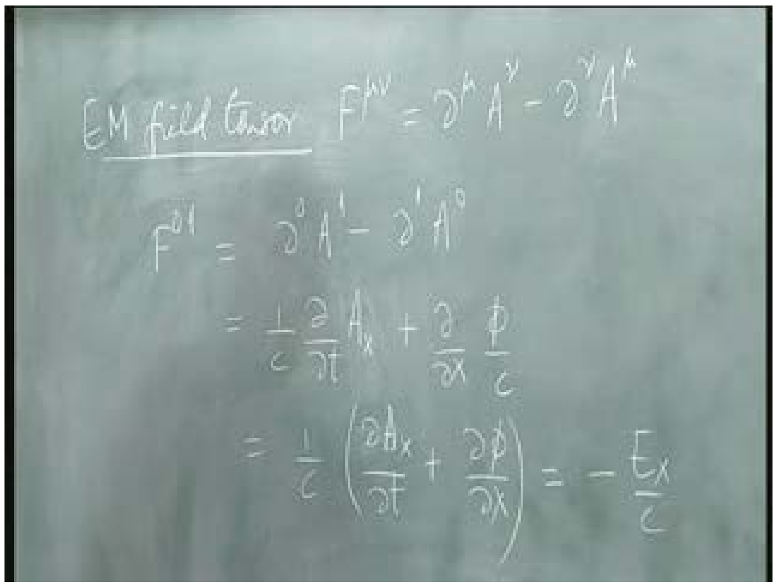
Classical Physics
Prof. V. Balakrishnan
Department of Physics
Indian Institute of Technology Madras

Lecture No. # 37

(Refer Slide Time: 01:09)



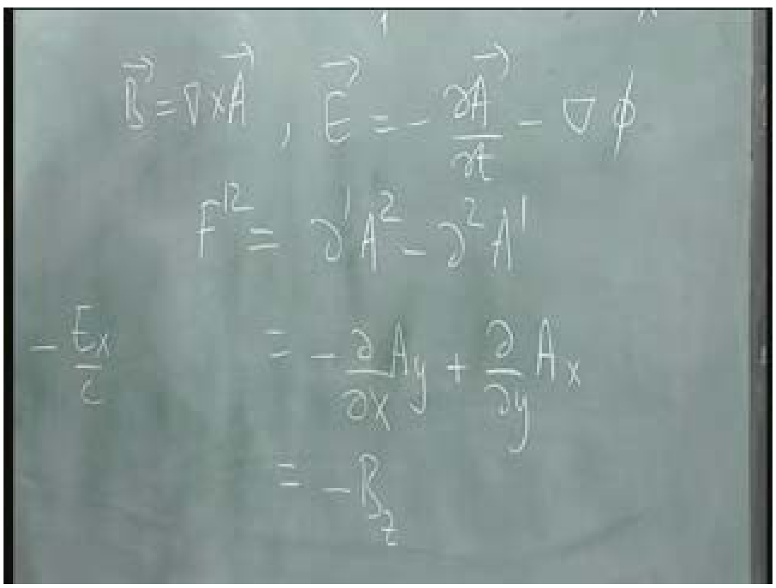
There was a question raise as to whether, when wrote the boxes del mu del mu I could write it us del mu del mu also just as valid does not matter. In fact, in general if you have 2 4 vectors, I have a mu b mu this could be written in either order you could also be written as a mu b mu does not matter in this case, these are all exactly the same. Now, we were in the middle of our discussion of the electromagnetic field and I introduce the tensor called the electromagnetic field tensor F mu nu and I defined it us del mu, A mu minus del mu A mu.

And it has six independent components being an anti symmetric tensor of rank 2 and the question is how the related to the electric are and magnetic fields. If work this out let us start by doing this let us look at F 0 1 for example, this is del 0 A 1 minus del 1 A 0. Please remember that del 1 stands for delta over delta x 1 which is equal to minus delta over delta x 1 which is the same as minus delta over delta x. Because x superscripts 1 is my Cartesian component x

So, if I put that in this is equal to 1 over C delta over delta t that is del 0 and a 1 of course, was phi over C minus on this side is there next 1 over C. A 1 sorry A 1 is A subscript x the x component of the vector potential A minus del 1, but that becomes a plus delta over delta x A 0 which is phi over C. I am losing minus sign here, that is fine that is fine, both signs are taken care of del 0 is 1 over C delta over delta t A 1 is A x and del 1 has a minus sign.

So, that cancels against this gives me a plus here so, this is equal to 1 over C delta phi 1 over C delta A x over delta t plus delta phi over delta x and this is equal to minus E x over C because the electric field is delta A over delta t plus grde phi that minus signs both terms.

(Refer Slide Time: 04:11)

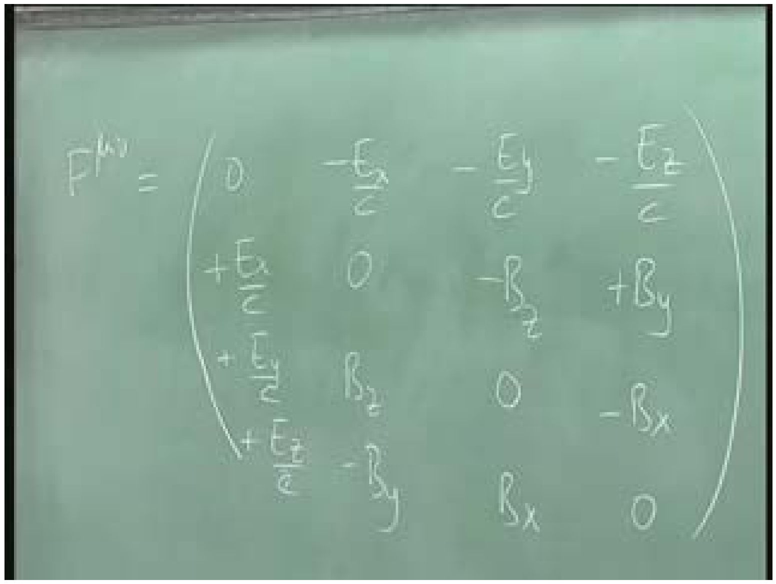


So, recall that E was equal to minus delta A over delta t minus grde phi by definition and the x component of that is precisely this with a minus sign; so this minus E x over C. Similarly A of 0 2 would be minus E y over C and A of 0 3 will be minus E z over C.

What would the phase space components look like? So, what would F 1 2 look like for instance, this is del 1 A 2minus del 2 A 1, and that is equal to minus this a minus delta over delta x A y plus delta over delta y A x and what is that equal to recall also that B equal to cull A. So, what is this equal to it is minus the z component of the magnetic field. Because the z component is magnetic field B z is del x delta over delta x A y minus delta over delta y

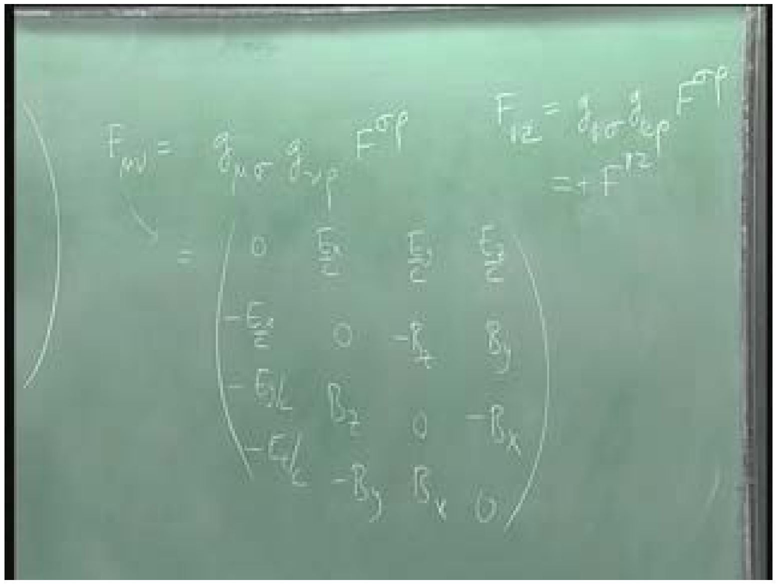
A x. So, it is a minus sign there, similarly f 2 3 and f 3 1 would be the magnetic field components.

