Biomedical Ultrasound: Fundamentals of Imaging and Micromachined Transducers

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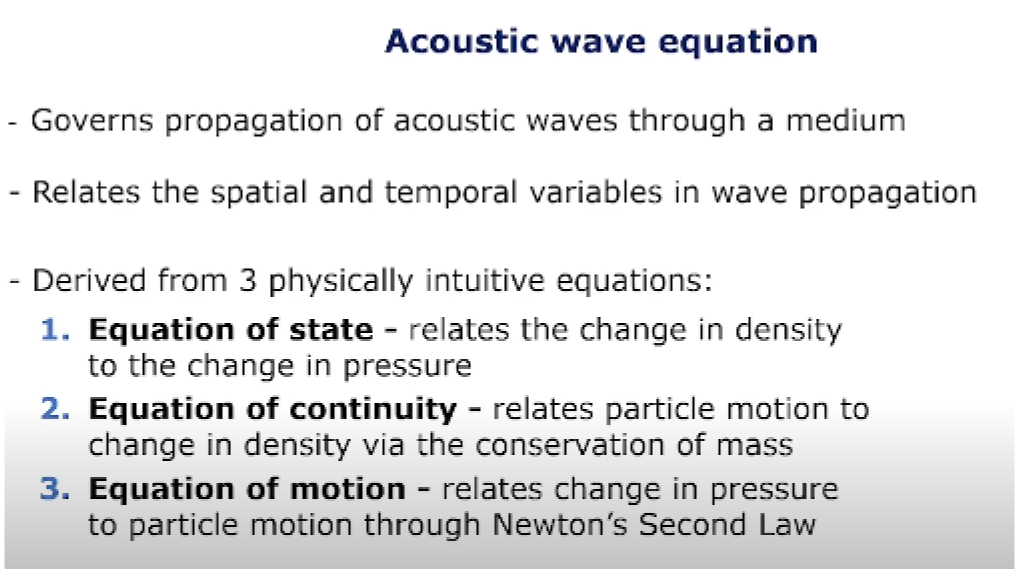
Lecture: 03

Basics of wave propagation

Welcome to Biomedical Fundamentals of Imaging and Micromachined Transducers. I'm Professor Karla Mercado-Shekhar. In this lecture, we will describe the basics of wave equation. What is the acoustic wave equation? This equation governs the propagation of acoustic waves through a medium, such as a fluid. It relates to spatial and temporal variables and wave propagation. We don't have time to derive the entire wave equation, but I will go over three physically intuitive equations that are important into deriving these acoustic wave equations.

You will recall that in the previous lecture, I showed longitudinal propagation and animation of this propagation via particles moving inside a fluid. You would recall that there are particles that are going close to each other in the compression region of the wave and also particles that are moving away from each other in the rarefaction part of the wave. So in these parts of compression, the density of the tissues is much more compared to the rarefaction part of the wave. So what the equation of state does is it relates the change in the density to the change in the pressure of the wave.



We also look into the equation of continuity, which relates the motion of these small particles to the change in the density via the conservation of mass. Here, we assume that there are no sinks or sources in the medium. Therefore, mass is conserved within the medium. The third equation is the equation of motion, which relates the change in the pressure to the particle motion through Newton's second law. You would recall in your previous physics courses that force is related to the pressure times the area.

$$Force, F = Pressure * Area$$

And this pressure will cause the force that causes the particles to move. So these are the three major governing equations that are used to derive the acoustic wave equation.

There are several assumptions in the acoustic wave equation. First, there's a constant sound speed, "c". Mass density is also constant, as well as the ambient pressure. We also assume that there's no mean bulk flow in the medium, and there are no losses, such as no viscous dissipation of the wave.

