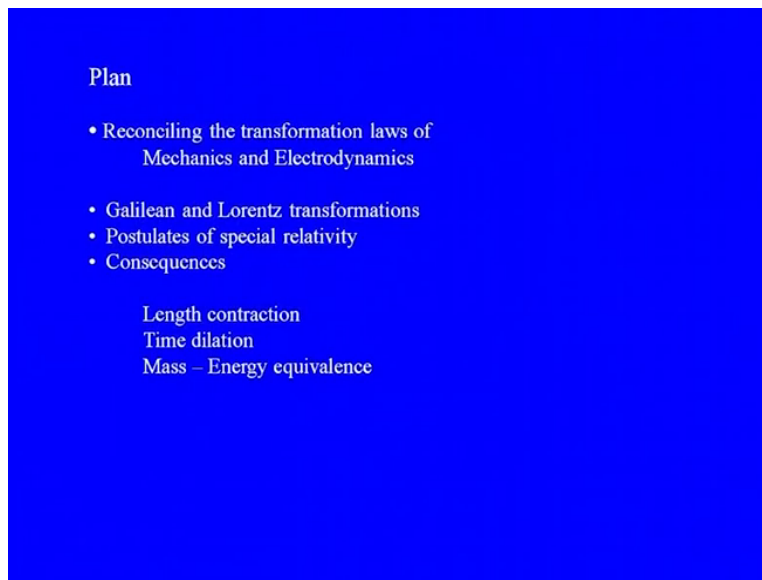


Engineering Physics 1
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Module-07
Lecture-01
Introduction of Special Relativity

Hello, everybody I am Rajdeep Chatterjee from the Department of Physics, IIT, Roorkee and I shall be talking on the Special Theory of Relativity.

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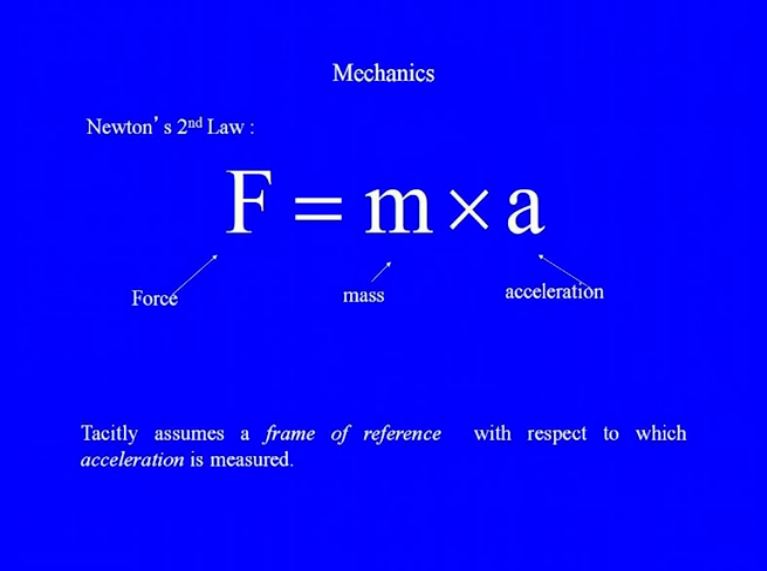


Well we plan for the next series of lectures is something like this I shall talk on how relativity arose while reconciling the laws of mechanics and electrodynamics but to be more precise reconciling the transformation laws of mechanics and electrodynamics. In that context I will be talking of Galilean and Lorentz transformations, moving over to be all important postulates of special relativity okay.

And then I shall of course go over to the consequences which are quite interesting we will be talking of length contraction, time dilatation, mass energy equivalence and this is one thing perhaps many of you are quite familiar with $E = mc^2$. Let us try to see we will try to see at a certain point of time how it all arose and in explaining all these things what I shall do is that as and when necessary we will talk of certain problems.

We will try to do some problems so as to illustrate the principles involved okay. So, let us start at the beginning mechanics.

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Mechanics

Newton's 2nd Law :

$$F = m \times a$$

Force mass acceleration

Tacitly assumes a *frame of reference* with respect to which *acceleration* is measured.

This $F = ma$, this perhaps is the most famous equation in mechanics if I may say so. You all know that if the force is applied on a particle of mass m it is going to accelerate with acceleration a you can even tell me that this is actually Newton's second law of motion okay. But what is assumed here apart from of course in this particular case that we treat m that is the mass of a particle that is that is always constant okay.

Well if you have any can argue that if you have a variable mass you can talk of force as rate of change of momentum but let us stick to this form of Newton's second law for the motion okay. But what is important here is that we always assume here that somehow we have a frame of reference where we are able to measure this acceleration okay. Now frame of reference is actually a very fancy name for a simple coordinate system.

I mean coordinate system you know simplest one of course it is the Cartesian coordinate system I mean if you see where the walls of this room meet, I mean the room you are in if it meets with the floor and then you see the axis of the coordinate system. So, we have quite familiar with the word coordinate system. So, that is a fancy name the frame of reference okay.

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Inertial Frame : Frame of reference where Newton's Laws are valid.

So, where to *look* for it?

A frame whose coordinate axes are fixed relative to the average position of a 'fixed star'

or moving with uniform linear velocity relative to the star.

Now another thing is we sometimes hear of this word inertial frame. So, what is an inertial frame well a very solid definition is an inertial frame is one in which Newton's laws of motion are valid okay. You know Newton's laws of motion are valid you know inertial frames. So, what does this explain where explain a person explain and tell an engineer or a scientist where to look for this inertial frame.

We need to be a little bit more precise than this idealistic definition and in doing that in defining that we can say that it is a frame whose coordinate axis are fixed relative to the to the average position of a fixed are fixed star in space of course. Or it is that frame is moving with an uniform linear velocity that is a constant velocity relative to the star and of course there should not be acceleration of this star otherwise the definition is not valid.