(Refer Slide Time: 05:28)



So, we are now ready to write down what F mu nu actually look like its got a 0 here, being a diagonal element minus E x over C minus E y over C minus E z over C and it is a anti symmetric tensor to therefore, these components here, would be just the positive corresponding positive values over C and 0 in the diagonal and then the 0 1 component. So, this is 0 0 0 1 0 2 0 3 then 1 0 1 1 1 2 and so on. It is 1 and 1 2 you already figure out was minus B z this becomes a plus by and then since its anti symmetric you have a B z here diagonal 0. This is minus by here since its anti symmetric diagonal 0 and this is minus B x B x. So, that is what the field tensor looks like its components give you the components of the electrical and magnetic fields.

(Refer Slide Time: 06:44)



What does the lower thing look like what is F mu nu look like. This is equal to you have to lower these two entities. So, it is g mu sigma g nu rho F sigma rho. So, you see I want to bring that index sigma down I do a g out here I want to bring the index rho down. So, I do g here .

Sigma and rho are dummy indices and that what this covariant version of this tensors and we could write that down as a matrix also this thing here will of course, have 0's everywhere in the diagonals. It is also an anti symmetric tensor and the question is what does for example, what does F 0 1 look like this, this element here 0 1. What would it be well it is clear that these g's. So, you would like to find F 0 1 and that is equals to g 0 sigma g 0 rho f sigma rho.

You have to substitute for these and then some over the remaining two indices the domain indices the sigma and rho. But it is clear that this vanishes unless sigma is 0. And this vanishes unless rho is equal to 1. But g 1 1 is the minus 1 and g 0 0 is a plus 1.So, the answer is that this is equal to component y minus sigma 0 1 everything as vanished. So, all you have to do is to put a minus sign there and then this is E x over C E y over C E z over C and this side you have a minus E x over C minus E y over C minus E z over C what does F 1 to do.

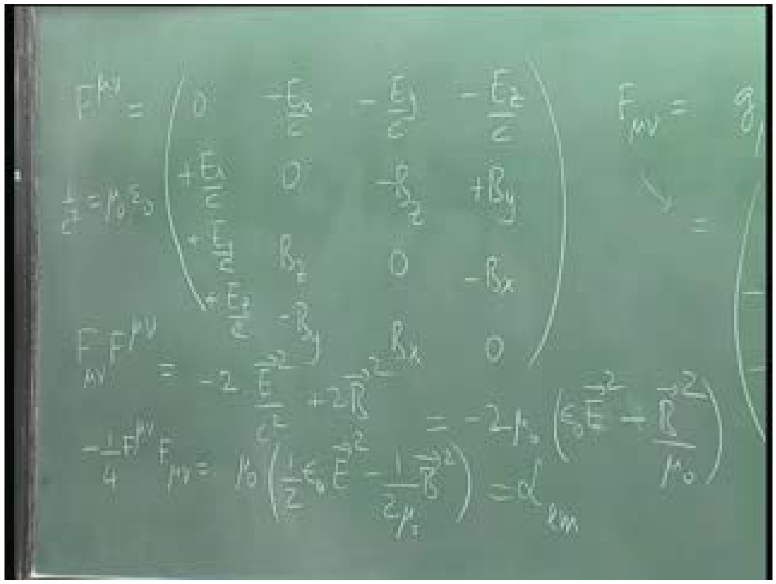
Well this is 1 2 now. So, this is 1 sigma and a 2 rho and you have to some over it and again the only thing that contribute q 1 1 and only thing the contributes there is g 2 2 and each of them is minus minus 1 and the product is plus 1 which tells you that this does not change at all. Numerically F 1 2 is equal to plus F 1 2 upstairs in which case we can simply write it down minus B z B z minus B x and this side is by and minus By plus B x.

So, that is the covariant version of this tensor it is clear that the entire set of fields the E x and E y all the components are include it here in this field tensor. So, in a sense what does happen you can check this out easily is that this definition that we have is like a 4 dimensional curve it is the equivalent of the except that this is run from 0 to 4 0 to 3.

And while these two relations look completely unlike each other they really are the same the kind of relation and they both very symmetrically written out in terms of a four dimensional curve this itself you tell you that electricity and magnetism electric and magnetic fields are intimately connected to each other. Really they just want field electromagnetic field. It is just that when you write it in terms of three dimensional vectors you artificially end up with expression for them which look very very different; but this expression which is a time derivative and gradient; and this expression here which the curle of vector field.

Really once you put them together in a 4 vector potential really is the curle of this 4 vector potential this is exactly the same relation. So, this tells you that these fields are intimately related to each other we will see how much closer when we do lot of transformations on them. What about the field equations themselves.

(Refer Slide Time: 11:04)

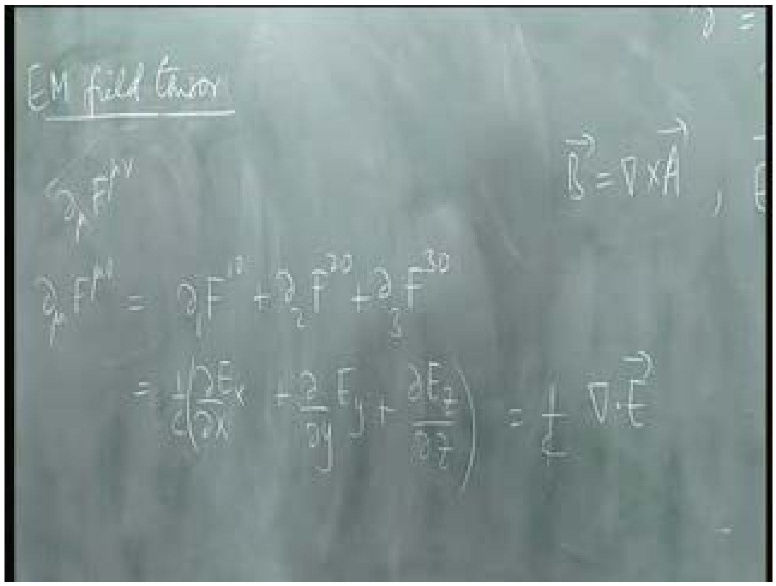


First of all before I do that what kind of invariant can you get from this well what is we have to do is to take the scalar F mu nu F mu nu write this out and what is that give you. This is equal to you put things in you take this and contractive with that. So, it is just the sums of squares of all the terms, because this is going to completely contracted.

So, all you have to do is to take this take this take this etcetera and multiply and add the whole thing and what you get you get minus twice E vector square over C square from the electric terms and the magnetic terms would be B square By square B x square. So, these things give you twice the magnetic fields. So, this is plus Y B vector square and lets things out. So, this is equal to unit which you are familiar with 1 over C square by the way recall one over C square is mu not epsilon not. So, units which you are familiar with this are equal to minus twice mu not E squared epsilon not E squared plus B minus minus B squared over mu not and if you put them together and take out minus 1 4. So, minus 1 quarter F mu nu F mu nu is equal to Mu not times one-half epsilon not E vector square minus 1 over 2 mu not B squared.

This is the lagrangian density of the free electromagnetic field. So, although we have in proved it in this course if you took this quantity this is a density now as dimensions of an energy density. These objects if you do Euler lagrangian equation on it for fields you are you recover the Maxwell equations. So, this is really the lagrangian density L for the electromagnetic field. The energy density of this field is the same thing with a plus sign here in these units. Now, let us go and look at what the Maxwell equations themselves look like and how to write them.

