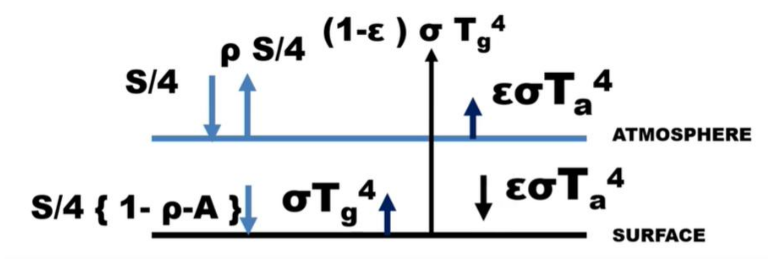
Climate Change Science
Prof. J. Srinivasan
Department of Environmental Science
Indian Institute of Science, Bengaluru

Lecture 07
Multiple-Equilibrium States

In the previous lecture, a simple analytical model was developed to estimate the global mean temperature of the Earth. This model incorporates the effects of solar reflectivity (albedo) from both the surface and the atmosphere, solar absorption by the atmosphere, and thermal emission by the atmosphere in both upward and downward directions. While the model assumes equal upward and downward infrared radiation from the atmosphere, this is not strictly accurate because the atmosphere's temperature decreases with altitude, resulting in less upward emission. However, since the model assumes an isothermal atmosphere, this variation is not captured.



Another simplification in the model is the neglect of energy transfer from the surface to the atmosphere through evaporation and turbulent fluxes. Interestingly, these two simplifications—overestimating downward emission and ignoring turbulent energy transfer—tend to cancel each other out, making the final temperature estimate reasonably accurate.

The final outcome of the model shows that surface emission balances the net incoming solar radiation, taking into account planetary albedo, solar absorptivity, and infrared emissivity of the atmosphere. Despite its simplicity, the model elegantly relates global mean temperature to just four key parameters: incoming solar radiation (S), planetary albedo (ρ), solar absorptivity (A) and infrared emissivity (ε) By using satellite data and modelled values for these four parameters, the model yields a global mean temperature close to the observed value of 288 K.

The simplicity of the model allows for a sensitivity analysis, evaluating how temperature changes in response to small (1%) variations in each of the four parameters. The results show that:

- An increase in solar radiation (S) or emissivity (ε) leads to a rise in temperature.
- An increase in solar absorptivity (α) or planetary albedo (A) results in a decrease in temperature.

Each 1% change in these parameters leads to a temperature change of approximately 0.5 K, with some variation. This general finding that the Earth's temperature is moderately sensitive to these key parameters provides a valuable framework for understanding how changes in atmospheric composition and solar input affect the Earth's climate system.

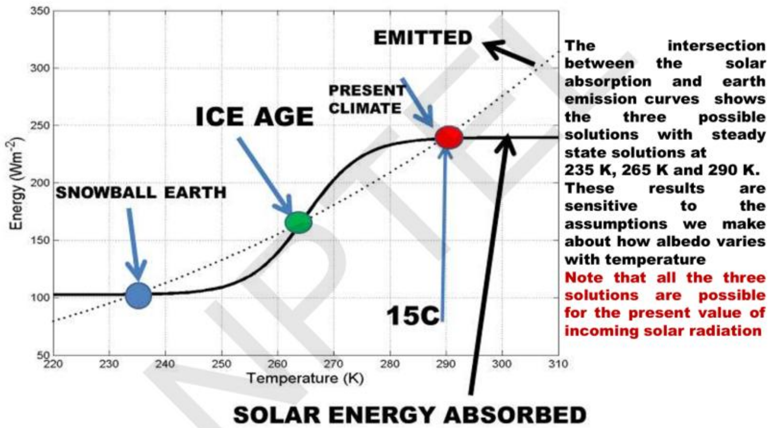
While the simple analytical model provides useful insights into how Earth's temperature responds to changes in individual parameters, it is important to recognize its limitations. Specifically, when the incoming solar radiation increases and causes a rise in temperature, this leads to secondary changes in other parameters that were previously assumed constant. For instance, an increase in temperature enhances atmospheric water vapour, as dictated by thermodynamics, which in turn increases the emissivity of the atmosphere. Similarly, cloud cover and ice cover change with temperature, altering the planetary albedo. Melting ice reduces albedo, allowing the Earth to absorb more solar energy that reinforces warming.

