Mine Automation and Data Analytics

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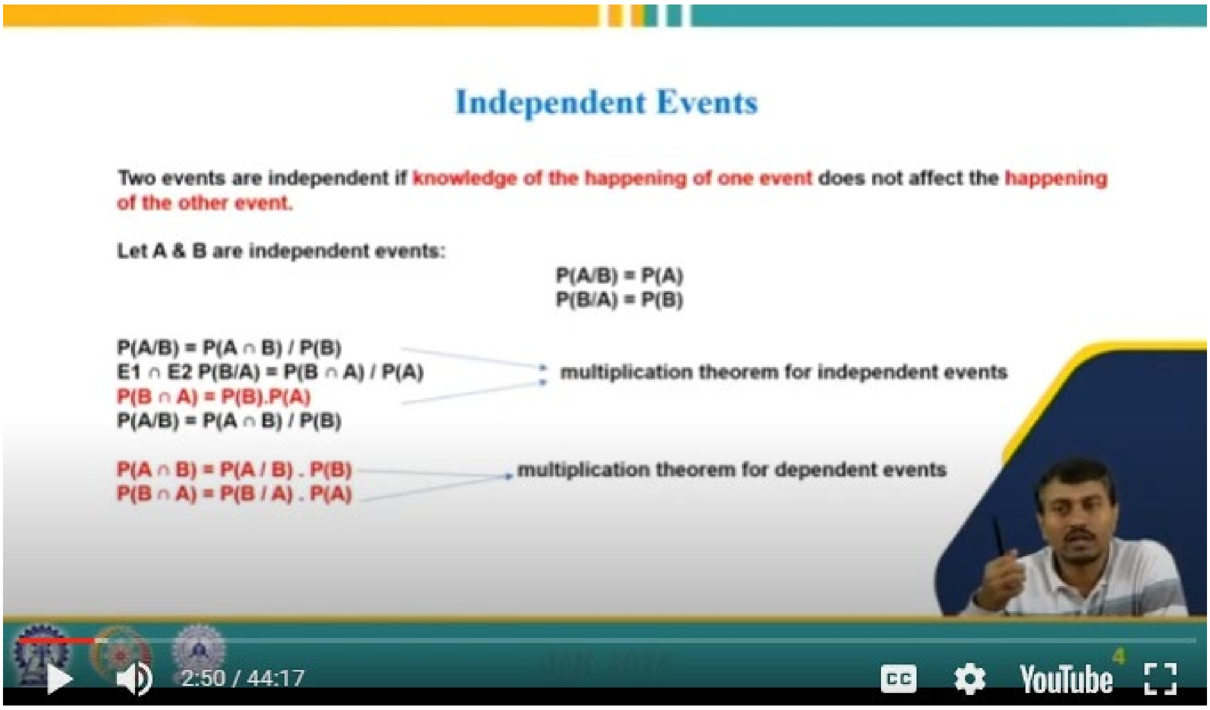
Week - 08

Lecture 36: Introduction to Probability and its associated terms

Music Welcome back to my course, Mine Automation and Data Analytics. Today, we will deal with more concepts of probability and some of the primary conditions under which we can apply the probability theorem in our mining cases. So these are the topics we will cover. What independent event we will deal with them, the theorem multiplication theorem, the theorem of total probability, and the Bayes theorem are essential theorems in probability. Lastly, we will discuss the applications of this probability theorem in the mining industry. So, let us examine what an independent event is. An Independent event is an event in which event A is independent of event B, which means event B is happening independently. There is no role for event A. Also, when event A happens, there is no role for event B, so this is an independent event.

For example, I was walking suddenly, and my shoelace was broken, or my shoe got damaged on the road. So this is an event. For example, another event is somebody in the department who fell ill due to the cold. So that person is located at least a few hundred meters geographically distance away, and they did not have any role in my damage to the shoe when I was walking on the road.

So this is not a related event, okay? For example, Brazil is playing a match against Argentina, and India is playing against England in the test, so the win-loss situation or probability of win-loss for England and India and the likelihood of win-loss for Brazil and Argentina is not dependent. These events are independent. So, this can be represented by the theorem, which is very required. There are many independent events in the mining context, and it is essential to understand what events are related and are not associated with each other. So, based on that, different formulations are there in the probability theorem that we will discuss briefly. So here, the example is given: A and B are the independent events.



