Mathematics - I
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Lecture - 29
Surface Integrals

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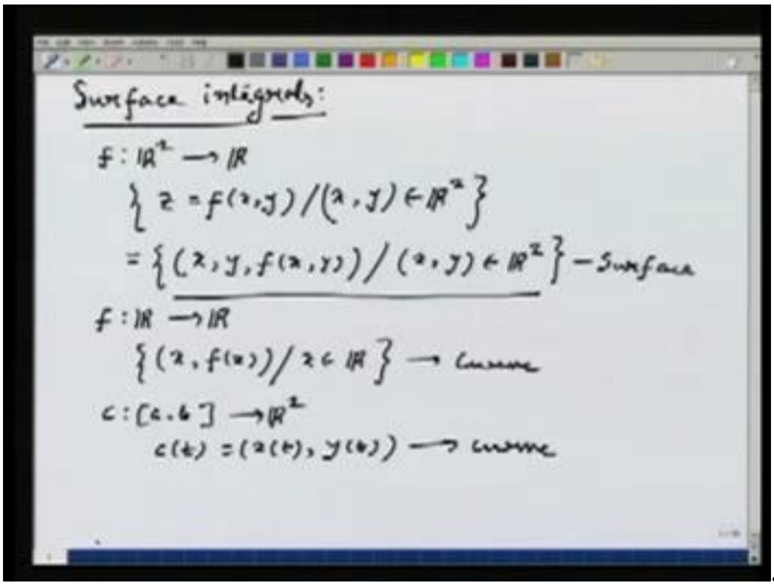

In today's lecture, we are going to start with Surface Integrals. So, the aim is that if I have given a function on a surface. For example let us say, I look at the sphere, which we can believe is a surface. I will come to the rigorous definition of surface later. Suppose I am a function defined on a sphere, how to integrate a function over a sphere. So, first let us start with definitions of surfaces, what do you exactly mean by surfaces. Well most of the surfaces which we encounter can occur as graphs of some functions like; the function f from R 2 to R, let us look at…

And then let us look at the set z equals to f of x, y. Where x, y belongs to R 2, what we mean by this. We actually mean, this subset of R 3, that is x, y, f x, y, where x, y is in R 2. Well this is just the graph of the function f is defined from R 2 to R. Similarly, if I come down to dimension two, if I look at some equations of R's. They are also given sometime, not always by graphs of certain function f from R to R. Suppose I have a function f from R 2 to R, then if I look at this set x, f x. Then you already know that this is a curve where x in r, this is a curve.

Similarly, if I look at this kind of a set, where f is a function from R 2 to R, this is a surface, these are certain examples of surfaces. We will see later, that all surfaces are actually not given by graphs. So, we generalize the definitions also. Well, another definition of curve we have given as some map c, from closed interval a, b to R 2, So, c t written as x t, y t this was a curve, this is also a curve. So, we can have an analogous definition to this, for surfaces also. Well that is given as follows.

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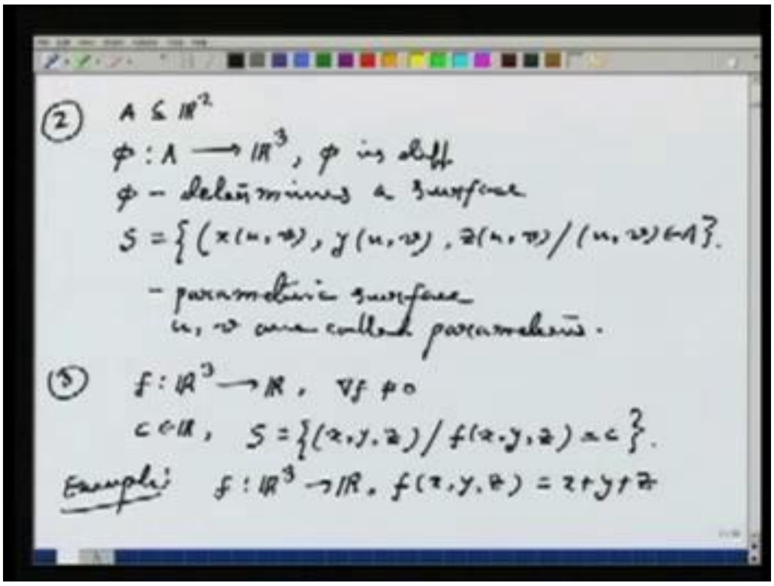

Let suppose A is a subset of R 2. Suppose I have a map phi, from A to R 3, we will assume phi is differentiable function. So, the domain A is such that, the derivative of phi make sense, then this phi actually determines the surface. And it is given by S, what is the set, the set is x, u, v comma y, u, v comma z, u, v, where u, v belongs to a. So, it is essentially means, that on surfaces you have essentially have two degrees of freedom, u and v.

When for curves, there is only 1 degrees of freedom that is t. And what do I mean by x, y and z, well phi is a function from A to R 3. So, the image of phi, we will have a x coordinate, it will have a y coordinate and a z coordinate. Those coordinates I have written as x u, v, y u v, z u, v. Because, this coordinates are going to be the functions of u, y. Now, this is the most general kind of surfaces, we are going to deal with. These are called parametric surfaces and u, v these are called parameters.

Now, another kind of surface, we will deal with some time, is the level set of a function. So, this is the most general kind of definition, which I have. Third is level set, suppose I have a function f from R 3 to R. I will assume a technical condition, whose role is very difficult to explain at this point of time. Just assume this condition the condition is the gradient of f is not equal to 0, for all points. Then I look at this set, I fix a constant c in R. And I look at this set S, that is all x, y, z such that f of x, y, z is equal to c.

Now, why this is a surface, the white image in that this is a surface is. That suppose x and y is given to you. Then from the rule f x, y, z equals to c anyway, I will be knowing the definition of f, I can actually find out what is f. For example, let us take the function. I take a very simple function f from R 3 to R, given by f of x, y, z. Let us say, it is equals to x plus y plus z. If that is the case, let me take c to be equals to 1.

