Network Analysis for Mines and Mineral Engineering
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Lecture – 08
Late start and late finish times algorithm

Let me welcome you to the NPTEL online certification course Network Analysis for Mines and Mineral Engineering. Today's topic is Late start and late finish algorithm for critical path network analysis.

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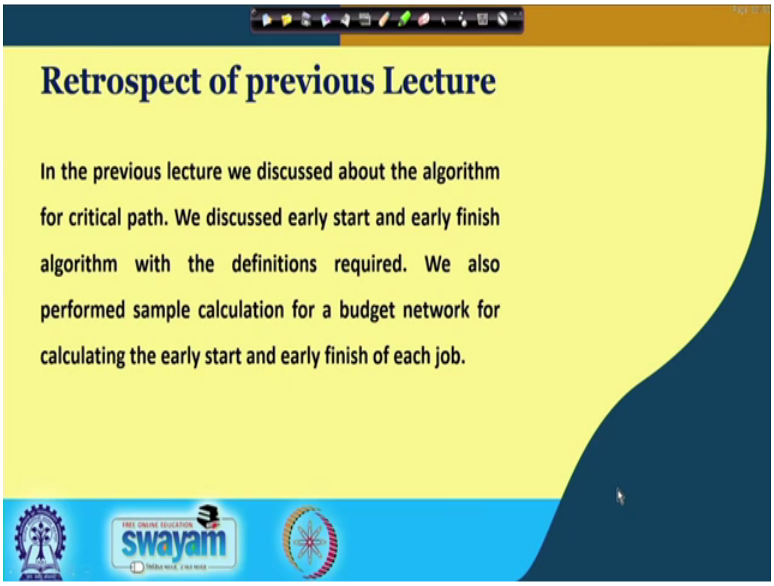

So, let us discuss what we have studied previously, already the network formation is known to you. I believe that any a network is jobs are given to you, you can construct the network. When you have completed the network we have to analyze this to find out which path is the important one, where you have to give more man power machine to complete it on schedule time, where you are having some slack hours for some manuring or alterations.

So, that analysis is required and for that critical path network has to be analyzed, and critical path in a network has to be analyzed. And, we have already discussed two methods: one manually how you can find out the critical path from calculating the total job duration. And, in another was that from early start, early finish algorithm how you can determine the critical path.

So, we will discuss late start late finish algorithm in this lecture. So, that you understand how these are interlinked. And, basically when you developed a computer network that time we wish to find out both early start early finish late start late finish algorithm; so, that only we can identify clearly which one is the critical path.

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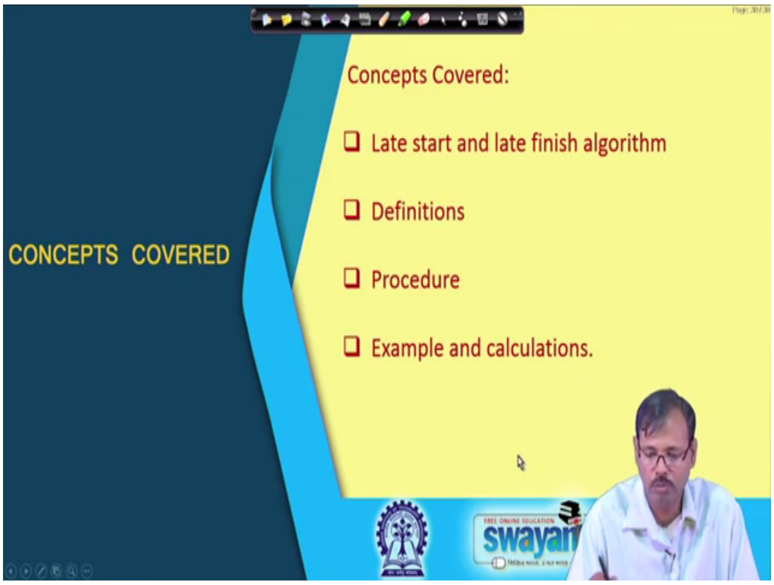

So, basically let us see what we will discuss in this, in this we will complete the late start late finish algorithm, then definitions and procedure with the example calculations.