So, in the last class, if you remember the probability of A, B means B is happening under the condition What is the probability of A? So when these are these two are independent, whether B is happening or not, probability A happening will remain the probability happening of probability A, so it is equal to p A. Similarly, for the probability that happening B under the condition that A is happening of event B does not depends on the probability of happening B does not depend on a probability of happening A. So here it is no matter, so it is probability of B. So for the independent event, these comprise that the intersection between two okay intersections between two means probability happening A and B intersection is the multiplication of probability A and B.

$$\boldsymbol{P(A \cap B) = P(A) \cdot P(B)}$$

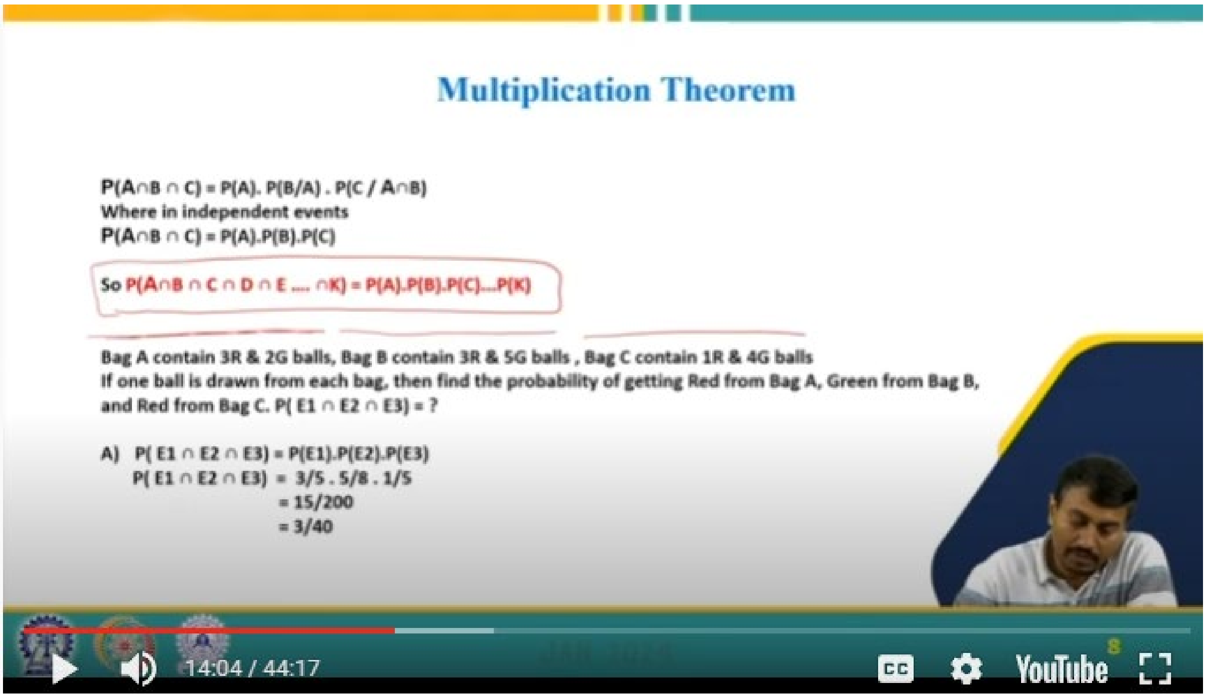
This is a significant conclusion, and when they are together, they mutually depend on each other. This is the theorem, so A intersection B is the probability of happening A under the condition of B multiplied by probability B. So this is already what we have covered in the last lesson. So, using this concept, we will use it for our field applications and try to understand what events are correlated to each other and what events are not strictly associated with each other or we do not know. So, in those cases, we do not know. Still, there are some interrelationships, so here, the probability concept and the total probability concept will be essential, so we will subsequently cover that. For an independent event, event A does not depend on event B to occur, so the probability of intersection A and B is a multiplication of probability A and probability b.

