Classical Physics
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Lecture No. # 21

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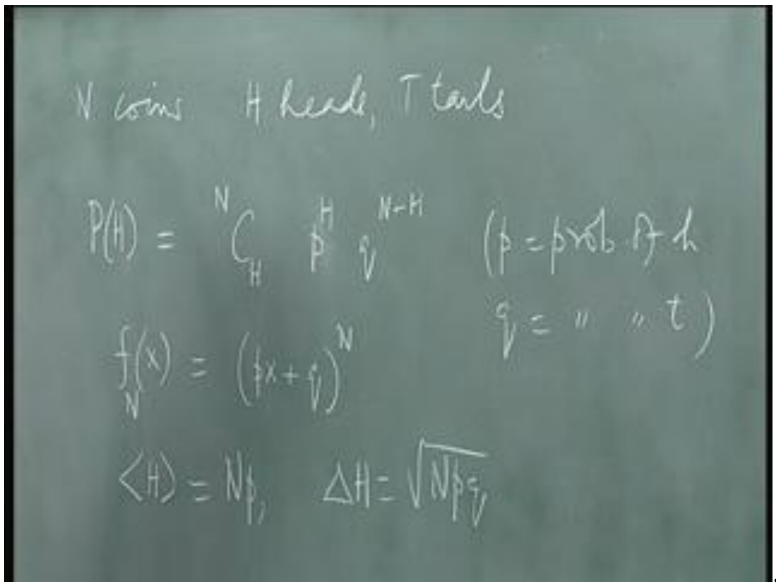

Let us begin where we left off last time. You recall that we were looking at this toy model of N coin tosses and just to recall to you; N coins and then you had H heads, T tails and the probability of getting H heads was given by N C H 1 over 2 to the N. For a fair coin otherwise it was something like P to the power h, q to the power N minus H, p equal to probability of H, q is equal to probability of t. So, this was the expression we had in the last time. And we also discovered that the generating function f of x which by the ways a function of n also because it is for a fixed number of coins. This was equal to px plus q to the power n, and it followed immediately that H was equal to N p, delta H was equal to square root of Npq is the standard properties of the binomial distribution.

Now, what we would like to do now is to ask, what does this distribution look like, what does this thing look like and in particular what dominates for very large values of N. You would expect that as N increases you are going to get a distribution which gets more and more sharply peak about the mean value and make it less and less probable for large deviations to occur this is what one would expect. If you toss two coins it is quite a chance that you are going to get both heads or both tails quarter in each case and then a probability half that you are going to get on head and one tail. On the other hand if you toss a 100 coins the probability that you are going to get all 100 tails or all 100 heads is one over two to the hundred which is extremely small.

On the other hand the probability that you are going to get say approximately 50 heads and 50 tails is overwhelmingly high the question is how overwhelming is this?

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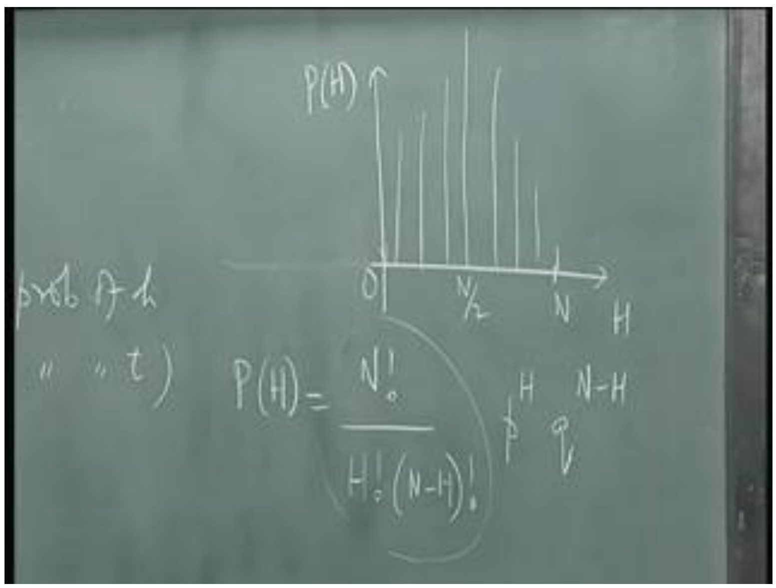

How peak is this distribution going to be and if I plot this, if I plot the probability of P of H versus H, you start with 0 here and of course, could have N, you get a histogram and the idea is that as as you get this is approximately N over 2. For even values of N N over 2 is a possibility. This is going to very sharply peaked and then you are going to get something which comes down, fairly, rapidly till it becomes extremely small here.

What I would like to show is that, this is exponentially smaller than this, as N increases for this we need to have an approximation for the binomial coefficient N C H, because this thing here P of H equal to N factorial over H factorial N minus H factorial and then p to the power

H q to the power N minus H. And we would like to find out what does this number do as N becomes very, very large; we need an approximation for this.

Well many approximations exist, but the point the best one of them the one that is practically a formula is the following. If you took your pocket calculators which have possibility of showing your numbers up to 10 to the power 99 and you press this factorial button, what happens where does it stop?

Around 69, so 70 factorial shows you an error. This means you cannot compute even a small the factorial of a reasonably small number like 70 it is already big greater than 10 to the power 99. So, the factorial increases extremely large extremely rapidly and the question is how fast does it increase, how do you make an approximation.

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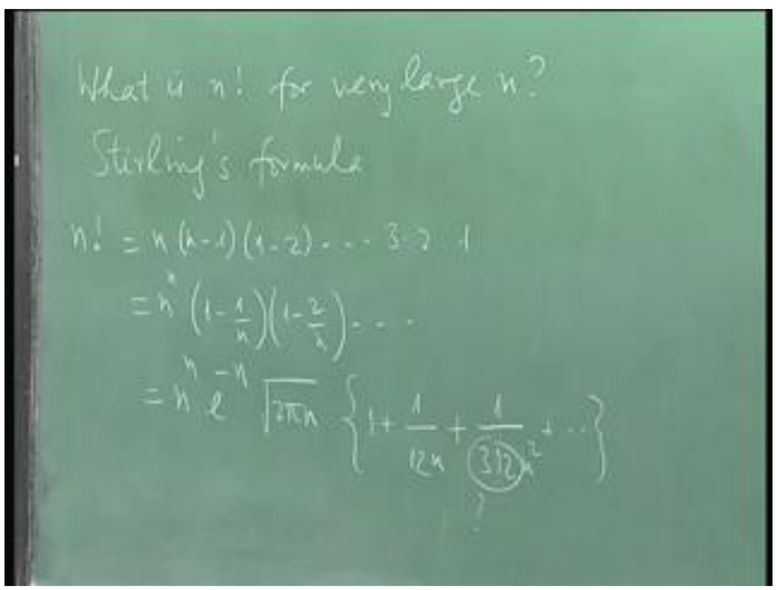

So, what is a what is n factorial for very large n? And the answer is provided by a formula called sterling’s formula sterling’s approximation. It is an extremely good formula and this is where what does? It provides an asymptotic expansion for n factorial in terms of quantities which you can calculate quite easily. And of course, one would like to know what the leading term is it is clear n factorial by it is very definition is N times N minus 1, N minus 2 etcetera. So, if you took out a factor n from each of them, we wrote n factorial equal to n times n minus 1, n minus 2 up to 3, 2, 1. And you wrote this as n to the power n times 1 minus 1 over n, 1 minus 2 over n dot, dot, dot. Then, it is clear that in the leading approximation if n is very large should left out these terms then it would go like n to the n.

