Network Analysis for Mines and Mineral Engineering
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Lecture - 20
Algorithm and computer program

Let me welcome you to the 20th and final lecture of this NPTEL online certification course of Network Analysis for Mines and Mineral Engineering. In the last class, we will discuss one example of network analysis using program evaluation and review technique. This is a very big and complete example and apart from that we will discuss algorithm and computer program also of the network analysis.

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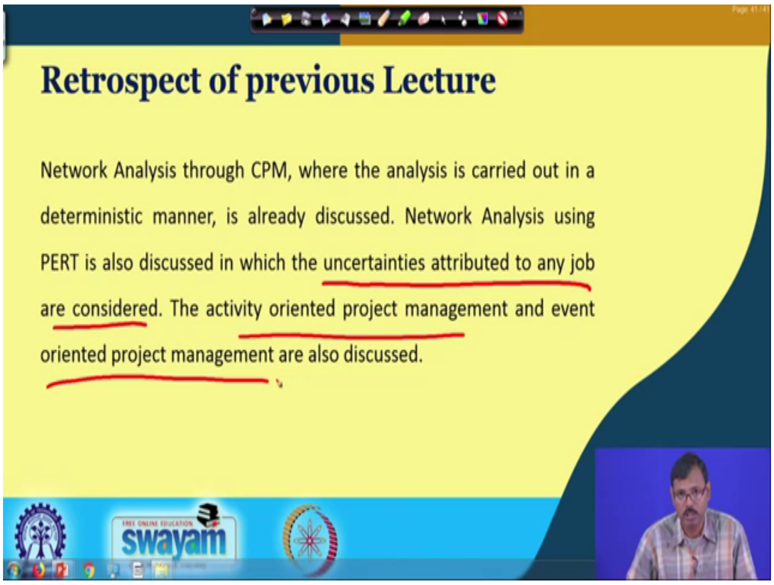

But before that let us summarize so far, we have discussed network analysis through critical path method which is deterministic approach. We have also discussed network analysis using program evaluation and review technique which is a probabilistic approach and in which we have considered the uncertainties attributed to any job which are considered here. And, we have discussed in our last class activity oriented project management and different oriented project management so that the instead of having the progress on a which particular job, we can consider only the starting of the job or the completion of the job as the event oriented project management.

So, this is already we have discussed and at this stage, you can consider the complete syllabus is cover. So, basically we are trying to summarizing this with a big example so that you can calculate your program evaluation and review technique for network analysis.

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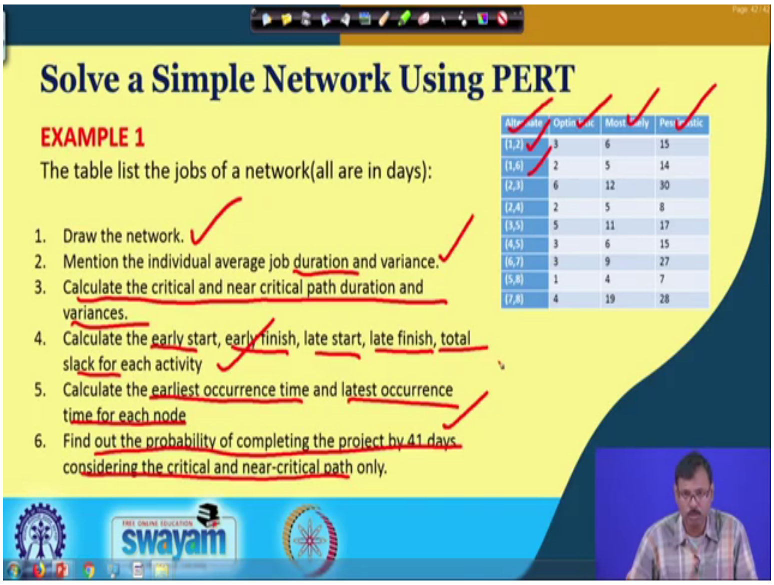

So, let us consider one example. This is the table given the network, now alternates are given here. Optimistic completion time, most likely completion time and pessimistic completion times are given. So, we have given our name A, B, C, D are attributed to these jobs. The problem is stated you have to draw the network. You have to mention the individual and average job duration and variance of each and every activity.

You have to calculate the critical and near critical path durations and their variances. You have to calculate early start, early finish, late start, late finish, total slack and free slack of each activities. You have to calculate the early occurrence time and late occurrence time of each node and you have to find out the probability of completion of the project by 41 days in consideration of the critical and near critical path only.

In last class also you have discussed, you need to go for the complete analysis of the network and a bigger probabilistic approach if you are considering all uncertainties attributed to all the jobs and that leads to the uncertainty on the completion of the project because interdependency are there. So, that is why it is forming more complex network or more complex statistical analysis for calculation of the probability for completion of the project. So, in this case also, we are consider such case where the critical path and near critical path are not interdependent; that means, they are paths these paths are completely different and that is why again, we can we can go for the analyzing the probability of completion of the job in consideration of the critical and near critical path.

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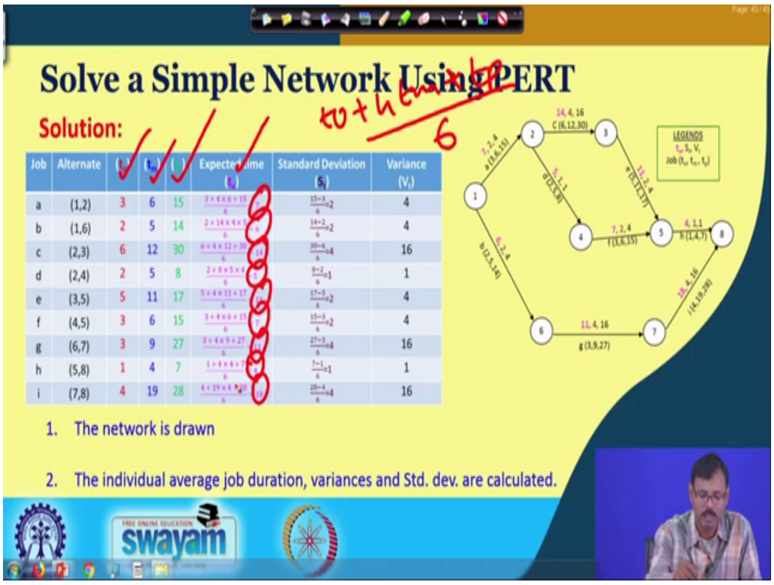

So, now let us solve this problem. This is the network we have drawn and we have attributed the names to all the jobs A, B, C, D, E, F, G, H and I. These are the jobs and we have calculated their optimistic most likely and pessimistic values are given. We have tabulated the expected time of this project where the formula is given t o plus 4 into t m plus t p divided by 6. So, using that formula we have come out with the expected time or mean time of each and every activities here in this place.