For example, if I die a roll dice if I roll dice so, there are four, this is five, this is three, so this is a die, and this is another coin, this is head, for example, so whether these heads will come or whether it is coming four showing four. Hence, these two independent events coming ahead remain the exact outcome of her four three five like that, so these are independent events. So here we can see that if four are coming here, what is the probability of four probability of coming? There are one, two, three, four, five, six. All are equally likely for a fair die, so the likelihood of four coming is one by six. So this coin is also fair, so the probability of a coming head is half. Now, I want to predict that four will appear in a die, and here, a head will appear, so the probability is one by six into one by two is equal to one by twelve. This is the outcome of this theorem of the independent event.

$$P(A \cap B \cap C \cap D \cap E \ldots \cap K) = P(A).P(B).P(C)...P(K)$$

So here we have tried to elaborate on with example another example so a coin is toss this coin is fair you have to remember and die is roll this is also fair there is no partiality so E1 is the event now that event is say that appearing head on the toss E1 and E2 appearing three in the rolling die outcome so we have to check now we want to test it I have already elaborated that it is independent even we can test it using this formula as well that E1 and E2 are independent event or not so let's see the probability of E1 E2 intersection so let apply this theorem P of E1 and P of E2 so this is basically coming 1 by 12 as same we have already calculated so when one coin is toss what will happen it will happen that it will come head then here may appear one in the rolling die here may appear head here may appear two here may appear here here three here may appear head here it is four here it is head it is five here it is head it is six similarly for tail one tail two tail three tail four tail five tail six so appearing head on the coin is this and three here it is the probability so out of 12 space 12 times the total outcome number of outcome and this is the only one it is appearing so probability is 1 by 12 it is actual so the theorem we have applied here multiplication theorem is correct and that is why now based on this theorem by using this theorem I proved that this event E1 and event E2 are independent event they does not depends on each other so event 1 does not influence the occurrence of the event B.

Another example of independent event one is the person hitting a target, which is three-fourths, which means 0.75 people. A person B's probability of hitting the object is four to five. Hence, these two percent are independent people. One person's capability does not affect another person's ability, particularly when reaching a target or hitting the target, so now the probability of both A and B hitting the target is okay, and since both events are independent, they do not depend on each other. Hence, the likelihood of hitting A and B both is the multiplication of 3 by 4 and 4 by five. It is three by five in total. This way, we can apply this theorem straightway to calculate the probability or chances of occurrence of those two events that are unrelated to each other. So we can assess that based on that, we can take a certain amount of precaution or measures to avoid unwanted situations in the mind.