But of course, as you go get closer to 3, 2, 1, etcetera, you are leaving out terms like the last term is going to be 1 minus n minus one over n and it is not fair to leave out n minus 1 over n relative to one of course. So, this is an over estimate n to the power n is the overestimate and you have to compensate for it; and this form of this suggest what the compensation is and in fact, this thing turns out to be n to the power n e to the minus n.

So, there is a compensating factor, this grows extremely fast it is grows like e to the n log n, this decays like e to the minus n. Of course this dominates over that, but still there is a substantial shrinkage here. And then it is multiplied by 2 pi n that is a very very small factor. If you took a log in the log scale you can see this better log n factorial is dominate term is n log n and then you subtract n and then you add half log 2 pi n, so there is a correction of order 2 pi n. The actual formula itself goes like this its one plus one over oh I forgot now what this factor out here is, it is 1 over 12 n plus 1 over 300 and something or the other n square plus etcetera, I am not too sure about this factor and work it out with…

So, this is what it actually looks like it is a big series, it is an infinite series, it is an asymptotic series and you can see that there is the first correction here if n is of order 100. It is already only point one percent one part in a 1000. So, you can immediately see that this is an extremely good formula, very good formula. And it gets better and better as n increases, what is the smallest value of n for which we could try to apply this? Well if you want 10 percent accuracy 92 percent accuracy, we would leave this out because 1 12. It is going to be correct to 92 percent, if you put n equal to 1 in this formula, it is already going to be extremely good, so let us do that just for fun.

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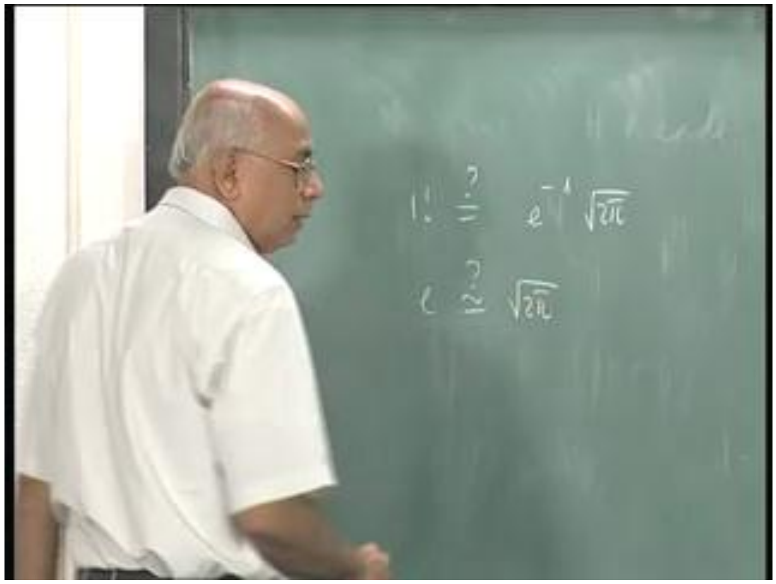

If n equal to 1, then the question is 1 factorial equal to 1 to the power 1 e to the minus one square root of 2 pi. So, the questions you are asking is e approximately equal to square root of 2 pi.

And it is so correct to 92 percent, so even at 1, this is pretty good at 10 it gets better you get a 1 percent error. At 100 you get point one percent error and at 10 to the 24, you can forget about this can completely forget about this. So, sterling’s formula is very very good even for mediocre values of n and of course, becomes truly large then you can completely forget about.

That is the reason why a lot of formulas in statistical mechanics will work because of the power of this factorial; the fact that this thing here is an extremely good approximation to n factorial. So, good that for most of the practical purposes I take the log of this, I can forget about this after all for a million log million n base 2 is base 10 is only 6.

You can just ignore its very, very slowly growing function and the rest of it is completely negligible. So, you could in fact, write n factorial is approximately n to the n e to the minus n, we will leave it like this. By the way how do you get this formula, how do you get this formula, where do you get this formula? Remember we are in our imaginary dessert island

and we forget sterling's formula, we would like to derive this formula; it is a good thing to know how to derive this it would be digress to do this, because there are other formulas which you can find by a similar trick.

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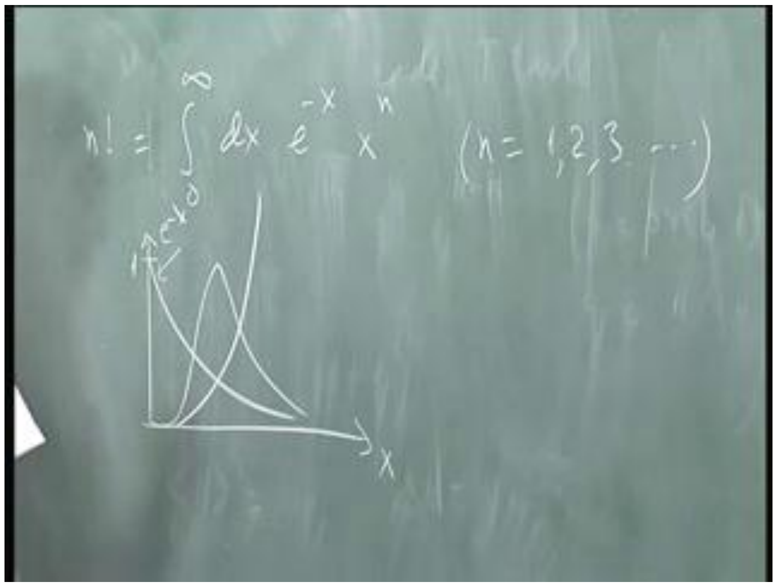

And it goes like this, so I start by saying n factorial is defined as 0 to infinity dx, e to the minus x x to the power n when n equal to 1, 2, 3, etcetera certainly.

By definition this is true, this integral is equal to n factorial trivially. Now if I put n equal to 0, here in this formula you still get one on this side, so in fact, this formula could be used to define 0 factorial as 1. Otherwise you do not know what how to define 0 factorial I define it as one because of this. Incidentally as you know the gamma function interpolates between the integers and provides a function of which the factorial is a special case for positive integer values, non negative integer values.

But if I now took this and I wanted to know what does this do for a very large n, then you see the argument goes like this, if I plot this integral e to the minus x is a function that comes down in this fashion, function of x this is e to the minus x. On the other hand x to the power n is a function, which increases rapidly in this form. This is finite here this is 1 therefore, the product is 0 at the origin and it goes down exponentially fast because, this dominates for very large values of x.