This interconnected behaviour where a change in one parameter triggers changes in several others is referred to as a chain reaction. In climate science, such interactions are formally known as feedbacks. The simple partial derivative approach used in the earlier model fails to capture these feedback effects because it assumes all other parameters are held constant when one changes. However, in the Earth system, this assumption does not hold true.

One of the most important feedback mechanisms involves ice cover. When ice melts into water, the solar absorptivity of the surface increases significantly by up to a factor of 10 in the affected regions. This greatly amplifies the initial warming. Therefore, a more comprehensive understanding of climate change requires models that allow all parameters to change simultaneously, rather than in isolation.

To address this complexity, the next step involves studying multiple equilibrium states in Earth's climate system. Unlike the previous model, which assumed constant values for albedo, solar absorptivity, and emissivity, these variables are now treated as functions of temperature. For example, cloudiness and ice-cover both of which affect albedo, respond to temperature changes. Regions with temperatures below 273 K are more likely to be ice-covered, while warmer regions have less ice, leading to further reductions in albedo and more warming. Hence, temperature-dependent feedbacks are essential for a more realistic understanding of how the Earth's climate can evolve.

This section introduces the concept of multiple climate equilibria using a schematic representation. The plot shows two key quantities: the absorbed solar radiation (solid black line) and the thermal emission from the Earth's atmosphere to space (dotted line). The intersection points of these two curves represent conditions where the energy balance is achieved—i.e., the incoming energy equals the outgoing energy, leading to an equilibrium state.



The amount of solar radiation absorbed by Earth depends strongly on the surface temperature, because temperature influences ice cover and hence the planetary albedo. At very low temperatures, Earth would be almost entirely ice-covered, reflecting much of the incoming solar radiation—this is referred to as the Snowball Earth state. At very high temperatures, the Earth would be nearly ice-free, leading to greater solar absorption and a different climate equilibrium.

Between these extremes lies a third intermediate state, where the Earth is partially ice-covered, resembling conditions during the Last Glacial Maximum (~20,000 years ago). For the current solar constant of 1364 W/m², the model suggests the existence of three equilibrium states:

1. A fully ice-covered Earth (Snowball Earth),
2. The current climate (~15°C global mean temperature),
3. An ice age state, with significantly more ice cover than today.

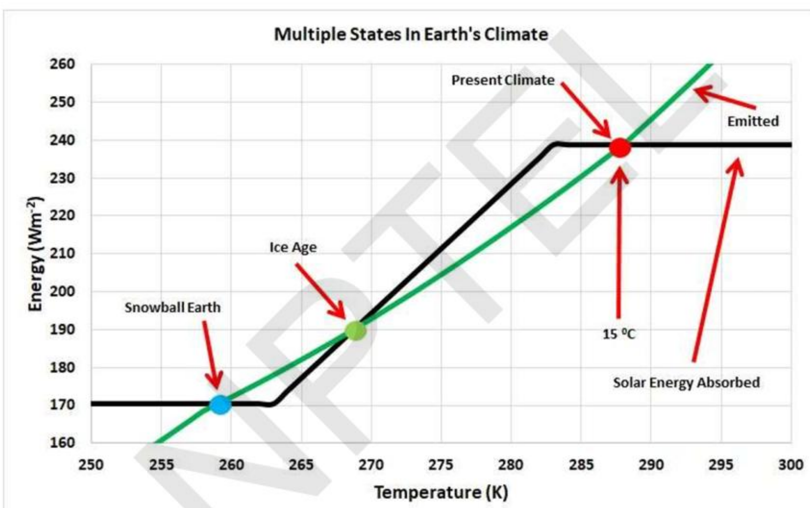
Given the existence of three possible equilibrium climate states for the same incoming solar radiation, a natural question arises: Why is Earth in its current climate state and not in one of the other two equilibria, the Snowball Earth or the Ice Age state? The answer lies in the role of initial conditions and historical pathways. The Earth's current climate state is the result of its climate evolution over time, and the specific equilibrium it settles into depends on where it started from.

If the Earth previously had a warmer climate, it would likely have transitioned into and stabilized at the present warm state, due to feedback mechanisms reinforcing that regime.

Conversely, if the Earth had emerged from a very cold, ice-covered state like the Snowball Earth, it could remain in that state indefinitely, unless a major perturbation—such as increased volcanic activity or changes in atmospheric composition—drives it toward a warmer equilibrium.