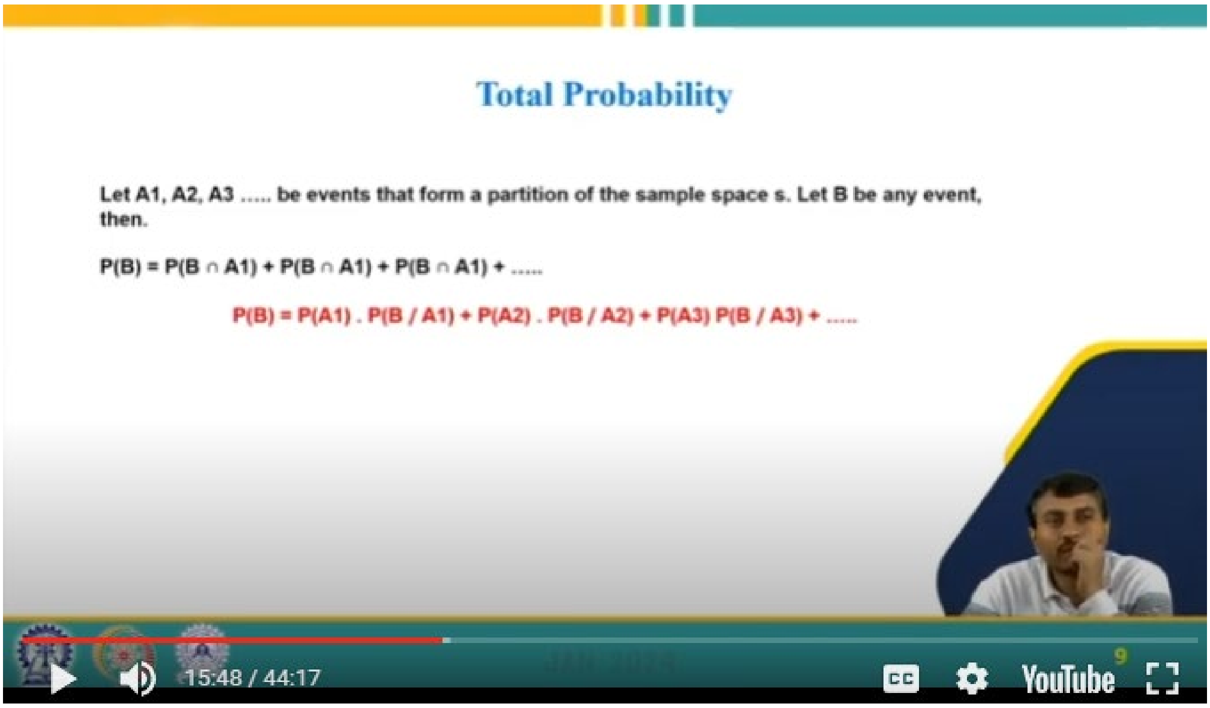


Another essential concept is multiplication theorem okay this multiplication theorem is basically when number of event are occurring and number of event are there there are sequence of event 1 2 3 4 like that and in those event are not dependent on each other okay then also we can apply these independent event concept that intersection of all this event probability of that is basically multiplication of possibility of coming out of that event you can multiply one by one one by one one that that is P A P B P C P D whatever so you can do that so this formula is extendable for all independent event so for example a b c d e f g h i j k is up to the event and these event are independent so intersection of a b c d e f g h i j k is basically multiplication of probability a probability b probability c dot dot dot dot dot probability k so here we have given one example that bag A content three red ball and two green ball bag B content three red ball and five green ball and bag C content one red ball one green four green ball now probability of coming rate probability of coming rate from bag B bag A bag B bag A is basically three by five from bag B it is three by eight from bag C it is one by five so the question is if one ball is drawn from each bag then find the probability of getting red from bag a green from B and red from bag C so green probability of B is basically five by eight green probability coming green ball five by eight and red ball C it is already calculated one by five red ball for the bag a is already calculated three by five so probability of intersection E1 E2 and E3 is basically multiplication of probability E1 probability E2 and probability E3 ok so this is coming around 3 by 40 so this theorem is applicable for all those event that is not dependent on each other so that is why it's called multiplication theorem we are extending the same theorem for various events.

Now another critical concept is total probability this total probability is such that there are event A 1 A 2 A 3 that event that form a partition of the sample space a s ok so let B let B be any event then other event so what is the probability of event B under all these that is the event space it is in it is there in the event space A 1 A 2 and A 3 so this is basically probability of B is equal to probability of A 1 multiplication by probability B under condition of A 1 probability of A 2 multiplication probability B condition of happening a 2 probability of a 3 multiplied by probability of B happening under condition of a 3 dot dot dot dot extendable to finite or infinite number of space or event