And you have one very large increasing factor and very large decreasing factor, when you multiply the 2 the answer is that both n functions is going to be essentially be 0, and in the middle there is going to be a maximum of some kind. So, this product is going to look like this and as n increases this is going to get steeper and steeper.

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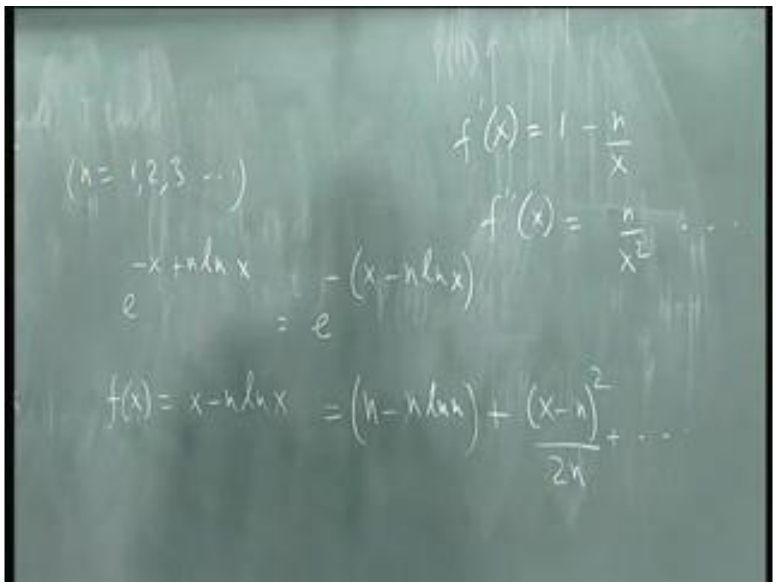

So, let us do the following let us write e to the minus x, x to the power n is e to the n log x plus n plus n log x write it in this form. And that is equal to e to the minus x minus n log x and let us take this function let us put f of x is equal to x minus n log x and ask what does it do, where does it have a maximum.

I expand this function and what would it do as a function of x and this is equal to where does where does it is derivative vanish by the way, where does it have a extreme? x equal to x equal to n x equal to n because this derivative is 1 minus n over x and you put that equal to 0 you get x equal to n. So, let us expand it and at x equal to n it is n minus n log n plus, so f of x is that f prime of x equal to 1 minus n over x f double prime of x is equal to n over x square and so on. So, the first term proportional to x minus n is going to be 0, because the derivative vanishes at that point since derivative is exactly 0 at x equal to n.

And then next term is x minus n the whole square over 2 factorial multiplied by this f double prime at x equal to n and that is equal to one over n.

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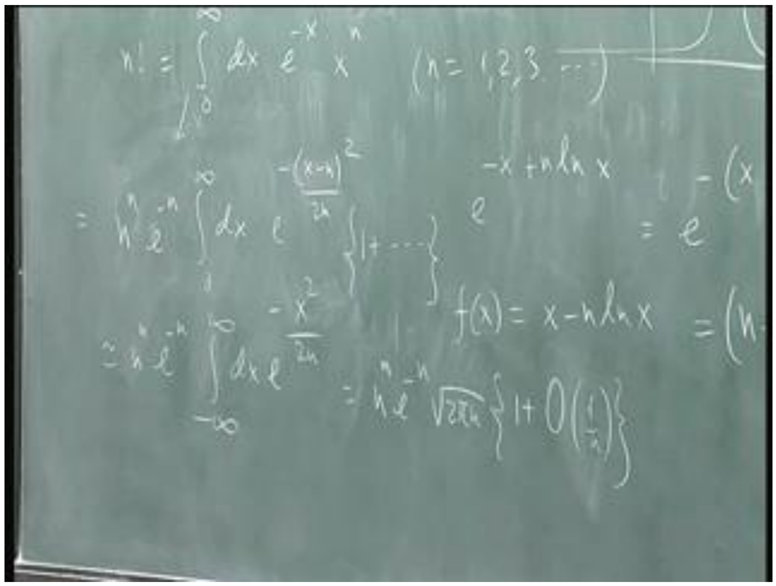

So, let us put that into this integral and this becomes equal to 0 to infinity, dx e to the power minus n minus n log n that that whole factor is just a constant, so let us take this whole thing out. It is n to the n e to the minus n integral 0 to infinity dx e to the power minus x minus n the whole square over 2 x and its corrections. You can pull those corrections down and write this as 1 plus dot, dot, dot, I leave that to you as an exercise to find the next corrections.

Now, this function here x it is like a Gaussian peaked at x equal to n; and as n becomes larger and larger this guy here sitting in the denominator becomes larger and larger and becomes peak becomes sharper and sharper. And therefore, for a very large n we could actually approximate this integral which look like this the function looks like this. You could extend the range of integration from minus infinity to infinity and make exponentially small errors.

So, in the next step this becomes equal to n to the n, e to the minus n into the minus infinity to infinity dx e to the minus x minus n whole square by 2 n and its correction and its 1 plus correction. This is a Gaussian integral and I can shift this the centre of this integration, I can shift variables to x equal to n because it is running for minus infinity to infinity. So, this integral is a same as e to the minus x square, that is a standard Gaussian integral e to the minus a x square minus infinity to infinity is square root of pi over a, it is a Gaussian integral. So, therefore, this is equal to n to the n e to the minus n square root of 2 pi n into 1 plus corrections. This is order one over n actually that is not trivial to show, but I leave you to show this that is sterling's formula.

What I used there is a form of Gaussian integral simple basic Gaussian integral and I have assumed that and then of course, the rest follows them. So, this is going to be used over and over again in statistical physics, this formula sterling's formula for the log of factorial of a very large number in that form. Incidentally we used the Gaussian integral here, you know how to derive that when using, yes everyone know this.

What do you do?

Square the integral and go to the polar coordinates is a brilliant trick, it is a brilliant trick square the integral is one would think of that one would think of integration by parts and then you get a linear relation of some kind. But, this is square and too this is a non trivial trick it takes a non trivial person to find it Poisson found this in 1815 or something like that, so it is long, long ago.

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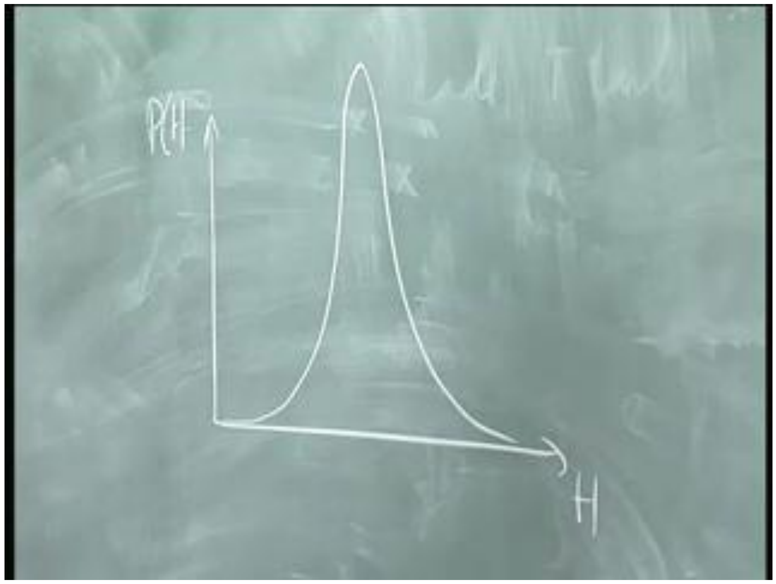

So, much for sterling's formula and if you use that then I leave you to figure out the rest of this distribution to show that, that if you plot H versus P of H, then indeed when n becomes extremely large, this distribution goes sharper and sharper and has something like this.