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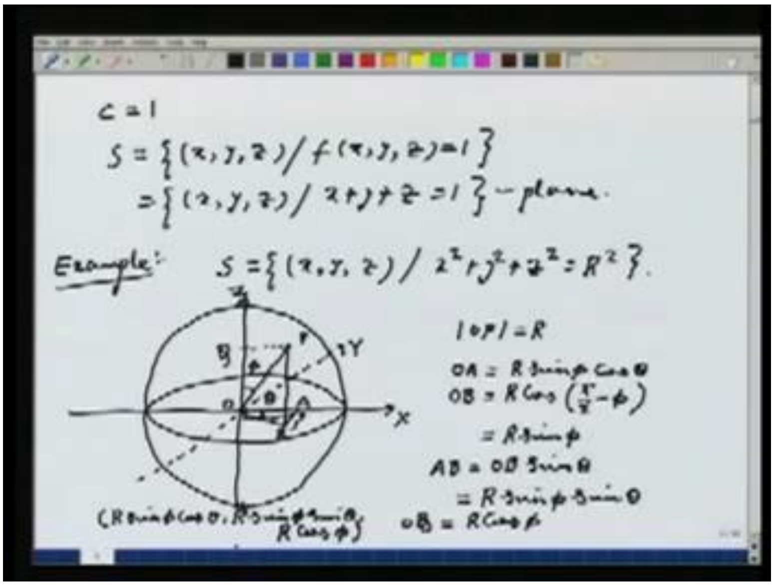

Then my surface is all x, y, z such that, f of x, y, z is equal to 1. That is all x, y, z such that, x plus y plus z is equals to 1. Now once I know x and y, I can calculate z from this equation. And you know, what exactly this equation represents, it should represents a plane, so that is a surface. Now, let us look at some more examples. I look at the sphere S, that is all x, y, z. Such that, x square plus y square plus z square equals to r square, so it is a sphere of radius r.

Now, you can imagine, that this sphere must be a surface, because if you stand on the sphere, then it is a kind of two dimensional things. You can have only two degrees of freedom there. But, how do we rigorously explain that, that S is the sphere. Well for S, what I want to do is, I want to show that, this the parametric surface. That means, I have to find the two parameters by which the sphere can be determined. So, let us try to draw the sphere first, from that it will be clear. So, this is my sphere.

So, question is how do you, how do I express any point on the sphere as a function of two parameters. If I can do it in a nice way that will show that S is the surface represented by a two parameters. So, let us take a point P here, this is my origin, this is the point P. Now, I want to find the coordinates of this point P, in terms of two parameters. Now, if I want to find the coordinates, I want the height, length and breath. So, this is the point P on the sphere.

There are distances which I want to find one is this distance, other is this distance and the total height. Now, what I do is I join the point P, by a line with the origin. Then says the sphere has radius r, I know this line OP, length of OP is equals to R, that much I know. Well, now suppose I look at this angle, I call this angle phi. And then, I have to use some other angle, well let us draw this line. Let us say this angle here is theta, so I have used two angles.

Now, these two angles are parameters. Using these two angles I should be able to represents the point P in three coordinates. Question is what should be the x coordinate of p. Now, let me give some names, this point is A, let us say this point is B. First what I want to know is, what is OA, that is this length. For that first I tried to find what is OB? That is easy to find OB. If you understand which angles I am looking at, OB is nothing but R cosine of pi by 2, minus phi.

Phi is the angle, which the line OP make the z axis, this is my z axis, this is x axis. And then, here is my y axis. So, I have found out what is OB. Well what is R cos pi by 2 minus phi, this is nothing but, r sin phi. Then what is OA, OA is nothing but, OB cos theta. That means, R sin phi cos theta, so I found out what is OA, that is the x coordinate of p. So, once again what I am doing is, I know the length of OP, which I have drawn here, that is R. Now, I want to find what is OA, OA is going to be the x coordinate of the point P.

Now to find OA what I do is, I first find what is OB? Now, since I is the angle between OP and the z axis. OB stands out to be R cosine phi by 2 minus phi, which is R sin phi.

Using that I can find what is OA, OA is precisely R sin phi cos theta. So, OA has been found out. Next I want to find, what is the y coordinate of the point. That means, I want to find what is AB, AB will give me the y coordinate of the point P. Now, let us look at the triangle.

What I have here, this angle between OA and AB is pi by 2. And the opposite angle is given by theta, so AB that is nothing but, OB sin theta. But I know what is OB, OB is R sin phi, so AB is R sin phi sin theta. Next thing remain is what is the height, that is I want to find, what is OP. OP is the height, that is the z axis, z coordinate of the point P, which is easy to calculate the z coordinate OP. Well, usually by OP, I was meaning the, line joining the O and P. Here what I mean is, well let me call this point Q.

That is the projection of the point P. And then, what I want to know is, what is OQ, well OQ is nothing but R cosine phi. So, what are all the three coordinates of the point P. The x, y, z coordinate of the point P, that I can easily calculate now. The coordinates are turning out to be first, the x coordinate, that is R sin phi cos theta. Then the y coordinate that is R sin phi, sin theta and then the z coordinate, that is R cos phi. So, what exactly is the map now.

Now, let us see first what is the variation phi and theta. Now, if you want to get any point P, the point is either on the upper hemisphere. Or it is in the lower hemisphere. In any case, if it is in the upper hemisphere, then the phi, variation of phi can be from 0 to phi by 2, that is very clear. And if it is on the lower hemisphere, then the variation of parameter phi is from pi by 2 to pi. So, as a whole my phi varies from 0 to pi. And what about theta, theta is actually measuring the angle between the x axis and the point. But, it can have a full round. That is this theta, it can vary from 0 to 2 pi. So, finally I get the map, the map is that I look at the map.