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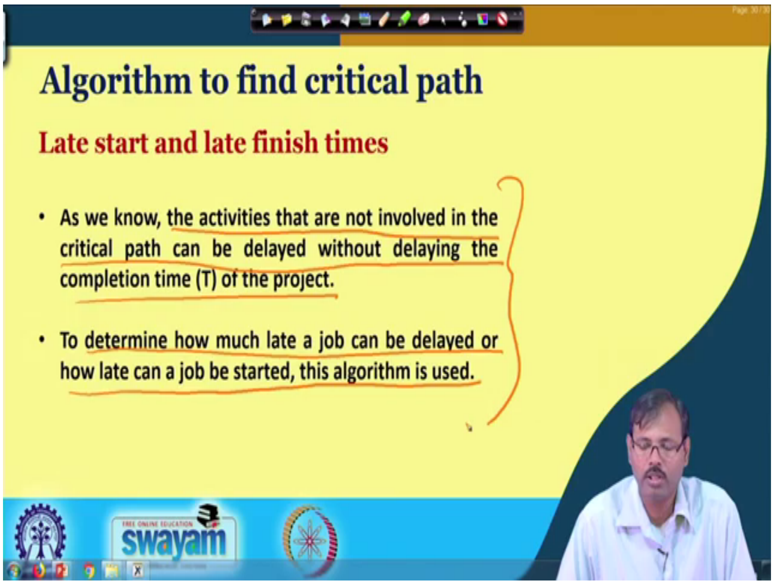

So, what is late start and late finish algorithm? So, in this say case the activities that are not involved in the critical path can be delayed without delaying the completion time of the project; this is first considerations. And, to determine how much late a job can be delayed or a how late a job can be started this algorithm is used. So, basically late start late finish algorithm is just reversed to the early start early finish algorithm, where late finish algorithm is the late finish is basically defined as the last possible time by which the job has to be finished is considered as the late finish time.

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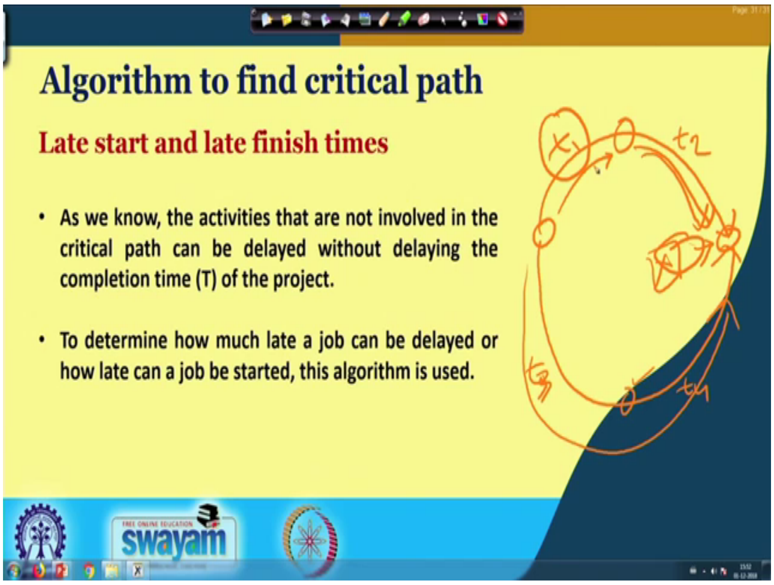

Late start time is considered as the most possible last time or latest time at which the job has to be started to complete the network on time. That means, if you consider a little bit our previous consideration; where a person was going from some Francisco to California another person is taking another path from some Francisco to California. One person is having some stoppage are there, another person is having some stoppage are there at this point.

So, they are having different part time durations at t 1 and t 2 and t 3 and t 4 and it was found that this person has a huge time as a slack time because, this path is longer and was consuming more time. So, in that case this person would have take a later time of starting, from this place to this place or this place to this place to reach at this point at a schedule time of x. So, that is why to without affecting the completion time, the time at which a job maybe finished a job maybe started is called late start late finish time and based on that the algorithm is also can be made.

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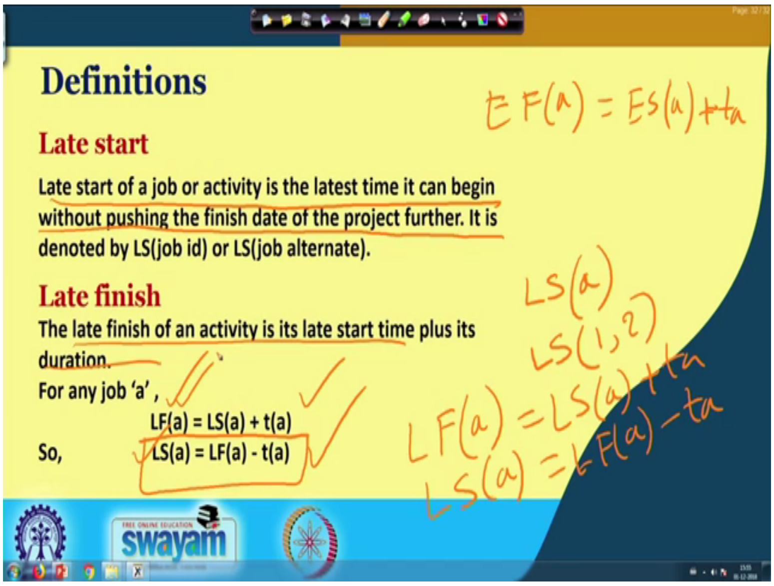

So, late start of a job or activity is the latest time it can begin without pushing the finish date or finish time of the project further. And, this is denoted as LS with the name of the activity inside that or we use LS or the alternate of the activities under the bracket. And, late finish is the latest possible finish time; obviously, late finish of job a is equal to late start of job a plus t a and late start of a is equal to late finish of a minus t a.