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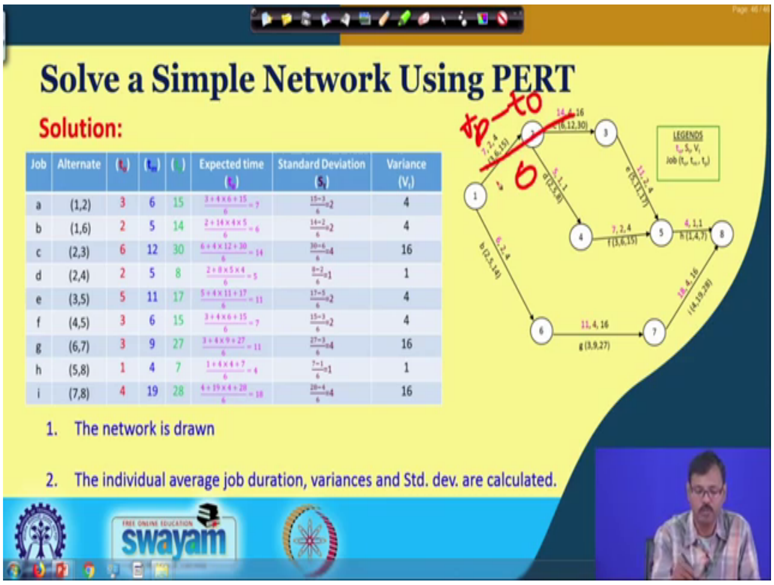

You know the variance of each job can be determined by t p minus t o divided by 6. So, using this formula we have calculated the standard deviation for each and every activities and those are presented in this table and variance is nothing but the standard square of the standard deviation.

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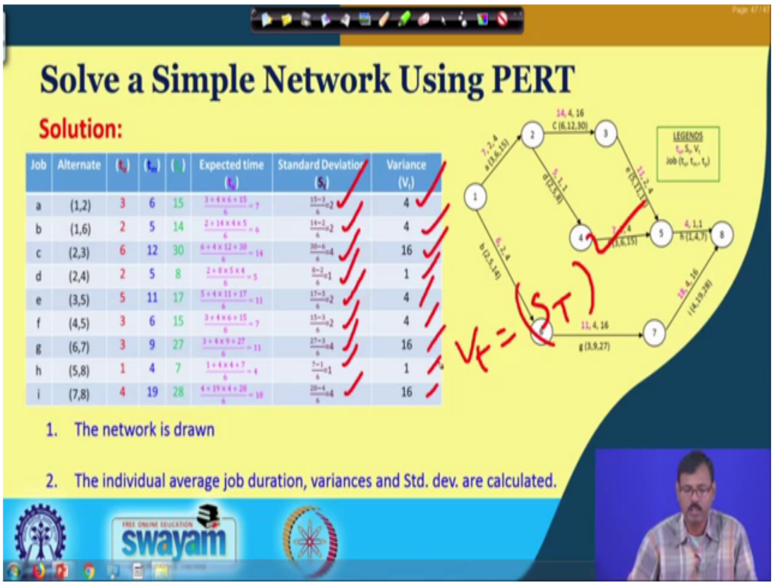

So, is by squaring this, we have obtained the variances of each and every activities which are tabulated here. So, our first two problems which are given there, we have to draw the network. So, we have drawn the network as then alternates are given. Using these alternates, we have drawn the network. So, we have placed these nodes and then we connected them with the arrow to show the different jobs and second is individual average job duration; that means, expected time or mean time, variances and standard deviations are calculated and they are presented here in the job where this is the expected time, this is the standard deviation and this is the variance attributed to each job. So, this is the first two objectives or whatever given to us we have completed.

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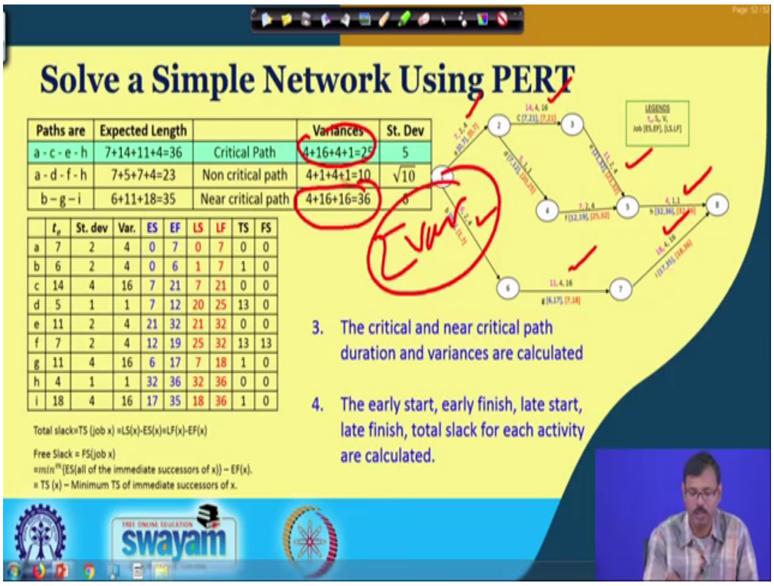

Now, next is that we have to find out the critical path, near critical path. So, let us find out how many paths are there. We are having one path, we are having another path and we are having that third path. So, all these paths are there all these three paths are there and A C E H, A C E H is having the critical length of this one where we mathematically summed up. We mathematically, summed up the expected or mean time of the jobs associated with that path and finally, it is coming 7 plus 14 plus 11 plus 4, so that is 36. So, the expected path length of this is 36.

Similarly, we have determined for path 2, that is A, D, F and H and this path durations are 7, 5, 7, 4. So, it is coming 23 for this path. The third path it is B G and I. So, 6 plus 11 plus 18 that is 35 and as this is 36 is the maximum 1; obviously, this is the critical path and this is 35 is very close to 36; that means, it is near critical path and this one is not a critical path; neither near critical path it is a non critical path having duration much lesser than the critical path.

So, this is giving us the idea about the critical path or non critical path we have calculated their variances also. You know the variances is nothing but the summation of the variances of it is member jobs. So, it did this variances are 4, 16, 4, 1. So, 4, 16, 4, 1 is 25. So, variances of path one is 25. Similarly, we have find the variances of this one is 4, 16, 16. So, that is why it is coming 36 and this one is 4 plus 1 plus 4 plus 1. So, it is 10.