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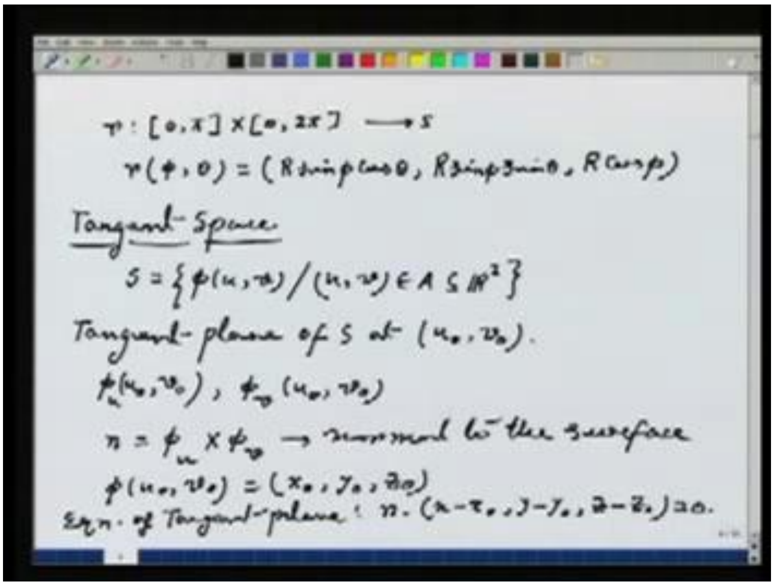

Let us call it as r, little r. It is a map from 0 pi, cross 0 to 2 pi to the sphere S. That r of phi, theta that is now comes the first coordinate. I go back to the picture again, ((Refer Time: 16:56)), if I look at the first coordinate, that is r sin phi cos theta. So, this is r sin phi cos theta, then R sin phi sin theta and then, R cos phi. Notice here that the capital R which I am writing, it denotes the radius of the sphere. So, it is not varying, the only variation which we having here are phi and theta.

So, you can see that any point of the sphere, now have this coordinates. These are called as spherical polar coordinates. In that, the only variation which can happen are in phi and theta. So, using those two parameters, I could represent the sphere S. So, it is a surface and this is the parametric equation of the sphere of radius r. Now, after understanding the basics of surfaces, let us go to the tangent basis. That is the one concept we will be needing. You already know it, so I will just recall it and try to be very short.

So, suppose as a parametric representation of a surface S, so S equals to phi u, v. Where u, v belongs to some sub set of R 2. I want to find, since I am concentrating on the surface S, I want to find the tangent plain of S at the point. Let us say, u naught, v naught. So, what is the idea, what you do is, you first fix v naught. And then, look at this quantity phi u naught v naught. This gives you detection of the tangent to the surface at the point u naught v naught. And the orthogonal detection, actually can be found then by phi v, u naught v naught.

So, what you do is, in the equation of phi, you first fix v naught, then it becomes a curve. Then you take the tangent of that curve, find the tangent line, you need the detection of the tangent line only. In the next case, what you do is you fix u naught and find look at the curve, phi of u naught v and find the detection of tangent line. It gives you two orthogonal detections. Then you look at n that is phi u cross phi v. Then this is normal to the surface.

And then, equation of the tangent plane, if let us say phi u naught, v naught is equal to x naught, y naught, z naught. Then the equation of tangent plane is nothing but, n dot, x minus x naught, y minus y naught, z minus z naught equals to 0. So, this is the equation of the tangent plane of a surface S, at a point x naught, y naught, z naught. Now, I want to give two examples, where this construction will be even more clear.

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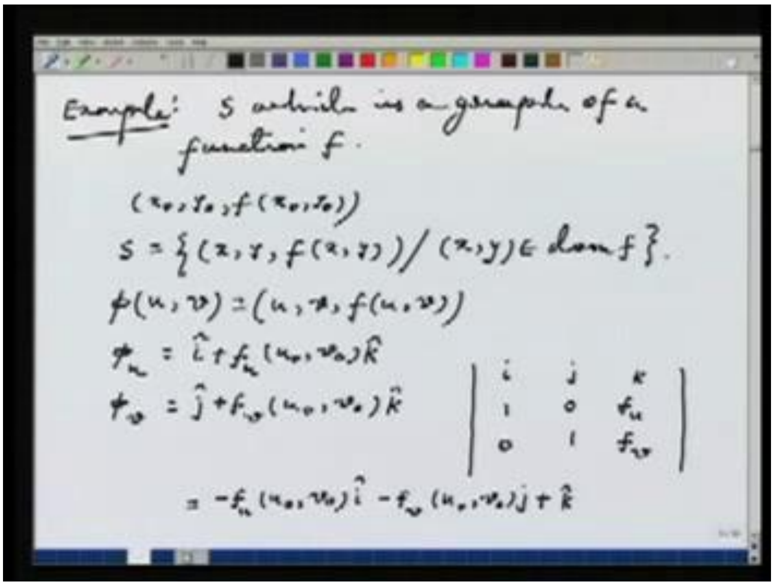

Now, the first example, let us look at a surface S, which is a graph of a function f. Certainly I will assume that S is a differential function, otherwise I cannot work with it. So, let us take a point x naught, y naught, f x naught y naught. At this point I want to find the graph of the tangent plane of the surface. So, what is the surface S, S is nothing but x, y, f x y, where x, y belongs to the domain of the function.

Now, that means what is my parametric representation of the surface. It is given by this function, that phi of u, v is equal to u, v, f u, v. This is the parametric representations of the surface. That means, given any graph of a function, I already get the parametric representation of the surface. And the precise parameter, parametric function is phi of u, v; phi of u, v, u, v, f u v. So, I first calculate phi u, so that trans out to be then, i plus f u at u naught, v naught, k.