Why this is mentioned probably, if you recall in our last class when we have discussed early start early finish we have given this formula. Early finish time early finish time is equal to early start time of a plus t a, not the early start time is equal to early finish time minus t a. The reason is that basically late finish late start algorithm is the backward algorithm, where we start from the completion time and from the completion time we find out which can be the latest possible award to start the different activities.

So, basically as this is the backward analysis so, that is why a we always calculate the LS because, LF is already received from the successor activities latest start time. So, that algorithm earlier algorithm mentioned algorithm is just reverse in this case.

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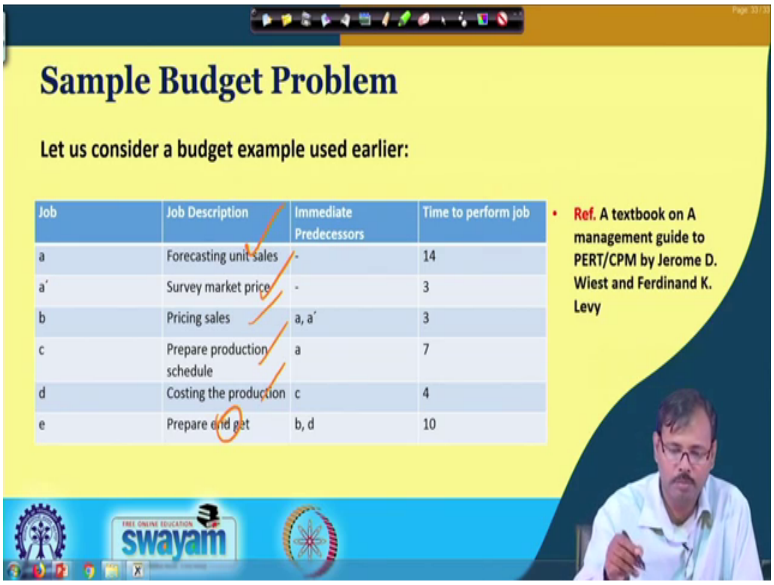

And let us again consider the same example, where we have carried out our early start early finish that forecasting of cells. Forecasting of cells, market survey, pricing, production scheduling, costing and again this is the budget; some printing mistake is there, prepare of the budget. So, that is why this is the project network which we are considering in this case.

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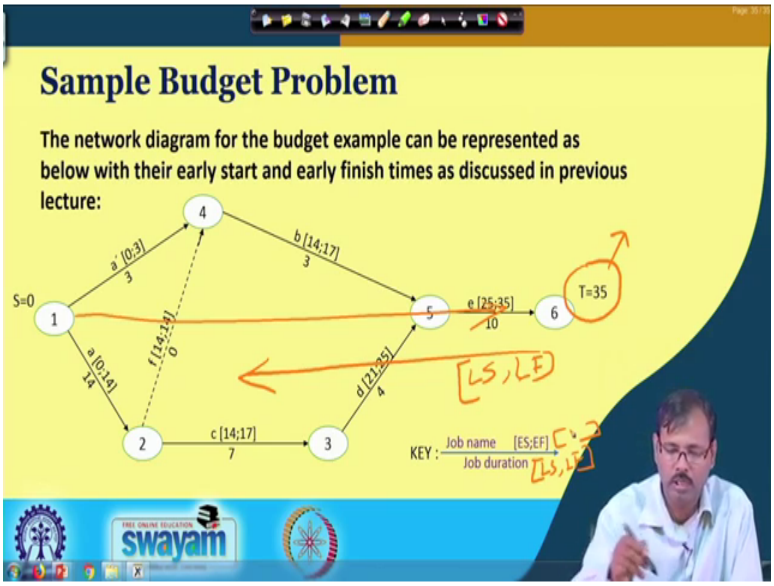

This is the already developed network where, early start early finish is already carried out and now it is available with you. Now, let us consider that early start early finish network which is already available here. And, now we have calculated from forward from this to this and now we will calculate it in a reverse direction, that we will start with T is equal to 35; the proposed completion time.

And, this proposed completion time without affecting the completion time we will calculate it in the reverse back to find out which one is the late start and late finish time; again it also denotes in under a third bracket. But not on the top, on the bottom it may also be given here also with another bracket. So, it is up to you, you can have a given it in the numerator or in the denominator, but it has to be proposed as the, a late start late finish times of the activities.