Acoustic pressure is typically denoted by the parameter P in its spatial, x, y, z, as well as the time coordinate.

$$p(x, y, z, t)$$

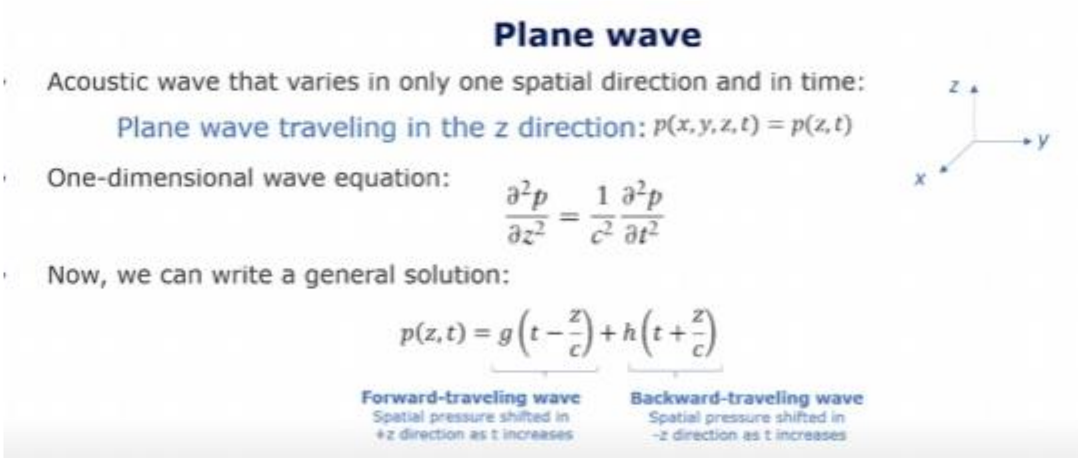
The three-dimensional wave equation is expressed as a partial differential equation here.

$$\nabla^2 p = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}$$

On the left side of the equation is the Laplacian operator, which you might have recalled from your calculus lectures. And it describes the spatial partial derivatives of the pressure.

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}$$

On the right side, we have the temporal partial derivative of the pressure. Now you would have noticed from the previous lecture where there is a time-space coordinate relationship of the wave. And this is a mathematical description that describes how the space-time is related in the wave equation. Now this is challenging to solve, but one can simplify using several cases, including plane wave and a spherical wave.



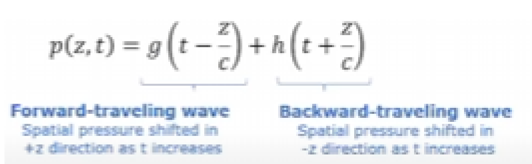
Now what is a plane wave? A plane wave is an acoustic wave that varies only in one spatial direction and time.So here is what a plane wave traveling in the z direction is denoted as.

$$p(x, y, z, t) = p(z, t)$$

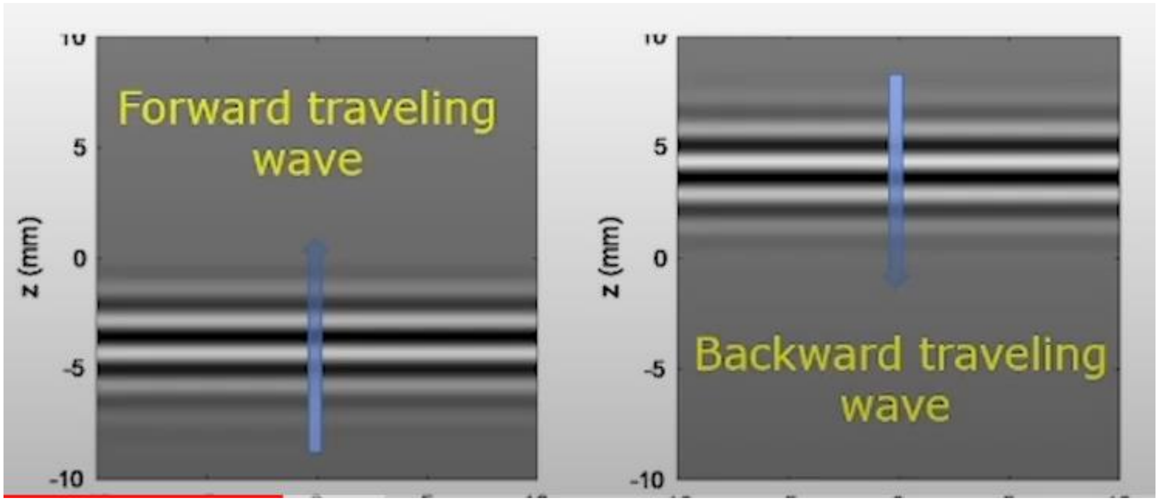
We have this one-dimensional wave equation only as a function of the spatial derivative of z on the left side.