I am just differentiating term by term. The first time u, if I differentiate, I get 1, v is 0 it produces the partial derivative. In the vector notation, if I use i, j, k, what I get, I have written here. Now, if I look at phi v, of course it means at u naught, v naught, what I get is j plus f v, u naught, v naught k. Now I can calculate, what is phi u cross phi v, so I do the calculation here. I write it as i j k, then 1 0 f u then 0 1 f v, just to calculate and this is 0. So, what do I get, what I get is minus f u, u naught, v naught, i. Then j, with j what I get is minus f v u naught, v naught, j plus k. So, then the equation of the tangent plane is easy to calculate.

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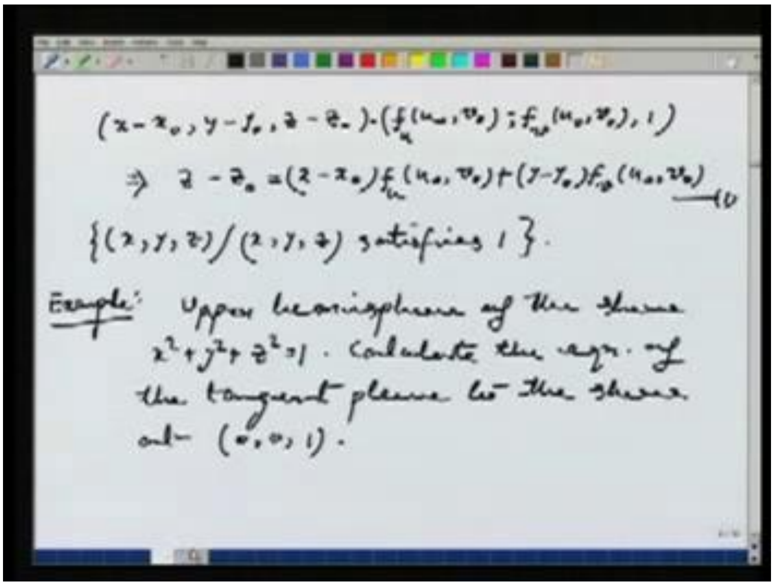

So, equation of the tangent plane is x minus x naught, y minus y naught, z minus z naught, dot. ((Refer Time: 24:56)) I need to know the normal vector, that is given by this, this is a normal vector. So, I choose that this is f u, so if I do the dot product, what I get is z minus z naught. So, this is the equation of the tangent plane, that is all those x, y, z. So, the equation of the tangent plane is x, y, z. If I call this equation 1 such that, x, y, z satisfies 1. This is the equation of the tangent plane of the surface at the point x naught, y naught, z naught.

Now, let us look at another example. Suppose, I take upper hemisphere of the sphere x square plus y square plus z square equals to 1. So, I want to calculate the equation of the tangent plane to the sphere, at the point 0, 0, 1. Before applying the formula, let us see what it means.

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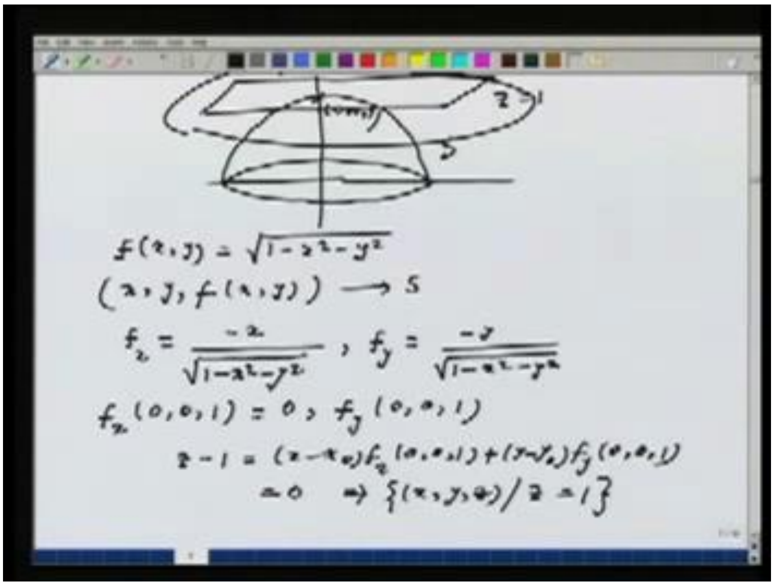

So, I draw the sphere here, I draw the upper hemisphere actually. This is the upper hemisphere, this is the point 0, 0, 1. We can very easily visualize, what exactly is the tangent plane there. You just put a post card on the top, that is this plane. This should be the tangent plane to the sphere, at the point 0, 0, 1, because the plane is tangential to the sphere. And the equation of this plane is I know what it is, it is just z equals to 1. z is equals to 1 is the plane parallel to x, y plane, at the height 1.

That looks like the tangent plane. Now, I have to see, whether it follows the formula, which I have derived. So for that, what I do first is that I visualize this upper hemisphere as the graph of a function. So, which function I am looking at, well I look at the function f of x square root of 1 minus, x square minus y square. I am taking the positive square root. And then x, y, f x y, this is a sphere, upper hemisphere, this is my surface S.

Now, if I want to calculate the tangent plane at 0, 0, 1, I can use my previous formula. For that I will need are f x, that trans out to be minus x by square root of 1 minus x square minus y square. I will also need f y, that is minus y by square root of. But, note that I will need the f x at the point, where I want to find the tangent plane. That is 0, 0, 1, if I put x equals to 0, then f x stands out to be equal to 0, so is f y.

Then the previous formula of the equation of a tangent plane at a point x naught, y naught suggests. That I get z minus z naught, which is 1 that is equals to x minus x naught into f x at 0, 0, 1 plus y minus y naught into f y at 0, 0, 1 but f x and f y at 0. So, what I get is 0. This implies the equation of the tangent plane is x, y, z such that, z equals to 1. But, this is precisely the plane which I have drawn here, this is the plane z equals to 1, so it matches our intuition. Now, with this ideas, now I am going to move towards the surface area, which will actually used in doing surface integrals. So, we start with the surface, now to calculate the surface area, which is a graph of a function.

