Biomedical Ultrasound: Fundamentals of Imaging and Micromachined Transducers

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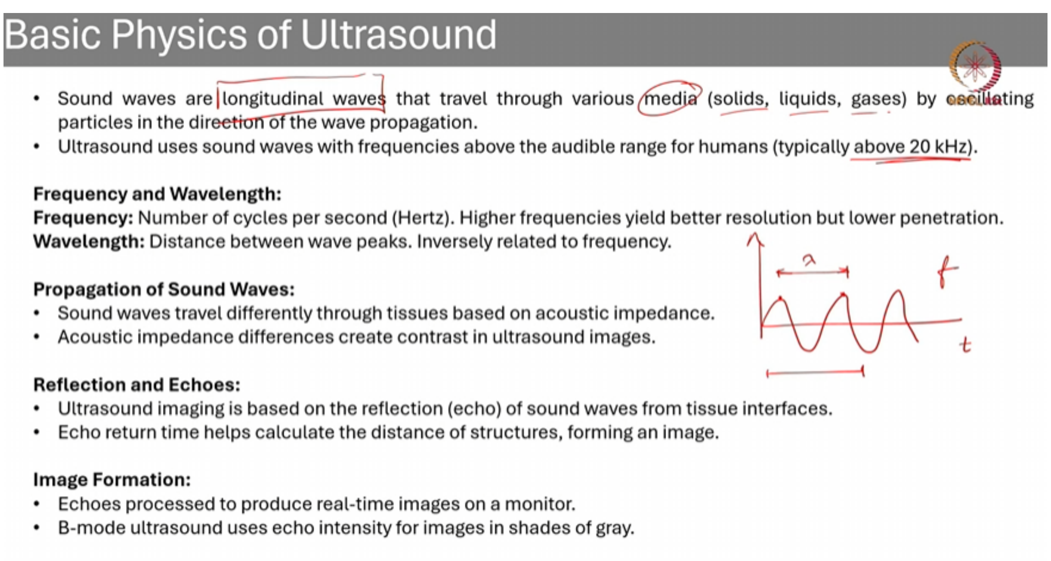
Lecture: 14

Numericals on Ultrasound

Hello and welcome all to today's session. Today we will be discussing certain numericals on the ultrasound. So, let us get started. So, before starting with the numericals, I would like to discuss certain aspects about what is ultrasound. So, as the name suggest there are two parts ultra and sound. And first if we define sound they are basically longitudinal waves that travel through various media.

The point to be noted here is that a medium is required. Now, this media can be a solid, a liquid or a gas and in the absence of media these sound waves cannot travel. And another thing to notice is that these are longitudinal waves. Means, the vibration of the particles are in the direction of propagation. There are other kind of waves. We have transverse waves where the motion of particles is perpendicular to the direction of propagation.