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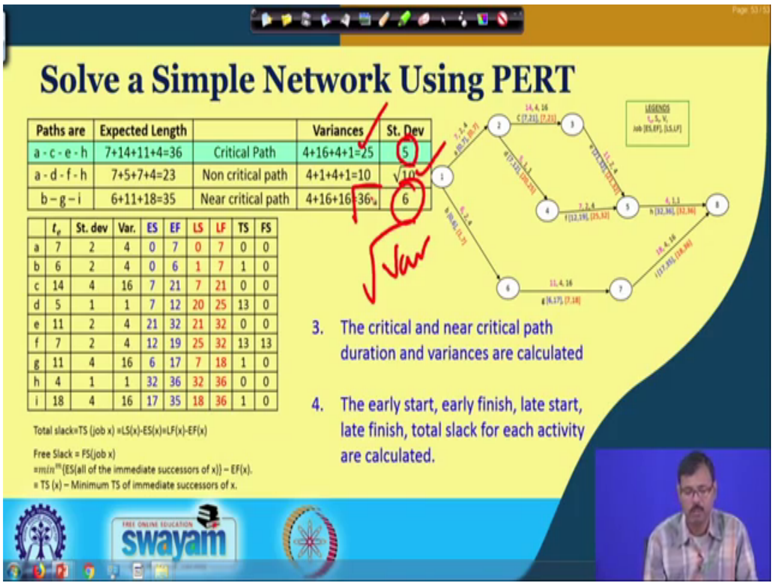

And you know the standard deviation is nothing but the under root of the variance. So, we have obtained standard deviation of this one is under root of 25. So, that is 5, this is under root of 10 and this is under root of 36 is giving us 6. So, the standard deviation for critical path is 5. For near critical path, it is coming 6. So, our job objective 3 was that to determine the critical and near critical path duration, their variances and we have complied that one along with that we have also determined the standard deviation.

So, the next one is to determine the early start, early finish, late start, late finish time, total slack and free slack. So, for that again, we have calculated this one we have considered this is the S is equal to 0, we have started and we know the T is equal to 36 from the critical path. So, based on that we have determined the early start early start is nothing but the latest time of the predecessors, early finish of the predecessors. So, this early start time, this early start time is mentioned in the blue arrow.

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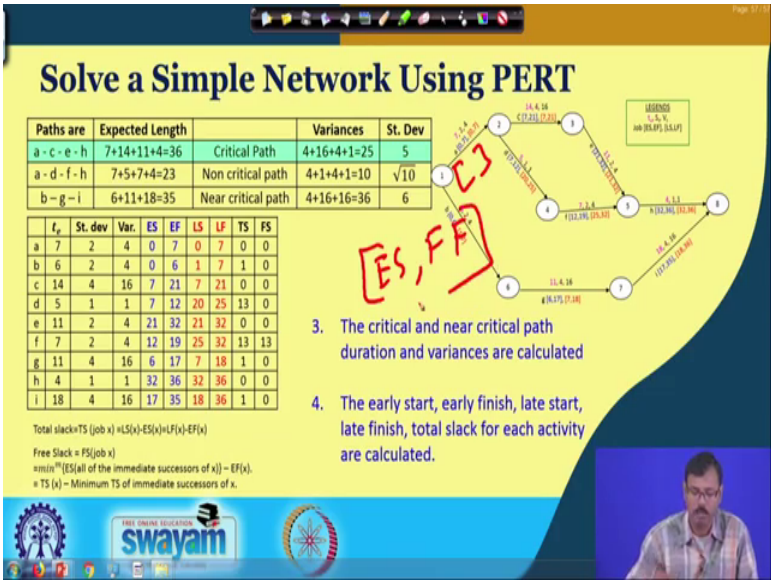

And early finish, early start and early finish are mentioned in a blue arrow, early start, early finish by separated by comma. So, you can see for job A it is 07, job B it is 0 and 6 and job g it is 6 plus 7, 11, 17 and job a I it is 17 plus 18, 35 is the early start, early finish and the predecessors of this one C is A. So, it can start at 7, 7 plus 14 is 21. For this it is 7 plus 5 is 12. So, this is 12 plus 7 is 19, 12, 19. Here it is 21 plus 11 is 32. So, latest here between 19, 19 and 32 is 32. So, it is the early start time of this one is 32 plus 4 is 36. So, that is why, this early start early finish times are given in the blue arrow separated by commas in the jobs.

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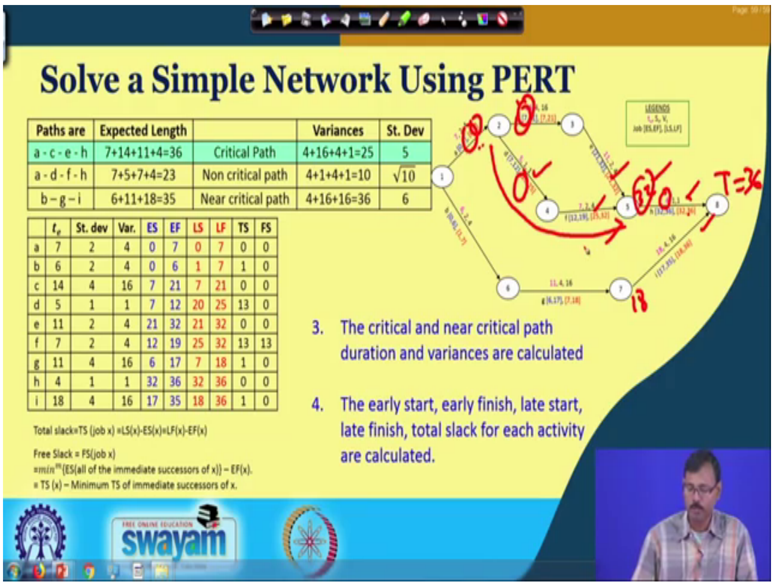

Similarly, considering T is equal to 36 if we go for the backward pass in backward pass. So, let us finish time of the both the jobs are 36. So, subtracting 4 there is the normal duration of that from 36 keeping us 32. So, this is the latest possible time of starting this job is 32. Similarly, here we are getting 36 minus 18 is 18. And considering 32 for both the cases, you are finding that this is 21, this is 25, 32 minus 7 is 25, 32 minus 11 is 21. 21 minus 14 is 7 and here it is 25 minus 5 is 20.

So, among this 14 and 20, the latest one is 14. So, you have considered 14 minus 7 it is coming 0. It is coming 7 here. So, that is why, this 7 sorry this is 21 minus 14 is 7. So, this 7 minus 7 is 0. So, this is coming 7, this is 0 here instead of this 20. And consideration of 20 minus 7 is not considered. It is having some slack we can understand. So, in backward pass considering T is equal to 36. You have gone and those let us late finish, late starts are given in the red color separated by commas under the third bracket associated to each and every job here.