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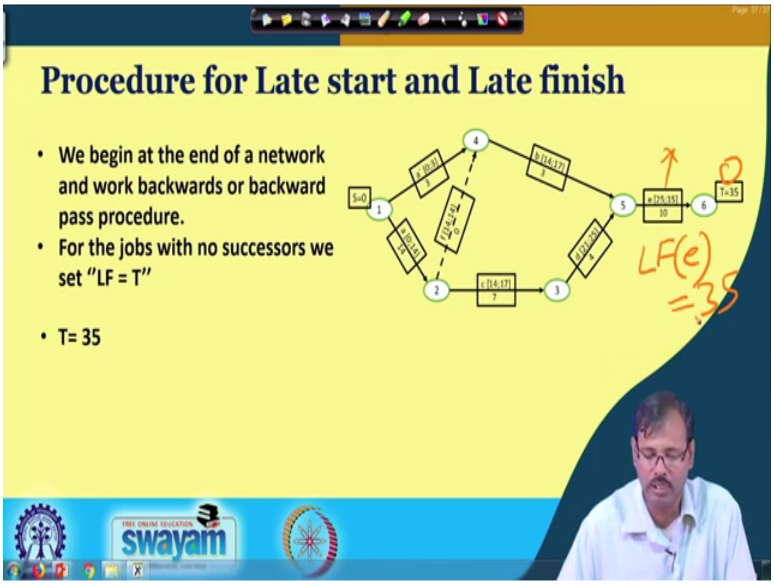

So, now let us go for the backward analysis of this one; say this is our proposed completion time. And, this proposed completion time we want that our last activity that is the e activity must be completed in this proposed completion time. That means, the late finish time of e latest possible time by which the e activity can be completed is equal to 35.

So, now you understood that we have to start from the final point and last activity which we are considering is that, that is the e activity which must be completed at the time of 35. So that means, T is equal to 35 is considered for the last activities.

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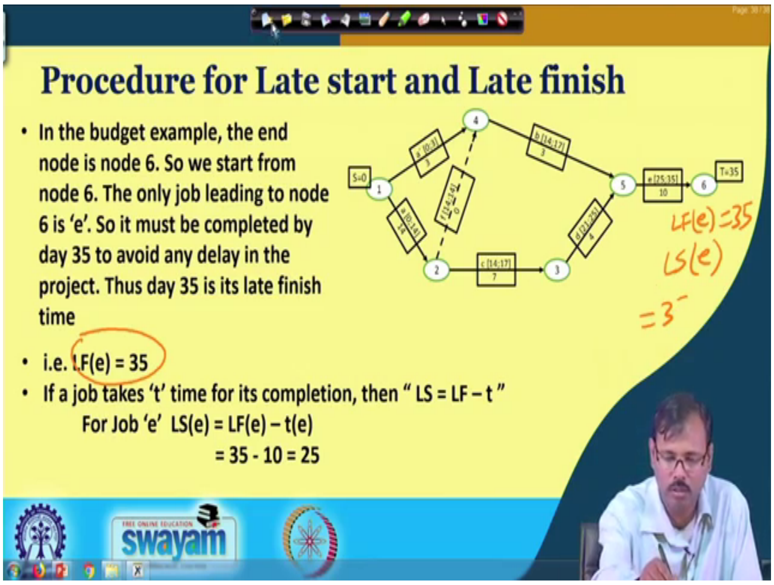

So, from last activity the 6 job e has to be finished, LF e is equal to 35.

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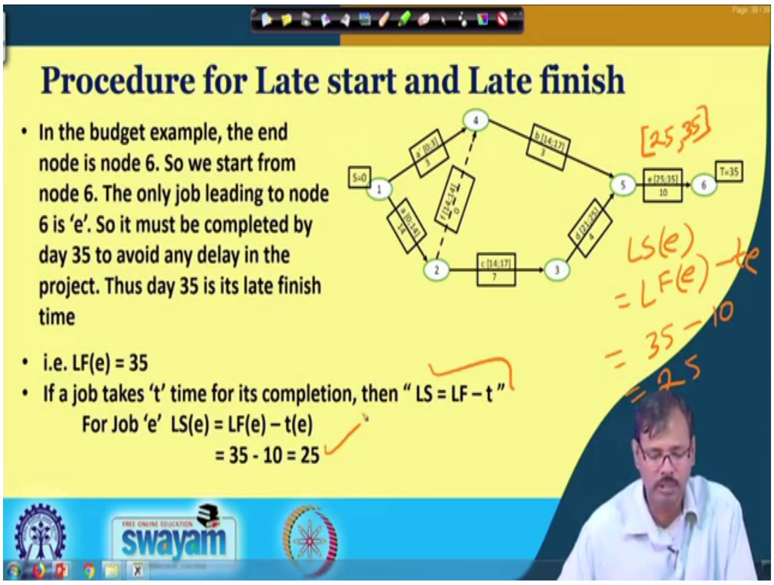

So, to get the late start of e we have to get the late start of e we have to make the LF of e let finish minus t e. So, that is 35 minus 10 is equal to 25. So, for e our late start is 25 and late finish is 35 which is expressed in this case and also you can see this is the formula which is used. And, we can get LS of e late start of e is equal to 25.