This concept illustrates a key idea in climate science: the climate system is path-dependent, and multiple stable states are possible under the same external forcing. The state that is realized in practice depends not only on the current conditions but also on the climate history and any past disturbances that may have pushed the system from one equilibrium to another.

We can also understand multiple climate equilibrium states by assigning three discrete values to the Earth's albedo based on temperature regimes. Although this approach is somewhat arbitrary, it serves as a useful conceptual illustration. If the temperature falls below 258 K, well below the freezing point of water, the Earth is assumed to be almost completely ice-covered, and the albedo is set to 0.66, reflecting high solar radiation. If the temperature is close to or above present-day values, the albedo is taken as 0.3, representing a relatively ice-free state. For intermediate temperatures, the albedo is assumed to vary linearly between 0.66 and 0.3.



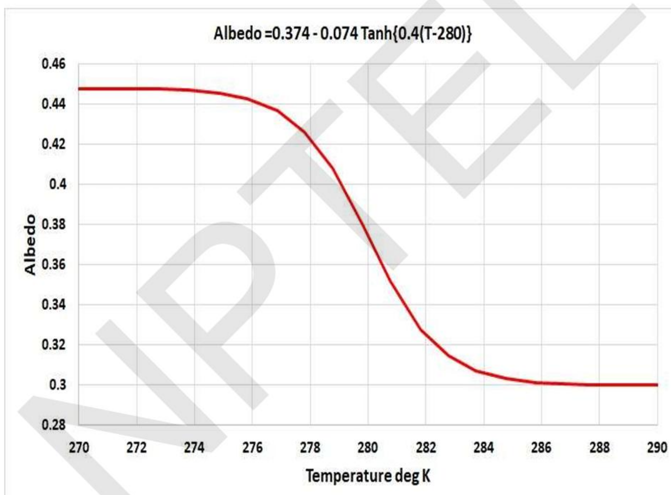
When these albedo values are used in the energy balance equation, three distinct solutions for equilibrium temperature emerge:

- 227 K for a snowball Earth,
- 278 K for the last glacial maximum (Ice Age), and
- 288 K for the current climate.

These solutions can be visualized the graph where the green line represents the outgoing longwave radiation emitted to space (which increases smoothly with temperature), while the black line represents absorbed solar radiation (which undergoes discrete jumps at around 263 K and 283 K due to the imposed albedo changes). The intersections of these two curves indicate the possible equilibrium states for the Earth's climate under the same solar input.

In this previous approach we have used arbitrary, piecewise values for albedo to derive the Earth's possible climate equilibrium states, which leads to an underestimation of the Ice Age temperature compared to observations. A more realistic approach involves modelling the albedo as a smooth function of temperature, rather than as step changes. A common and effective method is to use a hyperbolic tangent (tanh) function for albedo variation. In this formulation, the albedo starts at a higher value around 0.45 for cold, ice-covered conditions and gradually decreases as temperature increases. The transition is rapid but continuous, and as the climate approaches present-day temperatures, the albedo smoothly approaches a value of 0.3, consistent with observed modern conditions.

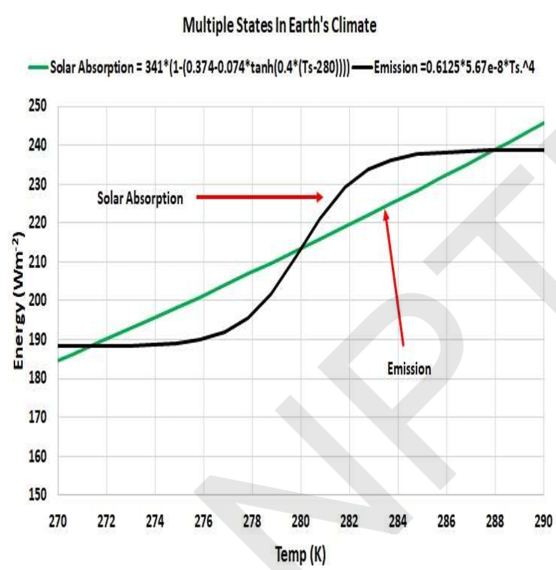
A different assumption about the variation of albedo with temperature



This continuous model better captures the nonlinear and gradual nature of changes in Earth's reflectivity due to melting ice and changing cloud cover. However, the use of a tanh function complicates the mathematics, making an analytical solution difficult. As a result, the model now requires a numerical solution to determine the equilibrium temperatures, unlike the simpler piecewise model that allowed direct analytical interpretation.