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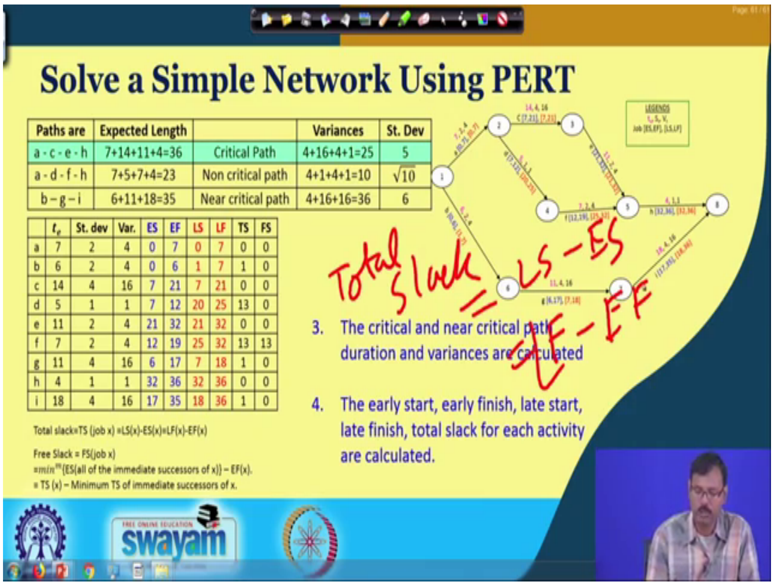

And you know the total slack is nothing but total slack or slack is nothing but the late start minus early start or late finish minus early finish and that is why those are listed accordingly those are listed here.

So, these formula are given total slack, this formula are given. So, early start, early finish, late start, late finish are computed for each and every activities their mean times are given, standard deviations are given, variances are given, late start late finish are also given. And consider some of that total slack are also decided. Here, it is 0, here it is 1 minus 0 or 7 minus 6 is equal to 1. Here, again it is 0, here 20 minus 7 or 25 minus 12 both are equal to 13. So, in that case also, it is 0 in that case it is again 13. So, both are having these two jobs are having slacks of 13 and this is again we are having one slack which is associated to these jobs. All these jobs are associated with one unit slack associated to everywhere. This two are having slack of 13, 13, this is are having 1.

So, this is the determination of the total slack and free slack basically you know it is the minimum of early start time all the of the all the immediate successors of job x and early finish time of job x. It is only associated with this f job as it is only associated with f job. That is why this free slack is available with the f jobs only that is of equal to the total slack. So, this is the formula given here and you which are used in the deterministic approach also or critical path method analysis also and that are used considering the meantime, this early start, early finish., late start, late finish, total slack and free slacks are computed and presented.

So, our fourth option which are required here that is the computation of early start, early finish, late start, late finish, total slack and also free slack for each activities are calculated and presented.

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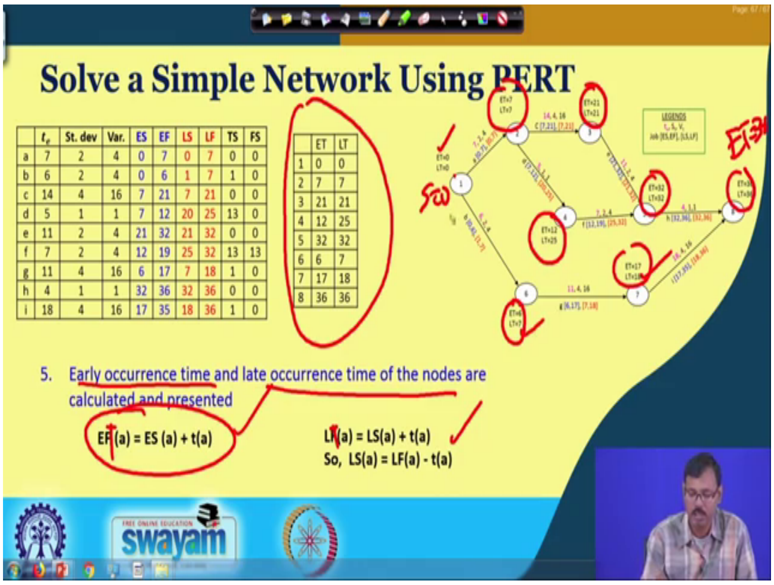

Now, our next job is to find out the early occurrence time and late occurrence time of the nodes. So, based on this table, we have prepared this table. Early occurrence time is nothing but the early start time and the and the T o of that. So, considering s is equal to 0, the earliest occurrence time of this one is 0. We have determined the early starting time of this one, early starting time of this one, early starting time of this nodes, early occurrence time of this nodes at this position and find out finally, we have come out with the e T is equal to 36 for the final node; that means, gives us it can be completed by only 36 hours.

So, that is the requirement. So, using this formula we have come out with the decision that this is actually this will be the T s will be placed here. So, based on that, it is considered and finally, this formula is presented for all the nodes. Here it is 6, here it is 17 and in the return path, we have found the late finish a time is coming here 18 and here 7 here, 0. So, if you look into this in a little bit detail, you can find this is early occurrence time is 7 here because by completing this job will reach here at 7.

Similarly, 7 plus 14 gives us 21 here, 7 plus 12 give 7 plus 5 gives us 12 here and this 12 plus 7, this 12 plus 7 is basically 19 coming here, but this 21 plus 11 this is 32 coming here. So, early occurrence time at this position by completing both and this these two jobs is possible only at 30 second days or 30 second hours which it whichever it is. So, that is why early occurrence time of this node is possible only at 32. Similarly, while we are considering the late occurrence time, we have considered this is the final time of 36.

So, 36 minus 4 it is coming 32 and 36 minus 18 this is coming 18 whereas, it is early occurrence time was 17. Considering this is 32, you can see this 32 minus 11 is giving this is the only job. So, we are considering this is 21, but in this case it is 32 minus 7 is giving us 25 with the late occurrence time. 25 minus 5 is giving us the latest occurrence time here at 20, but this is 21 minus 14 giving us latest occurrence time at 7. So, minimum of this two has to be considered here and that is why late occurrence time is considered 7 here.