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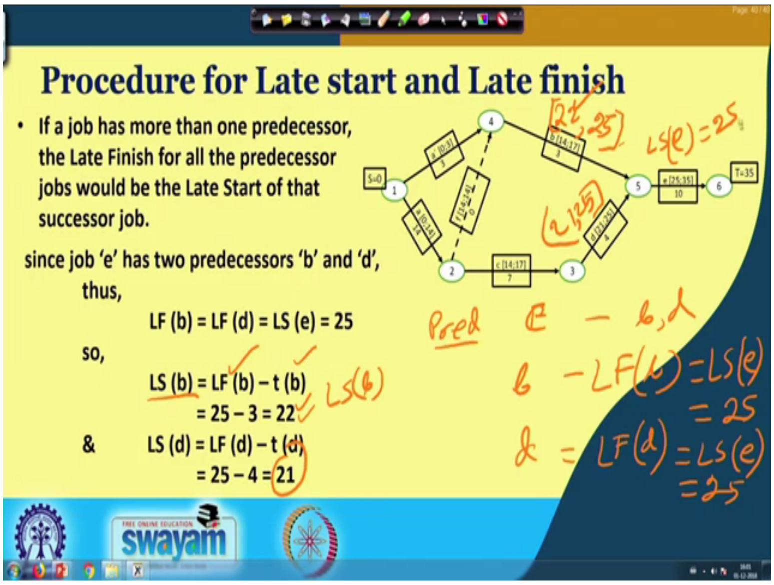

Now, let us calculate for the subsequent activities. The predecessors of e predecessors of e, e activity is b and d. So that means, if the latest start required for e latest start required for e is equal to 25 ; that means, the latest possible finishing for b job, latest possible finishing for b job is equal to latest possible start time of a e job; that means, it is 25. Similarly, for d job also the latest possible starting time finishing time of d job is equal to latest possible starting time of e job is equal to 25. That means, for both these two job the finishing time proposed finishing time proposed finishing time is 25. So that means, we have to start it such a way; so, that it will be finished by 25.

So, as again in the using the same formula we have to find out LS of b LS of b is equal to LF of b minus t b; so, that is 22 so that means, this job maybe started latest by a 22 so, that it will take 3 hours to complete it by 25. So, that as it is being completed at 25 LS can be started at 25. So that means, it is the interdependency of the jobs that allow us that the b job has to be started in 22 and the d job has to be started in 21; so, that we can achieve the target time of 35. So, this is very easy activity is easy calculation.

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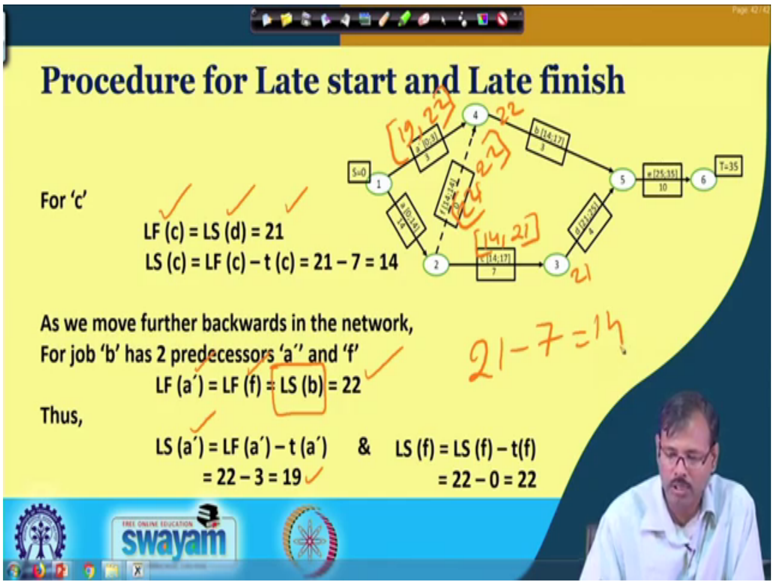

Let us go for the next one which maybe little bit different way. If you see the latest finishing time a latest starting time is 21 here, sorry this is 22 here, 22 here. It is 21 here latest finishing time for d activities a latest starting time. So, for c the latest finishing time is equal to latest starting time of d is equal to 21. So, it is LF is 21 and for a dot for f, the latest for LF latest finishing time of f for a dot f is equal to the latest starting time of b that is equal to 22.