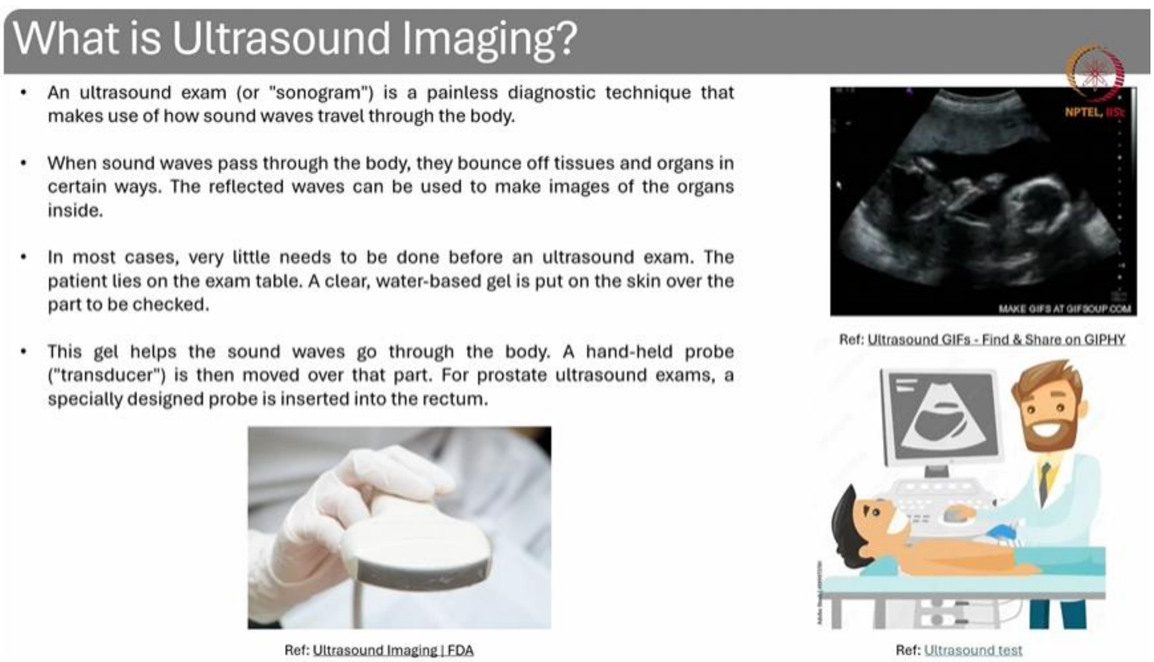
So, all the humans can perceive sound in a specific frequency range. To be precise an average human can hear any sound between 20 hertz to 20 kilohertz. Now any sound which is above 20 kilohertz we call it as ultrasound. Now, like any waves even ultrasound has certain characteristics such as frequency, wavelength and speed. Frequency is the number of cycles per unit time, and if you have any two specific identical points basically let us say we have peaks and the distance between any of these two peaks would give you the wavelength.



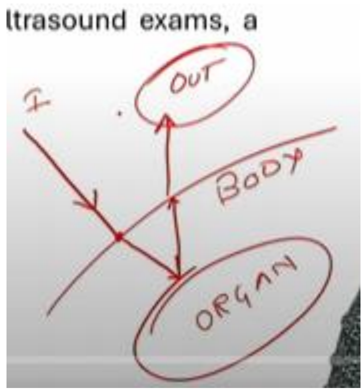
Let us talk about the propagation. Depending upon the medium the sound waves propagate differently. For example, in air the sound has a different speed. In an other media such as water or a solid say such as wood iron sound waves have a different speed.

And, then there is something called as reflection. So, when you say reflection the first thing that comes to your mind might be reflection of light by a mirror. Similarly, we can also have reflective surfaces for ultrasound. So, if we have a boundary which separates two different medium then there will be some reflection of this these ultrasounds. Today we will be diving a little deeper into how this reflection takes place based upon some numericals.

And when we talk about ultrasound the first thing that comes to our mind is imaging, to be precise, medical imaging. Ultrasound imaging is also called as a sonogram or sonography. If you visit any hospital there might be certain signs which will direct you to a sonography room or an x-ray room and there you might see some certain terms such as Doppler, 3D Doppler, sonogram etc. So, basically it is a diagnostic technique and most importantly it is a non-invasive and a painless technique which makes use of ultrasound travelling through your body to image the internal organs.



So, what happens is suppose this is your body or a part of your body and inside there is an organ. So, what would happen is some ultrasound will be directed towards your body and at the boundary, some ultrasound will be reflected back and some will be allowed to penetrate into the body. The incident wave and the output signal is marked in the image. Now, using this output signal you can form some images which would give you an idea of what is the condition of your organ.



In the image you can see a sample of an ultrasound image, and the transducer used to get this image.

Now if you have been through an ultrasound scan, you would see that the technician takes a transducer and put some gel on it and then performs the process on your specified body part. Now what this gel does is, it reduces the acoustic impedance or it matches the impedance between the transducer and your body and today we will be looking into that.

Lets look at the first numerical, here we have the problem says to calculate the distance of the object if an acoustic burst is sent in the air and received after 1.5 seconds. So, that is the first part of the problem.

Problem: Calculate the distance of the object if an acoustic burst is sent in the air and received after 1.5 seconds. Also, what would be the propagation delay if the medium was water and the distance was the same? Assume the speed of sound in air and water is 340 m/s and 1480 m/s, respectively.

Now, like I said sound waves require a medium to propagate and here this medium is air and it is given that the speed of sound in air is 340 meters per second. And it is told that the acoustic burst which is sent is received after 1.5 seconds. So, here this 1.5 seconds becomes the propagation delay and the since the medium is air the speed of sound let us say v = 340 m/s.