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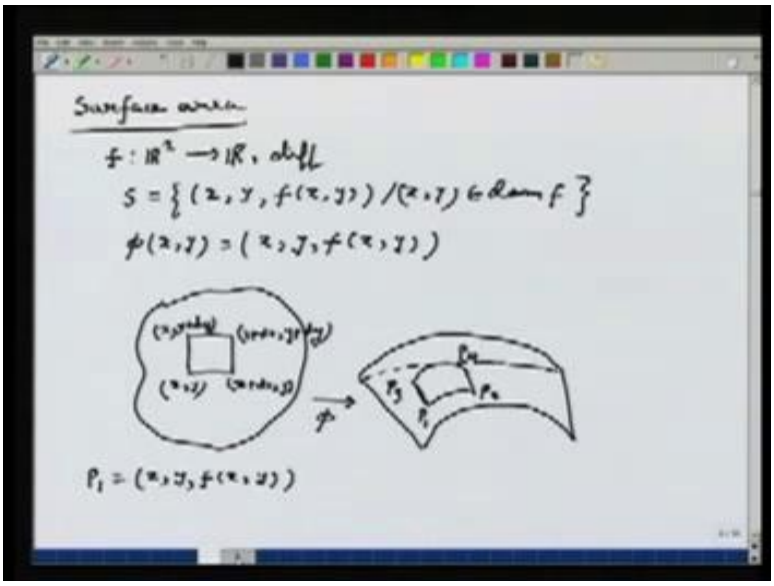

So, let us say, I have a function f from R 2 to R differentiable. And I look at the surface S, that is x, y, f x y. And I also know that, this gives me the parametric representations of the surface also. So, call that function phi, so phi of x, y, then is x, y, f x y, so pictorial it would mean, suppose this is the domain. And then the surface, it looks something like this. This is the surface and there is the mapped phi, which is given in terms of f. Because, I am dealing with surfaces, which are graph surfaces.

Now, what I am do is, I start with very small rectangles here. So, why I am drawing, it does not look very small, but what I mean is a small rectangle. And then, I level the coordinates say this is x, y, that is one edge of the rectangle. Then this one has an infinitesimal increment in the x direction. So, I write this, x plus d x, y. And then, this fellow at the top is x, y plus d y and the last one, that is x plus d x, then y plus d y. Now, this small rectangle, under the map phi is mapped. Let us say to a curved rectangle, it goes to something like this. Let us say this is the point P 1, P 2, P 3, P 4. So, what is P 1, P 1 is nothing but, x, y, f x, y, then what is P 2.

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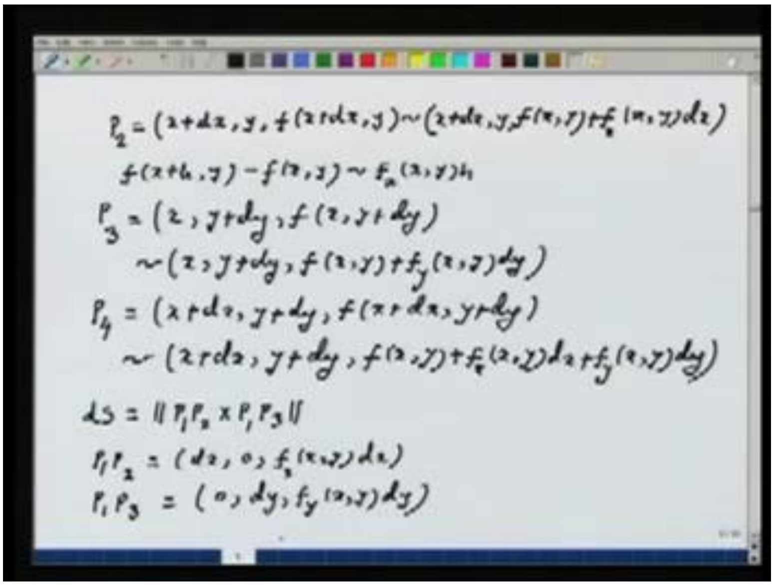

P 2 is x plus d x, y, f of x plus d x, y. Now, what I am going to use is the following. That if I look at f of x plus h y minus f of x, y. I say this is approximately equal to f, x at x, y times, h that is the linearization of this curve. Now, if I use this, then this P 2 turns out to be equal to x plus d x, y then f x at x, y d x. Now, I am going to use the fact, that f of x plus h, y minus f x, y is approximately equal to f x, x, y, h, where f x, x, y stands for the partial derivative.

So, I am using the linearization of the curve. Now, if I use this, then I can actually write that P 2, x plus d x, y, f x, y plus f x, x, y, d x. Let us look at what is P 3, P 3 was x, y plus d y, then f of x, y plus d y, which I say approximately equal to x, y plus d y. Again I want to use the partial derivative with respect to y of f. This is f x, y plus f y at x, y d y, then comes P 4, which is x plus d x, y plus d y, f of x plus d x, y plus d y, which is approximately equal to x plus d x, y plus d y, f x, y plus f x at x, y, d x plus f y at x, y, d y.

So, now I got a genuine rectangle, with this as the sides P 1, P 2, P 3, P 4. Then what is the area of that rectangle, if I call that d s. Then d s is nothing but, norm of the vector P 1, P 2 cross product P 1, P 3. And then, I am going to sum over all this rectangles to get the area of the surface, that is the idea. So, here the only idea involved is to find the area of the curved rectangle d s, but it is so small. That I am approximate it by the linearization of the bounding curves, which I am doing using the partial derivatives. Now, what is P 1, P 2 in question, well P 1, P 2 if I write down. That is d x, 0, f x at x, y d x, it is just P 2 minus P 1. And then P 1, P 3, that is 0, d y, f y at x, y, d y and then d s, I can calculate.