Well once I have said this and you realize that any frame the earth itself is evolving and it is there is day and night there is this rotation due to which you have of course they are night and revolution around the Sun that is the that gives the change of season.

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Motion of the earth (rotation/revolution) : coordinate system attached on its surface *non-inertial*.

But, this acceleration is so slight that for many purposes, this frame is considered *inertial*.

Non rotating frame with origin at earth's center (axes pointed towards the same fixed star) : *approximately inertial*

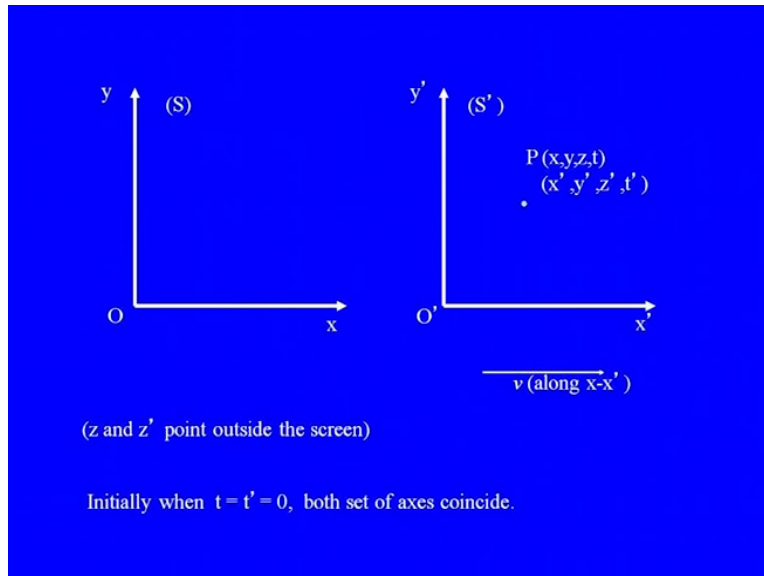
Caveat : Non-inertial frames are very common in mechanics, e.g. Coriolis forces

So, there is some amount of acceleration yes ok physically any coordinate system attached to its surface is known inertia. But for many purposes this acceleration is slight and then by many purposes I course do not mean for all purposes if this acceleration can be considered slight. Here this particular frame that is the frame on earth can be considered inertia a little better would be a non rotating frame with a origin fixed at the Earth's center.

And axes appointed towards the fixed star. So, that is we can say it is approximately interaction well even if you have this frame to be fixed at the center of the Sun for example that will be more inertia than compared to the frame I just described ok. But having said this we should be clear that non inertial frames are actually quite common in mechanics I mean must have heard of Coriolis forces okay.

So, these are not inertial forces ok. So, let us try to have a visual explanation of what we have been trying to say in more definitive terms ok.

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So, here we have frame S ok, so this is a three dimensional Cartesian the system a right handed Cartesian system I have not written the z-axis here but you can all guess and you can all you can all figure out that z-axis here actually points outside the screen okay. So, this is frame S and then we have another frame S Prime let us say and then this frame S Prime is moving with a constant velocity or let us say uniform velocity v along the common X X prime axis okay.

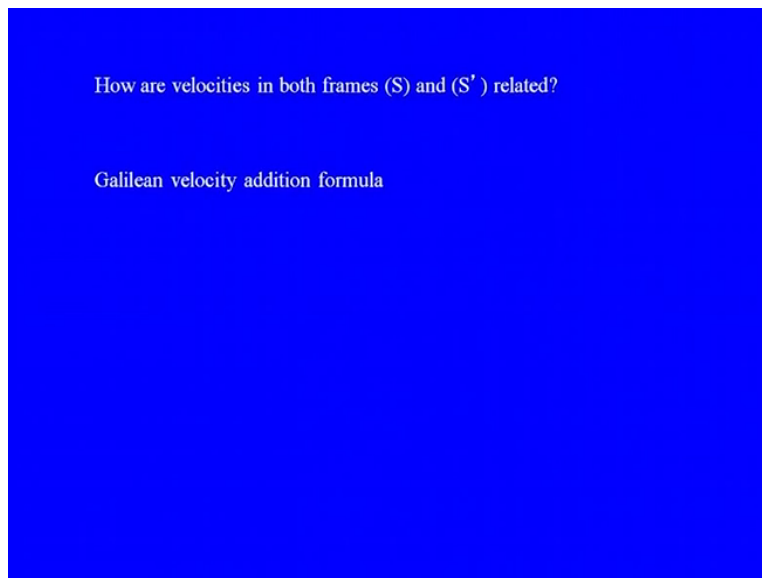
Now the coordinates here are the X prime, Y prime and Z prime and here is that prime again points outside the screen. Now at the beginning at the very beginning general let us we have two observers who are let us say at the origin so both these frames and at the beginning on both these flames coincide that is S and S Prime they have a common origin at the beginning at time $t =$ and t and t prime $= 0$.

So, I will talk of this chord at some time a little bit later. So, how to do that I mean at simpler and you start with two observers are there and they look at the watches and say that okay fine so our watches agree and this is the time we set as $t =$ our t prime $= 0$ and then S prime frame starts moving with uniform velocity v with respect along the common X, X prime axis. So, the only line if we have a point P which has coordinates X, Y, Z and then its measured at a certain time t remember clock is moving.

I mean time clock is working it is moving I mean that time is flowing at a certain time t , so at that same instant let us say observer in S prime measures the coordinate as X prime, Y Prime and Z prime okay. So, how are they related well you can say that it is simple actually. So, how are they related X prime is related by this relation $X \text{ prime} = X - v \text{ of } t \text{ v and } t$, $Y = Y \text{ Prime}$ and $Z = Z \text{ prime}$ okay and it is important that both these time coordinates agree okay.