So that means, latest finishing time is 22 here, latest finishing time is 22 here and we can calculate the latest starting time for a is 19. So, latest starting time for a is coming 19. This is as 0, this is again coming 22 and as this is 7 the latest starting time is coming 14 because, 21 minus 7 is equal to 14.

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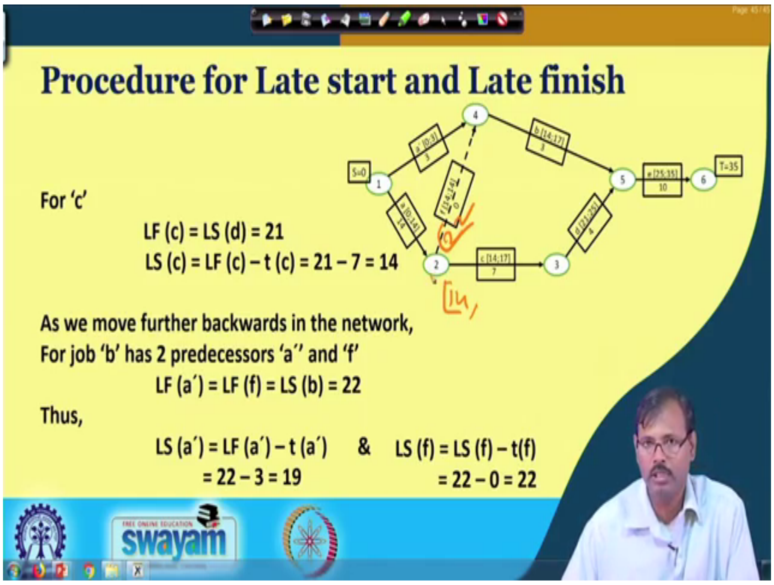

So, let us look into the calculations again; see latest finish time for c is equal to latest finish time for d is 21. So, the latest time of c is equal to latest finish time of c minus t c is 21 minus 7 that is 14. And, if you consider it for the a dot job for this job then L latest finish time of a dot is the latest starting time of b that is 22. So, the latest starting time of a is latest finish time of a minus t a that is 22 minus 3 19.

Let us consider it for dummy job f. So, latest finish time of this dummy job f is again 22 because, it is the latest starting time of b and the dummy job this time duration is 0. So, the latest starting time of dummy job is 22 minus 0 is coming again 22. So now, in this case you can find out that up to this we have received. And, we can find out this job has to be started at 22 latest by 22, this job has to be started latest by 14. So that means, at this particular point these two jobs are to be considered. So, let us see how we can calculate it for job a.

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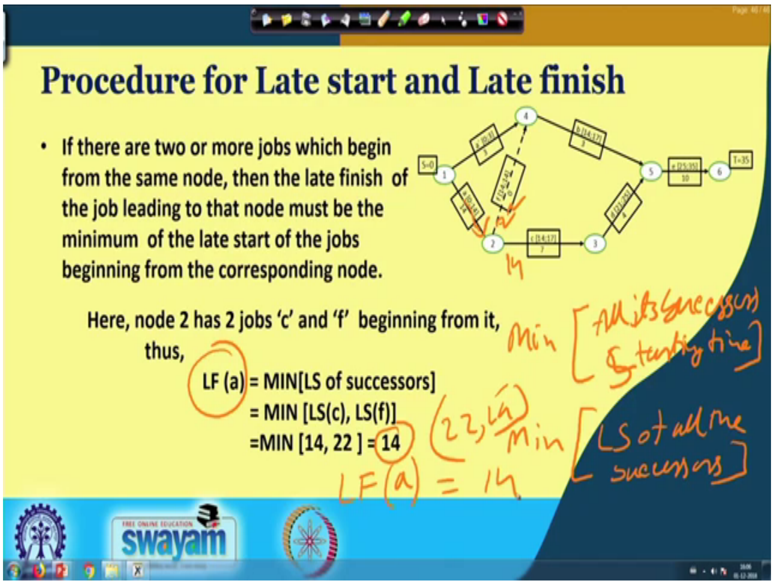

So, if you consider for job a, this one is supposed to be started at 22, this was to be latest started by 14. So, the latest finish of a thus, job has to be finished latest by minimum of all its successors starting time; that means, minimum of LS of all the successors. So, in this case there are minimum between 22 and 14, the minimum is 14 that means, the latest finish of a latest finish of a should be 14. Again look at this for some better understanding say unless and until this job is finished, we cannot start this job and this job.