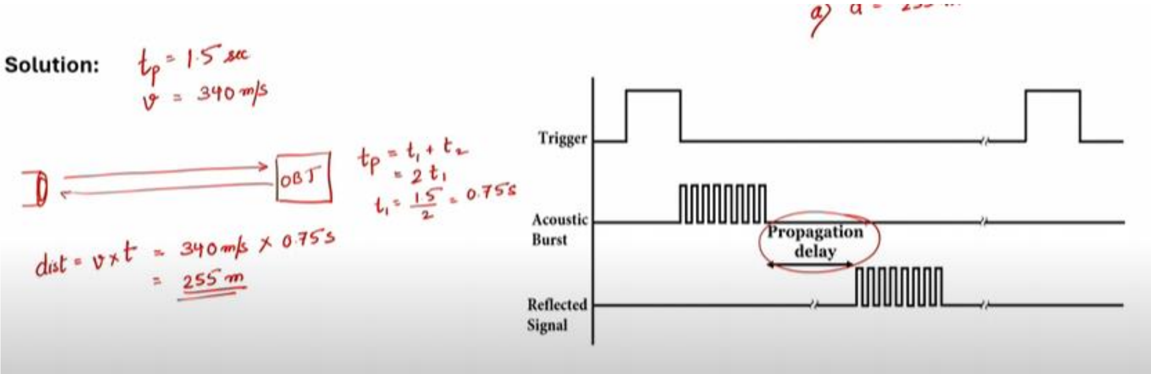
$$propagation\ delay, t_p = 1.5\ s$$

In this case, let us take a transducer which is giving some acoustic signal. The signal comes and hit the object and gets reflected back. Now, this whole time the sound is taking to go to the object and return back is your propagation delay or the time of propagation and which is a sum of the time it takes to go to the object and time it takes to come back. Now, since the medium is same for the same distance, the time taken to go to the object and come back would be same. So, let us say it is 2 times t1.

Now the total propagation of time is 1.5 seconds therefore,

$$t_1 = \frac{1.5}{2} = 0.75\ s$$

Also we know that the distance is equal to speed into time.



Now, since we know the speed that is 340 meters per second and we know the time that is 0.75 seconds, the answer comes out to be 255 meters. So, the first part is distance is equal to 255 meters.

Now the second part of the problem says that what would be the propagation delay if the medium was water and the distance was same. So, here using the same formula we will rewrite it to calculate t,

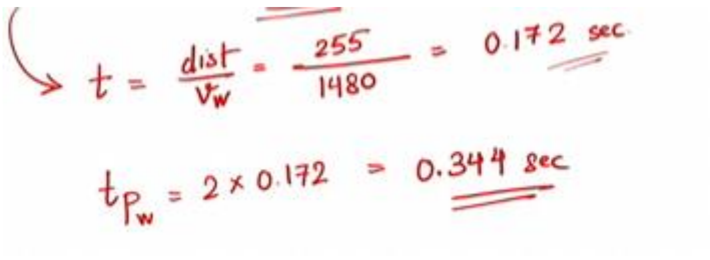
$$Time = distance/speed$$

Here, the speed is the speed in water, which is 1480 meters per second .

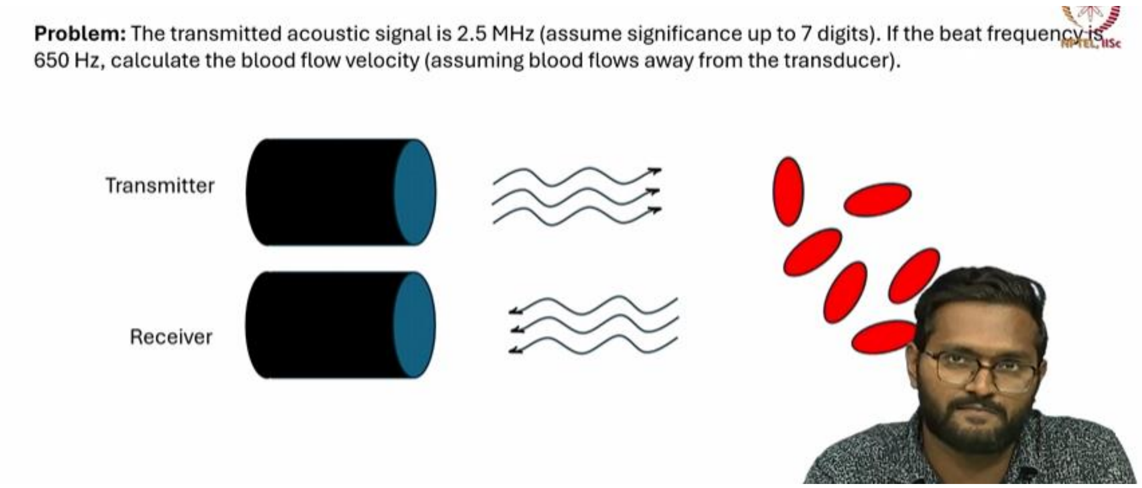
So, the distance is same as 255 and the speed is 1480 which gives us 0.172 seconds.