(Refer Slide Time: 14:16)



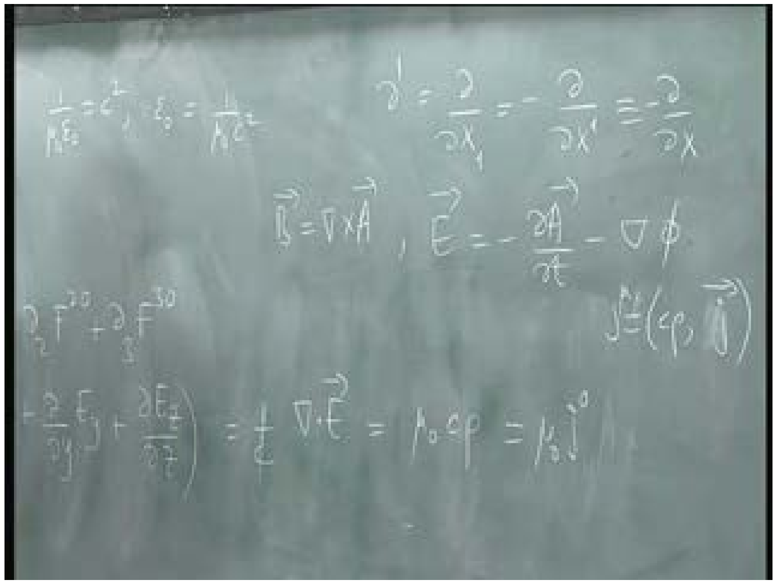
Consider what happens to this quantity so, let us look at del mu F mu nu what this does let us see what this quantity gives.

What kind of object is this is it a scalar vector tensors what is it?

This index mu is summed over and there is 1 free index. So, it is a vector with an upstairs index single vector. So, let us look at del mu F mu 0 for example, it is got 4 components let us look at this 0 component the time component. Lets this object this is equal to del 0 F 0 0, but I have 0 0 is 0 because it is an anti symmetric tensor and so, this is equal to del 1 F 1 0 plus del 2 F 2 0 plus del 3 F 3 0. Now, what is that give you del 1 is delta over delta x and what is F 1 0. Is this quantity here minus E x over C? So, this is equal to minus 1 over C Ex plus del 2 F 0 that delta over delta y.

Plus C x did I make a mistake 1 0 sorry it is here that write it plus 1 0 and then E y plus delta E z over delta z. That is equal to but this quantity is a divergence of E as you can see.

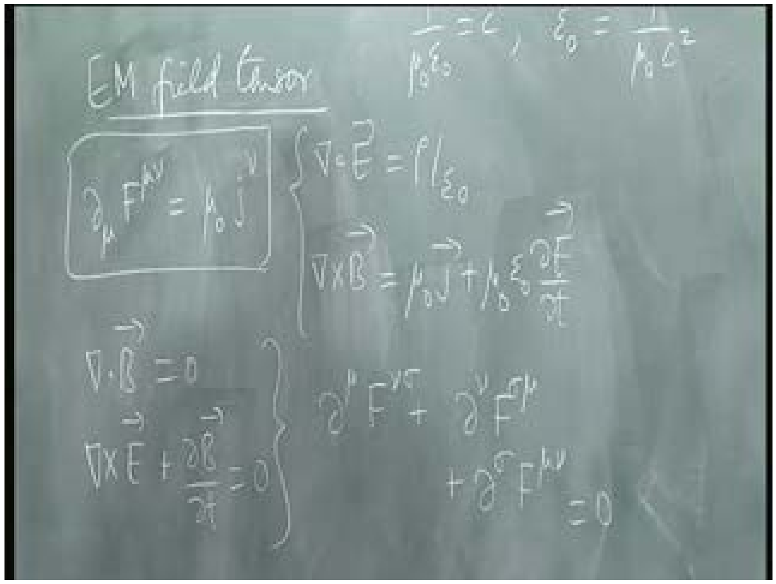
(Refer Slide Time: 16:21)



So, this equal to 1 over C del dot E but this is equal to 1 over C rho over epsilon not that is equal to mu not 1 over C epsilon not where the z give you notice that if I going to use this. We going to use this we going to use term 1 over mu not epsilon not equal to C squared or C squared or epsilon not equal to 1 over mu not C square.

Well just instead of roots I just want to write instead of epsilon not. So, let me write this as C square mu not. So, this is mu not C rho, which is equal to mu not j 0 remember that j mu consisted of 0 and the, and the vector current, current density. So, you have del mu F mu not is mu not apart from this unit j 0. Similarly, I do 1 2 and 3 .

(Refer Slide Time: 17:42)



And the result would be that I end up with the 2 Maxwell equations. I end up with del mu F mu nu equal to mu not j nu and this encompasses the Maxwell equations del dot E equal to rho over epsilon not and del cross B equal to mu not j plus mu not epsilon not del F over del t.

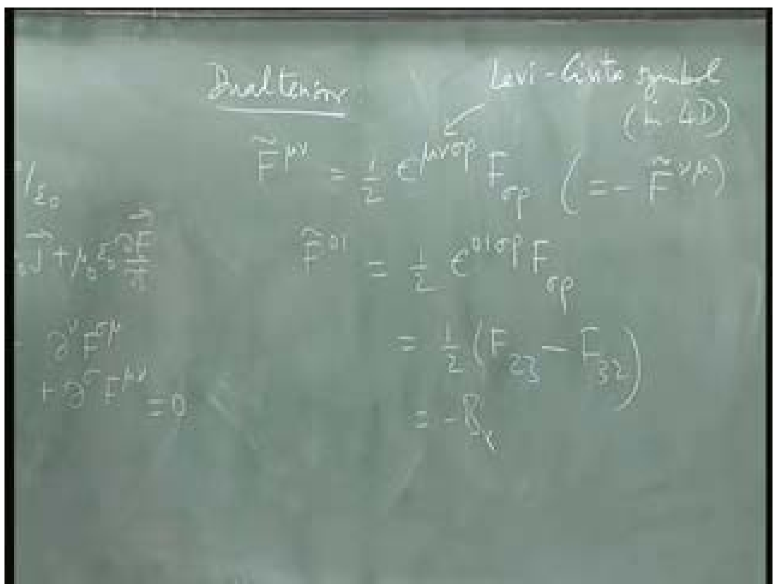
The 2 in homogeneous Maxwell equations or re-expressed in this form. So, this is the much more compact way of writing these 2 equations. So, now you begin to see that these 2 equations the in homogeneous equations really 2 parts of the same equation. They really say that the divergence this is the divergence of this tensor of the field tensor is the current. So, that is what in homogeneous equation.

There are still 2 equations left the homogeneous equations and you have to check out they say they are; obviously, going to say what the two equations say. But what is the way of doing this.

Well I leave you to verify, verify that del dot B equal to 0 and del cross E equal to minus delta B plus delta B delta t equal to 0, these two equations together. They correspond to a statement that del del let us use some notation nu F nu sigma plus del nu F sigma mu plus del sigma mu nu equal to 0. So, this is a cyclic permutation of these three indices and says this plus that plus that is equal to 0.

That is look like a strain sort of relation this set of equations is a deep identity in tense a calculus it is called the identity and it occurs in general validity it occurs in many other contact as well. But let me rewrite it in another form which is a little more convenient to understand and that is the following. Whenever you have a tensor of this kind I need these 2 expressions, I want to retain them whenever you have tensor of this kind you can also define what is called the dual tensor.

(Refer Slide Time 20:25)



Now, let me do that here and it is defined as F tilde mu nu equal to epsilon mu nu sigma rho F sigma rho there is a half. So, you introduce the totally anti symmetric symbol the symbol a tensor which I introduce in three dimension; the epsilon i j k the analog of that in four dimensions here. This quantity is equal to plus 1 if you have and even permutation of the natural order 0 1 2 3 minus 1 if you have an odd permutation and 0 whenever any 2 indices are equal. How many components does this object have 4 powers 4 yeah 4 to the power 4 that are 256? But lot of them are 0 how many are non-zero 24 four factorial that is the number of permutations of 0 1 2 3 of them 12 would be plus 1, 12 would be minus 1 and the rest would all be 0. So, this symbol in four dimensions has 24 non-zero components and you take that and contract 2 of these indices here you end up with this tensor

Is this symmetric or anti symmetric this tensor it is anti symmetric because the epsilon symbol is completely anti symmetric under the interchange of any 2 of its indices. So, this is also equal to minus F tilde nu mu automatically.