$$P(B) = P(A1) . P(B / A1) + P(A2) . P(B / A2) + P(A3) P(B / A3) + \ldots$$

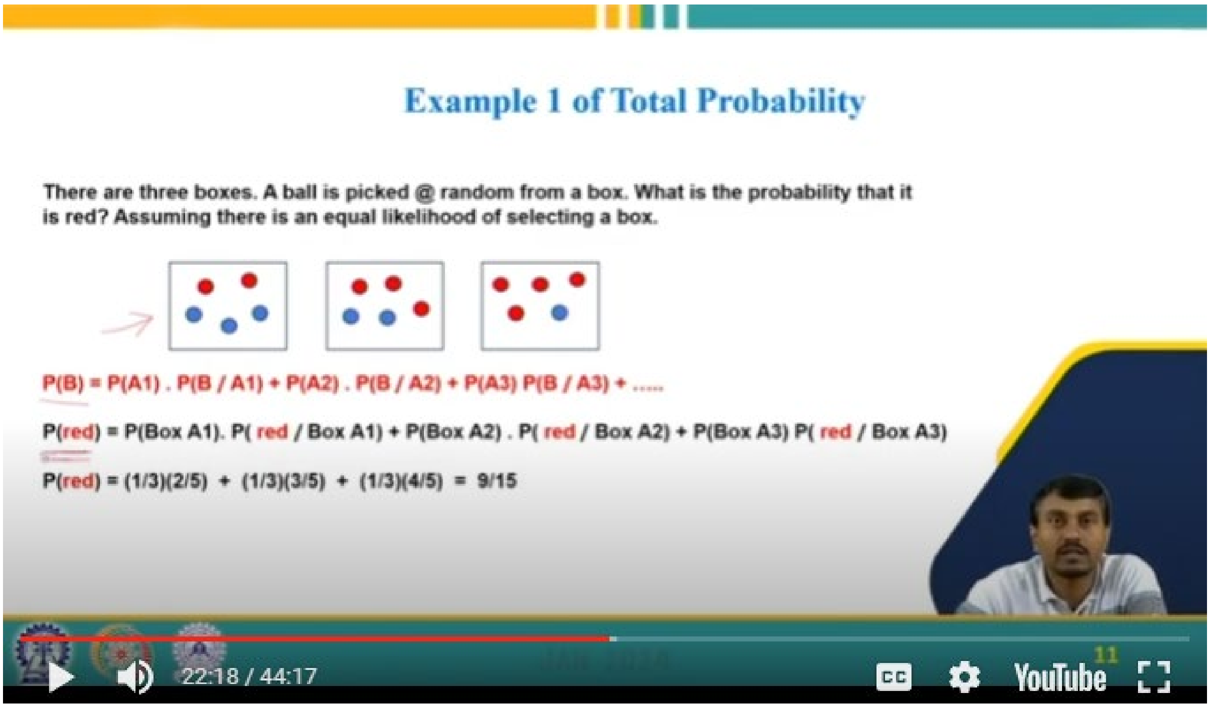
so this concept is very useful when you want to find out precisely what is the probability of happening B and this is similar to the concept probability that we have discussed in the earlier class that is marginal probability ok marginal probability looks after the probability of happening that event under all the circumstances ok for all the circumstances for example if I discuss one event that some task to be completed in time ok and that probability that if there is no disturbances ok no disturbances now disturbances I will tell that if there is no disturbances of happening the completion of the task so that is a probability 0.



9 for example or 0.95 now a 1 type disturbance may delay the completion of the of the of the target of the task so A 1 is the kind of disturbance so probability of completion of the work is for example 0.4 now disturbance type A 2 0.3 the probability of completion under that disturbance probability that that kind of event will happen or complete the task in time under disturbance A 3 suppose it is 0.45 so now you as an engineer of the at the side that you know all these sequence or all this role or all this influence and the and their interrelationship now you want to be pretty sure now under all these conditions what is the guarantee that the task will be completed in time so that based on that completion of the task is basically related to the other event that to be continued and to be related and to be convened through the different task and that will basically start another task that is chain of operation is related so you want to assess that what is the probability of completion of the event under these circumstances so in those kind of situation this total probability concept is very very useful concept

So, let us deal with some of the theoretical assumptions. The total probability theorem is applicable for both dependent and independent events. Yes, we can apply the total probability concept for both dependent and independent events. Another point is that the number of subsets of the samples that form a partition can be finite or infinite; yes, it can be finite, or it can be endless, so the total probability concept is a beneficial concept when there are several operations, or several things influence a particular task to be completed or the particular task to be done so to make the successfulness of that event to occur we you have interrelation of the relation with other consequence or other phenomena so finding the successfulness of that event out of all these complexities when you want to calculate this total probability concept would be very very useful so let us start with one example.