So, so that is we this point is measured at the same instant of time okay so they all started with synchronized watches. So, $t \text{ prime} = t$ here okay. Now this transformation we call this transformation as Galilean transformation okay. Let us delve a little bit further.

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How are the velocities if you measure the velocities in in both these frames how are they related well that actually will be given by the Galilean velocity addition formula. So, let us see how it how it can be derived.

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Galilean velocity addition formula

$$x' = x - vt$$

$$\begin{aligned}\frac{dx'}{dt'} &= \frac{dx}{dt} - v \frac{dt}{dt'} \\ &= \frac{dx}{dt} \frac{dt}{dt'} - v \frac{dt}{dt'}\end{aligned}$$

$$\frac{dt}{dt'} = 1$$

$$u' = u - v$$

We have this we have this equation $X \text{ prime} = X - vt$ see that now you differentiate $X \text{ Prime}$ with respect to the time in its own frame. So that is $dx \text{ prime} dt \text{ Prime}$ and on the right hand side all you got to have dx by $dt \text{ prime} - v dt dt \text{ prime}$ okay. And now you realize that on the right hand side you have a dx by $dt \text{ prime}$. Now X is the coordinate in the S frame but $t \text{ prime}$ that is the time that is measured in the primed frame that is $S \text{ prime}$ frame.

So, you need, so if you need velocities, so you need coordinates of the same frame okay, so a coordinate and the time of course in the same frame I should say. So, the third step clarifies how to do that so you have $dx dt$ and then you take this $dt dt \text{ prime} - v$ of $dt dt \text{ prime}$ okay. So, then you realize that $dx \text{ prime} dt \text{ prime}$ that is the $u \text{ prime}$ that is the velocity, velocity that is being measured in the primed frame okay.

Now you realize since $t = t \text{ prime}$ in Galilean transformation, so dt by $dt \text{ prime}$ that is $= 1$. So, you realize and then $dx dt$ that is u and so dt by $dt \text{ prime}$ that is 1 , so $- v$ of $- v$, so you have $u \text{ prime} = u - v$. Now I mean there is a subtraction sign here so do not be too bothered about that when I use the word addition formula because you can very well write the velocity which is in the S frame in terms of the $S \text{ prime}$ frame by saying that $u = u \text{ prime} + v$.

And what is v by the moment by the way so it is just the velocity with which the $S \text{ prime}$ frame is moving uniform that is uniform velocity with which the $S \text{ prime}$ frame is moving with respect

to the S frame along the common X prime axis ok. So, that goes for the velocity addition formula, what about the acceleration.

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What about acceleration ?

$$u' = u - v$$

Acceleration unaffected by uniform relative motion of frames

$$\frac{du'}{dt'} = \frac{du}{dt}$$
$$a' = a$$

Mass unaffected by motion of reference frame

Newton's Laws hold in both frames

$$F = m \times a = m \times a'$$

Well we start with what we have obtained for the velocity. So, that is $u' = u - v$, so if you differentiate this once again so you are going to have $du' / dt' = du / dt$ of course you know how to get this now. So, this is du / dt you know how it would be so if du / dt frame take du / dt Prime and then you have dt' / dt that is $= 1$. So, the acceleration is going to be the same in both frames.

So, these frames are moving with uniform velocity S with respect to one another so what we have is acceleration is being unaffected by if you have frames which are moving with uniform relative velocities okay. Now on top of that if you consider that mass is unaffected by motion of reference frames you come to a very interesting conclusion. We see that the form of Newton's second law is valid well actually Newton's second law is valid in both these frames in both these inertial frames okay.

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Consequence : By doing experiments entirely in one inertial frame one cannot distinguish it from any other inertial frame

Newton's laws of motion - equations of motion – conservation laws : same in all inertial frames

Laws of mechanics : same in all inertial frames

So, so what does this mean well this means that by doing experiments entirely in one of these frames you cannot distinguish it from the other okay? So, if you are doing experiments entirely in one of the frames, so and this frame is moving with a uniform velocity v with respect to another frame. You will you cannot distinguish this particular frame from any other inertial frame okay. So, by mechanical experiments alone that is what I am going to say here.

So, what you can ask so, so what happens is that since in Newton's laws of motion are being valid are valid in these two frames so our equations of motion okay. We charge right from them and consequently the conservation laws. So, going to have the conservation laws same in these entire inertial frames okay. So, if you do your mechanics in one of one of these frames and you derive a conservation law can be rest assured that another inertial frame it is going to be the same it is going to be valid okay.

So, we could say that the laws of mechanics are being invariant in all inertial frames okay. This is an important conclusion. So, next we move over to what is going to happen in electrodynamics okay.

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What about Electrodynamics ?

Changing Electric field (\mathbf{E}) ---- Magnetic field (\mathbf{B})

Changing Magnetic field (\mathbf{B}) ---- Electric field (\mathbf{E})

Are the laws of Electrodynamics *invariant* under Galilean transformation ?

So, what is the electro dynamics so you see it is an interesting thing? So, here you have what does it give you, it gives you that if you have a changing electric field you going to have a magnetic field and then if you have a changing magnetic field, you are going to have an electric field okay. Now we ask this question that is electro dynamics or the laws of electro dynamics invariant under Galilean transformation.

Remember laws of mechanics they were invariant under Galilean transformation. So, we asked another branch of physics electro dynamics, are the laws they are invariant under Galilean transformation. Well for that let us see what those laws are okay with the basic laws they are the encapsulated all in Maxwell's equations.

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So, what are these laws?