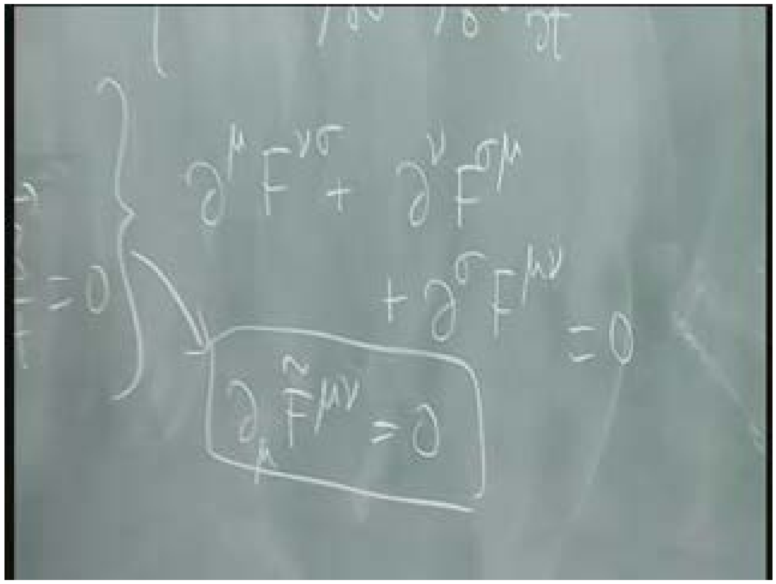
y. Now, you have to sit down and compute those quantities. So, let us see what F tilde 0 1 looks like this is equal to 1 half epsilon 0 1 sigma rho F sigma rho and what can this B what can sigma and rho possibly B 2 and 3 it just have to be 2 and 3 everything else is 0.

So, this is equal to 1 half F 2 3 minus F 3 2 minus because you interchange and make this in order which is in odd permutations of 0 1 2 3 you get a minus sign. But what is F 2 3 this is F 2 and then a 3 here some minus B x F 2 3 downstairs it is a minus B x and 3 2 of course, is a plus B x.

So, what does this give you this minus B x so, what this dual tensor does and you can easily check out the rest of it. Is that while the original filed tensor has the time space components are the electric fields? The space time components of the electric fields the phase space components are the magnetic fields in the dual tensor these are the magnetic fields and these are the electric fields. So, it exchanges the electric and magnetic fields

And in a sense you know going back to them Maxwell equations, you know this is giving you the divergence of the electric field and the curled of the magnetic fields so, it is really telling you what are the equations are depend on the sources. In the magnetic case there are no sources. There is no magnetic mono pole there is no magnetic changes isolated in magnetic charges. So, that is why you get 0 on the right hand side here.

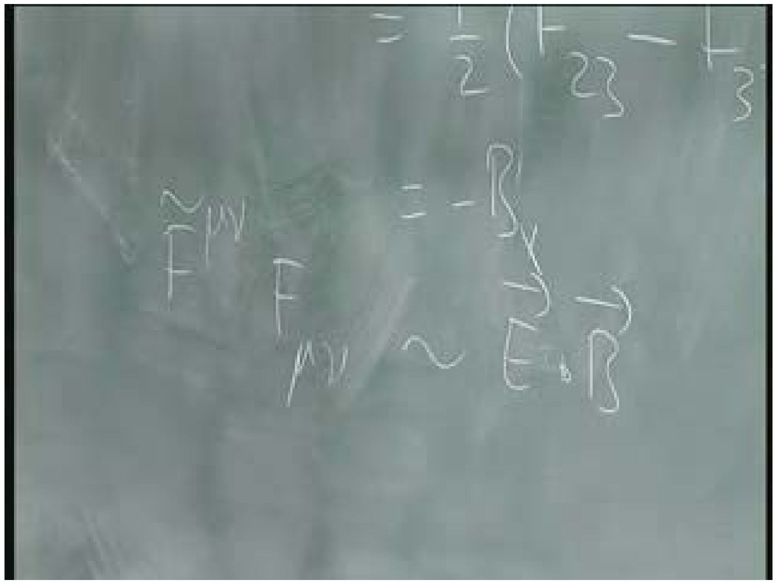
(Refer Slide Time 24:32)



And so, you expect to get a 0 here and you expect the relation pretty much like this relation here and not surprisingly this set of equations can actually be written as del mu F mu nu tilde equal to 0. So, these two equations and this equation is this. So, it is a very compact way of writing the electromagnetic equations. Not the most comeback way this is sort of joke about this, that one Maxwell first load equation down you wrote all the components does it not true. That the calculation not become very popular he wrote it everything down in Cartesian components and then vector calculation came along and then people wrote these 4 down here. After that tensor calculation came along then people wrote these 2 equations on here and that is Maxwell equations.

Today when we use differential geometry we write it much simpler we do not even write. So, much you really write only 2 symbols that is it. DF and d star F that is it and it those are the equations. So, it is a very natural geometrical structure sitting here inside here and as time goes along you write it more and more compact view more information written down symbols. In particular this thing here is actually an identity consistency conditions need it explicitly need it. We not going to do that, but what I would like to do is having written the Maxwell equations down in terms of the field tensor. We could ask is there any other quantity that is all show Lawrence scalar.

(Refer Slide Time: 26:12)



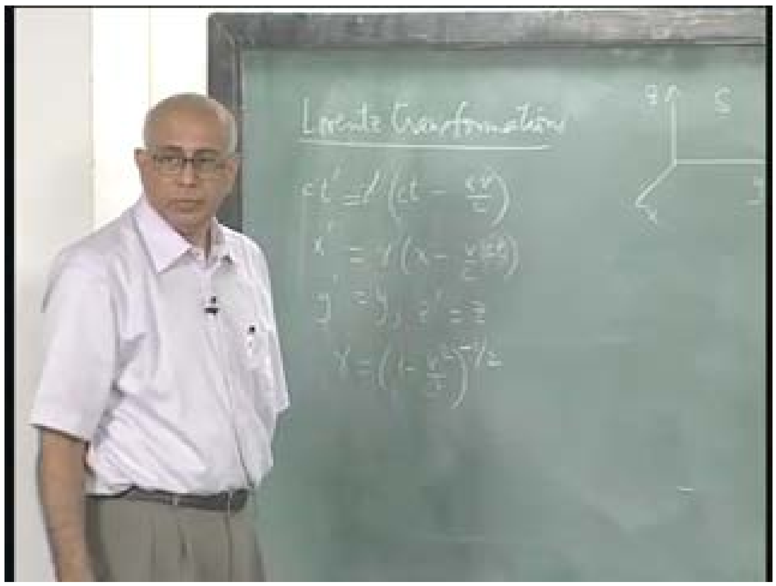
It is easy to see that if you took F tilde mu nu F tilde mu, nu you would not get anything nu, because all that is happen between the F’s and the F tilde is that the electrical magnetic fields have got exchange.

And you would get exactly the same thing as you got apart from some factor essentially A square minus B square. But you could ask what about this quantity I contract the dual tensor with this object here with the original field tensor and what you would get. Well you can actually see what you are going to get this thing here had an E x had an E x, but in the magnetic case you got a B x here and you multiply you get an E dot B so, this whole thing is going to look like E dot P I leave you to work this out this is going to start with E dot.

So, you have two invariants of a electromagnetic field one of which is F mu nu contracted with itself and the other is contracted with its dual But there is a little here this is not truly and invariant under all possible transformations. Because this object is not a real tensor this object is called a tensor density it is a pseudo tensor in the sense that it go from right handed to a left hander coordinates system this changes sign this object changes sign. Therefore what you have here is a pseudo scalar and if you say electromagnetic fractions are invariant electromagnetism is invariant and parity transformations. Then this is not an invariant you need to square this object its invariant and proper Lawrence transformations I am going to come back to Lawrence transformation, but you need to square this object here.

And it is not hard to show that there are no other invariants just these two guys, next step is to ask what happens in the Lawrence transformations for this I have to go back now and rewrite Lawrence transformations more efficient way and let us do back.