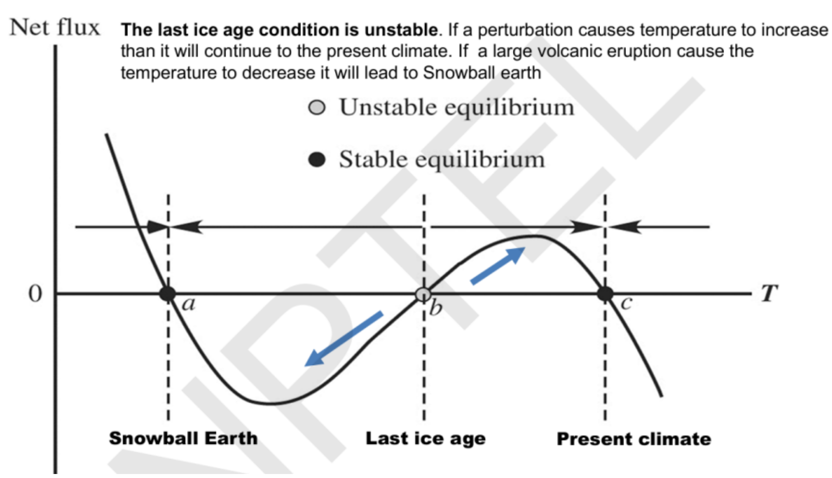
Using the more realistic Liou model, which employs a smooth albedo function, three equilibrium climate states emerge: the Snowball Earth (≈271 K), the Last Ice Age (≈280 K), and the present climate (≈288 K). While the temperature of the Snowball Earth is uncertain due to the lack of accurate historical data, the Ice Age and current climate values align closely with observations. A natural question arises regarding the stability of these three states - whether a small perturbation, such as a volcanic eruption, could push the Earth away from one state toward another.

Stability can be assessed either through complex climate models or a linearized perturbation approach, assuming the disturbances are small. Even without detailed mathematics, qualitative reasoning reveals that the present climate and Snowball Earth states are stable, while the Last Ice Age state is unstable. This conclusion is based on the balance between absorbed solar radiation (black line) and emitted thermal radiation (green line). For a stable state, if a perturbation reduces the temperature, the resulting imbalance between the absorbed and emitted fluxes acts to restore the original temperature.



Which of these solutions are stable to small perturbations?
These perturbations can be volcanic eruptions are human induced changes in outgoing longwave radiation(due to increase in greenhouse gases)
The temperature changes due to perturbations can be written as
C dT/dt = Net radiative flux
Where C is the heat capacity and
Net flux
= Absorbed solar - Earth's emission to space
If the perturbation causes a positive net flux it will be unstable because the temperature will increase continuously until anew steady state is reached

In the present climate state, a temporary cooling from volcanic aerosols reduces emitted radiation more than absorbed solar radiation, producing a positive net flux that pushes the system back to equilibrium. A similar restorative effect occurs in the Snowball Earth state. In contrast, the Ice Age state is unstable: a small perturbation can cause a runaway change because the imbalance between absorbed and emitted fluxes amplifies the disturbance. This behaviour is represented below schematically.