Maxwell's equations :

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \quad \text{Gauss' Law}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \text{Faraday's Law}$$

$$\nabla \cdot \mathbf{B} = 0 \quad \text{No magnetic monopoles}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \quad \text{Ampere's Law with Maxwell's correction}$$

μ_0, ϵ_0 : permeability, permittivity of free space

Well divergence of E that that is = Rho by epsilon 0 and E as you know is the electric field, a Rho is the charge density and epsilon 0 that is the permittivity of free space and then the curl of E that is - del B del t B is the magnetic field and then the divergence of v that is 0 the curl of v that is Mu0 J, Mu0 that is the permeability of free space and J that is the current density that plus of course Mu0 epsilon 0 del E del t.

Now do not be bothered too much about this mathematical details so, what do they stand for I mean that is what I have written on the right hand side obvious equations. The first one is actually Gauss's law. Well so, it is a very important law it actually allows you to calculate the electric field if you have symmetries in the in the problem symmetry the charge distribution that is okay.

The second one curl of E that is = - del v del t that is actually Faraday's law and I am surely aware of it because had this law not been there I mean did not have motors, you electric motors that is. You must have heard of Faraday is important experiment in which you had this he was moving us a magnet within a solenoid and then he detected an EMF within the leads of the solenoid okay.

Then the third thing I mean a third equation that I have written here and should not be talked as a third law that is divergence of B = 0 which well does not have a law it does not have an name as

such that its physical implication is that there are no magnetic monopoles, so like that like you have a you have a positive and negative charges you do not have charges in a magnetic poles in isolation okay, do not have magnetic monopoles.

The curl of B that is $\mu_0 J + \mu_0 \epsilon_0 \text{del } u \text{ del } t$ that is actually a curl of $B = \mu_0 J$ that is actually Amperes law okay. And then added to that is Maxwell's correction, well that Maxwell's Corrections are actually quite important you going to see later on because it had rather quite rather very interesting implications in showing that these the Maxwell's equations well by the way so what you see here is of all E and B are coupled.

These are coupled partial differential equations, so well this correction is very important because he was able to show that well he put he introduced this concept of displacement current and then was able to show that these equations when written in terms of only E or only B could be framed in terms of the wave equation the more of that a little later okay.

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Q: Are Maxwell's equations invariant under Galilean transformation ?

Answer : NO

$x' = x - vt$
 $y' = y, z' = z$
 $t' = t$

So, our Maxwell's equations invariant under Galilean transformations under the transformations mechanics is invariant on. So, what were these transformations once again, so you have an S prime frame moving with an uniform velocity v along the common x, x prime axis okay. Again z and that Prime this axis are moving are actually out of the screen okay they are pointing out

outside the screen. And this transformations $x \text{ prime} = x - vt$, $y \text{ prime} = y$, $z \text{ prime} = z$ and $t \text{ prime} = t$.

So, our Maxwell's equation invariant under this? The answer is no okay. They are invariant under a different transformation. Maxwell's equations are not invariant on the Galilean transformation.

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The slide contains the following text and equations:

(S) (S') $P(x,y,z,t)$
 (x',y',z',t')

x x'
 y y'
 o o'

v

Lorentz transformation

Maxwell's equations are invariant under Lorentz transformation

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$

$$y' = y, z' = z$$

$$t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}$$

But Maxwell's equations are invariant under Lorentz transformation okay. So, what is that so again we have the frames S Prime moving the uniform relative velocity v along the common $x, x \text{ prime}$ axis? But here we need $x \text{ prime}$ to be given by not only $x - vt$ divided by root over of $1 - v \text{ square by } c \text{ square}$ okay and of course here a $y \text{ prime} = y$ $z \text{ prime} = z$ is = remember we are moving along common $x, x \text{ prime}$ axis.

What is interesting here is that see that the times are not matching. In Galilean transformation we had $t \text{ is } = t \text{ prime}$ but here $t \text{ prime} = t - vx \text{ by } c \text{ square divided by root over of } 1 - v \text{ square by } c \text{ square}$ ok. So, this v is actually the velocity with which the frame S prime is moving with respective S frame ok. What is this see here well if you have guessed that is the speed of light but all of a sudden how come this speed of light is there, so remember this is the transformation under which Maxwell's equations are invariant ok.

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$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

We do not see 'c' explicitly in Maxwell's equations.

So where is it coming from?

c = speed of light in vacuum

So, do we see c and that is the velocity of light I am sorry the speed of light in vacuum explicitly in Maxwell's equation I mean on the left hand side I have written that once again just for your convenience. Well it is not present explicitly so we asked this question so where is this coming from okay. So, is c ingrained somewhere within Maxwell's equation itself ok for that what you have to do as I said is that Maxwell's equation these are coupled partial differential equations.

Now if you uncouple them ok we have if there is a price to pay. You see that you have a second order equation then ok.

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Maxwell's equations transform as :

$$\nabla^2 \mathbf{E} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$$\nabla^2 \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2}$$

Wave equation

$$\nabla^2 \mathbf{f} = \frac{1}{v^2} \frac{\partial^2 \mathbf{f}}{\partial t^2}$$

Velocity of the wave

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \text{ m/s}$$

So, you have $\nabla^2 E = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}$ and similarly for the magnetic field also. You have the Laplacian of B or $\nabla^2 B = \mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2}$. Now this has an uncanny resemblance with the wave equation. You know waves, water waves, sound waves so it is wave equation here. So, the Laplacian of f that is $\nabla^2 f = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2}$.

So, t is the time here and what is v ? v is the velocity of the wave. Now you see in these two sets of equation if you compare Maxwell's equation of B and E with the wave equation what you are going to see is that this term $\mu_0 \epsilon_0$ can be compared with $1/v^2$. So, which means that if it resembles the wave equation $\mu_0 \epsilon_0$ somehow has some sort of relation with velocity okay.

It is actually you are going to see that $1/\sqrt{\mu_0 \epsilon_0}$ does indeed turn out to be and $1/\sqrt{\mu_0 \epsilon_0}$ does indeed have the dimension of velocity. It is actually 3×10^8 meter per second okay. So, on that value later on when you put in values okay but also from physical principles in hindsight you can also check that $\mu_0 \epsilon_0$ should have the dimensions of $1/\text{velocity}^2$. Well how to do that, well check any one of any one of Maxwell's equations in E or B .