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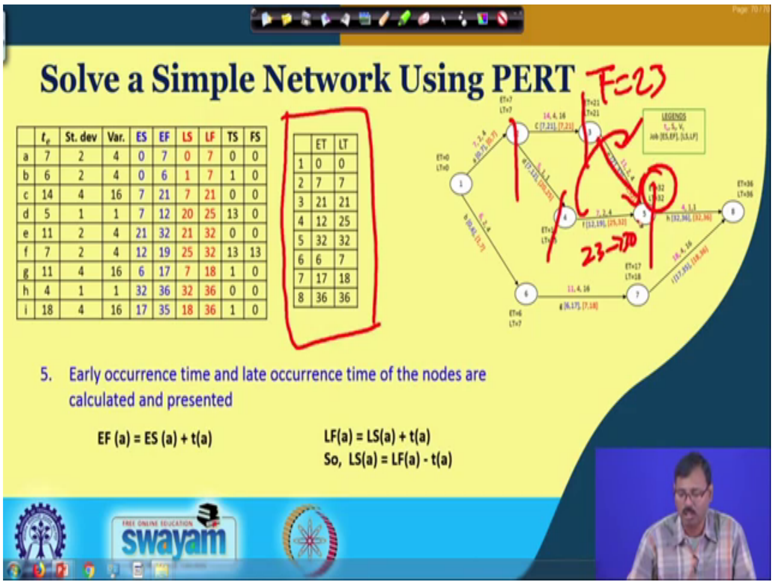

So, by that way we have computed early occurrence time and late occurrence time of each and every nodes. So, that the event oriented manage project management can be carried out; that means, manager can think that whether they have reached at this position at 7 or there is some delay in that, whether they have reached at this position at 32 or there is some delay in that. So, in those case if they found that while they are observing this two and found that this one instead of 21 here, the arrival time is become 23. It is on time they have reached.

Then, they may if they start this job at 23, then also they can complete it by 30. So, what they manager will do? They will deploy some of this man manpower and resources to this so that, they can expedite this job to be completed by 32 and that can be completed beforehand of that one. So, this is the event oriented objective that can be carried out by the management to check whether the resources has to be diverted or not. So, our fifth objective of determination of early occurrence time, late occurrence time of the nodes are calculated and presented here.

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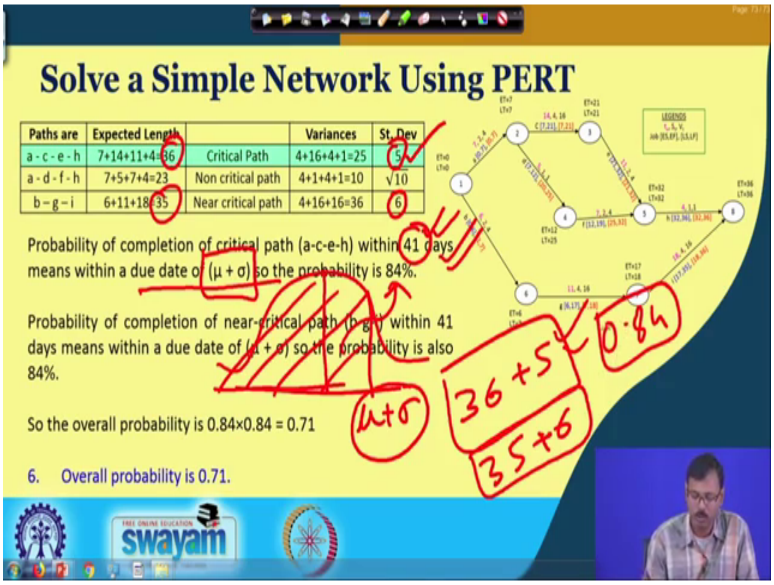

So, now, let us see solve this one and to find out how, what is the probability of completing it by 41 days. So, we have seen this is the critical path with path length of 36 standard deviation of 5. This is the near critical path of path length of 35 and near standard deviation of 6.

Now, when we are talking about completing this path, this complete project by forty one days; that means, we have given a due date of mu plus sigma for both the cases; 36 plus 5 for the critical path case, 35 plus 6 for the near critical path case. So, for both the cases, it is coming 41 days and we know the probability of completion of this one is 0.84 from the standard normal distribution table where if this is the mu plus sigma, the area under all this is comprising 84 percent area. So, that is why it is basically 0.84 for critical case also, for this one also 0.84, for this one also 0.84.

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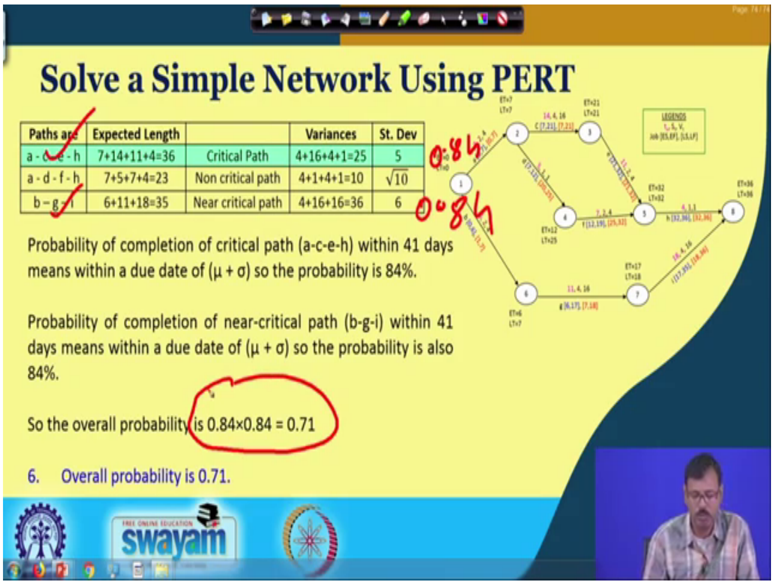

So, basically overall probability is 0.84 into 0.84 and that is coming 0.71, but the condition again here you can see the first path, critical path is this one, near critical path is this one. So, they are not interdependent. So, that is why both are independent and they are that is why they can be considered separately, their probabilities are considered separately and they can be multiplied. But instead of that, if critical path is this one near critical path is this one, in that case there are dependency in this job and this job.

And that is why they cannot be a multi their probability cannot be multiplied in those cases the common job has to be considered separately and the separate jobs are has to be considered separately so that, the overall probability can be calculated. So, this is the advanced part advanced portion of the network analysis. This is not within the syllabus and in future that in that course it can be discussed further. So, in this case, our sixth objective overall probability has been found 0.71 for the completion of the job by 41 days.