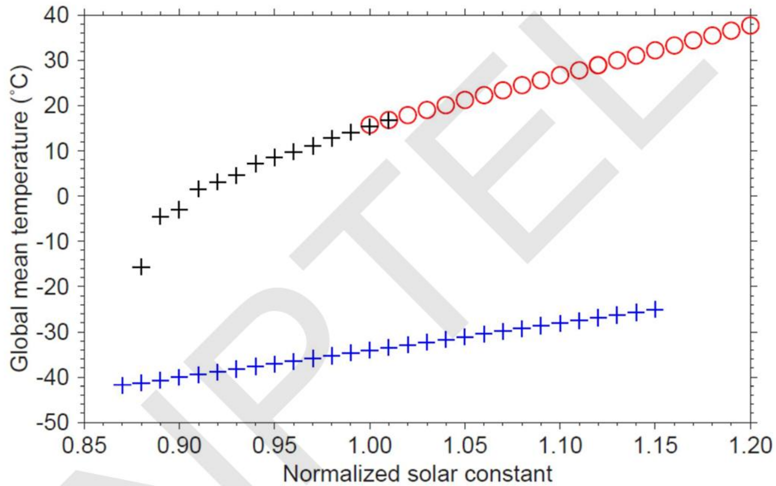


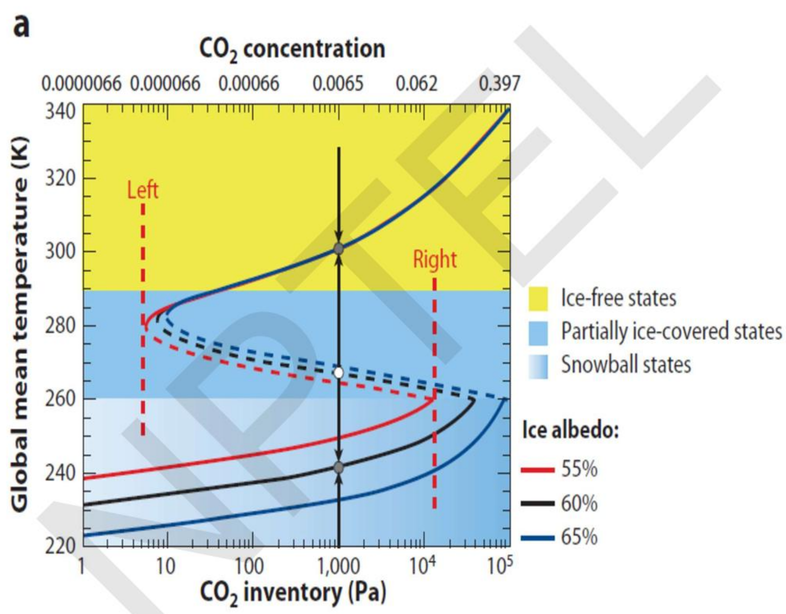
Figure 1: Characteristic solution of the EBM, plotted as global mean equilibrium temperature against the fraction of the present-day solar constant (cf. McGuffie and Henderson-Sellers 1997). Red circles denote *ice-free solutions*, black crosses an *ice cap solution branch* and blue crosses *fully ice-covered solutions*.

Following the Liou's energy balance model with varying planetary albedo as a function of global mean temperature, the different possible stable equilibrium solutions as a function of Solar constant is indicated in the above figure. The Solar constant values denoted above are normalized with respect to present-day value of Solar constant. It can be observed in the figure that, a completely ice-free (red circles), a partially ice-capped (black crosses), or a fully ice-covered (blue crosses) – all three states being possible with respect to present-day value of Solar constant. However, when the Solar constant is decreased anywhere between 1% to 10%, we observe that Earth's climate cannot have a solution where it is completely ice-free. Beyond a Solar constant decrease of 12%, we notice that only Snowball Earth solution holds true. So, if the Earth's climate is in a partially ice-capped state, any further decrease in Solar constant beyond 12% would result in a sudden jump towards a fully ice-covered Snowball Earth state as there are no intermediary stable solutions.

Similarly, it can be observed in the figure that an increase in Solar constant between 2% to 15% cannot have a partially ice-capped solution. Also, an increase in Solar constant by more than 15% for a Snowball Earth climate will lead to a sudden transition to completely ice-free state due to the absence of any other stable solution in-between. These discontinuous jumps in the climate response confirm that the system cannot be described by a simple linear model; instead, it behaves in a highly non-linear manner.

The energy balance equation developed earlier still applies but will yield no solution in the unstable region. The implication is that for certain ranges of input radiation, only specific temperature states are physically realizable, highlighting the non-linear behaviour of the Earth's climate system.

This phenomenon can also be understood not only through variations in incoming solar radiation, but also via changes in atmospheric carbon dioxide concentration, whether due to human activity or natural processes such as volcanic eruptions. A reduction in CO_2 levels pushes the Earth's energy balance toward colder equilibrium states. In such cases, only one stable solution might exist: the Snowball Earth state. The exact temperature of this state depends strongly on the assumed ice albedo, which can vary significantly depending on surface conditions such as the presence of bubbles in ice, snow cover, and impurities. Typical ice albedo values range from 0.55 to 0.65, with higher albedo leading to lower equilibrium temperatures due to greater reflection of solar radiation.