Now, you might make a mistake here that this actual 0.172 second is the actual propagation delay or the time of propagation, This is the time it takes for the sound to travel from the transducer to the object that is half of the propagation delay.

So the actual propagation delay in water would be 2 times 0.172 second which comes out to be 0.344 seconds that is the second part of your answer.



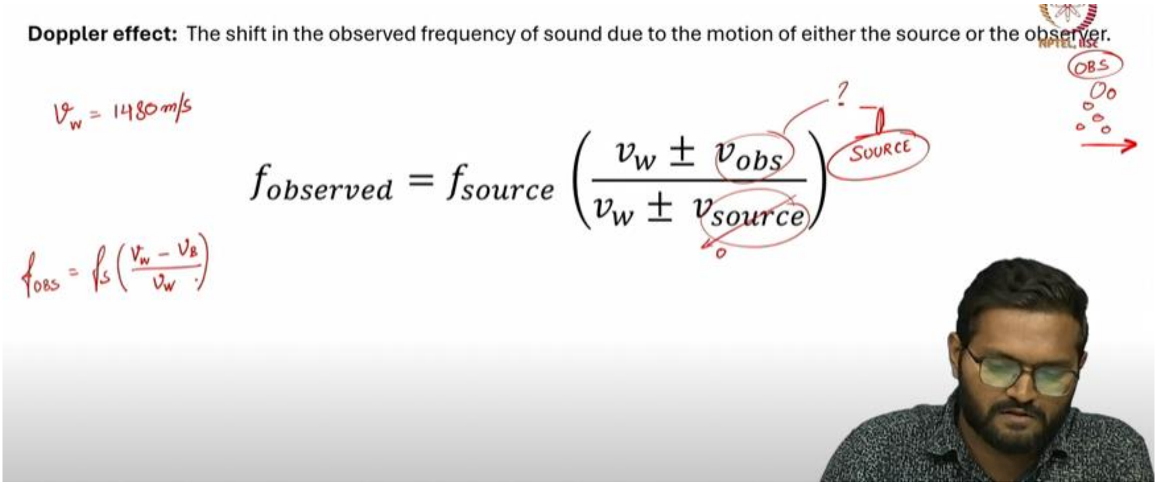
So, now let us move to numerical number 2. So, the problem here says that the in the transmitted acoustic signal the frequency is given as 2. 5 MHz and it is given that the beat frequency that you receive is 650 hertz. Now, we are told to calculate the blood flow velocity.



So let us see the diagram where we see a transmitting transducer and the sound waves are travelling at 2.5 MHz and from the blood particles the sound is reflected and there will be some frequency.

So, let us see the concept which we will be using right now in this case is Doppler effect. The Doppler effect is defined as a shift in the observed frequency of sound due to motion of either the observer or the source. In this case let us divide this problem into two parts. So, the first case would be where it is transmitted. The frequency is 2.5 megahertz and some frequency will be received or seen by the blood So what happens in this case?

So,the transducer is the source in this case and the blood is the observer. So let us say the blood is in this case moving away from the transducer and we have some velocity of sound here. So, let us assume that the velocity of sound is here 1480 meters per second which is same as the velocity of sound in water.