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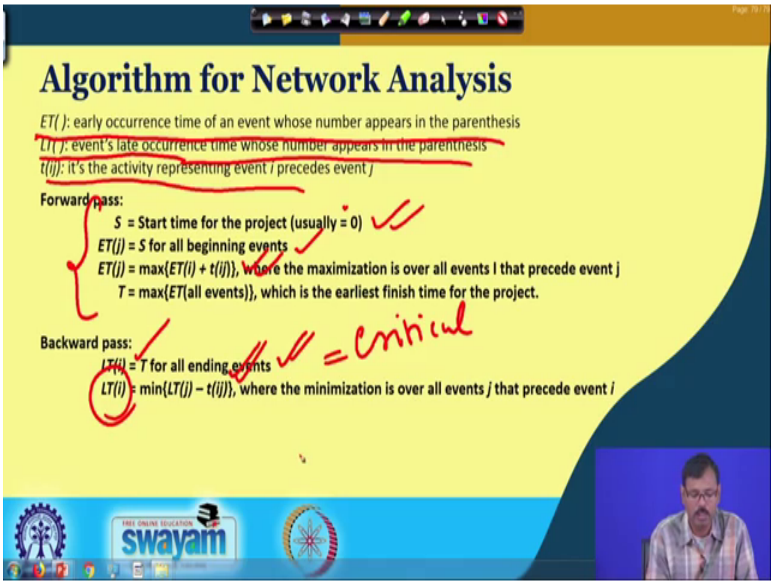

So, that is why this is very very important and this has to be considered. Finally, let us look onto the algorithm of the network analysis. If you are considering, it is the early occurrence time of an event, L T is the late occurrence time of the event for a particular node and T i j is the active duration activities duration comprising between that to the two corresponding nodes. And in forward pass we consider starting time S is equal to 0 or you may initialize anything E T j is basically early occurrence time is basically given for the first node and then for the next node, we consider it and to the time of that activity for getting the early occurrence time of the following nodes.

In backward pass, we consider the time of completion T which is we are getting from the critical path as the completion time and that is given to the final node first. Then we backward pass it minus the duration from the latest occurrence time which are given to get the early occur, the latest occurrence time of the preceding node of that particular node.

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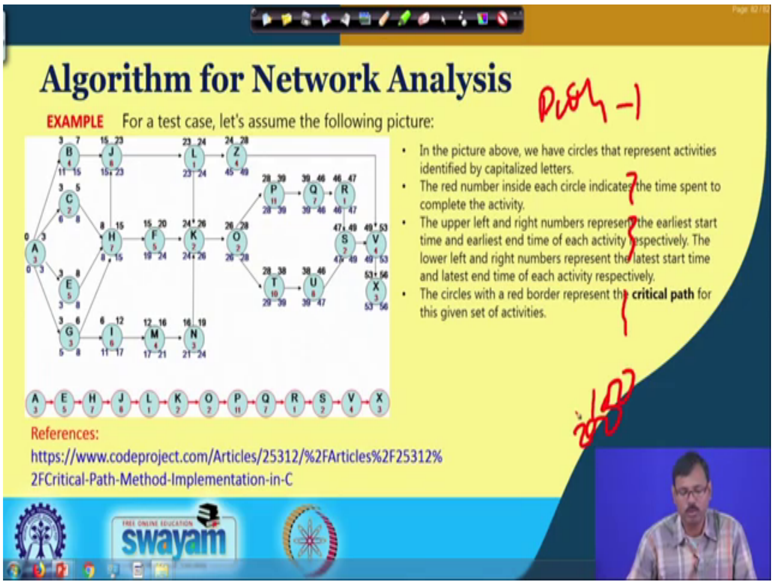

So, based on this considerations this one example calculation is given this is available in internet in this reference because the complete C programming or that network analysis is given, that is why I am giving you this reference you can go through this reference and find out what is the programming there in details. If you see this, this is the network, this is the network given and there are n number of paths are available those you can see these are the different paths available. So, n number of paths are available. So, that is why this is very difficult for hand calculation of this one. So, path number of paths are too much, number of paths are too much in this case.

So, if instead of using a computer programming, someone would like to do it manually then he has to write maybe probably path 1, 2, 3, 4 or 5 by that way maybe 100 paths or 2000 paths will come out for analyzing this one. So, it is very difficult to completely analyze this type of critical networks and computer programming is required. So, you can see in the picture above, we have circles representing activities identified by capitalized letter the red numbers inside the each circle indicates the time the upper left and right members representing the early starting time of the job nodes and early end time of the nodes.

Similarly, the lower one is the late start late finish time. This is basically activity on node diagram and circles with red borders are basically representing the critical path. So, this is the red borders are given in the critical path where which is represented separately here. So, how we have come out in this critical path by calculating this one, there is one program given in this we have given in given in this reference. You can see this reference also.

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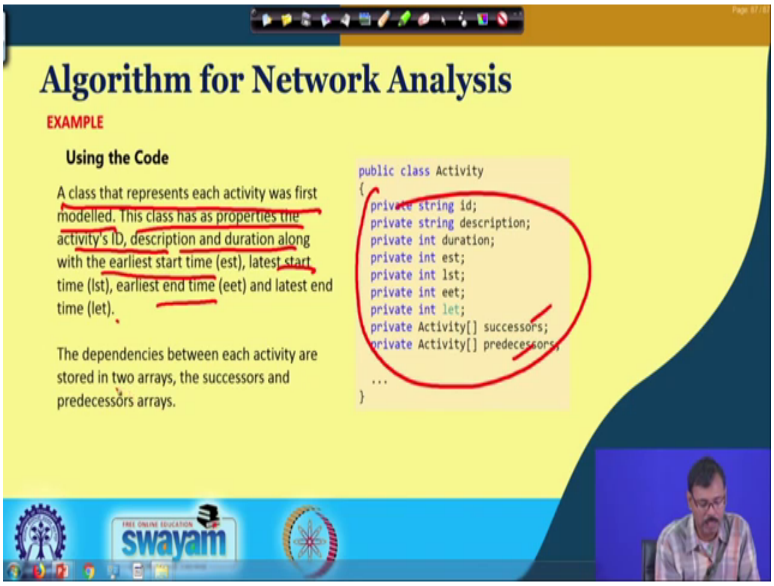

So, what is the basic of this, first it is basically classify using the code a class that represent each activities was first modeled. And this class has a as properties the activity id description duration early starting time late starting time end time these are given these are basically defined in this case and that is why this is activity successors and predecessors are also defined for each activities as the two separate arrays.

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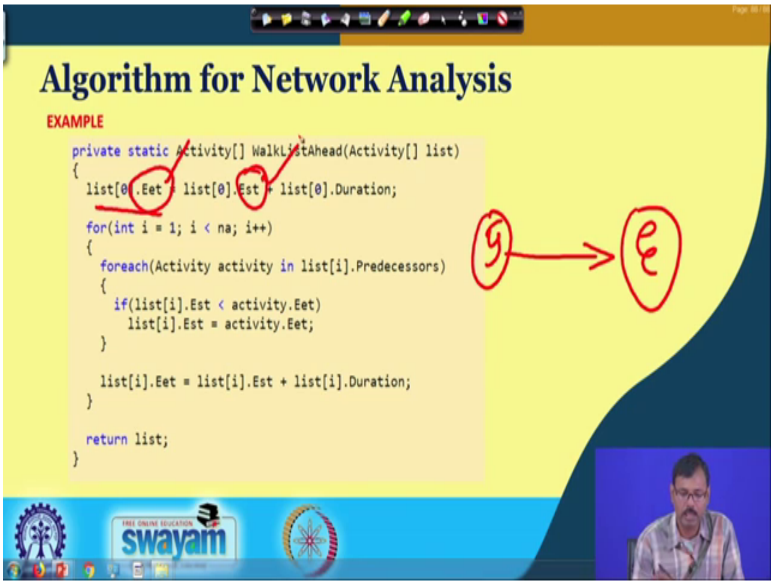

So, this is basically the way the inputs are given. Then the first the forward pass is made first, first or forward pass is made to identify the early ending time early starting time of each jobs.