$$\frac{\partial^2 p}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}$$

And we can write a general solution for a wave that's traveling. Here we have the forward traveling wave component here, which is a function of the spatial direction z, the sound speed as well as time. And we also have a backward traveling wave here.



So what this describes is that the wave has two components, a forward traveling and a backward traveling wave. But in ultrasound imaging, we're typically more interested in the forward traveling wave. In the figures below, here's just some example of a forward traveling wave in the z direction, the plus z direction. And here is an example of a backward traveling wave in the minus z direction.



In ultrasound images, we typically look at a single ultrasound pulse that can have a wide range of frequencies. But here is one solution in which we have a single frequency plane wave. Typically pulses are assumed to be sinusoidal signals. And here's an example of a sinusoidal signal of a forward traveling wave equation right here.

Sinusoidal signal (forward-traveling): $p(z,t) = g(z - ct) = A\cos(z - ct)$

Now the function here, g, it's typically has to be twice differentiable and that will satisfy the wave equation. So this equation here is a cosine function.

It's a solution to the 1D lossless wave equation. So at any given z, the pressure varies sinusoidally with frequency of f, which is defined by

$$f = \frac{\omega}{2\pi} = \frac{kc}{2\pi}$$

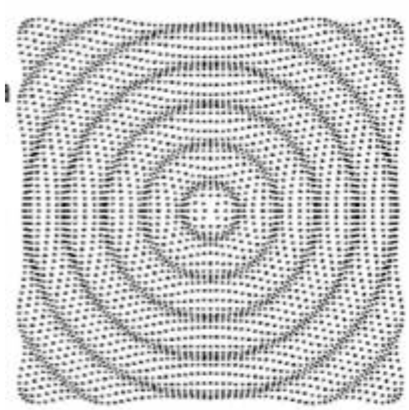
the angular frequency here divided by 2 pi. Or you can rewrite this equation, the angular frequency can be in terms of the wave number k and the sound speed divided by 2 pi.

And at any given t, the pressure of any oscillating particle varies sinusoidally with this spatial frequency, k, which is also known as the wave number and is defined by

$$k = \frac{2\pi}{\lambda}$$

lambda, the wavelength here.

Let's talk about spherical waves. If you assume an isotropic media, meaning that all the material properties are the same in all directions, and you perturb the medium at a particular point source, so this produces a pressure wave that is spherical in nature, such as the one shown here.



So a spherical wave depends on the time as well as the radial distance from the source, which can be noted by the three spatial coordinates.

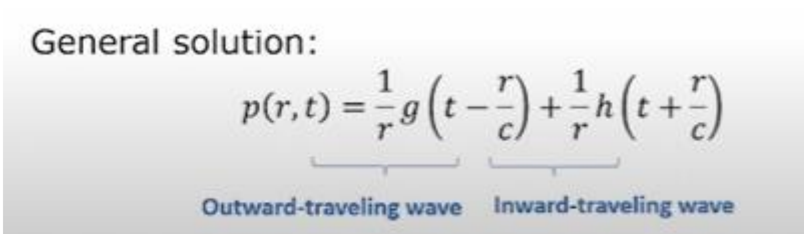
$$r = \sqrt{x^2 + y^2 + z^2}$$

And the wave equation in spherical coordinates is denoted by the following here.

$$\frac{1}{r}\frac{d^2(rp)}{dr^2} = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}$$

On the left side, you have the equation in terms of the radial coordinate, and in the right side is the temporal coordinate.