It is almost a continue because n is very large and H is goes to takes on very large values then the discrete nature of the integer H is becomes relevant and starts approximating more and more distribution, which looks like this goes up and comes down and is very, very sharply peaked.

We will do this at a later stage we will start with a binomial distribution and I will show you that a binomial distribution goes over into what is called a Poisson distribution, which then the deviation from the mean goes over in to a Gaussian distribution. Incidentally we could do this right away we could do this right away and let us do this in a physical context instead of P of H.

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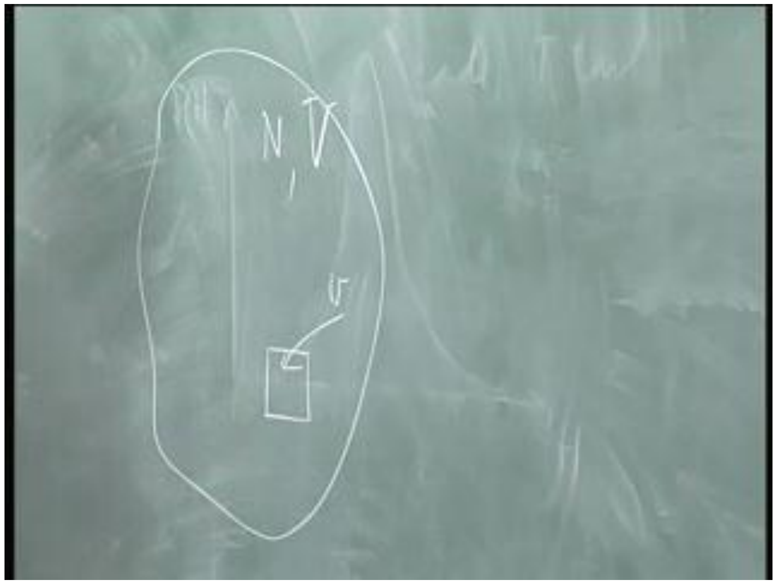

I asked the following question, I have a gas in a container of volume v, so let us take an ideal gas in a container of volume v. And the gas has a large number of particles n and of course, the particles are moving about inside here it is at fixed temperature where everything is constant.

Then, I could ask what is the probability that if I took a small sub volume v, what is the probability that I have a number n of particle inside this sub volume v; if I took an instantaneous snapshot of all the molecules in this room froze them and just looked at them, then what is the probability that the number of particles inside the sub volume is little n. And that as you will see is a binomial distribution, because I want the probability that inside the volume v, I have n particles.

Probability that v contains n particles, what is this equal to, well what is the probability that a given particle is inside the sub volume v and these are assumed to be uniformly distributed, they are equally likely to be anywhere. So, the probability that a single given particle is inside the sub volume v is in fact, v over v. And you assuming that all the particles are moving independently of each other therefore, the probability that there are n of them inside is indeed this. But that is not enough the rest must be outside.

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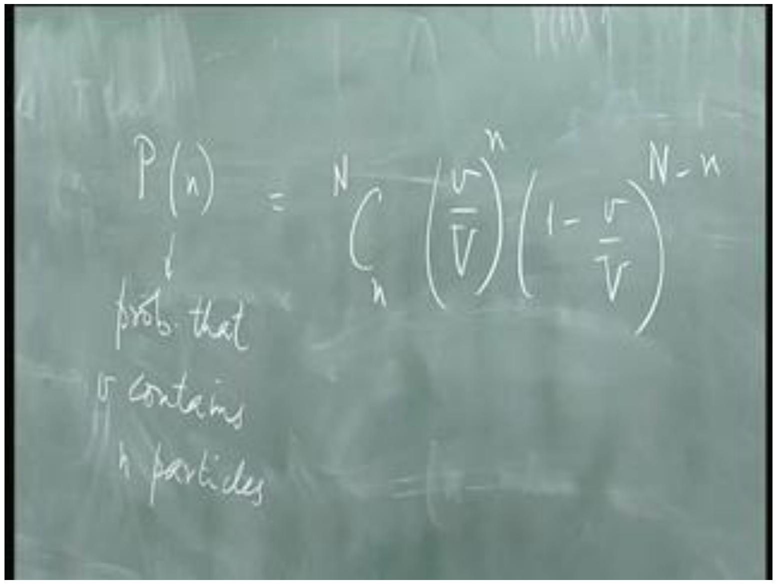

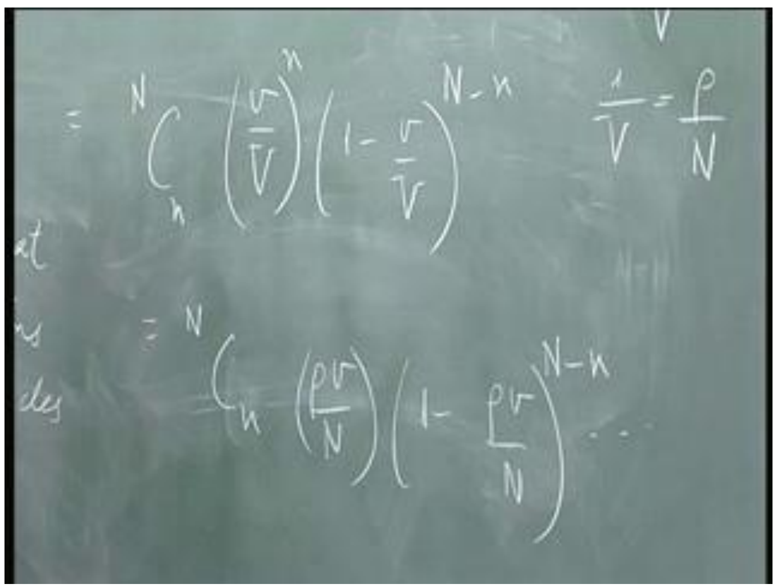

So, they cannot be inside because I want the probability that you have exactly n particles inside, so the other particles must be failures. They must be tails instead of heads that is equal to 1 minus v over v to the power n minus n and of course, it does not matter which of the particles is inside you are not asking that question.

And therefore, there is n c n and that is precisely the binomial distribution once again with little p replaced by capital V over V and of course, we could get rid of this little bit.

Because, I would like to see what happens to this for very large n. So, let us put let rho equal to n over v the number of particles per unit volume in this huge gas and that some constant is given to you.

Since capital N and capital V are both constants with some fixed thing therefore, this is equal to N C n and I could write this as rho rho V N. So, this says 1 over V equal to rho rho by N and I am going to use that here it is right 1 minus rho V by N to the power N minus N where we are and that is a binomial distribution.