So, starting from the start to end start to end, all the jobs are considered along their path and early ending time early start time of each jobs are determined and kept in the memory of the computer. And this is the for C program, this is the C program, this is the C program for the forward pass.

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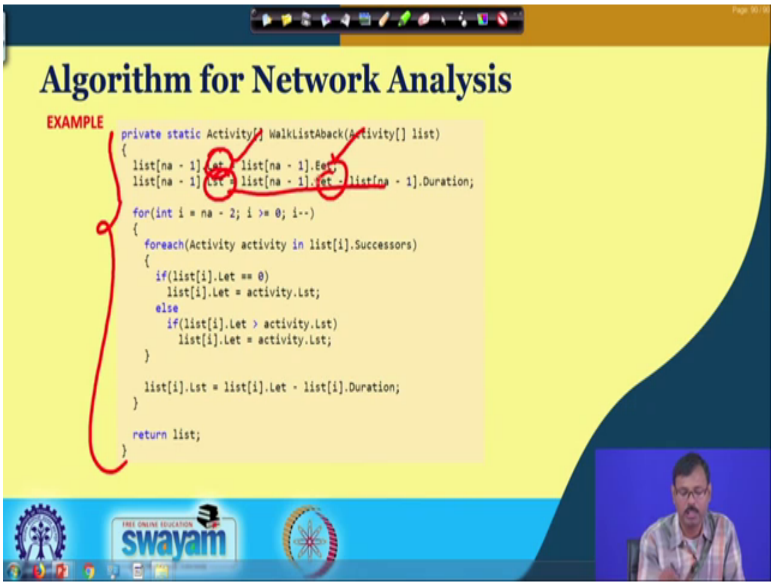

Similarly, the program is made for the backward pass also. And see, this is the program for the backward pass. That is why the late ending time and late start time are being calculated. So, late ending time is assigned with the early ending time, then it is back calculated from the late starting time from the late ending time.

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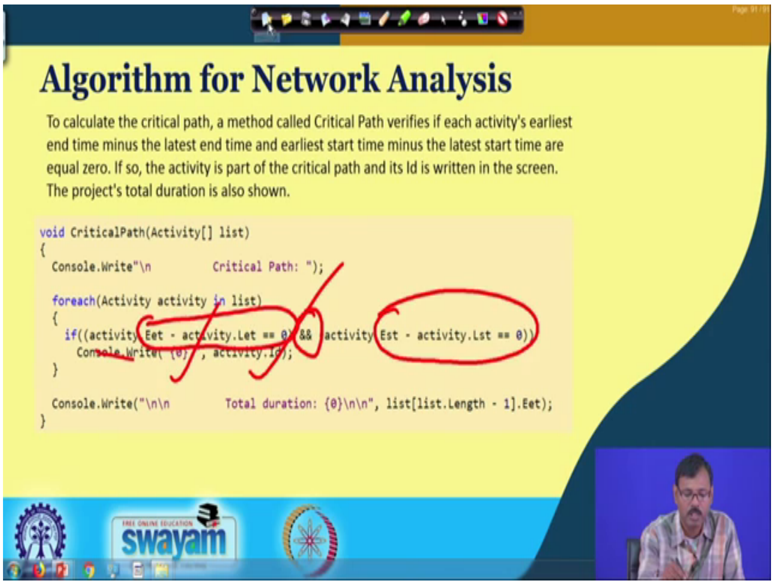

So, this is the backward pass program for the backward pass calculation. And finally, the critical path is calculated consideration that early ending time minus late ending time must be equal to 0 and early start time, late start time must be equal to 0 and that activity has to be memorized and that duration has to be memorized.

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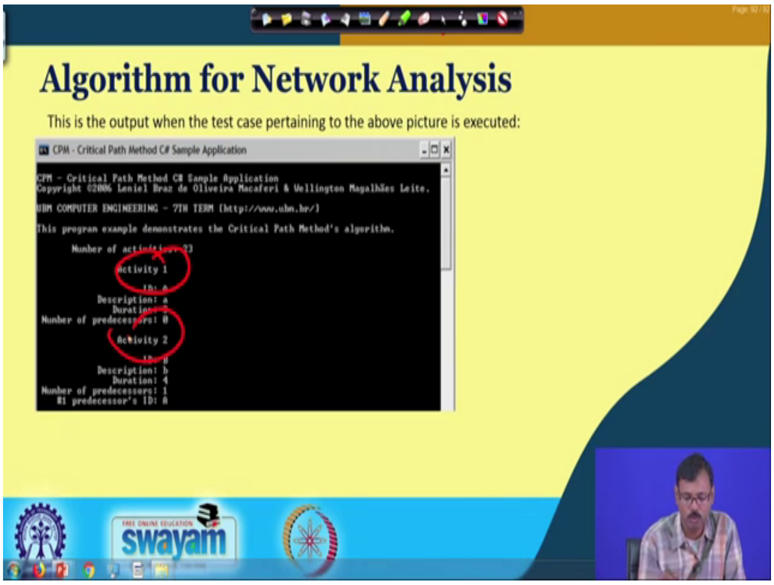

So, in consideration with that they have found what are the activities having 0 slack or is there. In consideration with that, they have calculated the activities for that. This is the output you can see, this is given for activity 1, activity 2 and for all the activities which are having 0 slack are placed identified by here.