example is there are three boxes and here is the picture and here you can see in the box one we have two red ball and three blue ball okay and box second box having three red ball and two green ball and third box having four red ball and one blue ball okay so in the first two red ball in the second three red ball and on the third that is four red ball now a box is fixed at random the ball is picked from a random box any box I can choose so what is the probability that it is red assuming that an equal likelihood of selecting box so randomly by closing my eye I just pick a ball from any box so if I assume that I do not have any partiality of choosing which box to so let's see what is the likelihood or the chance that you have basically picked a red ball so how to calculate that we have already seen the theorem that is probability B is basically probability of A1 into probability of B under condition of condition that is event happening A1 probability A2 multiplied by probability B under happening of the condition of or event that is A2 and so and so forth so probability of R that is rate picking rate is basically choosing the box A1 probability and in that probability that in that box choosing the box what is a probability of the picking the red ball so that is period under condition of happening box A1 similarly box A2 multiplied by probability of red box to boss A2 and so and so forth so picking the box is equally likely for all probability of picking box A1 probability of picking box A2 probability of picking box A3 is equally likely so all are equally likely 1 by 3 and picking red ball for the first is 2 by 5 picking red ball from the second is 3 by 5 picking red ball from the third is 4 by 5 so you just summing add and multiply then finally it is 9 by 15 so now you are pretty sure that if you conduct this experiment 15 number of times at least 9 number of times you have the probability that you are coming out with a red ball this is a very useful a concept and useful outcome that would be used time to time in the in the different decision making process



Let us see another example a box containing five fair coins five fair coins and there are three unfair coins so unfair coins have the probability of coming head is 1 by 3 probability of coming tail is 2 by 3 for this fair coin the probability of coming head probability of coming tail is half up because it is a fair coin so now if coin is picked any coin out of the 8 total is the number of 8 is there on the box and you toss and find the probability of getting a head from a pick coin so probability of head now depends on probability of picking the fair head and you have picked the fair head so what is the probability of coming head that is half now you pick the probability of unfair coin unfair coin what is the probability and multiplied by of that what is the outcome or what is the possibility that in that particular coin what is the chances of coming heads that is one third so this is basically this so 5 by 8 into 1 by 2 3 by 8 is 1 by 2 is equal to 5 by 16 plus 3 by 24 so this is equal to 48 15 plus 6 is equal to 21 by 48 so if you conduct this experiment for 48 number of times or 96 number of times so it will the it will come head 21 times and 42 times respectively possibly under these circumstances nearly now let us see another example that a task to be completed in time without any disturbances okay that task to be completed probability of a task to be completed for instance probability a we have to calculate what is the success rate now now under this a task to be completed in time without any disturbance without any disturbance is equal to 0.

Nine okay with disturbance probability of completion of the task in time is equal to 0.42 now based on the data you have found the probability of these disturbances that is B probability of these disturbances is 0.45 so you know that 45% there is a chance of these disturbances interfering in the process or this disturbance may come so what is the opportunity that these disturbances will not go for it your B complement is equal to 1 minus PB equal to 0.55 so now if we apply the concept of total probability probability of probability a is equal to probability B under happening probability of A. Now, without disturbances, this means what probability of A under B complement and probability with A by B? So B is the probability of A B and probability of not happening that disturbance of probability B complement multiplied by probability a divided by B complement. Hence, probability B is 0.545 for the B is 0.

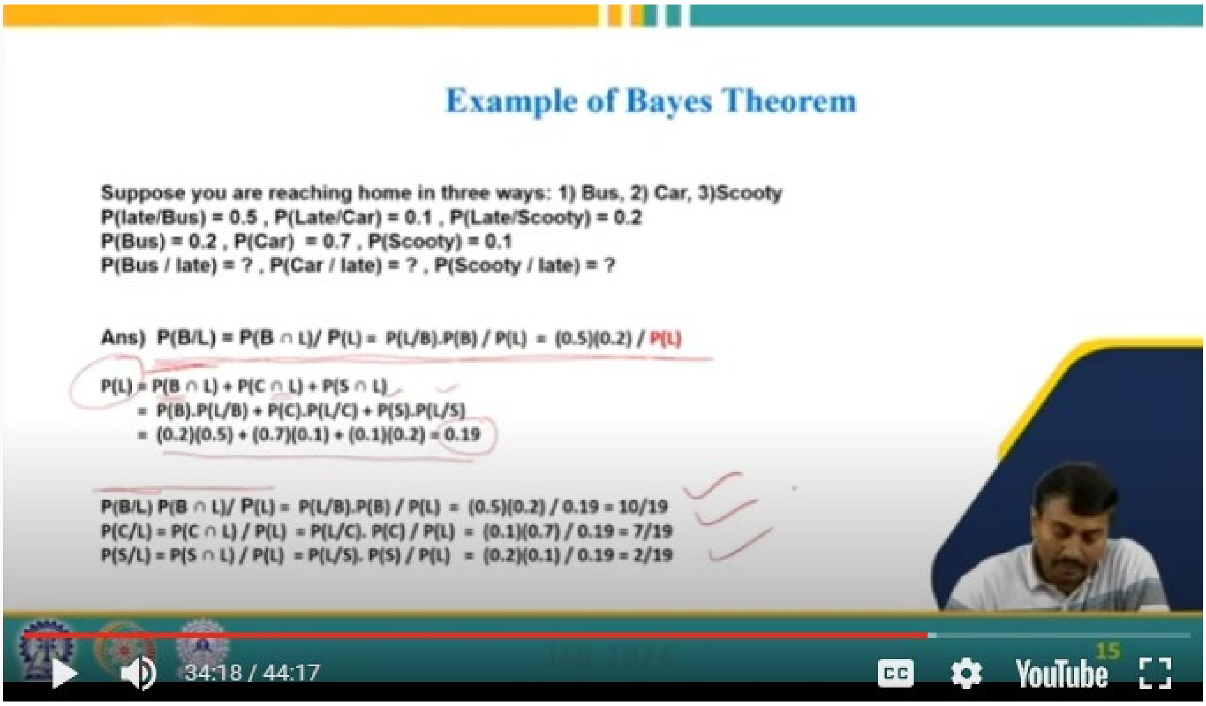
42 plus probability B complement is 0.55, the probability of a B complement equals 0.9, so this is around 0.45. It is 0.189 and 0.