Conversely, if CO_2 concentrations rise significantly, such as from intense volcanic outgassing, the Earth system can undergo a sudden transition to a much warmer state, bypassing intermediate climate regimes. In this schematic framework described in the above figure, there are again three potential equilibrium solutions:

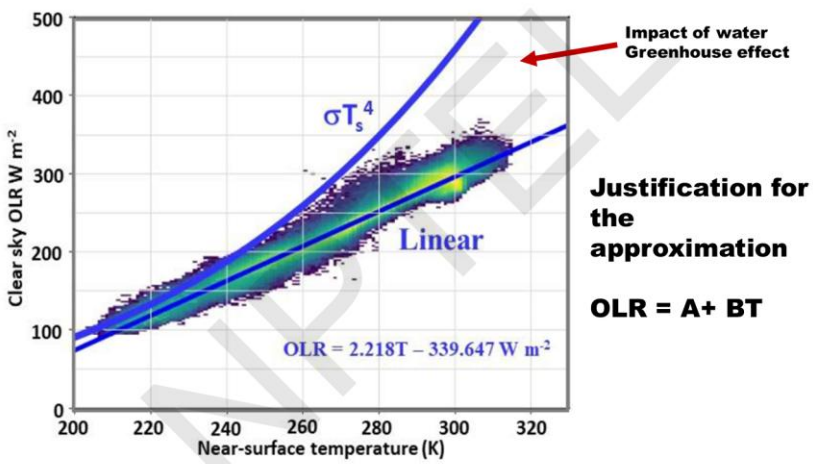
- A stable cold state with complete ice cover (Snowball Earth),
- A stable warm state with minimal to no ice (Ice-free states),
- And an unstable intermediate state with partial ice cover, analogous to an ice age.

The intermediate state is unstable, as small perturbations can push the system toward one of the two stable states. This reinforces the inherently non-linear nature of the Earth's climate system and the potential for abrupt climate transitions triggered by gradual changes in key variables like CO_2.

To analytically examine the stability of the climate system, a simplified model is employed. Unlike the earlier blackbody assumption where Earth's outgoing longwave radiation (OLR) is proportional to $\sigma T_s{}^4$, this analysis adopts a more realistic approximation valid for small temperature changes. Due to the presence of greenhouse gases, the Earth's emission observed from satellites deviates from the idealized blackbody curve and behaves almost linearly over a temperature range of 220 K to 300 K. Thus, OLR is approximated by the linear relation:

$$OLR = A + BT$$

Here, the constants $A = -339.647\, Wm^{-2}$ and $B = 2.218\, Wm^{-2}K^{-1}$ are derived from satellite data and depend on greenhouse gas concentrations and climatic assumptions.



A transient energy balance model is now considered. The energy equation is:

$$C\frac{dT}{dt} = \frac{S}{4}(1-\rho) - OLR$$

Where:

- C is the heat capacity of the Earth (mainly the ocean),
- S is the incoming solar radiation,
- ρ is albedo,
- T is the surface temperature.

To explore perturbations around the present equilibrium temperature T_0, a small deviation T' is introduced such that $T = T_0 + T'$. The change in albedo with temperature is expressed as:

$$\rho = \rho_0 + \frac{d\rho}{dT}T'$$

We assume the following equilibrium

$$\frac{S}{4}(1-\rho_0) = A + BT_0$$

On substituting the equilibrium state with the perturbed terms in the transient energy balance model, we get

$$C\frac{dT'}{dt} = -\frac{S}{4}\frac{d\rho}{dT}T' - BT'$$

This is a first-order linear differential equation whose solution is:

$$T' = T_0 e^{-\frac{1}{C}\left(\frac{S d\rho}{4 dT}+B\right)t}$$

The sign of the exponent determines stability:

- If the term inside the exponent $\left(\frac{S}{4}\frac{d\rho}{dT} + B\right)$ is positive, T' decays with time ⇒ the system returns to equilibrium ⇒ stable
- If $\left(\frac{S}{4}\frac{d\rho}{dT} + B\right)$ is negative, T' grows ⇒ the system diverges ⇒ unstable

Hence, stability requires:

$$B > -\frac{S}{4}\frac{d\rho}{dT}$$

Now, B depends on the greenhouse effect, while $\left(\frac{S}{4}\right)\frac{d\rho}{dT}$ depends upon at what rate the albedo of the earth varies with temperature. Note that the value of B depends on the amount of greenhouse gases and albedo changes depend on how temperature changes affect the ice and the ice model used. In the present climate, B is typically greater than $\left(-\left(\frac{S}{4}\right)\frac{d\rho}{dT}\right)$ for typical changes in albedo, making the system stable. The same is true for the Snowball Earth, where $\left(\frac{S}{4}\right)\frac{d\rho}{dT} \approx 0$, so the condition is trivially satisfied. However, during the last Ice Age, albedo sensitivity may have been high, potentially violating the condition, rendering the intermediate state unstable.