How did we get this well if a given particle is equally likely to be anywhere in this entire volume, then the probability that it is inside this sub volume is just the fraction of this sub volume to the total volume right. This is a not a trivial statement, but it is a sort of intuitively obvious proving this regressively is little harder you need some notions of geometric probability, you have talked about measures and. So, on which I do not want to do.

But other things being equal, if it is likely to be anywhere as likely to be in one part of the volume as in any other then I would say the probability apriori probability that its inside here. For example I divide this room into halves two half probability that is in the left or in the right is half and the reason is, it is the ratio of the sub volume to the total volume.

We are not talking about, we are ignoring all these things we are ignoring interactions we are ignoring we are simply saying it is an ideal gas, there are these points dots everywhere I take an snapshot an instantaneous snapshot and ask I count how many of them are there at some instant of time.

Of course that will change from instant to instant, it will fluctuate very rapidly where some fixed instant of time, what is in probability that there are exactly n of them inside it. And little n can go all the way from 0 up to capital N, so that is the sample space for this random variable. The random variable here is little n and I am talking about it is probability distribution and that is a binomial distribution here everything will be suppose to be given and you have given this the idea.

And now, I would like to know what happens to this as capital N tends to infinity and capital V tends to infinity, but keeping the density fixed that is important I should keep the density fixed, density is an intensive quantity.

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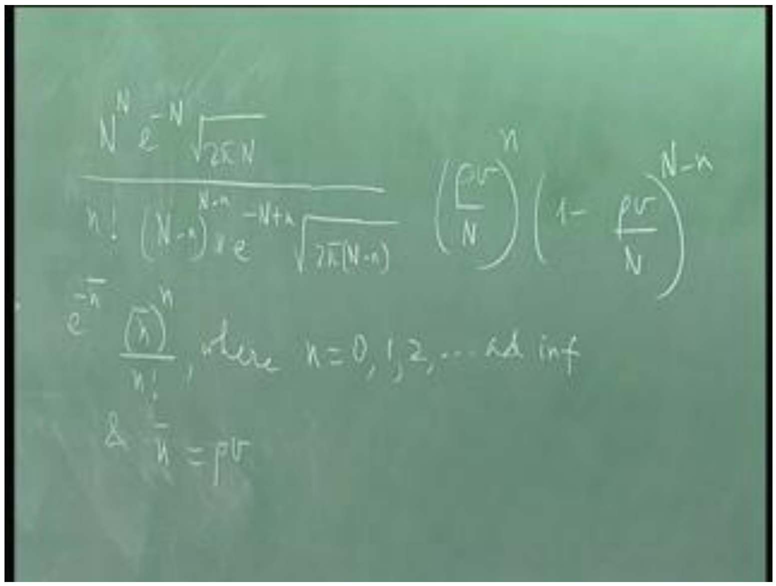

Then this becomes p of n becomes as n tends to infinity N C n, so N C n is n factorial over little N factorial n minus n factorial. And we will start putting a approximation here, so this is n to the power n e to the minus n if you write square root of 2 pi n divided by little n factorial, you cannot do a sterling for that, because little n could go all the way from 0 to infinity, but capital N minus n. So, one could do that one could do capital N minus n to the power N minus n e to the power minus N plus n root 2 pi N minus n.

These are irrelevant factors we put them in any way and then rho V over N to the power sorry there is an N here to the power n here. And then 1 minus rho V over N to the power N minus n and now do a sterling do simplify this simplify this expression here I leave that to you a an exercise. And this thing here will finally, go over to p of n goes over into e to the minus n that you can already see emerging from here sorry no this e to the minus N will cancel it will go to the following. So, let me write the answer down and then we will justify it e to the minus n bar n bar to the power n over n factorial, where n is 0, 1, 2, add infinite term because capital n has gone of to infinity and n bar equal to rho v.

So, I leave you to do the rest of the algebra and show that it reduces to this expression here, what is this expression called? It is called the Poisson distribution. N bar and what is the physical significance of n bar? Well, it is the average number density multiplied by the volume the sub volume therefore it is the average number of particles inside the sub volume.

The Poisson distribution is characterized by one parameter namely the average value itself and indeed it looks like this. And now little n has sample space running from 0 to infinity because it is an infinite volume, but with fixed density.

So, what you have to do, no no little n sample space goes from 0 to infinity now what you have to do is carefully use sterling's approximation, these factors that is why I did not do anything here, I left this term here. So, this thing is regress true in the limit when capital N goes to infinity and you must do the algebra carefully and make sure you do not make an invalid approximation of the kind you talking about and then this is indeed the formula that emerges I want you to do this exercise.

And of course, in that in that formula capital V would not appear anymore and capital N would not appear anymore, because they are both gone off to infinity their ratio is taken to be a number rho some fixed number. By the way this is called the thermodynamic limit of a statistical system when you let the number of particles go to infinity, the volume go to infinity such that the density is finite.

And statistical mechanics will reduce to thermodynamics in the thermodynamic limit, this is when fluctuations disappear completely. And there is a reason why we are going to pay a little more attention to this. The Poisson distribution is going to appear also in statistics very often. And I wanted to appreciate some properties of the Poisson distribution right away. So, let us do that.

We have now assumed that these guys are all very plain ordinary particles, no quantum statistics, no these things between none of those complications do not assumed any of them. This thing here this distribution is that distribution of density fluctuation of gas in this room in a classical ideal gas. So, it is telling you that the density locally the number density changes it fluctuates and it tells you what is the distribution in this simple situation. No correlations, no complicated interactions nothing is going to be assumed.

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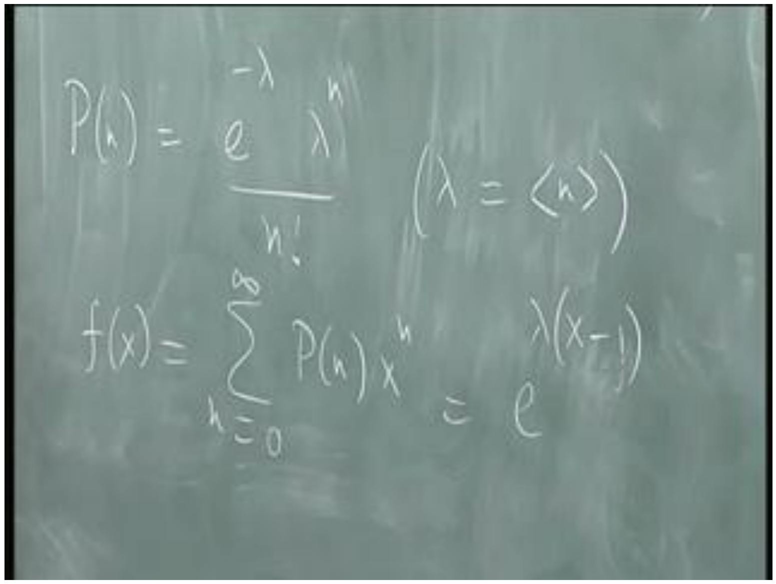

Just a couple of quick words about the Poisson distribution; for a random variable n which runs from 0, 1, 3 possible integers the Poisson distribution given by P of n is given by e to the minus n bar n bar to n let me use another symbol e to the minus lambda lambda to the power n over n factorial lambda is the average value of n and that is not hard to show. P of n summed from n equal to zero to infinity is equal to one because sigma lambda to the n over n factorial is e to the plus lambda and that cancels this here.