Now we have the velocity of the observer. In this case that is blood and this we need to calculate we do not know this. And, then we have the velocity of source. Now, source is your transducer which is stationary. So, this becomes 0. So, in this case what we would have is the frequency observed by the blood is equal to this is 2.5 megahertz. Since we assumed that the blood is moving away, the sign will be negative for the velocity of observer. Here we have Vw and this is the velocity of blood.

$$f_{BLOOD} = f_S\left(\frac{V_w - V_B}{V_w}\right) \quad \text{——} (1)$$

So, this becomes your first case. Now going to the second case, the sound is reflected back from the blood. So, now the receiver becomes the new observer and blood becomes the new source. So, let us see what happens in this case. So, in this case the receiver is stationary. So V_{obs} becomes 0 and the source, which is your blood is moving away. So, here in this case we will use a plus sign.

$$f_{observed} = f_{source}\left(\frac{v_w \pm v_{obs}}{v_w \pm v_{source}}\right), \quad v_{obs} \to 0$$

$$f_{OBS} = f_{BLOOD}\left(\frac{V_w}{V_w + V_B}\right) \quad \text{——} (2)$$

Now, if you see these two equations carefully we can see that

$$f_{OBS} = f_S \cdot \frac{V_w - V_B}{V_w} \cdot \frac{V_w}{V_w + V_B}$$

$$\frac{f_{OBS}}{f_S} = \frac{V_w - V_B}{V_w + V_B}$$

$$\Rightarrow \boxed{\frac{f_{OBS} - f_S}{f_{OBS} + f_S} = -\frac{V_B}{V_w}}$$

So, this is your final equation. Now, in the problem it was said that the beat frequency was 650 hertz. Now, what is beat frequency? It is the difference between the two frequencies that you are having. So, in this case the source frequency is 2.5 MHz and another was the observed frequency. So, the difference between the frequency that is transmitted and the frequency that is received is your beat frequency and in this case that it is 650 hertz ok.

So, in this case what would happen is your

$$f_{OBS} - f_S = 650\ Hz$$

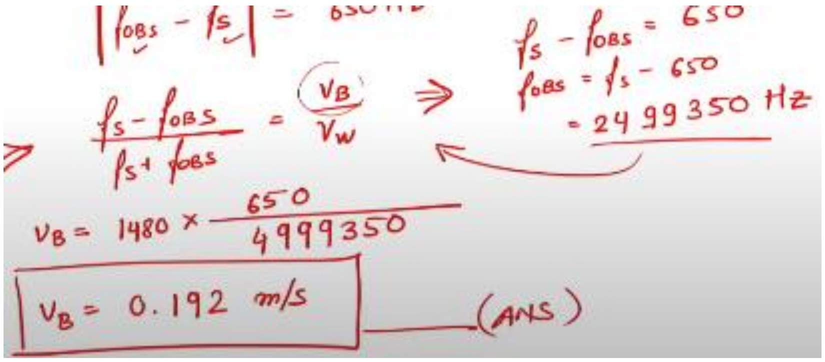
but in this case what would what might happen is that the f_{OBS} might be lower than f_s and the beat frequency cannot have a negative value because when it is measured it is just a magnitude. So, we will put this mod sign. So, if you see your final equation that we have boxed have a negative value ok. So, let us not have a negative value here we will make it positive.

$$|f_{OBS} - f_S| = 650\ Hz$$

So, we have

$$\frac{f_S - f_{OBS}}{f_S + f_{OBS}} = \frac{v_B}{v_w}$$

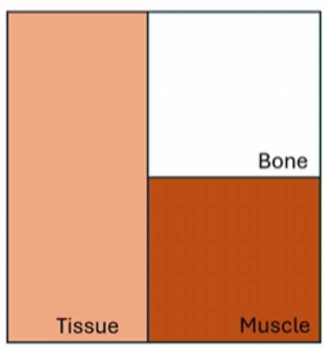
So, what you want to calculate here is V_B and we know this is the beat frequency, which is 650 Hz So, your f_{OBS} becomes 2.5 MHz. After substituting, you will get f_{OBS} to have a value of 2499350 Hertz. In the problem it was told that we can assume that the significant digits are up to 7 digits. Now, if you put substitute this value in this equation we would have the velocity of blood that you are imaging is 0.192 meters per second.



Now, let us move to numerical number 3. So, here it says that calculate the ratio of intensities reflected back to the transducer using the tables for calculation.