This linearized perturbation analysis provides valuable insight into why of the three equilibrium climate states — Snowball Earth, Ice Age, and present climate — only the first and third are stable under small perturbations.

At present, human activity is inducing climate change at an unprecedented rate, with global mean temperature increasing by approximately 1°C over the past 100 years. This rate of change is nearly ten times faster than natural climate variations observed in the Earth's history, where comparable temperature changes occurred over 1,000 years. Given this rapid pace, the simple linear perturbation approach discussed earlier, which assumes small and gradual changes, is insufficient to capture the full complexity of current and future climate dynamics.

In reality, multiple feedback mechanisms beyond just albedo may contribute to temperature change. These include changes in greenhouse gas concentrations, vegetation cover, cloud formation, and aerosol effects, among others. When these factors interact, they can lead to nonlinear and accelerated changes, making the climate response more complex.

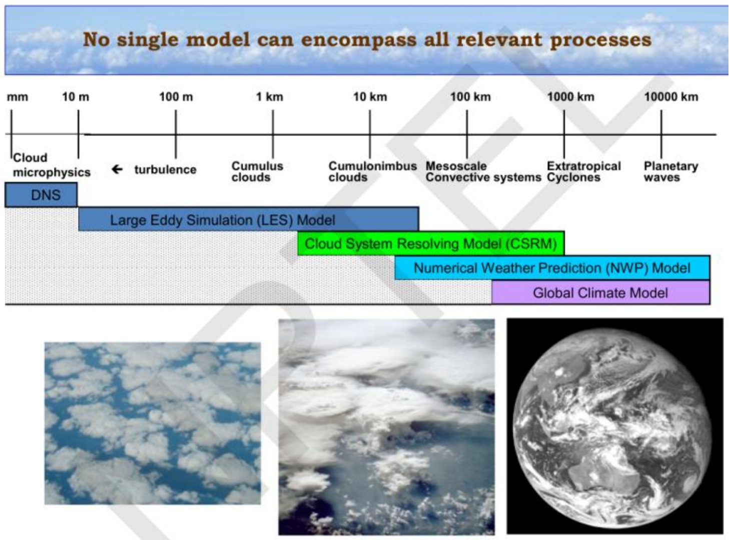
To analyse such scenarios, simple energy balance models are inadequate. Instead, comprehensive climate models are required. These models integrate a wide range of physical laws that govern the climate system, such as fluid dynamics, radiative transfer, thermodynamics, and biogeochemical cycles. These laws must be mathematically formulated, numerically solved, and implemented on high-performance computers. Thanks to technological advancements over the past 60 years, such complex simulations are now feasible. Consequently, state-of-the-art climate models are actively being developed and used worldwide to project future climate change with increasing realism and accuracy.

In 2021, the Nobel Prize in Physics was awarded to Syukuro Manabe and Klaus Hasselmann for their pioneering work in developing numerical models that simulate the Earth's climate system with remarkable realism. Their contributions have played a central role in enabling the Intergovernmental Panel on Climate Change (IPCC) to convincingly demonstrate the reality and urgency of human-induced climate change to policymakers and global leaders. A highly recommended book by Syukuro Manabe and Anthony J. Broccoli, titled *Beyond Global Warming: How Numerical Models Reveal the Secrets of Climate Change*, provides valuable insights into this field.