It is very easy to show that the average value is equal to n n there and this lambda here and what is the generating function f of x equal to summation n equal to 0 to infinity P of n x to the power n what does this work out to? Well, it just multiplies this and becomes lambda x to the n and then it is e to the plus lambda x. So, it is easy to see that its e to the lambda times x minus 1.

F of one is indeed equal to 1 for consideration of the total probability f prime at x equal to 1 is lambda what is the variance, what is the variance in this case? It is lambda itself.

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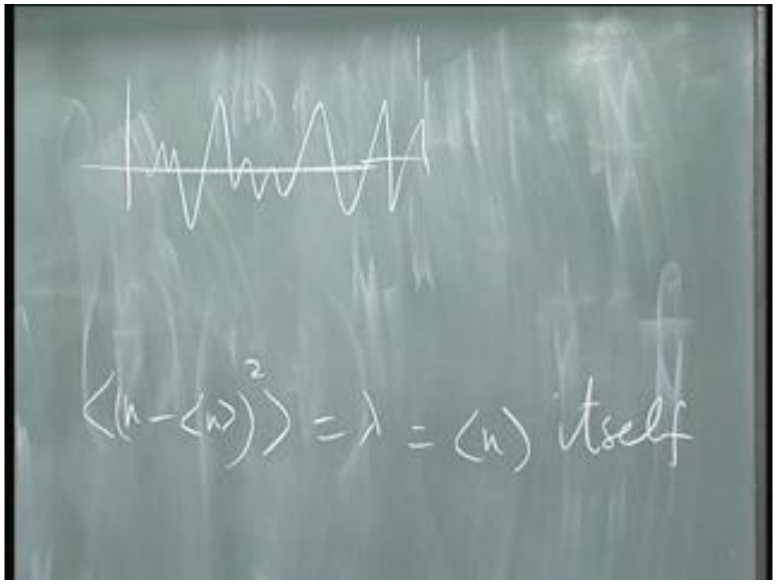

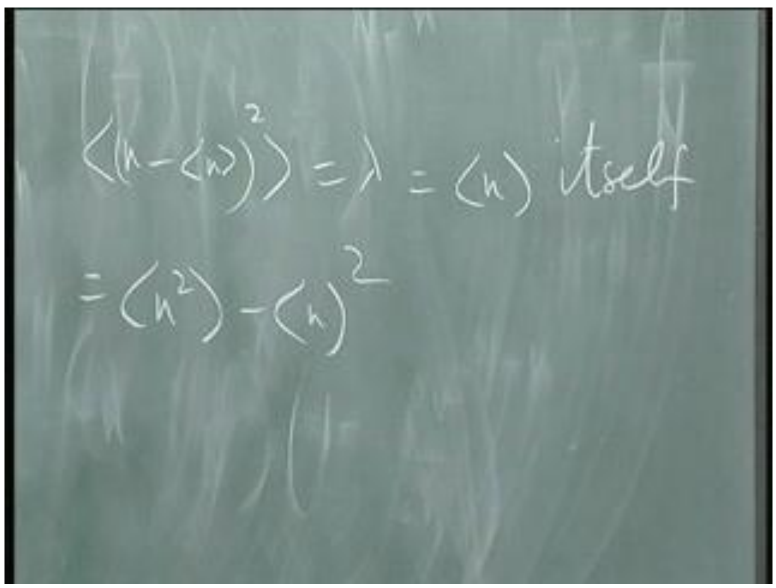

So, the Poisson distribution has its interesting property that the variance is equal to the mean; therefore, the relative fluctuation which is a standard deviation divided by the mean is one over the square root of the mean. So, you see yesterday we saw this one over square root of n appear in the denominator and it is characteristic of these distribution such distribution.

These are all uncorrelated and that is the reason why this is happening. So, it is a very interesting property of the Poisson distribution that the variance is equal to the mean. What is the generalization of the variance and what is the idea behind the variance in any case? Since we are doing statistical mechanics we are going to know little bit of statistics what is the idea behind the variance for random variable?

See, if you have a variable that goes up and down in this fashion it fluctuates about some mean value then of course, you would like to know, what is the strength of these fluctuations? And then if you add them up you might get you will get 0, because the guys on top cancel the guys below, so what you do is square it first and make the thing non negative and then take the average value and take its square root. And that is the standard deviation that will tell you the size of the fluctuations relative to the mean. So, that is the idea behind the variance and of course, you would like to make this a little more general and go to higher orders and so on. For instance if you look at the third moment then there are pieces of the third moment which come about, because of second moment in some sense just as there are pieces in the mean square value you remove.

So, what I do is remove this, I remove this thing. So, this is also equal to n square minus n whole square and then I get an idea of the true fluctuations in exactly the same way for the third moment you remove pieces which come from the lower moments for the fourth moment you remove pieces which come from first three moments and so on and so forth, those are called cumilants, they call the cumilants of the distribution. And one of the most interesting properties of the Poisson distribution is at all the higher moments or all the higher cumilants are equal to the mean and that is it; so just a single parameter distribution. In the case of Gaussian the third and higher cumilants vanish identically we have just the first two cumilants, we have the mean and the variance and that is how the Gaussian is defined in terms of the mean and the variance. We will come back to these things little later.

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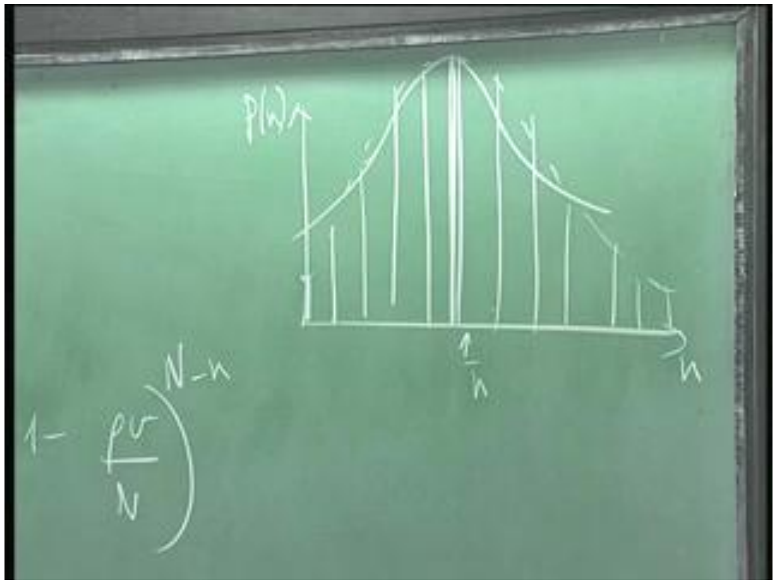

The point I want to make right now is that if you took this thing here and let n bar become very large then this Poisson distribution, which has the shape of this kind. It is a, it is a histogram actually p of n it has some probability that n is 0 some probability that n is 1 etcetera, etcetera. And then reaches some value and comes down sort of exponentially, it comes down in this fashion. Now, if you let an n bar is approximately somewhere in the middle. If you let n bar itself become very large, so this whole thing shifts when the deviation from the mean about this point here starts looking like this and takes on a Gaussian shape.