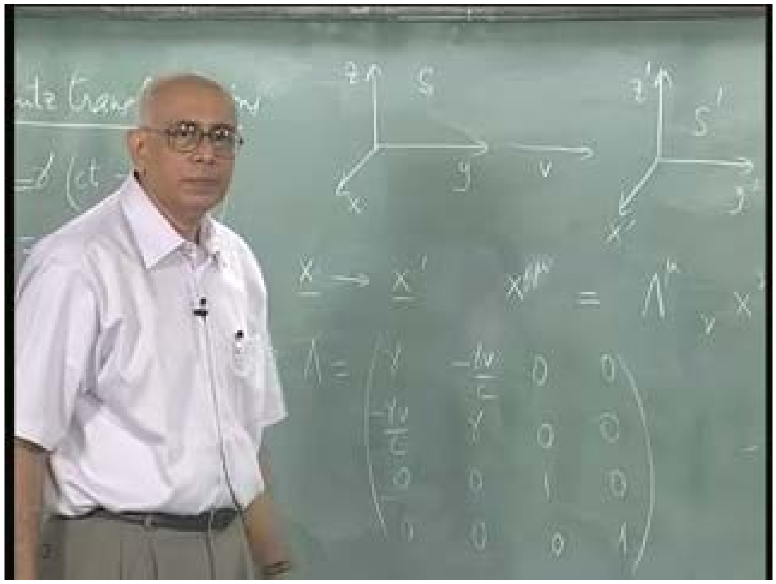
(Refer Slide Time: 28:28)



So, let us go right back and we right Lawrence. Let us focus not; so much on rotations, which we studied some intendancy and what they do, but rather on velocity transformations are , in particular let us looks at the simplest of these guys. The one, if I start with the frame of reference S with an x y z here, and I move to another frame of reference which is moving along the x axis during now X direction of the original frame at uniform speed.

So, this is x prime y prime z prime and this is terms of an is prime and we have the Lawrence equation transformation equations it says t prime is equal to C t prime is ct minus xv over C times what I have called gamma It is just call it gamma. X prime equal to gamma x minus V t c ct. Y prime is y z prime is z. And gamma is equal to 1 minus v square over c square power minus half, is a linear transformations and they want 2 n its invertible and we like to write this in a more efficient notation I like to write a general Lawrence transformation in a more efficient notation.

(Refer Slide Time: 30:02)



So, let me do that. Let us say x under a Lawrence transformation goes to x prime space time point x goes to a space time point x prime and in component form x mu prime this is really x prime mu

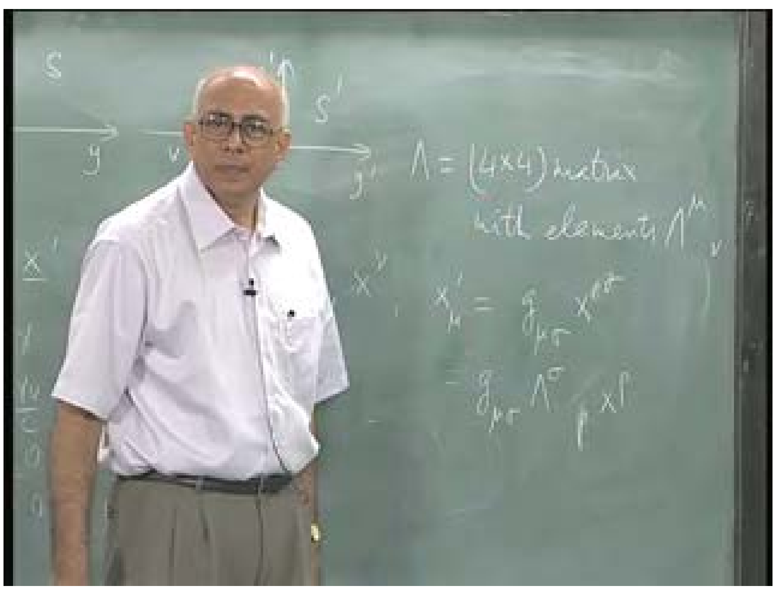
But x mu prime is equal to lets write it properly x prime the mu component is some matrix lambda mu x mu I want to keep track of the fact that whenever I contract an index one downstairs one upstairs index. So, lets me introduce this notation there is some matrix lambda of Lawrence representing the Lawrence transformation. It is mu nu element then contracted herewith respective nu gives you the new coordinates.

So, every component of the new set of coordinates is a linear combination of the old coordinates and space time coordinates by this rule here. So, this lambda is the matrix 4 by 4 matrixes with elements lambda mu and nu. Just keep track of the fact that I have these upstairs and downstairs indices.

So, I put a mu here and then view a space . In our case we can write this down very easily because lambda 0 0 would be equal to what just gamma. Because x not is ct you will just have a gamma and what would be lambda 0 1 0 and 1 it would be this coefficient gamma V over C and similarly on this side you have a minus gamma V over C and in the diagonal element would be just gamma. So, in our case this lambda matrix would look like this set of equations would look like out here it would be just the gamma minus gamma V over C minus gamma V over C and gamma. Because my space time point x mu is defined as x not equal to ct and then x y z that is my column that representing a space time point and what happens here and here there 0 in this particular case and then there is 1 0 0 1.

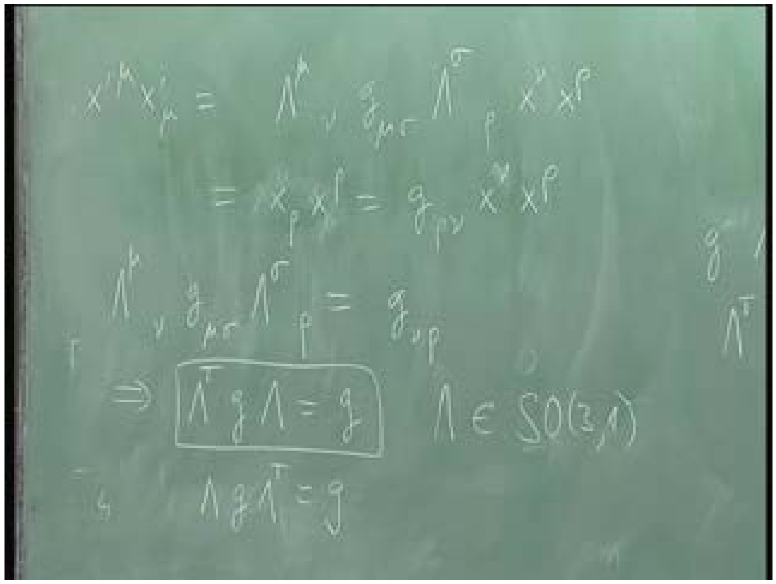
In this particular case this is what you get. Now, we would like to write this slightly better way. It happens to be in this (( )) in this particular case it happens to a symmetric matrix, but there is no reason why that should be the case in general we need to derive a condition in general on what happens. So, let us do that completely in general given this lets calculate what is x mu prime equal to what is that equal to?

(Refer Slide Time: 33:41)



g goes on this side. So, this is equal to g mu sigma X sigma prime x prime sigma. Because the g the metric tensor is the same in all frames of reference it is a nice (( )) of in flats space time in our case so, what does this give you this is equal to g mu sigma and then there is a lambda sigma there. So, this is equal to sigma rho x square because x prime sigma is lambda sigma rho it is that is the Lawrence transformation. And then I need a g to bring this guy down.

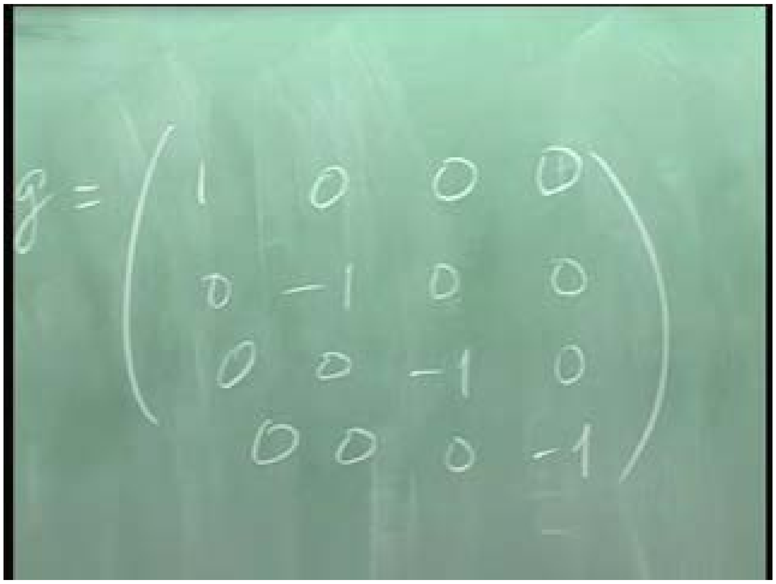
(Refer Slide Time: 34:31)



Therefore what is x nu prime x prime nu x prime nu equal to that is the product of these 2 fellows? So, its equal to lambda mu nu g mu sigma this may not be the best way to write it, but anyway its do not matter lambda sigma rho and then x nu x rho. Let us just check that with dummy indices are sum this is sum over it is a scalar. So, mu is gone the nu is gone the sigma is gone and the rho is gone. So, it is perfectly, and every index has 1 down station 1 upstairs. So, the contraction is correct.