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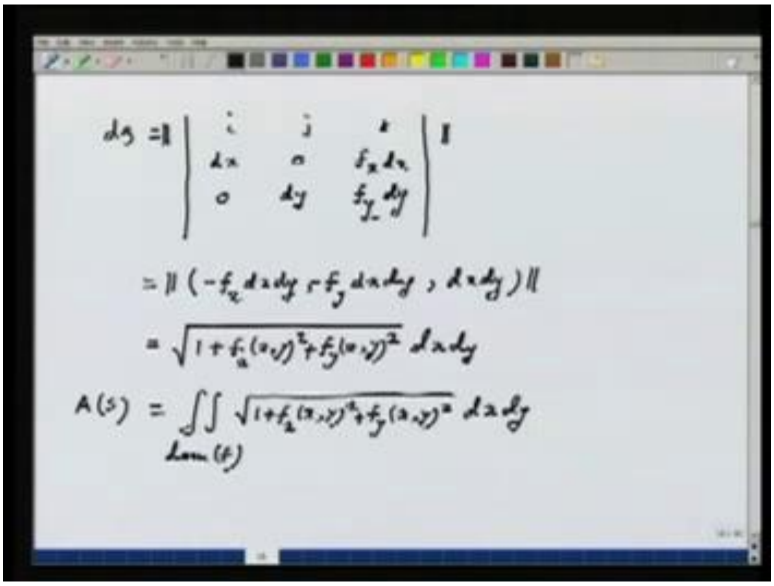

I just write it as, this i, j, k, d x, 0. I will simply write f x here, but what I mean is f x at x, y. Then 0, d y, f y, I have d x here and I have d y here, then the norm of this vector, because after all area is going to be a number. Now, what is the norm of this vector, it is the norm of the vector minus f x, d x, d y comma, then f y, d x, d y, then just d x, d y. Let us see, whether all the calculations are correct here. Then i times 0 into this minus d y into this, so minus f x, d x, d y then j into this minus, so.

So, here is a minus sign here, so this is what I get. Then this norm is easy to calculate. This is square root of 1 plus f x at x, y square, plus f y at x, y square, d x, d y, and then the total area of the surface, if I call it A S. That is I just going to sum up all these rectangles over the space of parameters, which is domain of f, so domain of f, square root of 1 plus f x at x, y square, plus f y at x, y square, d x, d y. This is the total area of the surface, now when I go to the genuine parametric representation of a surface. That is, it is not given by a curve, I can do the analogous, absolutely analogous analysis there. In that case, what I get is as follows, so I have a map now phi. So, it is a parametric surface.

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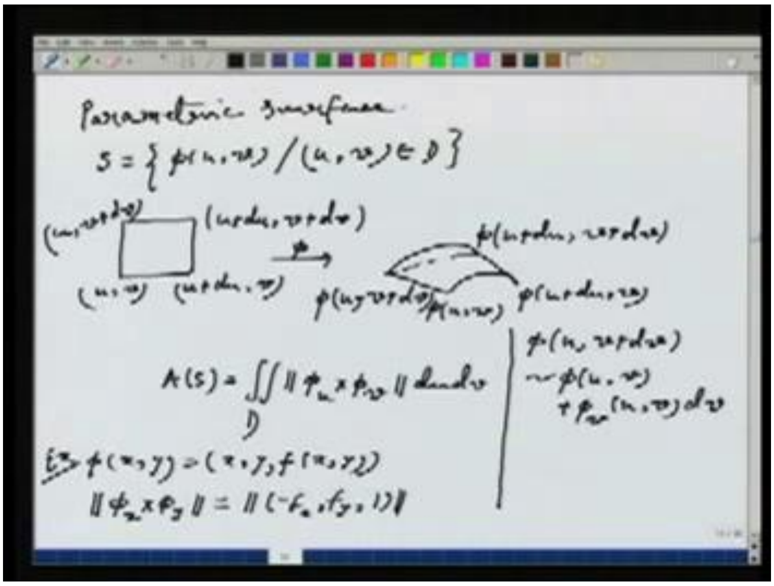

So, S equals to phi u, v, where u, v belongs to some set D. Suppose, this is the surface, I want to find the surface area of this surface. Again I start with some infinitesimal rectangles, which look like this. The points are u, v, u plus d u, v, then u, v plus d v, then u plus d u, v plus d, v, this goes by phi to a curved surface, goes to a curved rectangle. So, the points are phi u, v, phi u plus d u, v, phi u plus d u, v plus d v then phi u, v plus d v. Once I do this, then I can linearize by now phi u and phi v.

So, it stands out then, that AS, in this case trans out to be, double integral over D. Norm of phi u cross phi v, d u d v. The analysis exactly analogous, what you have to do is, you have to linearize phi u, v plus d v. Well, what you will be using is for example, phi u, v plus d v. Once you look at you will say, that this is approximately equal to phi u, v plus phi v at u, v. Then d v, go on doing this for all the coordinates, then do exactly what I have done for the case, when the surface is a graph, that you find P 1, P 2, P 3, P 4.

Find their lengths and then, apply the fact. That norm of the cross product of the two adjacent vectors gives you the area of the rectangle. And in that case, what it trans out to be is norm of phi u cross phi v. And then, to check I can give it as an exercise, that if you assume your phi, x, y to be equals to x, y, f x, y. Suppose, this is your phi, then show that norm phi x cross phi y is nothing but, norm of the vector minus f x, minus f y, 1. Just show this as an exercise. That will show, that the previous case whatever I have done has been generalized in the next case. Now, the next thing what I want to do is, applying this I want to find the area of the given surface. Let us look at the sphere again.