So, you can actually go from the variable n to the variable n minus n bar and call that a continuous variable if n bar is very large it is of the order of 25 million then plus or minus 1 does not matter it is practically a continuous variable. And then it turns out and this again uses sterling's formula that from this formula you can get an expression for the probability density of variable x which is n minus n bar the deviation from the mean of a Poisson variable.

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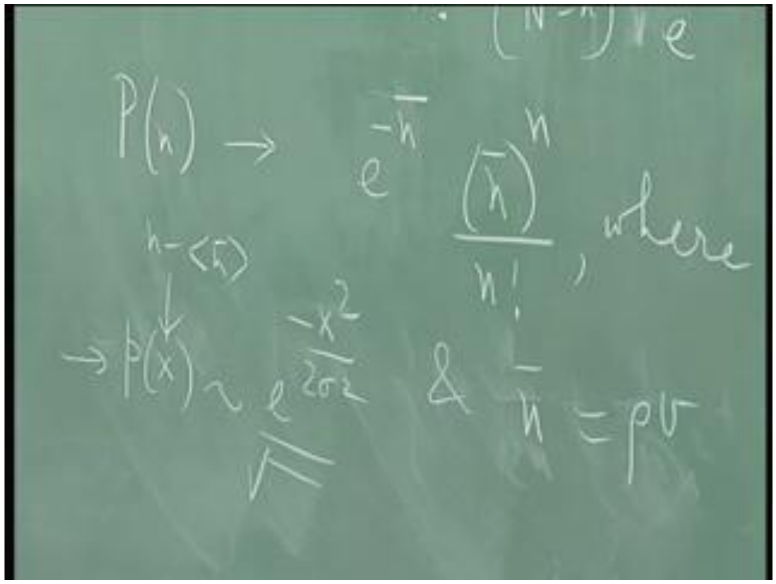

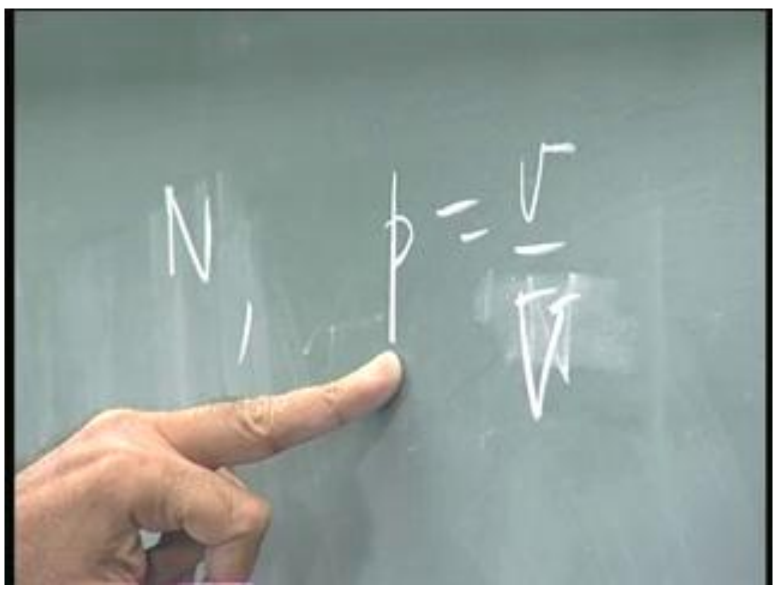

And this will take on an expression which looks like p to minus x square over 2 sigma square over normalization factor, it will start looking Gaussian. So, this is how in this simple instance, the distributions change from one shape to another you start with a binomial distribution of Bernoulli trials and then from that you take the number of particles number of trails to be very large. And you take the probability of success in a trail to be vanishing the small that was my limit n over v, because remember p was equal to v over v and n was the number of trials.

So, I start with like tossing a coin it is exactly like that; the probability of success in a trail the probability of being in the sub volume is going to be 0, because capital V is tending to infinity. The number of trails the number of particles is increasing such that the product of these two guys is finite and that is my rho times little v, then a Bernoulli trial goes over in true, Bernoulli binomial distribution goes over into a Poisson distribution.

Then, if the mean value of the Poisson distribution is very, very large compare to unity the deviation from the mean is approximately a continuous variable that has a Gaussian shape goes over into a Gaussian shape. So, this is how the Gaussian appears in these problems where we start with integers and then eventually it ends up with Gaussian distributions. There are other fancy ways of saying this invokes what is called the central limit theorem, but we do not do that right now let me do with what we are doing. But, I want you to appreciate this fact that the probability distribution can move into another probability distribution shifts over now with this preliminary.

Let us get down to where we had ended the last time. So, we go back to our problem in statistical mechanics of an isolated system in thermal equilibrium and I pointed out that we have a postulate now and we are going to use this fundamental postulate of equal apriori probabilities to see where we can get.

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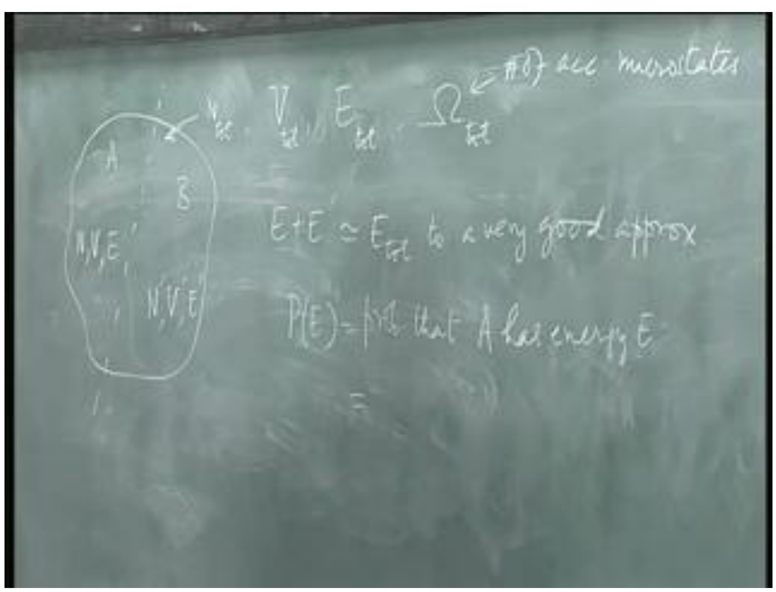

I already told you what kind of system we have a huge system with a total number of particles capital N. Let me call it n total for reason which will come here the volume of this system is V total and the energy of system is E total, right now I do not want to give other attributes to this system except there is some collection of particles may be interacting with each other classically.