But this must be equal to x mu x mu that is the point about a Lawrence transformation it leaves this scalar product the distance space time distance in 4 dimensions in variant, just as rotations left the ordinary Euclidean distance invariant. But this could also be written like write this as x rho x rho if you like. So, you could write this as x rho x rho or g lets put a g here rho nu x nu x rho. I want to be able to compare with that, so I rewrite this quantity till I write that x nu x rho and this is true for every x. that is only possible if lambda mu nu g mu sigma lambda sigma rho equal to g rho nu or nu rho does not matter. G is a symmetric tensor. So, its 0 nu I can write 0 now, please notice that this g can be written as a matrix.

(Refer Slide Time 36:34)



And this matrix g was equal to 1 0 0 0 0 minus 1 0 0 0 0 minus 1 0 0 0 0 minus 1. So, this looks like a matrix multiplication here. This follows a matrix matrix whose sigma rho the element is lambda sigma rho. So, looks like the matrix mu g is getting multiplied with the matrix 0. But that is not happening here, because unfortunately if nu was here and nu were there then this would be matrix multiplication. But what element is this it is the element of the transpose. So, this could be written therefore, this implies lambda transpose g lambda equal to g that is the orthogonality condition pseudo orthogonality. Look at how similar it looks to what happened in the case of simplistic transformations in classical mechanics. The matrix there was that matrix j that I wrote down the anti symmetric matrix and that was m transpose j m equal to in the case of ordinary Euclidean matrix and rotations.

You have r transpose r equal to the identity matrix. So, this is exactly the same structure except that now literally a matrix is g in this space and it is not hard to see from here that lambda inverse exist lambda inverse is the inverse transformation you go from s to s prime we can go from x prime to x always. So, lambda inverse certainly exist and you can write down what its property is. In fact, you can write lambda transpose in terms of lambda inverse

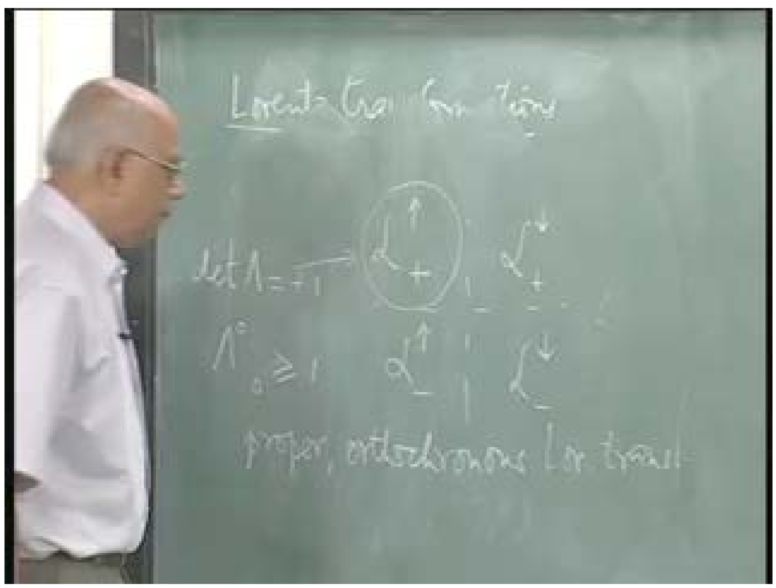
So, let us just play with that remember that g square is the identity matrix x square g you get the identity matrix. So, g inverses same as g and therefore, if you play with that here what are you going to get, I put g inverse on the left hand side you get g inverse lambda transpose the many ways of doing this g lambda equal to the identity matrix. Put a lambda inverse on either side and you get lambda inverse this side, but g inverse is the same as g is where you are so, lambda inverse is g lambda transpose g similarly you can write lambda transpose in terms of lambda inverse. So, what is lambda transpose do we have lambda inverse which is also Lawrence transformation and then put a g inverse on the right hand side that is the g you put a g inverse in the left that is g. And it is not hard to see, it is a trivial exercise to show that this also implies lambda g lambda transpose is also g.

In other words if lambda is a Lawrence transformation matrix lambda inverse of course, it is a Lawrence transformation matrix lambda transpose is also Lawrence transformation matrix. In this particular case lambda transpose turned out to be the same as lambda is that special to this particular transformation. They could be all sorts of rotations sitting here they could be complicated and so, on not true in general. In general lambda transpose is this in terms of lambda inverse retransformation form a group the Lawrence group and the group of these matrix is is a set of rotations it s like rotations it is almost like rotations.

And these lambdas are elements of the group called SO 3 comma 1. I have to explain little bit more about the determinant and so on. Let us take determinants and both sides you have determinant lambda whole square times determinant g is equal to determinant g therefore, determinant lambda can be either plus 1 or minus 1. Once again the proper Lawrence transformations the once you can get from the identity from no transformation at all could be the once with determinant plus 1. There is a little more to this and there has to do with the following. I will come back and talk about it little bit and need to do that now. So, let me do that, included then these lambda transformations where the time itself can be reversed also. They would still satisfy a condition of this kind.

So, you really have four classes of transformation you have those and these are denoted by the Lawrence group you have those for which the determinant is plus 1 and the direction of time is not reversed . Then you have determinant plus 1, but the direction of time is inverted and then you have those in which you have parity and stop like that. So, you have L minus determinant minus 1 time upwards and minus down here so, together this set of transformations forms the full set of proper Lawrence set of homogeneous Lawrence transformations of those the once that keep these are the 1 connected to the identity these keep the direction of time and your comeback and explain what the light cone is this implies this is the set of transformations for which the determinant lambda is equal to plus 1 and lambda 0 0 is greater than equal to 1. What was lambda 0 0 the coefficient of t t in the expansion of t prime what was it our previous case. That is gamma and that numbers certainly greater than 1 because it is 1 over square root 1 minus v square over c square it was greater than 1. So, these are called proper orthochronous Lawrence transformation.

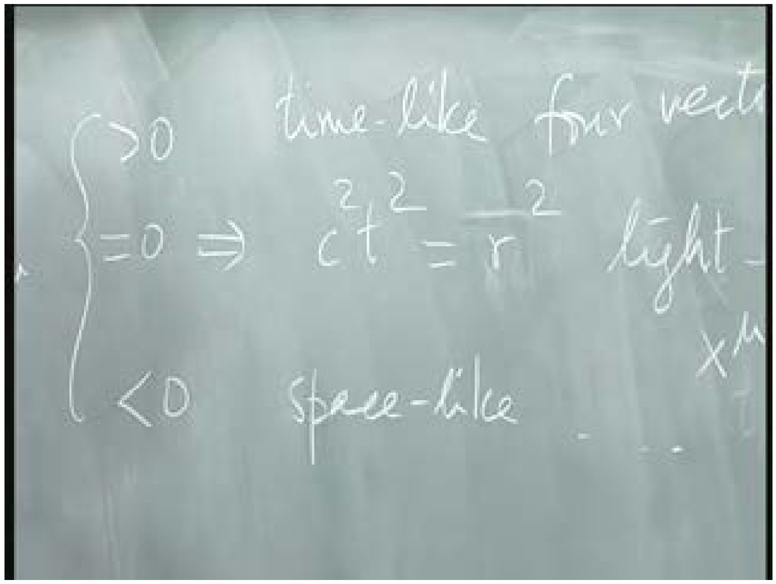
(Refer Slide Time 41:30)



We will see we will see in a minute why this is so as you can see.

This would mean determinant is plus 1 this would mean determinant is minus 1 and this would mean the direction of time is not reversed whereas, the downward arrow would be the direction is reversed. Now, as you know since we did not mention it let we mention it here.