495 equal to 0.684, so the probability of completing the task in time anticipating the disturbance may occur so it is the total probability that 68% is a chance 68.4% is a chance that this task will be completed in time, so this kind of estimation is very well required and very very much required in the mining

Another essential concept is the Bayes theorem. There are many applications of this Bayes theorem Bayes theorem says that the probability of a under condition happening of B is the probability of A intersection B divided by probability B or the probability of A multiplied by the probability of happening B under the condition of A divided by B so this is the theorem probability of A under condition of happening of B is a probability of A then multiplied by probability to happening B under condition of a divided by probability of B so this concept will be beneficial time to time so let's see what is the utility. Now, this is an efficient example that in metro cities or in a town where you are living, there are fair chances that several times it is observed that you are coming late or you are reaching home late or you are coming home in time so for a more extended period if the experiment is conducted. Hence, these are the few assumptions we have taken so it is a realistic situation we can simulate. Let's see that. So here you are reaching home in three ways: there are three possibilities through the bus, the second is through the car, and the third is through the scooter. Scooty, where scooty is available, where the vehicle is available, and several operators are applying in different metro cities, and the bus is also there.

The probability of coming through the bus late is 0.5, and the likelihood of taking the car is only 0.1. The probability of late taking under scooty that condition is 0.2. Now, the possibility of picking up the bus is 0.

The probability of picking a car is 0.7, and the probability of choosing a scooter is 0.1, so for a hundred cases, you have found that you mostly prefer 70% car, 20% bus, and only 10% shooty. Now we have to calculate if you assume there is a late okay and you are coming through the bus. You are basically what is the probability that the bus you are picking up and you are late, okay, and the car and the late and the P and the scooty, so then you are applying the same Bayes theorem here that is the probability of late we want to calculate. So, for the probability of late, we want to calculate the probability of bus intersection left and probability of C intersection L probability S intersection L S is scooty C for car B for bus so probability of B intersection L means what probability of picking B and under condition probability of happening late under B that is bus probability of picking C that is car and probability of late in the vehicle and probability of scooty and probability of late in car so multiplied that probability of late is coming 0.



19 okay so you can pretty well assume under these condition that what is the possibility that you may come late assuming all these things so in 20 to 19 percent chances that you are basically coming late okay under all these conditions so these basically can be extended this theorem may be extended so probability of bus late and probability of bus under condition late bus probability of B and L compliment then slash P late this can be calculated like this 10 by 11 19 17 by 19 and 2 by 19 okay so these are very useful concept and you can very well basically use this concept when number of event basically interrelated with each other total probability theorem that we have discussed and Bayes theorem is also a helpful concept probability distribution we will deal with that in later so it basically described the likely or different outcome in a random experiment okay so example is uniform distribution equal probability of all outcome or normal distribution that is built separate card or commonly many it basically represent the natural phenomena particularly in our Institute marks is fitted in a bell separate card to our the grade okay in an average bill separate card so there are very useful application of these distribution so now

Let us discuss some of the features of how this probability will be helpful in the mining industry. The first is resource estimation. Resource estimation is a critical task for mineral extraction as well, and taking the project or going ahead through this project depends on the confidence that this small amount of assured resource is there that calculation required a vast amount of applications of geostatistics kriging methods probabilistic model to interpolate and extrapolate the possibility of the ore body. It sprayed on the geological deposit okay, so it has vast applications.