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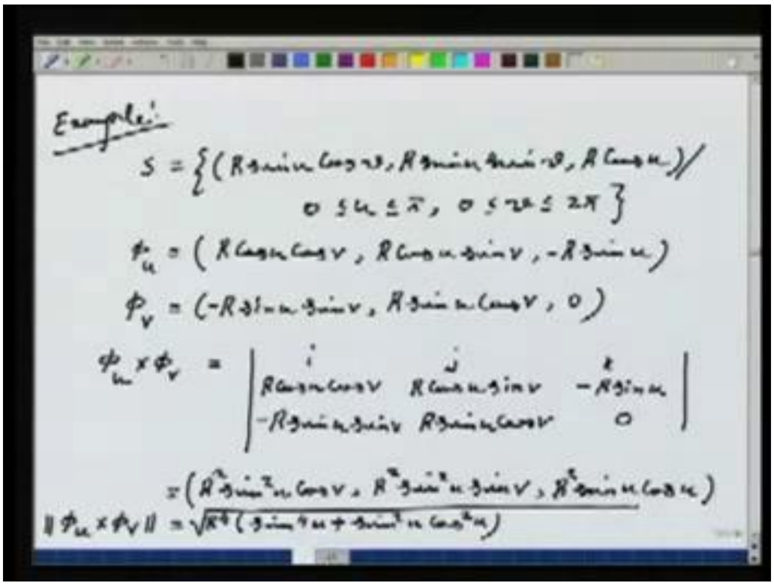

So, let us say S equals to, so I want to calculate the surface area of the sphere of radius R. Then I know that the parametric representation of the sphere is given by, whatever is written here. R sin u, cos v, R sin u, sin v, R cos u, where u varies between 0 to pi and v varies from 0 to 2 pi. I want to calculate the surface area of the sphere. So, for that what I need to calculate first is, what is phi u. Well, that is easily calculated, it is R cos u, cos v then R cos u, sin v then minus R, sin u, that is phi u.

Then, I want to calculate what is phi v, that is minus R, sin u, sin v, R sin u, cos v. Then 0 and now I have to calculate, phi u cross phi v. For that again I write down the determinant, that is i, j, k then minus R, sin u, sin v R sin u, cos v, then 0. Let us see, what it trans out to be, so what I get out of this, let me write it down. I get R cos u, sin v time 0, which is 0. Then, the next component is R square, sin square u, cos v, that is the first one.

Now, with j, what I get is it is R square, sin square u, sin v. Then comes the k component, which I am going to write in the next line. K component gives me R square, sin u, cosine u then cosine square v, plus r square sin u, cosine u, sin square u. Now, this last component, I can take this cosine square, sin u cosine u, R square to be common. Then what I am left with is, sin square v, cos square v which is 1. So, the final result then would be, R square sin u cosine u, that I will write down here.

So, this is R square sin u, cosine u. Now, I have to calculate the norm of this vector. That is the sum of the squares of this, then I have to take the square root, now once I do this. So, norm of phi u cross phi v, if I look at, what do I get. So, this is then equals to square root of, I first get R to the power 4. Because, everything is getting squared up, then I have sin to the power 4 u cos square v, plus sin to the power 4 u sin square v. That would produce sin to the power 4, u and then, the next thing is, sin square u, cosine square u. If I take sin square common, then I get 1.

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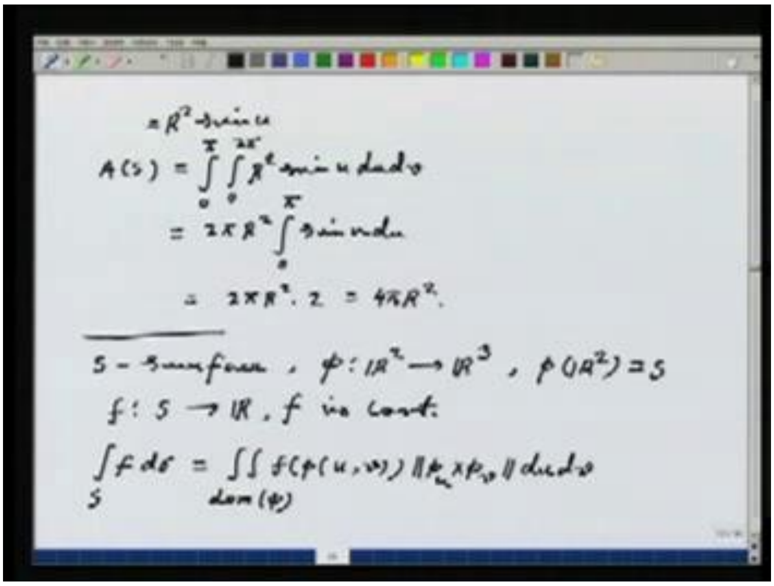

So, the end result is actually, equal to R square, sin u, so it does not depend on v. So, I have found out, what is d s, now I have to integrate it. So, total area AS, then is integral from 0 to pi, that is the variation of u. Then 0 to 2 pi, r square sin u, that is the area of the infinitesimal small rectangle, which I have found and called d s. So, this is d s and then I am summing them up, so it is d u, d v. Now, d v has nothing to do with integrant, so it is just 2 pi R square, then integral 0 to pi, sin u d u.

So, that means, it is 2 pi R square, since I am going to get cos u, that is again produce 2. So, the final answer is 4 pi R square, which we already know. So by this method also, we can find the surface area of a sphere. Now, after this, I am going to define surface integral of functions, so suppose S is a surface. Let us say it is given by the parametric representation phi. So, phi is a map from R 2 to R 3 and phi of the domain of phi, which is I am calling R 2. It might be some subset of R 2 also, I do not care, this is my S.

It is a surface and suppose, f is a function, from the surface to the real line. Assume that f is continuous, then I want to talk about the integral of this function over the surface S. So, how do I define this, well integral over S, f d sigma, that is the notation for the surface integral, I will tell you what exactly it means. Well it is given by the double integral, it is meaning is the double integral over domain of phi, f of phi u, v. Then the area of the infinitesimal small rectangle, that is phi u, cross phi v then d u d v.

This is what I mean, by the integral of the function over a surface. So, what you do is, given the function f, if the surface has the parametric representation phi. Look at the function f compose phi, now that is the function of two variables. That is the two parameters which I am using it to define the surface. Then multiply with it the surface area of this infinitesimal small rectangle that is d s, whose expression we have found out.