The general solution of this spherical wave equation is as follows, wherein similar to the previous plane wave equation, you have a outward traveling wave and an inward traveling wave.



And it's a function of the radial coordinate as well as the sound speed and the time.

So when do these outward and inward traveling waves exist? For a point source where the wave is originating from, typically outward traveling waves make physical sense. In a vibrating shell, for instance, you have something vibrating, it produces both outward and inward traveling waves.

Now let's describe a characteristic of a wave in terms of attenuation.What is attenuation? So it is the decrease in signal amplitude that can be caused by several factors including absorption. Now tissues can absorb ultrasound energy and hence convert this mechanical wave into thermal energy. Also, scattering can affect depending on differences in the material properties. We'll describe later the acoustic impedance of the medium, depending on which there could be some reflection or scattering of the ultrasound wave.

Therefore, the ultrasound will attenuate as it propagates through the tissue due to this scattering. Also, diffraction. Now, the ultrasound beam that we create is not uniform in nature. There could be some beam spreading, which we'll discuss more in later lectures. And this can cause what's called beam diffraction, wherein certain regions of the tissue will not be excited or exposed to ultrasound due to this beam diffraction.

Also refraction as well as mode conversion, which can occur and can also attenuate the sound. When sound goes through an interface, it can refract, right? And that can decrease the amplitude of the ultrasound propagation through the tissue. And there are several cases where the longitudinal waves can convert into shear waves. For instance, as longitudinal wave is traveling in a bone, depending on the angle of the longitudinal wave front, it can convert into a shear wave. And therefore, this is also a factor that can affect wave attenuation.

Attenuation denotes the decrease in signal amplitude caused by:

- Absorption
- Scattering
- Diffraction
- Refraction
- Mode conversion (e.g., longitudinal to shear)

Now the model of attenuation is defined experimentally. There is an empirical model of attenuation. And what we have here is an equation that describes the amplitude decay of attenuation.

$$A_z = A_{0e^{-\mu_a z}}$$

Where here we have attenuation as denoted by A as a function of distance here, and A_0 is the original amplitude of the signal. And as the signal is being sent through the medium, the amplitude is being decayed as a function of this exponential here, which μ_a is the amplitude attenuation factor and z is the distance within the tissue.

And this amplitude attenuation factor is defined by this equation here,

$$\mu_a = -\frac{1}{z}\ln\frac{A_z}{A_0}$$

And this amplitude attenuation factor has units of Nepers per centimeters, typically written as Np/cm.

And then we calculate, an attenuation coefficient that is usually denoted by α here.

$$\alpha = 20(log_{10}e)\mu_a \sim 8.7\ \mu_a\ \ (dB/cm)$$

Now, this term equation can be simplified as approximated as 8.7 times the amplitude attenuation factor.

So doing this allows the attenuation coefficient to be in units of decibel per centimeter. And this is what is typically reported in literature. It's also important to note that attenuation coefficient depends on the frequency and typically a power law relationship that is shown here. And this power law equation is defined by coefficients such as the a coefficient and the b coefficient, which is the power coefficient. One can also approximate b, typically to one.