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$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \text{ m/s} = c$$

c = speed of light in vacuum

$$\nabla^2 E = \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 B = \frac{1}{c^2} \frac{\partial^2 B}{\partial t^2}$$

Light is an electromagnetic *wave*

Check the first one the Laplacian of $E = \mu_0 \epsilon_0 \nabla^2 E \frac{1}{t^2}$. Now this Laplacian of E , Laplacian how does it look like $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ that kind of a thing. So, it has a dimension 1 by length squared okay. So, on the left hand side I mean for the moment look at these operators that is then that is more important now. Because E and E so the $\nabla^2 E$ they have the same dimension.

So, what we need to do is to balance the dimensions of rest of the operators and rest of the things here. But on the right hand side will be concerned with $\mu_0 \epsilon_0 \nabla^2 \frac{1}{t^2}$ okay. Now you have a t^2 in a denominator here, so which means that is time squared okay. So, on the left hand squared you have on the left hand side you have 1 by length square and then on the right hand side you have 1 by time square okay.

So, what should be then the dimension of $\mu_0 \epsilon_0$, so that you have this entire thing $\mu_0 \epsilon_0 \nabla^2 \frac{1}{t^2}$ to have the dimension of length square okay. Well it has to have then the dimension of 1 by velocity squared ok. So, then in hindsight we can actually we actually can figure out that $\mu_0 \epsilon_0$ should have the dimension of 1 by velocity square that similarly has the same thing the same conclusion you realize from the second equation Laplacian of V is $\mu_0 \epsilon_0 \nabla^2 \frac{1}{t^2}$ to be $\frac{1}{t^2}$ ok.

Now on this value 3×10^8 meter per second okay and you might have already guessed that this number is actually the speed of light okay. So, you see speed of light is actually ingrained within Maxwell's equation itself and then $\nabla^2 E = \frac{1}{\epsilon_0} \rho$ and the Laplacian of the magnetic field you see that it is $\frac{1}{c^2} \frac{d^2 I}{dt^2}$ and then here on top you have $\frac{1}{t^2}$ and for the magnetic field L to be $\frac{1}{t^2}$ okay.

Now since it follows the pattern since it follows the form of the wave equation okay. So, Maxwell concluded that then light must be an electromagnetic wave okay. Now this had a profound significance because light electromagnetic wave and you see that I have written wave in italics.

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19th century : *waves* require a *medium* to propagate

Sound '*waves*' need a '*medium*' to propagate

Likewise Light '*waves*' should require a '*medium*' to propagate

Medium : '*luminiferous ether*'

Because in those days the bend in the 19th century actually people thought that waves actually require a material medium to propagate, why was that? The reason that okay you have water waves which water to propagate you have sound waves you need medium air we need air or even sound waves can travel through another material. For example to a metal but in any case you need a medium to propagate.

So, the reason that perhaps they also light also should require a medium to propagate so and then they just name this medium as ether or actually used to call it the luminiferous ether okay.

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As light can travel through vacuum

Vacuum must contain the medium of light : ether

Challenge : To detect ether and its properties

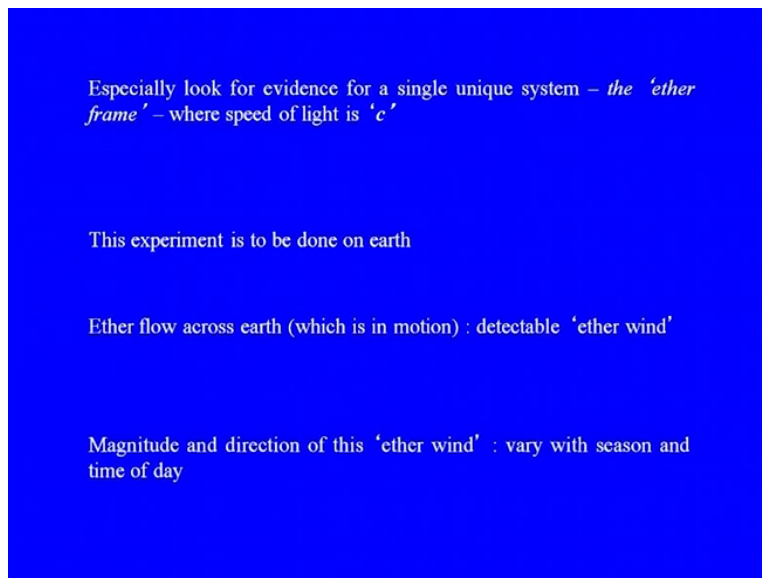
A possible experiment : Measure speed of light in different inertial frames

Note if speeds are different in different systems

And then further reason that as light can travel through vacuum, then vacuum must contain this medium of light which is ether. So, vacuum is full of ether. Okay, that is the medium of light okay. Now like every assertion in physics and if you even if you make a theory it has to be proved it has to be validated by experiments and so that is the challenge that confronted physicists at in those days in the late 19th century is to detect ether and its properties okay.

So, they were thinking of a possible experiment in which to measure the speed of light in different inertia frames okay and to see if these speeds were different in these different systems okay.