Problem: Calculate the ratio of intensities reflected back to the transducer. Use the table for calculations.

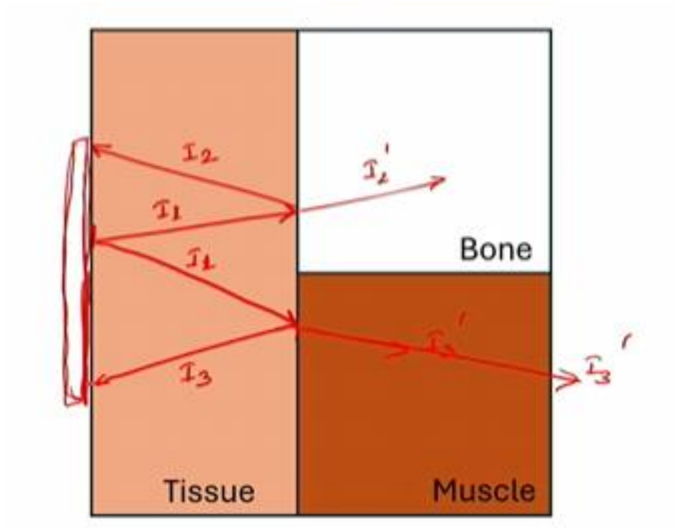
Medium	Density (kg/m^3)	Speed of sound (m/s)
Soft tissue	925	1450
Muscle	1075	1590
Bone	1600	4080



So, here in this case suppose the transducer was at the left extreme, and you will have some incident beam into the tissue medium. Now some of the incident energy would get reflected at the transducer-tissue interface, but we are not considering that.

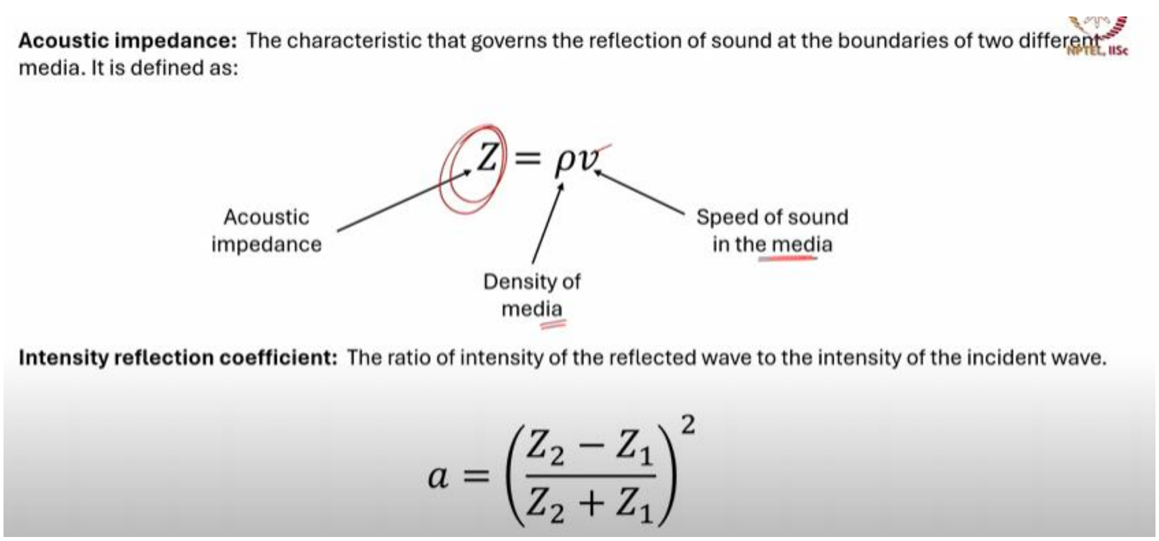
Now from the ultrasound wave passing through the tissue, some would go to the tissue-bone interface and some would go to the tissue and muscle interface.

Let us say the incident intensity is I_1. At the tissue-bone interface, some intensity will be reflected, say I_2 and the transmitted part to be I_2'. Similarly let us say from tissue and muscle interface also there will be some reflection let us call it I_3 and the reflected part would be I_3'.