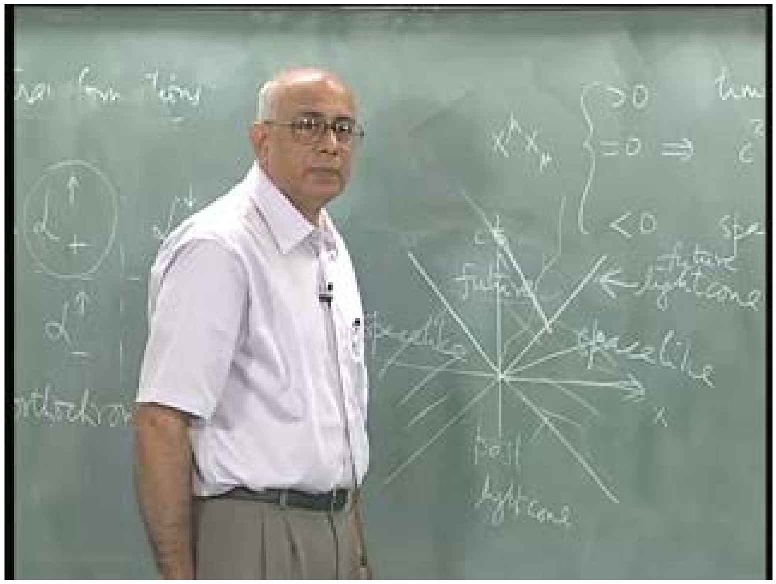
(Refer Slide Time: 43:36)



This quantity x mu, x mu is a Lawrence scalar and it has 3 possibilities it is a real number it could be positive negative or 0. Now, this number is equal to 0 this would imply c square t square equal to r r square. This is the way lie to propagate. So, actually this equation would tell you something about the way light propagates because for light r is equal to c t for material objects what would happen if I threw a ball a constant speed and it most in a straight line. What kind of c squared t squared minus r squared you get. C square t square greater than or less than r square greater than, because it does not travelers much as ct would therefore, this is called light like 4 vector x mu on the other hand. Greater than 0 is called a time like 4 vector x mu and less than 0 is called a space like.

And what the invariants and Lawrence transformation tells you is that a light like 4 vector is going to remain a light like 4 vector and Lawrence transformations because it is a scalar quantity does not change in value. Similarly, a time like 1 would remain time like space like 1 would remain space like therefore, every1 of an every instant of time is carrying a light cone in our frame of reference.

(Refer Slide Time: 45:16)



So, in my frame of reference if this is the origin and I draw a time here versus space here? I cannot draw 3 directions in space and a time I need four dimensions do that. Let us take 1 particular direction here. Then if I suite a beam of light in a particular direction along this space direction that beam of light. So, let us let us draw ct here versus x for example. As time goes on its going to trace a path which looks like this x is equal to c t where shoot in the negative x direction it is going to looks like this.

Its path it is going to look like this, this is called a word line of this object on the other hand if I receive a pulse of light coming from the left hand side at p equal to 0. That light would have traveled along this line meet me here at equal to 0 and similarly from the other direction would look like this. Now, really you have a y and z also. So, these are really like cones except that not even in 3 dimensions, but there in four dimensions. This thing here is called a light cone; something coming along from the negative x direction. So, that I receive it at the origin at e equal to 0. No why it is travelling backward its coming like this from that side and it is me at equal to 0. So, it came from by passed and it evident from here that this is my future and this is my past. So, everything that happen to me all the space time events which I am causally connected coming from here and that is going to my future. This is called the future light cone and this is the past light cone. Now, few stands there at e equal to 0 you both in the same of frame same frame of reference. Now, use stand there and you are here with respect to me and you throw a ball to me and I catch this ball sometime in the future.

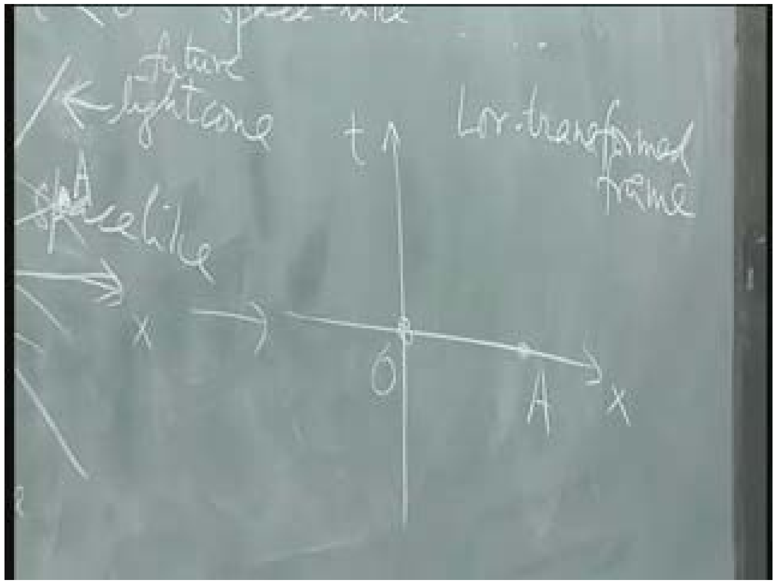
What would the path of this ball look like?

Clearly it is going to me somewhere here and you are going to throw it you have your own light cone and when you throw a ball at me at constant p let us assume this no acceleration etcetera in a straight line. This cannot have a speed greater than the speed of light. Therefore to travel a given distance it is going to take a longer time the light does, so the slop of the word line of the ball is going to be greater than 1 it is going to go like this and eventually at sometime in the future its meets me that is the word line of this model. So, now anything in my frame of reference this object etcetera, etcetera all these things have word lines I have a word line I do not do anything I just stand here. I still have a trajectory in the space time diagram I just stand here and get older and wiser up this home. I start moving about then I am doing things that would be path, but the fact is that the slope of that can never exceed 1. It is always got to be less than 1 so, you want rho catch play catch then this going to have exact path in this passion here, my entire past has come from here everything casually connected. So, material particles will have trajectories which are time like the word line should be always time like light would have things which travel along its speed 1 with the slope 1 here.

Then you can ask what is this guy what is its space time point here going to do this point and Greater than 0 is called a time like 4 vector x mu and less than 0 is called a space like are not casually connected. That elsewhere and else when no even there is casually connected to me there is no way in which I can send a signal to this person.

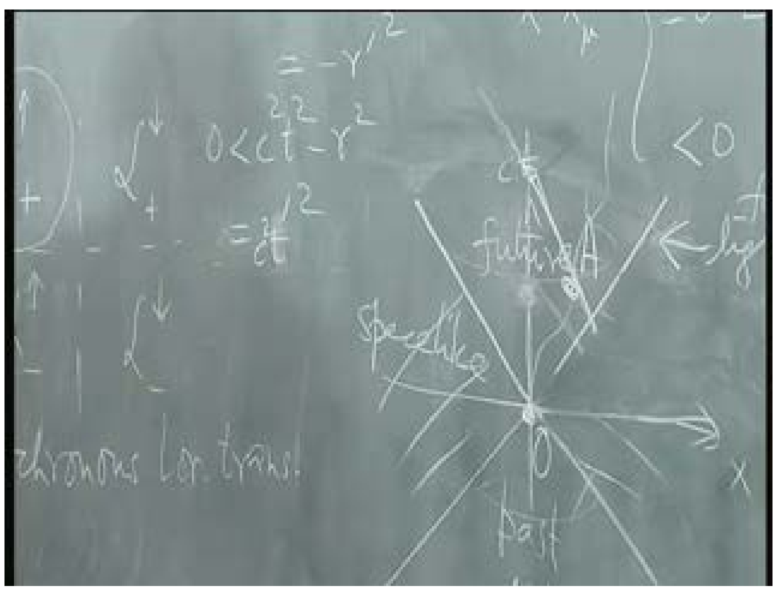
In fact, I can make a Lawrence transformation this vector this vector here is a space like vector because the squared of this object is negative the space time square four dimensional squared is negative. So, this whole region is space like for me and there is a simple way of demonstrating from the Lawrence transformation equations that if these 2 points are separated by a space like separation it is possible to go to a frame of reference where the difference is made purely space like.

(Refer Slide Time 51:03)



So, this picture can be changed to look like. So, here is O and here is an it can be change in a prime frame. So, this is in a given frame this is a Lawrence transformed frame, such that O is here and A is here. What is that implies?