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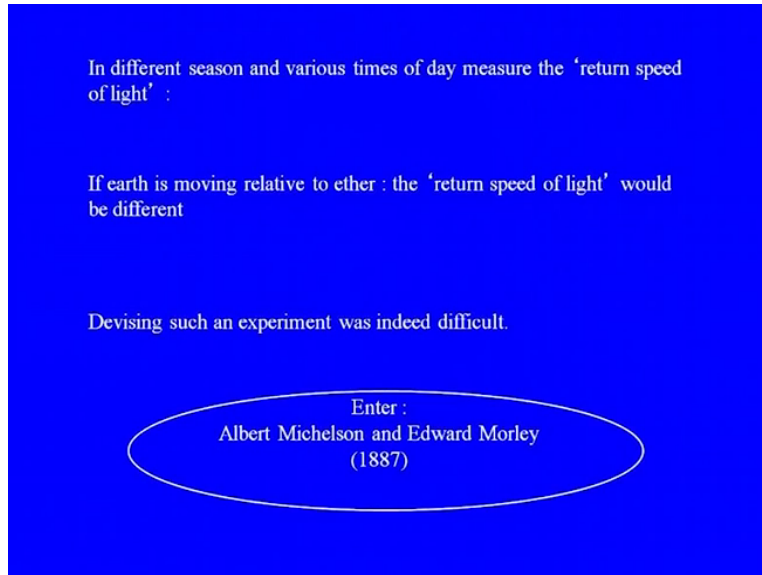


Now in case they were different they will look for evidence of a special frame where that is the ether frame and that that is going to be a preferential frame where the speed of light is seated that c into 10 to power 8 meter per second that is the speed of light in vacuum okay. So, they were they were looking for an ether frame and this experiment remember was to be done on earth. So, sitting or not they were supposed to detect ether.

Now consider the fact that earth is in motion okay, so if an experiment has been done on earth and then earth is in motion. So, you should be able to detect an ether wind in a sense quote unquote an ether wind okay. And then the magnitude and direction of ether of this ether and would vary with season and of course the time of the day because of rotation of the earth okay.

So, the point was to the suggested experiment was to measure the return speed of light okay. So, going and coming back okay, since ether was always gas in an ether frame in different seasons and in various times of the day okay.

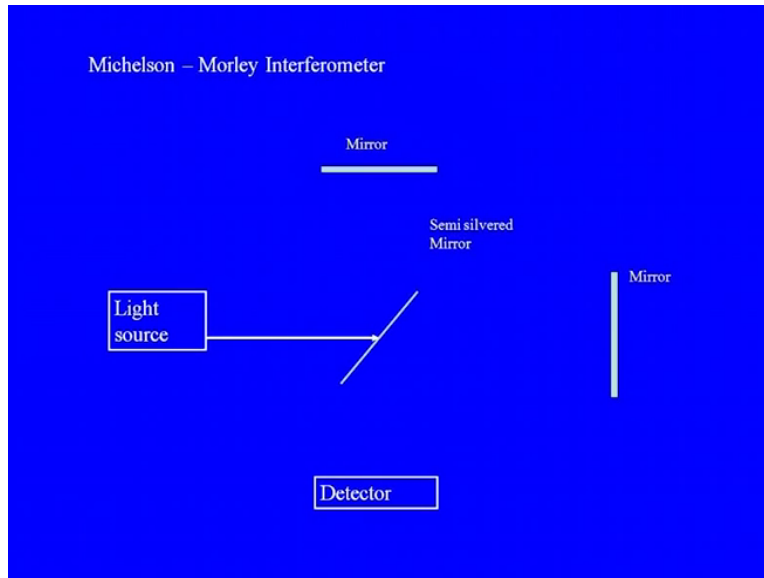
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Why because if earth is moving relative to the ether frame the return speed of light would be different and this difference could be detected then and that would be a test for the presence of ether. Remember that devising such an experiment was indeed very difficult okay. So, but there were smart people they were wherever as they were Michelson and Morley who in the later part of 19th century.

They device then interesting instrument they devised actually devised an interferometer which goes by their name.

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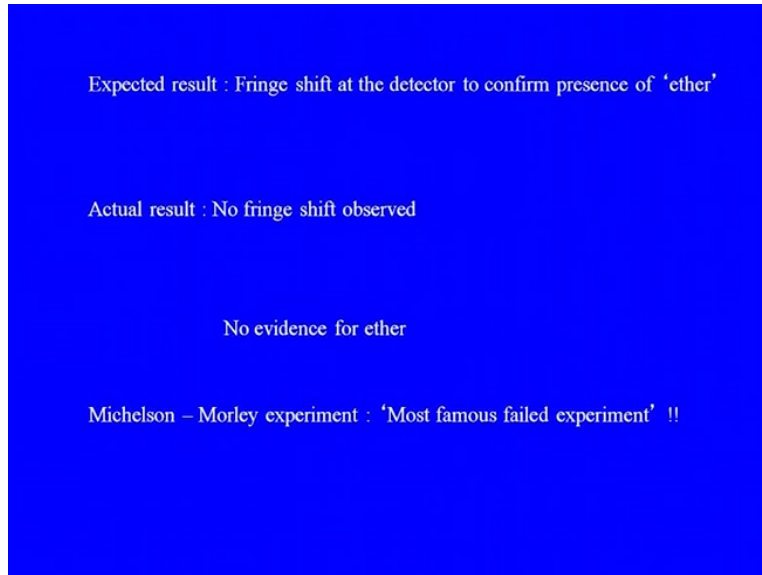
So, that had a light source okay, so light is emitted from the source it comes and hits the semi silvered better that is actually a beam splitter okay. So, then you have mirrors on two sides perpendicular and parallel to this light source and then and detector on the other side like it is shown here. So, what happens is that light comes and hits this semi silvered mirror it splits into two parts okay goes to the mirrors is reflected back okay.

So, that you see the science a little bit different symbols for this reflected rays and for this reflected ray then re combines and goes to the detector and there will be constructive and destructive interferences due to which there will be a fringe pattern at this detector okay. Now remember this experiment is being done on earth okay. Now as earth is moving in the ether frame okay.

And so if this flow of ether is parallel to one of the one of these beam directions let us say parallel to going from if the direction of ether is from light source towards the mirror on your right hand side. And then what will happen is that the returned speed of light will be different from the returned speed on the perpendicular to the ether flow. Why? Because if you spiral to the flow of ether and then once it goes parallel it is towards it is flowing with you know in the direction of ether.