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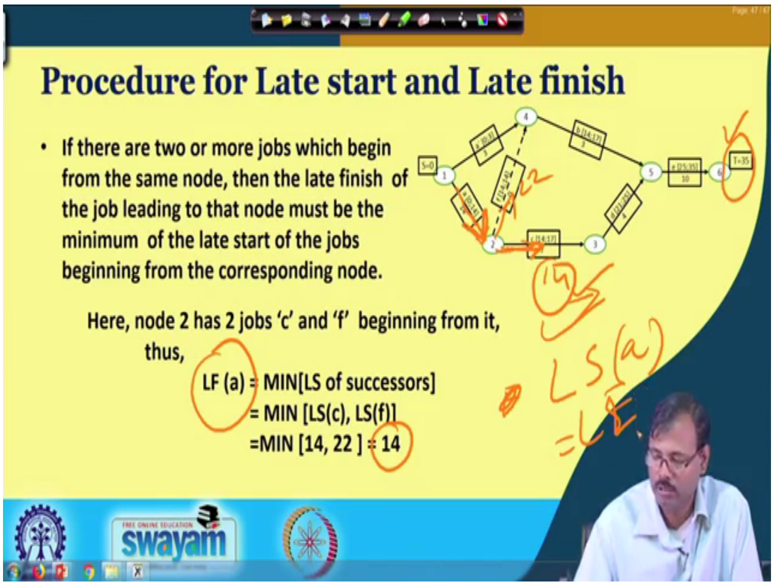

This job has to be started latest by 22, this job has to be started latest by a 14; that means, this job has to be finished latest by 14. So, that there will not be any problem for commencing of this job; so, that we can attain our target time of completion of 35. So, to target to achieve the target time of completion 35, we should reach we should start this job latest by a 14. And to achieve this target of starting this job latest by a 14, we have to finish this job latest by a 14. So, the late finish latest finish possible for job a is 14. So, the earliest so the latest starting time of job a is latest finishing.

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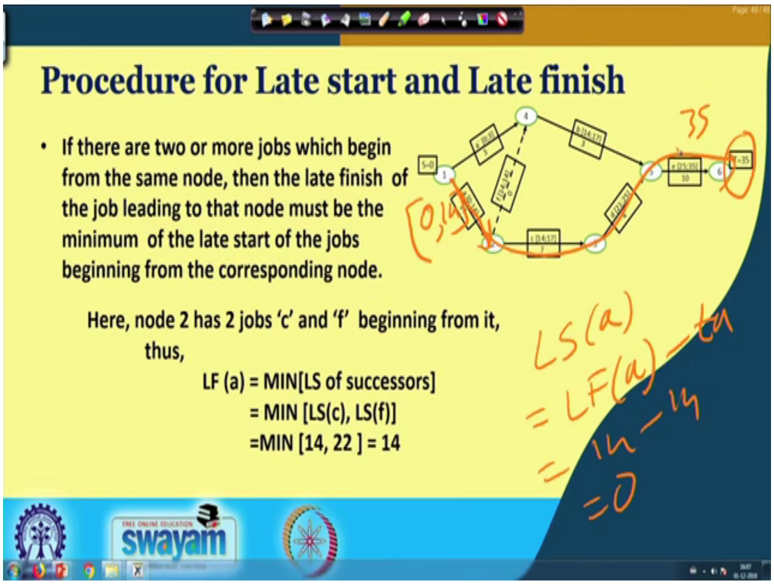

Latest starting of job a is equal to latest finishing of job a minus t a that equal to 14 minus 14 is equal to 0. So, this job must be started at 0 time and must be finished at 14 time; latest by a 0 time latest by a 14 time for completion of all this networks at a target time of 35. So, this is very very important and this algorithm required to be understood properly.

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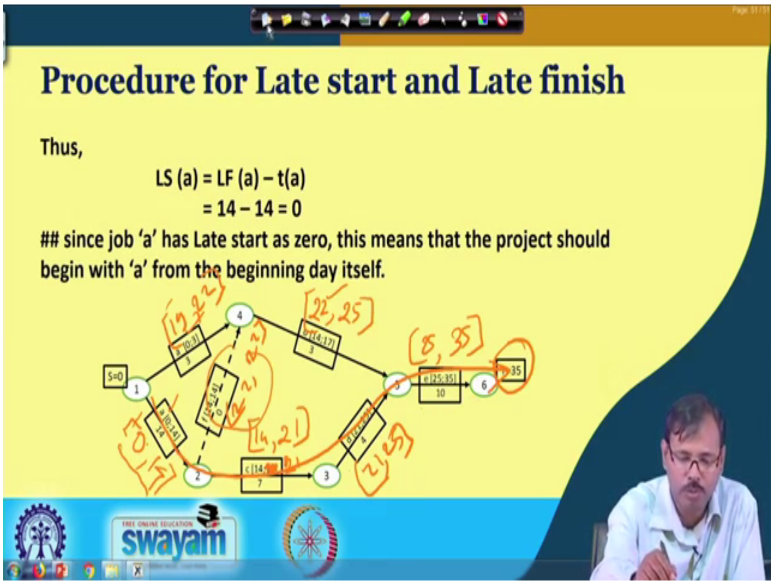