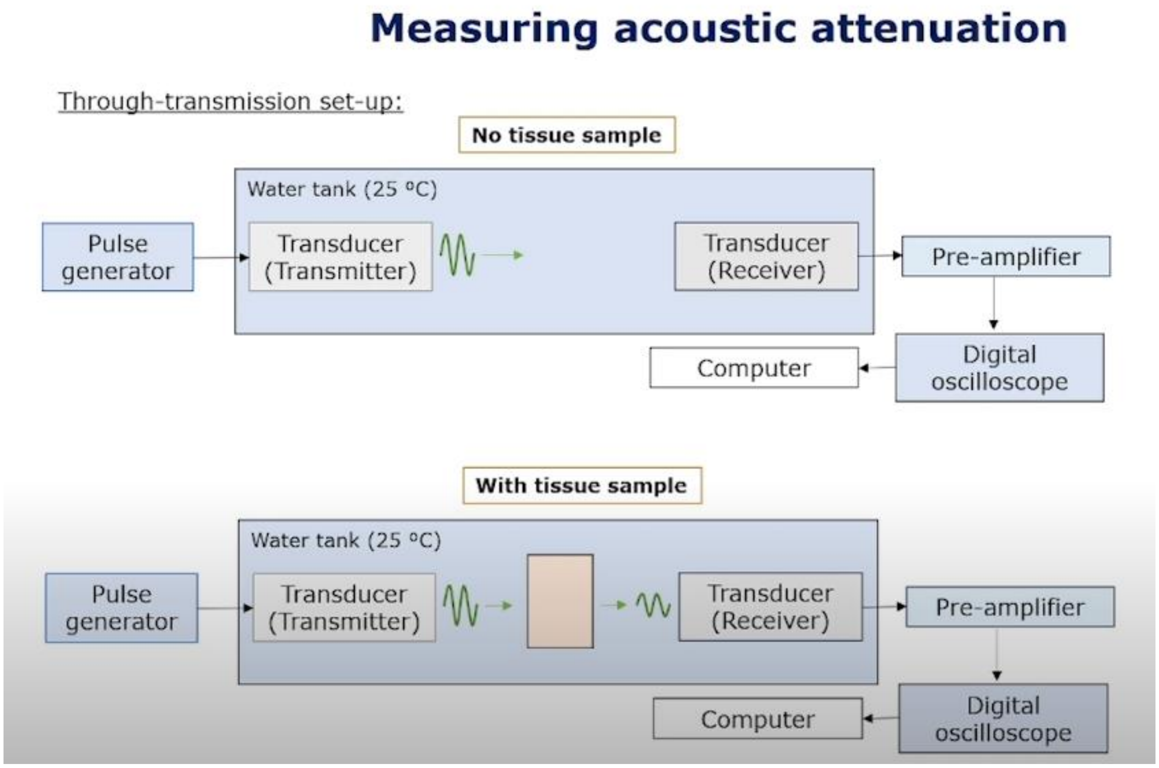
$$\alpha = af^b \qquad b \approx 1$$

Typically in soft tissues, this b parameter goes from 1-2. Most tissues are much closer to b parameter =1. So therefore, here's just an example of a couple of tissues that have their attenuation coefficients here in terms of MHz, wherein b is approximated as one. So you can see here, there are several tissues that fall within a range of each other.

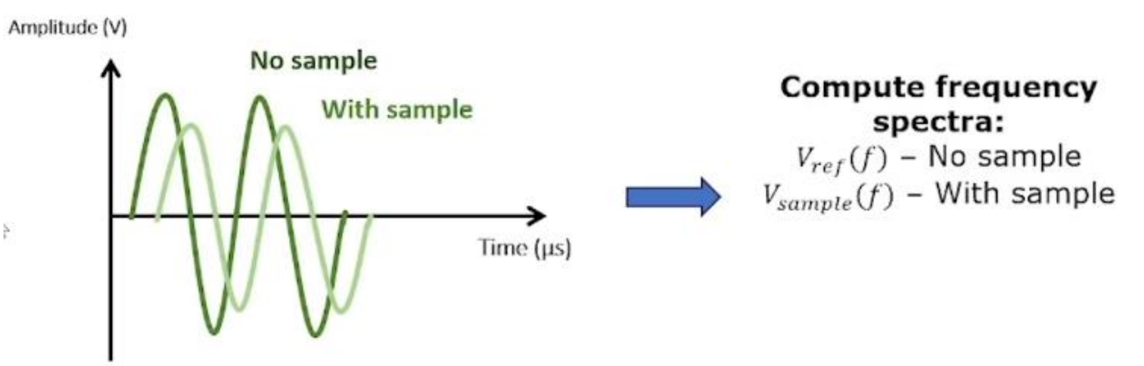
Tissue	Attenuation (dB/cm/MHz)
Breast	0.28-0.63
Liver	0.30-1.30
Kidney	0.85-1.00
Heart	0.24
Brain	0.60
Skin	1.84-3.50
Lung	15.83
Fat	1.50-1.70
Bone	13.10-26.00