But when it is been reflected back it is opposite to the flow of it okay. So, there is going to be a difference in time of the return speed of the return of light in both this axis and what you are going to have is that this difference is going to cause a shift in the fringe pattern at the detector okay.

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So, the expected result was that there would be a fringe shift at the detector which would confirm the presence of ether okay. But surprise the actual result was that although this was done on a different season, different times of the day no discernible fringe shift was observed I mean you could have argued that maybe a more sophisticated instrument or later on they could have rechecked.

It was checked even by other people and also by more sophisticated equipments and there was no evidence of this ether frame. Well jokingly of course sometimes people call and this is the most famous failed experiment okay, so, there was no ether.

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On the other hand

Albert Einstein was very concerned :
Classical Mechanics and Electrodynamics do not follow the same
transformation laws.

Galilean and Lorentz transformation

Does it mean that one inertial system can be distinguished from another
using 'optical methods'.
(Remember : Inertial systems indistinguishable by mechanical
experiments)

Now towards the end of the 19th century on the other hand Albert Einstein was also very concerned and he was also concerned on a different thing. He was concerned that the laws of classical mechanics and electrodynamics we are not following the same transformation laws. They were following Galilean and they all in transformation laws okay. So, this was quite troublesome to him.

He being a theoretician, so he is not that does it mean that an inertial system which is actually indistinguishable by mechanical experiments remember, we saw earlier that with the help of mechanical experiments you are not able to distinguish between inertial systems, different inertial systems. Because Newton's law is going to be valid in each one of them in the same form ok, so does it mean that okay I mean with mechanical experiments it is not being possible?

But by other means by other electromagnetic means maybe optical methods can you then distinguish between inertial systems. That to Einstein was a very why something because here you have then different branches of physics following different transformation laws okay. Now he reason that this need not be so this that there is there is somehow there is a there is a problem somewhere.

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Einstein figured out that Lorentz transformations were *more general* than Galilean transformations.

Need to modify 'Mechanics' accordingly, so that Electrodynamics and Mechanics follow the same transformation laws.

To do so Einstein had to make two assumptions.

So, he figured out that actually it is the Lorentz transformations which are more general than the Galilean transformations we will put the words more general in italics. Yes so explain that a little bit more later on and he talked of the need to modify a mechanics the laws of mechanics accordingly. So, that electrodynamics and mechanics follow the same transformation laws okay. Now to do this Einstein had to make two important assumptions okay.

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Postulates of special relativity

Postulate 1 : The principle of relativity

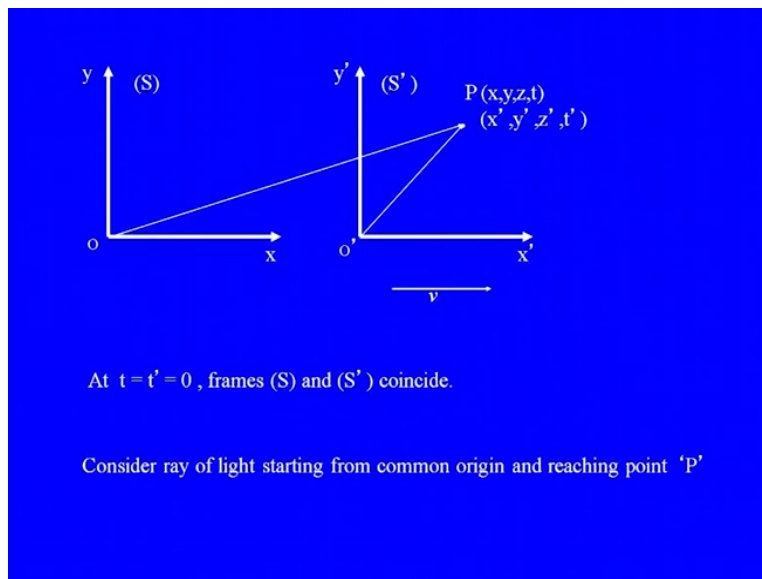
The laws of physics are the same in all inertial systems. No preferred inertial system exists

They are actually the postulates of special relativity. So, the first postulate that is the principle of relativity so which tells us that the laws of physics are going to be the same in all inertial frames okay. So, there is no that there should not be any preferred inertial frame okay there is no

preferred inertial and no preferred inertial frame axis okay. And then the second postulate which says which talks of the constancy of the speed of light.

The second assumption postulate and the speed of light in free space it has the same value c in all inertial frames okay. Now with these two postulates Einstein started his calculations and let us go let us check a little bit more on let us let us take this idea a little bit more on the second postulate.

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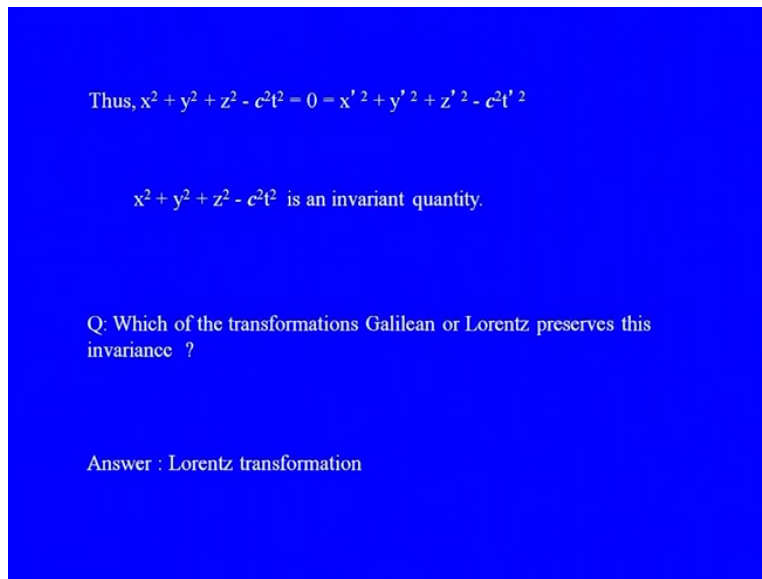


So, here we have the two frames S and S prime moving with velocities going with a velocity v with respect this prime frame is moving with a velocity v along the common x, x' axis then and then of course at $t = t'$ they started so the it coincide and we considered a ray of light starting from a common origin and reaching point P okay. And then let us measure the distance OP and O prime P in both these frames.