And you have these parameters given to you and it is an isolated system in thermal equilibrium. So, all its micro states are equally probable; and it has a large number of micro states omega total, this is the number of accessible microstates of this huge system. The probability of system being at any instant of time in any one of the micro states is one over this omega total that is the assumption that is gone in not derivable from mechanics it is gone.

Now, I ask suppose I come along and for reasons which will become clear, I imagine that this is system is really made up to two sub systems. So, let me call this total system, sub system A and sub system B, it is really made up of these two sub systems imagine an imaginary partitions in the middle of this huge container like and this is really made up of two sub systems.

Each of them is a very large system and they are evidently in equilibrium with each other, because they are isolated from the rest of the universe. Now, the energy of this let us say N V and E are the corresponding parameters for this sub system and on this side you have N prime V prime and E prime for that system.

And you have trivial relation, which says N plus N prime equal to N total, V plus V prime is equal to V total, E plus E prime is E total. E plus E prime is not quite E total, because now you should say look if these particles are interacting with each other, then this is very subtle that is where everything is buried like in all these things E plus E prime is not necessarily E total; because I cannot define E, I cannot define E. Even if you had one particle on this side and one particle on this side and they were interacting with each other I cannot define E and E prime why is that?

Yeah, because there could be a potential energy the kinetic energy of each particle are quite unique, but the potential energy is a common property and depends upon the coordinates of both particles. So, if I have two particles charge particles I can tell you the total energy of the system, I can tell you the kinetic energy of particle one the kinetic energy of particle two.

But I cannot tell you that one third of the potential energy belongs to this particle two third belongs to that I cannot tell you. So, you cannot do this portioning. So, there is that problem. But now I am going to argue that let us assume that the interactions are reasonably short ranged. Let us further assume that if you have a particle here and here near the boundary then definitely there is a potential energy of interaction between these two guys.

And there is exchange of energy between the two systems at some instant of time this side may have more energy more particles the other side may have less energy fewer particles and so on this is possible. So, there are rapid fluctuations on both sides the totals are kept constant. However, if this is the number of particles here and its very, very large and imagine really putting a take a three dimensional volume and put a partition here. Some kind of a screen or a mesh or whatever what do you think is the number of degrees of freedom which are actually interacting on either side.

If you have a volume with n particles then on a wall how many of these particles would be close to the wall? I have 10 to the 24 particles in this room. How many of them do you think are close to the walls of this room at any instant of time?

Of the order of 1 over n

No no, what do you think is the answer, I have capital N 10 to the 24 particles in this room, how many are in how many of them do you think are close to the walls at any instant of time.

N to the power n to the power two-thirds, because that is the surface to volume N to the 2 3rds. So, 10 to the 24 at any given instant of time 10 to the 16 of them are close to the walls of the order of. Therefore, this ratio of surface to volume is 1 over 10 to the 8 and is negligible in exactly the same way at any instant of time the number of degrees of freedom that are in interactions across this partition is of the order of n to the 2 3rds.

And I am going to neglect that compared to capital N itself. That is the level at which the fluctuations are in and as capital N becomes larger and larger remember we really talking about astronomically large systems here the numbers of particles. And that is why this is completely negligible am therefore, going to say that I can do a good approximation partition the total energy into E and E prime. So, without a very good approximation certainly I am going to write that. Then I ask the following question at any instant of time, what is the probability that the energy of A is equal to E. So, P of E equal to probability that A has energy E. Now, what is this equal to given no other information given just this, what is the probability that this is going to happen?

When I argue that this is equal to its equal to the probability it is equal to the number of microstates of this entire system such that A has energy E. See, once I write down the energy of each particle write down a microstate I know everything about the system. So now, exactly the same argument has little v over capital v all micro states are equally probable. Therefore, all I have to do is to count that fraction of microstates for which of the total system such that the sub system A has an energy E.

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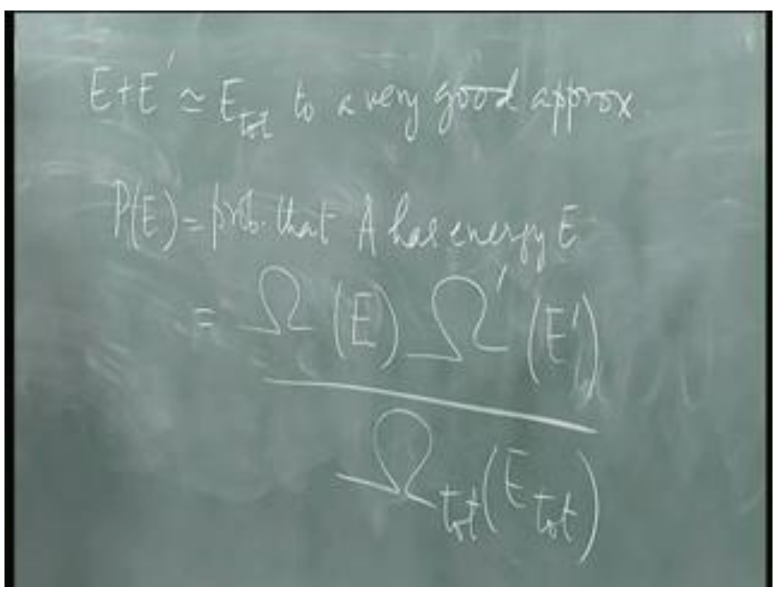

So, this is equal to omega total number of microstates of a system omega total such that A has energy E. This is divided by and of course, the rest such that A has E and the rest of them have E prime. So, let us use the symbol subscript total for the total system no subscript for the sub system A and a prime for the subsystem B that is my notation. So, this is omega of E this tells you the number of microstates for which a has energy E multiplied by omega prime E total omega prime E prime.

So, this is like saying particles in the sub volume, particles outside sub volume the product of these probabilities. But, it must be normalized this whole thing must be normalized. So, it must be normalized by omega total of E total why that is it that is it because, this is already factored. So, what I should have put is a symbol omega subscript total such that A has energy E a prime has energy B had energy E prime and that quantity the numerator and factored into a property of A times property of B.

So, there is no square, so this is directly follows directly from the postulate of equal apriori probabilities that is it. These functions are not known to me. I do not know these functions and I do not know that function in particular and this is possible if this huge system is a container of oil oil drum sitting inside an atmosphere of this kind a bigger room.

Then this and that may have very different degrees of freedom altogether. So, the function omega and the function omega prime may be very different functions. One describing the possible microstates of oil and the other describing possible microstates of air I do not care I still do not care. The only assumption is all the microstates of the total system are equally probable then it immediately follows that this is the probability.

And our next task is to analyze this probability we have to impose the condition of equilibrium this is something we are going to do. So, let me stop here today since some people may have a test may be day after this and we will start from this point and see where we go. Is this convenient to stop or we go, because I have no idea of what is it is it quiz for people is a test, so perhaps its test as well we will stop here today.

So, this is my starting point, I believe there is a holiday coming up?

We have a class on Wednesday next, next Tuesday is a holiday next Wednesday we meet and then we will discuss. Yes we will take it up from there.