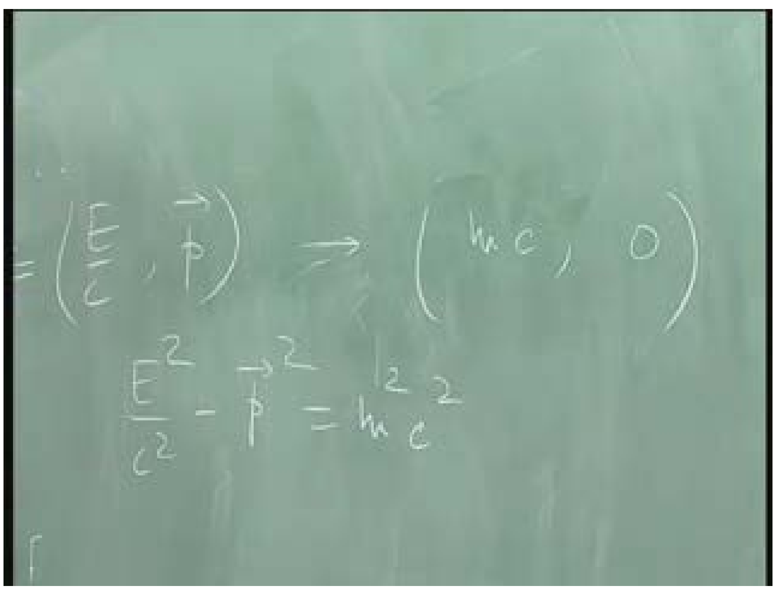
(Refer Slide Time: 51:36)



It says if you have this quantities c square t square minus r square then the quantity numerically cannot be change when you go from 1 frame of reference to another. But if this quantity is negative it is possible to go to a frame of reference such that this is equal to minus r prime square here only space like separation. In other words these two look simultaneous. So, this is why it is say in relativity simultaneously is relative. When the separation is space like you can always go to frame, in which these two events here to be simultaneous on the other hand, so this quantity is negative; so 0 greater than this all the other hand if 0 is less than c square t square minus r square. So, you have 1 event here and another space time event here this is an; It is always possible to go to a frame of reference where O is here and A is here.

In other words the separation can be made completely time like fully time like. So, this can also be written as c square t prime square. So, space like separations can be made pure space like time like separations can be made pure time like. How do I do this for a particle that is moving in space with some given velocity V? How would I make it 4 4 momentum time like what could I do?

(Refer Slide Time: 53:25)

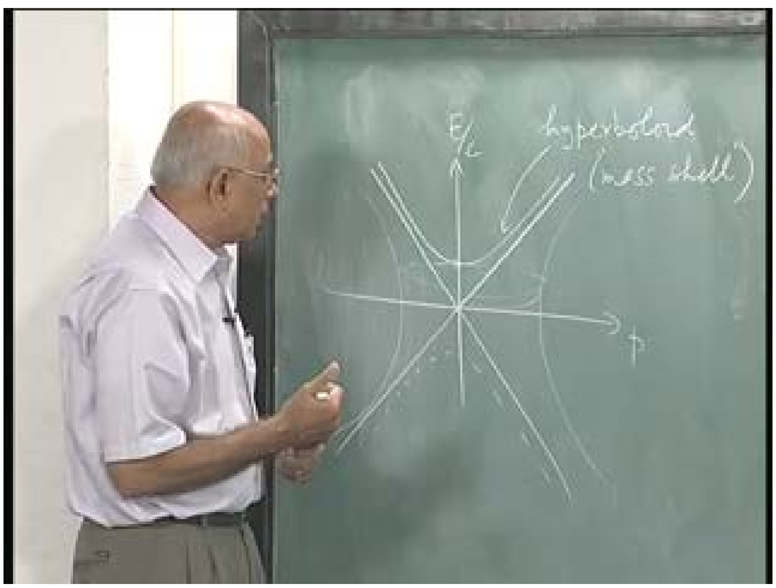


It is moving in some frame of reference I discover that its energy is E and its 3 momentum is P and I know that E square by c square minus p vector square is m square c square and a physical particle has a positive M non negative M and let us have a positive M then this quantity is invariant as I make Lawrence transformations from 1 frame to another and this is the 4 vector of the particle.

How do I make this purely time like what should I do picks a frame to the particle go to its rest frame. If I go to its rest frame it has only rest energy which is positive and the P is 0. So, from there you can go to another frame of reference in which this can go to another frame of reference in which this is equal to m square c 4 or the way it is reference m square c square and 0. It has no momentum linear momentum I am sitting along with it.

But it still has rest energy and that is m c square. Sorry e is m e over c. So, m c must have dimensions momentum. So, the 4 vector looks like m c and 0. Now, we can ask what this particle is where is it going to lie as it moves along. So, we would actually plot where is this particle is going to be in the momentum space.

(Refer Slide Time: 55:07)



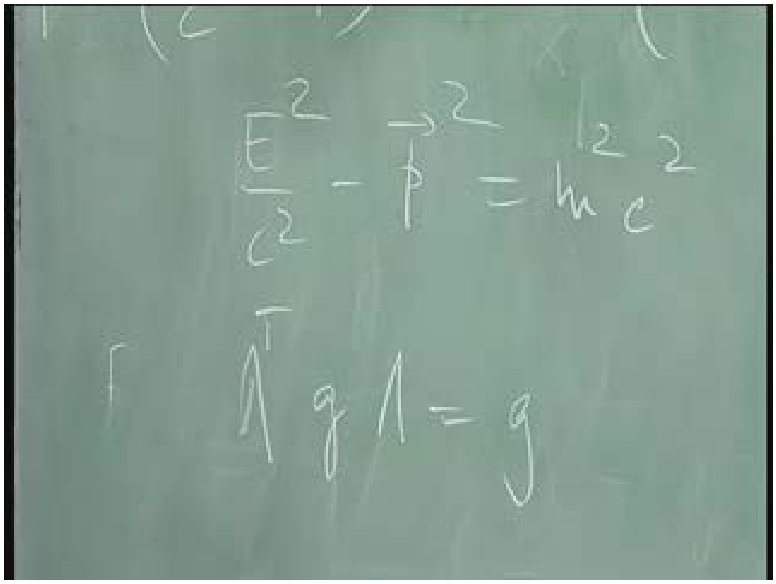
So, let us say here is P and here is the energy E over c. Light will be E equal to c p always. So, light would propagate like this by this Pi means x y z components they are sitting there. So, it is really on a cone. What would a massive particle look like what would the energy momentum relation for a massive particle look like? What does this relation look like? So, pre time for a moment that this is this is p x square plus p y square plus p z square and let me plot it to the function of p p x al1 example.

What would this look like if the motion is purely along the x direction say you have the curve always it must satisfy E square over C Square minus p x square equal to m square c square which is a positive number. So, when p x is 0 it is here, and therefore, it is on a hyperbola, but it is possible if its energy have been negative it would also have another sheet here the hyperbola would have another sheet these are disconnected. But this corresponds to negative energy solutions then classical physics you not bother about this. It on this object here this is a hyperbola it is called the mass shell. Because whatever E and p do it must remain on that as long it is a free part if it was space like if you had particles with negative res masses for example, imaginary res masses. So, that there can be negative. If you had such objects this was negative then what would this hyperbola look like.

It would really look like this and like this and remember you are in 3 dimensions. So, this is connected this guy here is connected. So, it is really. So, it is completely connected this hyperbola it is connected on this side by that is the space like hyperbola its physical particles do not clip there. Particles with imaginary mess could in quantum theory you have to deal with our situations. You have to deal with what happens outside also, but you have to also impose causality correct way here concern with this hyperbola positive.

And if you have in the rest frame of the particle it is just takes here, but we starts moving then it is on this shell all the time. Now, we have to ask not have time. So, tomorrow what we will do is we will go back and ask what the Lawrence transformations do to the electric and magnetic fields.

(Refer Slide Time 58:09)



Now, that we have this condition now that we have this exact condition lambda transpose g lambda is equal to g. We can write this lambda out in more efficient form in that what I will do, and then show you what happens to the electric and magnetic fields under these transformations, we will get a few more things this kind. So, let me stop here.