But there are also other tissues such as lungs. Lung is filled with air and we'll discuss later how that has higher attenuation compared to the other tissues. Also bone here. Bone is considered a hard tissue. It's fairly dense and it typically attenuates ultrasound more than the other soft tissues here. Now, how do we measure acoustic attenuation?

So here I give another practical example using the through transmission setup, which we had described earlier for measuring sound speed.



So we can use the same setup here, and I described this earlier in the previous lecture, where we can measure the ultrasound signal that is transmitted through water and detect it, and then put the tissue sample. When the signal is transmitted, it goes through the tissue sample, what is received is actually being attenuated. And we see the signal that is being received, such as this in the figure, where this dark green represents the signal without the sample, and it just propagates through the water. And the one when the sample was placed inside the experimental setup, you can see that it also has a decrease in the amplitude compared to the one without the sample. So here, we take these time-varying signals and then we compute the frequency spectra of this signal.



So here I denote the V_{ref} here as a function of f, where is f is the frequency. This is the case, the frequency spectra of the signal without the sample. And here V_{sample} corresponds to the frequency of the spectra with the sample. And we calculate attenuation using this equation right here, and this is the frequency-dependent attenuation coefficient.

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$$frequency\ dependent\ attenuation\ coefficient,$$

$$\alpha(f) = 10\log_{10}\left(\frac{|V_{ref}(f)|^2}{|V_{sample}(f)|^2}\right)/h$$

The thickness of the sample h, can be measured using a caliper or a ruler. This is how you can estimate acoustic attenuation.

Now, moving on to another parameter of the tissue, the acoustic impedance, which is a characteristic of the tissue itself. It is analogous to the Ohm's Law in an electronic circuit. In Ohm's Law, we have the impedance equals the voltage by the current.

We can relate the pressure P as equivalent to a voltage. We can relate the particle velocity u as related to the current and the acoustic impedance related to the impedance of the circuit

The acoustic impedance is, pressure divided by the particle velocity right. And we can rewrite this equation as a function of the density rho and the sound speed.

$$Z = \frac{P}{u} = \rho c$$

Typically for soft tissues, since it is mainly composed of water, the density is typically on the order of 1000 kg/m^3. And the sound speed on average is 1540 m/s. Now the units of acoustic impedance is known as the Rayleigh. Rayleigh is inspired by Lord Rayleigh who developed the or who published the theory of sound and it has units of kilograms by meters squared by per second(kgm^{-2}s^{-1})

And a Rayleigh is very, very small. So what we typically use is Mega Rayls. So one million Rayls, which is a more practical unit in ultrasound. The acoustic impedance has, a factor of the density of the tissues. So what we have here is the list of the different densities of various soft tissues in the body. As you can see here, most of them fall within very close to the density of water, 1,000 kg/m^3. We have bone here that is much, much more dense than these other soft tissues.