It is the norm of the vector phi u, cross product phi v, you get a number. Then look at d u, d v and do the double integration. That gives you the surface integrals of the function over that surface. Now take, to execute this, I will look at an example of a particular function and a particular surface. And I will try to integrate the function over the surface.

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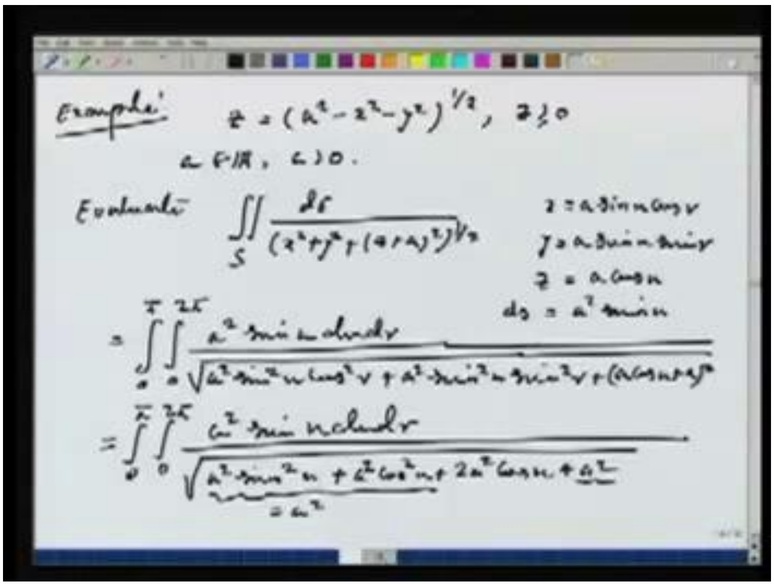

so in the example, let me look at the upper hemisphere of the sphere. That is z equals to, a square, minus x square minus y square whole to the power half and z is bigger than or equal to 0, a belonging to R is strictly bigger than 0. That is what I have done is my surface is, the upper hemisphere of a sphere of radius a. Where a is obviously positive and I want to evaluate, this integral, that integral over S, d sigma by x square, plus y square, plus z plus a whole square. So, this is of integrating the function and I want to integrate the function, over this sphere.

To evaluate this integral, I will use the parametric representation. That is x equals to, a sin u cos v, y equals to a sin u, sin v, z equals to a cos u. In that case I know that, in the element d s, that trans out to be, a square sin u. Then the double integral is, first the derivation of u, since I am looking at the upper hemisphere, u varies from 0 to pi. V varies from 0 to 2 pi suddenly, I get a square sin u d u, d v. That divided by the square root of a square, sin square u, cos square v, plus a square, sin square u, sin square v, plus a cos u, plus a whole square.

That is 0 to pi from 0 to 2 pi, a square sin u, d u, d v, that remains. Now, I concentrate on the denominator. If I had the first two terms, I get a square sin square u. Then from the third term, I get another a square cosine square u plus twice a square cosine u plus a square adding the first two terms. Again I will get a square, anyway there is another a square, so I will get twice a square.

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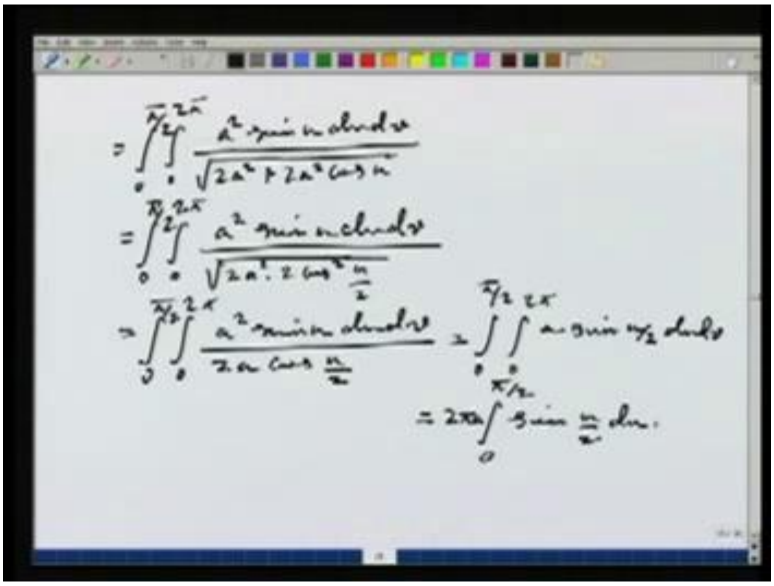

So finally, what I get is, 0 to pi, 0 to 2 pi, a square sin u, d u, d v divided by square root of twice a square plus twice a square, cosine u. This if I use the formula of cosine of u by 2, I get the following square root of twice a square into 2 cosine square u by 2. This is then, it is then 0 to pi, 0 to pi by 2. Because, variation of u is from pi by 2, what I get is a square sin u, d u d v divided by twice a cosine u by 2. That then is equal to, 0 to pi by 2, 0 to 2 pi, a sin u by 2, because sin u, I can write as twice sin u by 2 cosine u by 2.

So, cosine u by 2 cancels, 2 also cancels 1 a cancels, I get d u, d v. Now, there is no v in the integrant, so I just get 2 pi then 0 to pi by 2. A also I can take out, it has nothing to do with this integral, it is just sin u by 2 d u, which now I can easily calculate. Because, the integration of sin and such things you can do, so this how one does the integration. So, I repeat once again, how to integrate a function over a surface, if you have a parametric representation of the surface.

The first thing would be to find the area element. That is norm of a vector phi u cross phi v, that has to go with the d u, d v. Where u and v are parameters, then what is your integrant, your integrant is f of phi u, v. Then calculate the variation of the limits of u and v, put those limits in the integral write down the function, f of phi u, v multiply it with the area element. That is norm of phi u cross phi v d u d v, calculate the double integral. That is the integral of the function over the surface.