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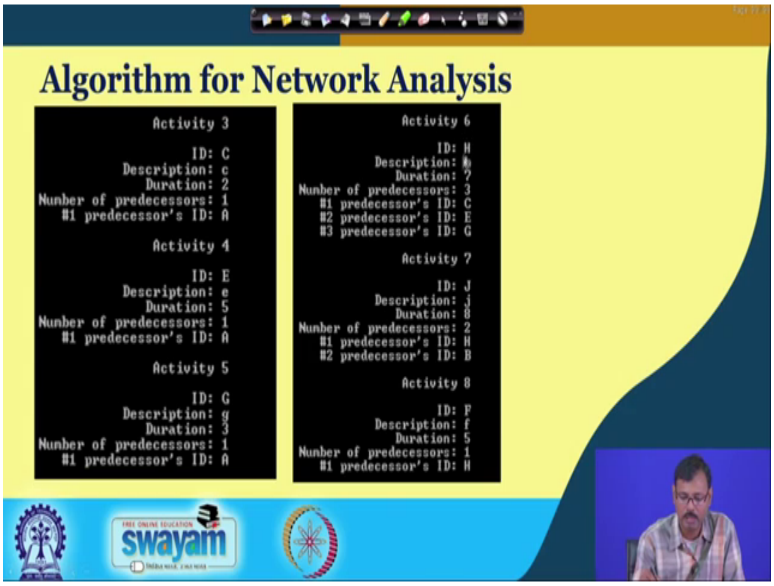

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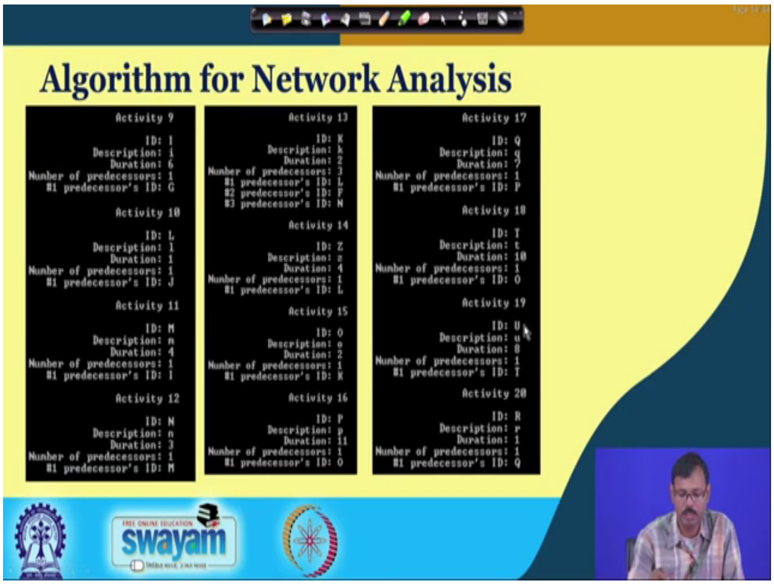

And finally, this activity 9, activity 13, activity 17, they are a part of critical network.

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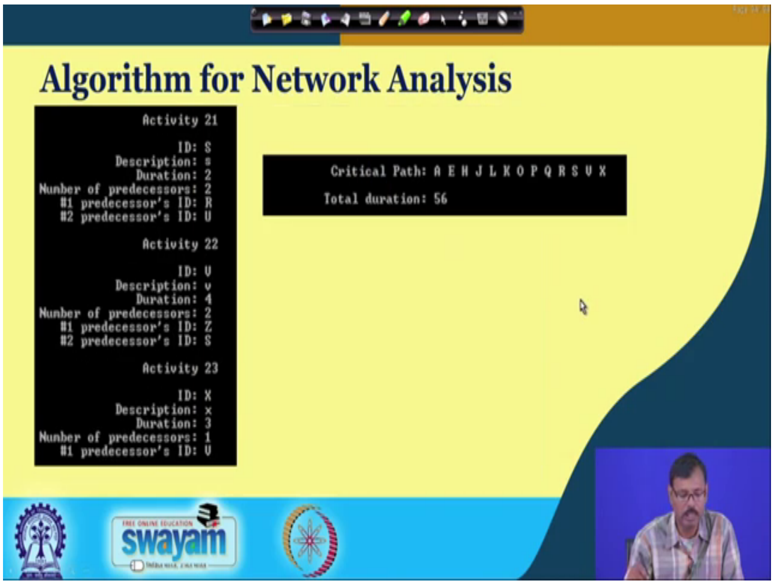

And that is why, their names are placed they are total duration are summed up and it has been found this is the this is the critical activity associated with the critical path and this is the total duration by summing up the T I j s of this durations you can get this one. So, this is the basic algorithm used in this case for determination of the critical path.

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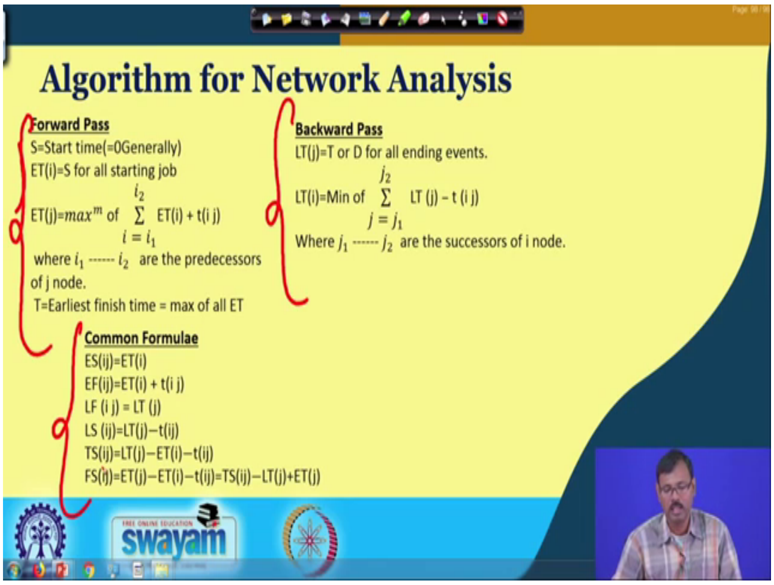

And let me once give you those algorithm once again presented in this yes. So, this is the final summary formula for network analysis. So, if we would like to summarize the total course, if we like to summarize the total course now, in this course initially you have you have learned that how the network will be formed. Then, after forming the network, how we can identify the jobs which are more critical and needs the most attention of the managers for their early completion. Considering this critical job, we have found that how the costing will be carried out the crossing or stretching of the jobs will be carried out to finally, come out with the most minimum cost or you can say the optimum network or optimum project scheduling.

So, that is the important thing. After that, we have founds before the hand whatever the deterministic approach we have followed. After that, the probabilistic approach must be there because of in consideration and in consideration of the uncertainties and so for that, what we did we consider the most optimistic most likely and most pessimistic times for the jobs for their completion and from there we have come out with some estimated or expected time for the completion of the jobs associated with their deviations and variances.

So, finally, we are able to come out that the possibilities or the probability of the completion of the network within a given date. We can have some mathematical expression for that. For we can we have learned that there are also different packages are also available, commercial packages are also available and you can three packages are also available to download and utilize those packages for your network analysis.

So, I hope you have already understood all the details of this net, basics of the network analysis and you can apply the same for your respective different problems.

Thank you.