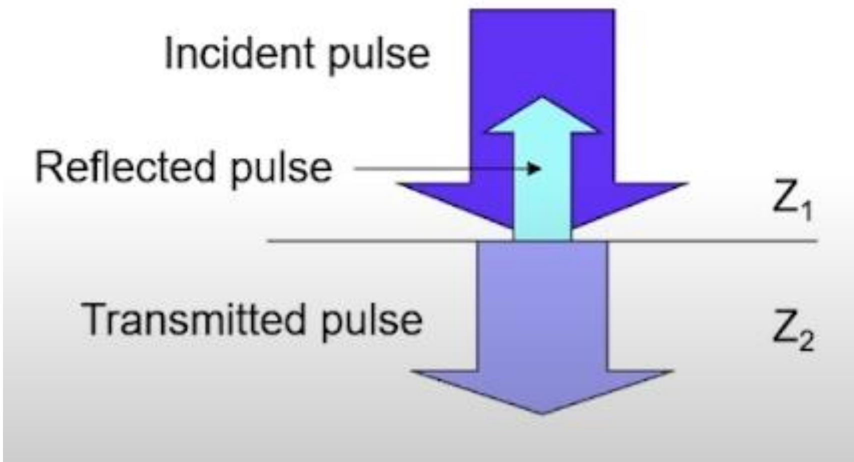
Tissue	Density (kg/m³)
Breast	990-1060
Liver	1050-1070
Skeletal muscle	1038-1056
Kidney	1044-1050
Heart	1060
Arteries	1050-1075
Brain	1030-1043
Skin	1093-1190
Lung	230-290
Spleen	1054
Cornea	1076
Uterus	1052
Fat	916
Tendons	1110-1220
Cartilage	1092-1104
Bone	1250-1750

We have the acoustic wave speed in tissues right here. So I have described some of these earlier and you can see how much they can vary in a range, with bone being the highest since it's a hard tissue.

Tissue	Longitudinal Wave Speed (m/s)
Breast	1450-1570
Breast cancer	1437-1584
Liver	1522-1623
Skeletal muscle	1500-1610
Kidney	1558-1562
Heart	1528-1602
Arteries	1559-1660
Brain	1460-1580
Skin	1498-1540
Lung	577-1472
Spleen	1515-1635
Cornea	1542-1639
Cervix/uterus	1625-1633
Thrombus	1586-1597
Tendons	1631-3530
Cartilage	1520-1665
Bone	1630-4170

Now putting these parameters together, we get the acoustic impedance. And the importance of acoustic impedance is that any reflection or scattering that you can get from the tissue is because there's a difference in the acoustic impedance between different structures of the tissue.

So if there are discontinuities in the material, that can cause a reflection or a scattering. So for instance if I have two media with different acoustic impedance, let's say the top medium here has acoustic impedance of Z_1 and the bottom one here as Z_2. If I send an incident pulse, once it reaches the interface between those two media, some of it will get reflected and some of it will get transmitted. And that's because there's a difference in the acoustic impedance between these two media. Now if the acoustic impedance of these two media are the same, or these two organs have the same acoustic impedance, then you won't receive any reflection. The ultrasound will just travel through the rest of the organs.



And here is an example of the average impedance of different materials.

Material	Average Impedance (MRayl)
Air	0.0004
Lung	0.18
Water	1.5
Blood	1.62
Fat	1.35
Liver	1.65
Muscle	1.70
Kidney	1.63
Bone	7.8

So air has a very low acoustic impedance compared to soft tissues of the body. And this is what also impacts the acoustic impedance of lung, because lung is typically inflated with air. Therefore, the acoustic impedance of lung is very, very low.

And you will see later in the subsequent lecture how this can impact the type of reflection, the type of signal or image that you will get from ultrasound. And you can also see from the table how bone has a really, really high acoustic impedance. So if I show you an ultrasound image of bone, then you can see how a really bright echo can be received from that bone interface.

In summary, we have discussed the acoustic wave equation that governs wave propagation. We've also looked at two cases wherein the acoustic wave equation was presented in case of plane waves and spherical waves. We also talked about a phenomenon of the medium called acoustic attenuation, which is a decrease in the wave amplitude as the sound is propagating through an attenuating medium, as well as an important characteristic of the medium that defines the echoes that are being reflected from the tissue, which is acoustic impedance. So in the next lecture, we will talk more about reflection and scattering and how these are used to form an image.