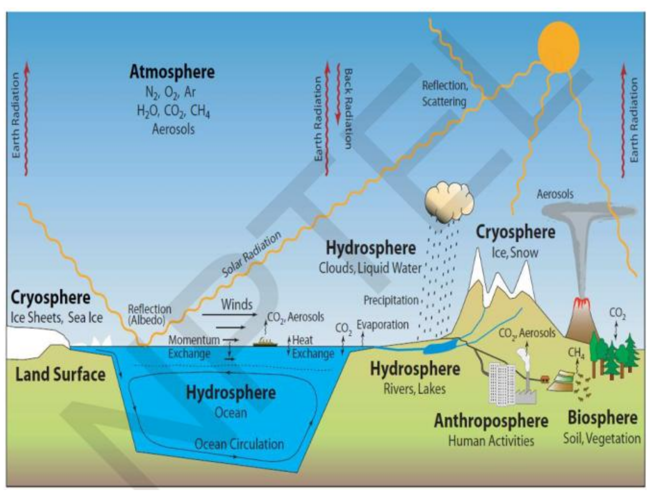
Manabe has emphasized a critical point: "The prediction of climate change without accompanying understanding of it is no better than the prediction of a fortune teller." This statement is particularly relevant in the current era, where machine learning (ML) and artificial intelligence (AI) are increasingly applied to climate and other scientific problems. While these tools can offer powerful predictive capabilities, their black-box nature can obscure the reasoning behind a given outcome. As a result, even if such models produce correct predictions most of the time, they can still fail unpredictably, and without understanding the underlying mechanisms, one cannot fully trust the results.

This underlines the importance of not merely running models but also diagnosing and interpreting their behaviour. In academic research, particularly in doctoral programs, a significant focus is placed on understanding how and why a climate model produces certain outputs. This involves analysing internal components, running multiple simulations, and reviewing a wide body of scientific literature. Without such rigorous scrutiny, reliance on model outputs—no matter how sophisticated the tools—risks becoming speculative rather than scientific.

Despite the advances in complex climate models, it is essential to recognize that the Earth system is inherently complex, and approximations are unavoidable in all models. These simplifications, though necessary, can introduce significant errors in simulations. One of the most challenging aspects in climate modelling today is the representation of clouds. Clouds operate at spatial scales of about 1 kilometer, but current global climate models typically use grid cells that are 50 kilometers by 50 kilometers or larger. This means that cloud processes occurring within each grid box cannot be directly resolved and are instead represented using empirical parameterizations. These empirical formulations introduce uncertainty, and while modelling capabilities may improve in the next 10 to 20 years, the present limitations necessitate caution in interpreting model results.



The complexity of the Earth's climate system spans a wide range of spatial and temporal scales, from cloud microphysics (e.g., ice and liquid particles, aerosols) to cloud systems that range from 1 km to over 1000 km, including large-scale features like cyclones and planetary waves. Presently, most models perform reliably only at scales greater than 100 kilometers. Simulations at smaller scales are still evolving, though researchers hope to reach 10-kilometer resolution in the next decade.



Furthermore, the climate system encompasses multiple interacting components—the hydrosphere (oceans, lakes), the land surface (rivers, vegetation, urban areas and heat islands), and the cryosphere (ice sheets, glaciers, snow). For a model to be comprehensive, it must realistically simulate all of these interconnected elements, which adds another layer of difficulty to building reliable predictive tools.

Representing all components and processes of the Earth system in a climate model is an enormous scientific challenge. Although significant progress has been made, the full complexity of the Earth's system is yet to be fully understood or accurately simulated. Climate models typically begin with energy balance models, which are the simplest models used to simulate climate. As discussed in earlier lectures, these models use the global mean temperature and consider only a single node for the entire planet. This type of model is termed a zero-dimensional model because it does not account for variations with latitude, longitude, or altitude.

A more advanced version is the radiative-convective model, which adds the vertical structure of the atmosphere to simulate how temperature changes with height. This addresses a major shortcoming of the zero-dimensional model, which assumes a uniform temperature profile. Further complexity is added by including both vertical and latitudinal variations. We have statistical-dynamical models, which will be discussed later. Models can also be classified as intermediate complexity models, which strike a balance between realism and computational efficiency.

The most comprehensive and widely used today are the General Circulation Models (GCMs). These models account for three-dimensional variations (latitude, longitude, and altitude), as well as temporal changes, and simulate a wide range of physical processes including wind, radiation, evaporation, cloud formation, and energy exchanges between different components of the Earth system. This progression of models, from simple to complex, is known as the climate model hierarchy. It starts from zero-dimensional models, then proceeds to one-dimensional (vertical or horizontal), two-dimensional (vertical plus either latitude or longitude), three-dimensional (vertical plus latitude and longitude), and finally to four-dimensional models (vertical, latitude, longitude, and time). The next lecture will delve into how these general circulation models represent climate change in detail.

Model Hierarchy

1 Zero dimension : global mean

2. One dimension:vert or horizontal

3. Two dimension: vert +one horz

4.Three dimesion : vert, lat, lon

5. Four dimesion: vert,lat,lon,time