Now, let us look into the, this is the calculation of the latest starting time; 14 minus 14 which we have already discussed. So now, it this is the early start early finish time is given. You can see in early start we have started this as a 0 so, both are 0 and accordingly we have calculated and we are finishing this activities are 35. But now, these are altered and you have find this is coming 35, 25 as the early start early finish early start. This is coming 25 21, this is coming 25 22, this is coming 22 19, this is coming 22 22 dummy job. This is coming 21 14 and this is coming 14 sorry, this is coming 0 14.

So, that is why you can see now this job has to be started at 0, this job may not be require to start to be started at 0. And this job may not be required to started at 14, we can start this job at 22 also. This job maybe at 14 may not be this is dummy job not required. So, this job has to be started essentially at 14 and to be finished at 21, this is 14 21, this is 21 printing mistake is there. And, this is again 21 25 this has to be as usual as the previous one which we have considered in the early start.

Basically, if you look into this you can find out there are some activities where early start early finish is same with the late start late finish. So, basically this jobs which are showing us the early start early finish late start late finish same basically that is giving us the critical path of the network.

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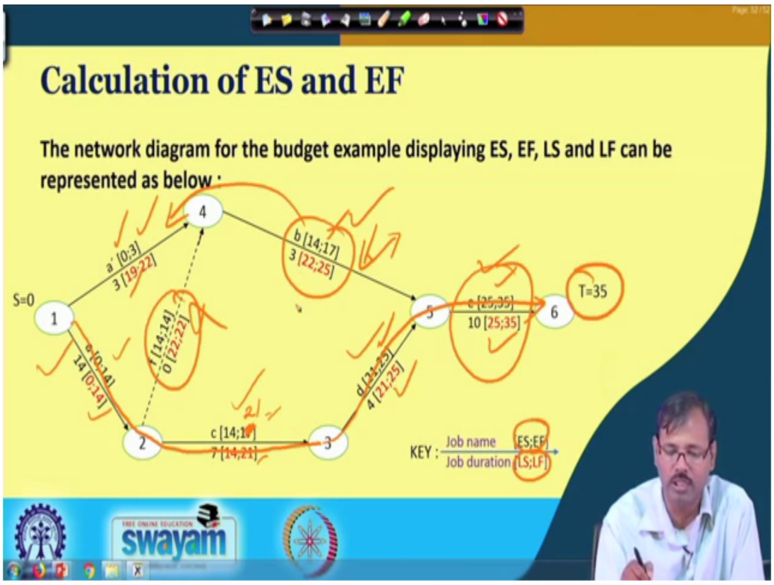

So, basically if you look into this here, here we have use the nomenclature that LS and LF are given in the denominator and expressed in red color; here it is in black color. So, you can see earlier starting time 0 here; however, it may be 19 here. So, in between 0 to 19 whenever you can start you can complete it, you can start it between 14 to 22 based on the completion of this activities beforehand.

Anytime if you start you can complete, still you can address the schedule. This is dummy job we need not to discuss about this, but if you look into this one, this one, this one and this one; you can see their early start early finish time late start late finish time is same.

This early start early finish time late start late finish time is also same here, actually this is the printing mistake. This should be 21 we are discussing it since the last class also unfortunately it was there. So, this late start late early start early finish and late start and late finish time is same here, here also it is same. So, that is why this early start early finish time late start late finish time, if these are same for those activities they are the part of the critical path.

And at most care must be taken for the completion of this job timely into the network otherwise this target time of T is equal to 35 maybe missed. However, this activities are having some slacks, this is technically it is called slacks. So, this slacks basically have a gives some breathing time for this activities and also management can think of withdrawing man power, reducing man power, reducing material maybe in this activities; so, that the costing can be reduced for this activities.

So, these are the possibilities where it can be carried out or this may be possible that we can adopt more man power etcetera, in this to reduce these activities. So, that we can match the give a optimization to this network. So, these managerial decisions can be considered and that is why it is a good practice to think of the early start early finish analysis, late start late finish analysis.

And, this will give us the complete idea which jobs are having slacks and can be considered in the analyze optimization of the network. And, which are the critical jobs has to be considered more seriously, more attention must be given to those jobs for their timely completion.

So, this is all for this class.

Thank you.