The transducer is on the left most edge, and there will be some matching layers with some gel in between. It will reduce the acoustic impedance mismatch between the transducer and the tissue interface. We need to calculate the ratio of intensities reflected back to the transducer.

So, to solve this problem what we need to know is a concept called as acoustic impedance. So, what it actually means is that it governs the reflection of sound at the boundaries of two different surfaces. Now, we saw there is a boundary of soft tissue and bone then there is a boundary of soft tissue and muscles. So, in this case we should know their respective acoustic impedances.



Acoustic impedance is denoted by Z,and

$$Z = \rho v$$

where v is the speed of sound in the given media and ρ is the density of the media. So, in the previous table we are given the density as well as the speed of sound in that particular media and Z can be calculated.

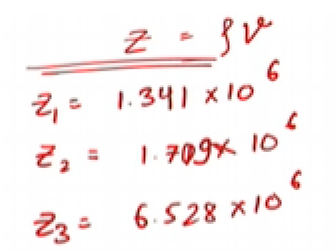
Now another thing that we need to know before approaching this problem is intensity reflection coefficient or the coefficient of reflection. So, this is the formula that gives you the ratio of the intensity. So, this coefficient defines the ratio of intensity of the reflected wave to the intensity of the incident wave.

So, let us suppose at a boundary, the intensity of incident beam was I_1 and the reflected beam intensity is I_2. So, this ratio of intensity would be given by I_2 / I_1.

$$coefficient\ of\ reflection\ ,\ \alpha = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1}\right)^2$$

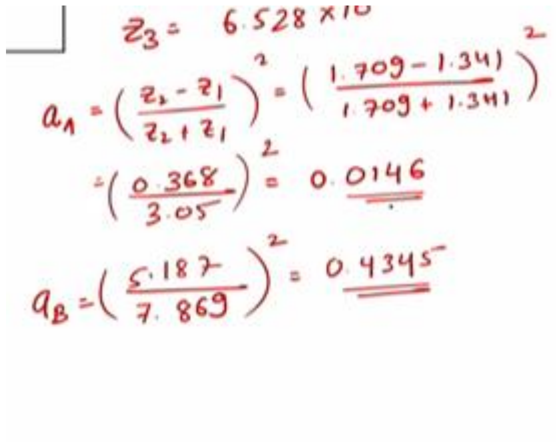
Z1 is the acoustic impedance of the first media and Z2 is the acoustic impedance of the second media. With this, we can start approaching the problem. Since we know what is the density and speed of sound we can use the formula to start calculating the acoustic impedance of the soft tissue.

For soft tissue it would be 1.341 $*10^6$. Let us not worry about the units right now, but it is calculated in Rayls and for the muscle we have Z= 1075 * 1590 which would give Z2 as 1.709 $*10^6$. For the Bone, we have Z3 = 6.528 $*10^6$



Now, since we have these values, we could start calculating the intensity coefficient of reflection. So, for the first interface that we have the tissue and muscle interface, let us say it is a_A. Here we have Z2 and Z1 that is 1.709 $*10^6$ and 1.341 $*10^6$. Now, if we calculate this the answer comes out to be 0.0146.

Similarly if we calculate for the tissue and bone interface a_B comes out to be. 0.4345.



So, here what is happening is we have these two values. Let us say the intensity I_1 was 100% So, let us talk about the tissue and muscle interface first. So, in this case

$$\frac{I_3}{I_1} = a_A = 0.0146$$

Similarly,

$$\frac{I_2}{I_1} = a_B = 0.4345$$

Now, we are told that we need to calculate the ratio of intensities reflected back to the transducers transducer that is I_2 /I_3.

$$\frac{I_2}{I_3} = 29.76$$

So, that is pretty much everything we had to discuss today and I am hope that by these three numericals we learnt something new. The first numerical we understood how can we calculate the distance by knowing the propagation delay of the ultrasound. In the second numerical we learnt something about the Doppler effect and how we can calculate the speed of the object we are trying to image. And in the third one we learned the importance of the acoustic impedance and how it plays an important role in the quality of image. If there is a very good impedance matching that means the acoustic impedance differences between the media is very low and hence the ultrasound waves would transmit with minimum losses, but if the acoustic impedance mismatch is very high, then only a small fraction of the incident wave will be allowed to transmit from the first medium to the second.