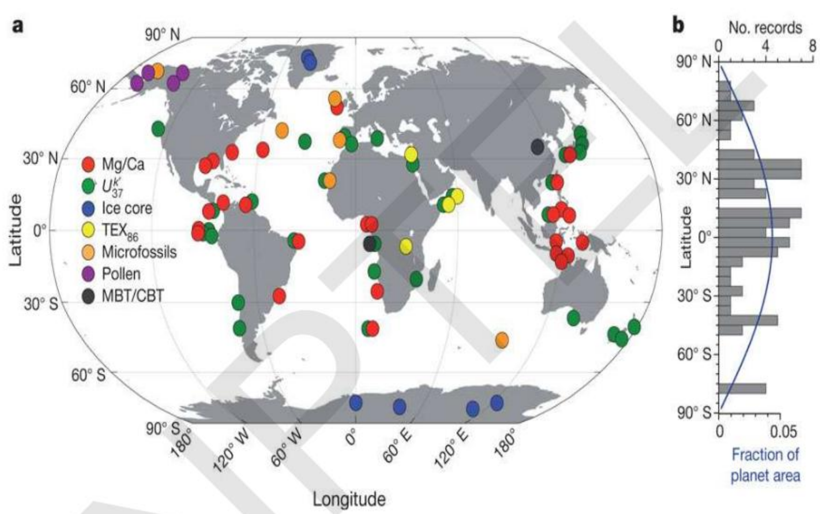
This is important to understand because many people have argued that the increase in carbon dioxide during the past 150 years could have been on account of an increase in temperature and not the other way around.

The resolution of this data is not good enough to determine whether temperature lags behind the increase in greenhouse gases

So, the question is: did the change in temperature occur due to the increase in greenhouse gases like CO_2 and methane, or was it the other way around?

This is important to understand because many people argue that the increase in CO_2 over the past 150 years could have been caused by rising temperature, not the other way around. They suggest that CO_2 increased because warming caused it to be released from the ocean.

To resolve this, we look at a very accurate dataset.



JD Shakun et al. Nature 484, 49-54 (2012) doi:10.1038/nature10915

This was a famous paper published by Jeremy Shakun in *Nature* about 12 years ago. It's a very important paper, and that's why I want to spend some time discussing it. They looked at all kinds of cores from around the world—Antarctic ice cores, Arctic ice cores, lots of ocean core data. Using various proxies—magnesium-to-calcium ratio for temperature, microfossils, pollen—they tried to determine both temperature variation and CO_2 variation from bubbles in the ice cores.

Now, notice that the cores are not uniformly distributed around the globe. There are large gaps in the Indian Ocean, Atlantic Ocean, and Pacific Ocean—there's no ice core data. We can't do much about that.

So, the challenge was to take all these different proxy data from the last 20,000 years and combine them to estimate a global mean temperature. That's tough, because you don't have much data in certain regions. You have to somehow convince people that averaging over the available cores gives you the correct global mean temperature. Today, we have the advantage of climate model simulations that extend back 20,000 years. These models help confirm whether these observations are sufficient to estimate global mean temperature accurately. That's the challenge we need to tackle.

We'll continue this discussion in the next lecture.