So, what would an observer in S frame measure OP as and what an observer in S prime frame measure O prime PS okay. So, the distance wise that would be OP that would be $x^2 + y^2 + z^2$, so that is $= c^2 t^2$. The member c is the speed of light and then O prime P that is $x'^2 + y'^2 + z'^2$ that is $= c^2 t'^2$.

Now I notice of course so get into the second postulate we have again the speed of light to be the same in both these frames okay.

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Thus, $x^2 + y^2 + z^2 - c^2t^2 = 0 = x'^2 + y'^2 + z'^2 - c^2t'^2$

$x^2 + y^2 + z^2 - c^2t^2$ is an invariant quantity.

Q: Which of the transformations Galilean or Lorentz preserves this invariance ?

Answer : Lorentz transformation

Now you are assured that $x^2 + y^2 + z^2 - c^2t^2 = 0 = x'^2 + y'^2 + z'^2 - c^2t'^2$. It is going to give you 0 and a similar thing where going to happen if you subtract out c^2t^2 from $x^2 + y^2 + z^2$ and $c^2t'^2$ from $x'^2 + y'^2 + z'^2$. So, now the same thing is going to happen if you go to another frame moving with certain other velocity v let us say or v' let us say ok.

So, the distance there could be $x^2 + y^2 + z^2 - c^2t^2$ and if the observer there has measured time $t - c^2t^2$ remember that this speed of light is taken the same in all inertial frames here but we point out that the quantity $x^2 + y^2 + z^2 - c^2t^2$ is an invariant quantity ok. So, that is the thing that is not changing.

Now for this invariant quantity so which of these transformations Galilean or Lorentz preserves this invariance ok the answer is and you can actually check this out you can put $x' = x - vt$, $y' = y$, $z' = z$, $t' = t$ and check if this invariance is preserved you are going to see that it is not so, it is only the Lorentz transformation which is going to preserve this invariance okay.

Now let us see what that is, so well we have we have been introduced to Lorentz transformation before, well I have been talking about the laws of electrodynamics, so but let us write down once again.

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Lorentz transformation

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$

$$y' = y, z' = z$$

$$t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}$$

Substituting,

$$\beta = v/c$$

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}$$

$$x' = \gamma(x - \beta ct)$$

$$y' = y, z' = z$$

$$ct' = \gamma(ct - \beta x)$$

So, that is $x' = x - vt$ by root over of $1 - v$ square by c square, $y' = y$, $z' = z$ and $t' = t - vx$ by c square root over $1 - v$ square by c square of course I mean if this looks a little bit more complicated sometimes people actually write it a more compact form by taking this ratio v by c as β and then writing γ as $1 - 1$ by root over of $1 - v$ square by c square which is the same as 1 by root over of $1 - \beta$ square.

So, Lorentz transformations can be very concisely written in terms of in this fashion written in this white box $x' = \gamma(x - \beta ct)$ ok, so why βc because you see β is v by c ok and here we had $x - vt$. So, we had to write βc here okay. So, the y 's and z 's are the same here but it is very interesting to write the time coordinates. So, if you multiply that by c , so it has the dimension of length again.

So, $ct' = \gamma(ct - \beta x)$ have you noticed one thing it is that notice the thing for this x' the transformation equation for the x' and the ct' okay. See that x' and the last you have β times ct ok but when you have ct' you have in the last you have β

times x okay and then so you see it is it looks very symmetrical. So, when you have the length coordinate you have the time coordinate.

And when you have the time coordinate you have the length coordinate. Now this is something new to us this is something which is not natural to us, I mean we are quite used to Galilean transformation in our real life okay. But here what you see is that in this time coordinate you have some you have you have the length coordinate as well okay. So, naturally this is going to have consequences and we are going to check all these things in subsequent lectures okay.

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(S) frame in terms of (S') frame

$$x = \gamma(x' + \beta ct')$$

$$y = y', z = z'$$

$$ct = \gamma(ct' + \beta x')$$

Just one more thing we were actually talking of all we were actually always talking of what is the quantity in the S prime frame in terms of quantities in the S frame. So, we can also talk of the opposite thing so what is the how can you say what are the quantities in S frame in terms of quantities in S prime frame. For that it is very easy to concern it is it is it is the same situation if you consider S frame to be moving with a velocity -v with respect to the S prime frame.

In that case you can simply write down $x = \gamma(x' + \beta ct')$ of course y and y prime are the same z and z prime of the same and then $ct = \gamma(ct' + \beta x')$. Now what I have done here as I said was to express the quantities in in in S frame. So, you want to understand calculate quantities in x frame and in the S frame in terms of quantities in the primed frame okay.

So, like we can take a break here and in the next talk we are going to focus on again on the postulates of relativity then the consequences of the Lorentz transformations where we are going to carry our discussions a little more okay. So, to summarize what we have been doing today we have been looking at the laws of mechanics and electrodynamics and we saw that actually they were not transformation laws of mechanics and electrodynamics.

They are not the same there were two Galilean and Lorentz because Einstein was so concerned and then he showed that you take if he took he actually showed that the Lorentz transformations were actually the more general transformation laws. And if you take that you want to you need to change mechanics okay. So, these are certain things that we will be considering in our future talk's okay thank you.