Second application is risk assessment because we all know that mining is a task that is associated with many event that you are basically attacking the nature okay and you are working with the nature and you are working with different kind of machinery you are working with different type of workforces and there are n number of tasks going on simultaneously okay so in a mine there are a lot of uncertainty in the in the in the in the geology key geology that where the fault fold may appear because it is very difficult to assess very clearly where the fault might be where the joint might be you suddenly you basically face that so that is basically probabilistic event and that is fluctuating market is also fluctuating and this is very very applicable for the mineral industry particularly gene copper for the precious minerals market fluctuation basically basically influence the production rate or production volume in the mine because ultimately you want to gain profit out of it so there are risk as well as the operational hazard anytime failure may happen some kind of movement some kind of root fall or damage in the machine that may occur so quantifying all these and and you are basically generating a scenario or creating a scenario that under considering all these what is the possible chances of happening these so for that risk assessment is very useful.

That is, the safety analysis of human safety is critical, and minor safety is also essential. So, based on the historical data, several data are generated over time, and you have been operating the mines for over a hundred years. You have experience with that under different mining conditions, so based on that, you can assess the chances of the accident under this kind of situation before basically deploying the miners or before basically actually doing the work you are doing with the idea that these are the chances and these are the potential so accordingly you can design your protocol you can create your steps that might be helpful to avoid or avert that kind of hazards in the mind.

Another application is for equipment reliability. You have seen that in this automation, a vast number of machines are used for different applications. Giant machinery is giving enormous benefits to the mining industry. Hence, equipment reliability is an issue, so you require time to maintain the machines. You do not know when the machine will break down, so you do have to estimate the downtime and the probability of these machines' breakage occurring. The machine, you should be pretty sure about the performance of the machine, so calculating that is a mechanical part, and you have to predict for a continuous period. Based on that, do you have to schedule the machine to be deployed at what site, or do you have to keep a reserve of that machine? So, to estimate equipment reliability, you need the help of probability.

Another is the environmental impact assessment because we know that by the mining process we are basically damaging some component and some part of the environment and there is a restriction that we must take some amount of action to basically refresh the environment or we do something so that it clean the environment and so that it basically mitigate the negative effect that mining is created on the environment so you have to make different kind of models and how these mining is going to impact the environment and what way it is impacting and the ecosystem so you have you need to predict and you have to analyze so there is a long process chain and you are basically employing some devices some mechanism some kind of steps you are going to take to reduce the effect on the environments and how they are machines that you have deployed you have taken some control step and how those control step are practical in basically addressing the environmental issue that you need to calculate and for that you need use of probability theory.

Another is the exploration of decision-making. Exploration is an instrumental part of the mining process chain; exploration is necessary to get an idea of how small a reserve we have, and for that, we require a vast amount of statistics and probability. Here, one is the Bayesian inference, and there are several engineering calculations related and based on the heterogeneity of the deposit and the different ways the deposit occurs sporadically and the ore body based on how much spread likely the chances that there a possibility of other ore body and nearby vicinity so calculating that there are the massive application of this probability theorem in the mining industry great control on the running in a run of mines or the mines is running controlling the grade and you are maintaining the grade over some time so that yeah there is no complaint from the part of the customer to whom you are supplying the ore so

For example, you have to check that maybe some foreign material that does not required or some of the customers have already specified that it should not go beyond that percentage, so you have to check from time to time so proper blending is needed from time to time adequate blending in the processing plant or the production chain mixing should be ensured so and on the from mineral processing plan the final output that is generated also that complies with the excellent control mechanism so to establish those set up you need data and based on that you run the process, and you stick to that standard and maintaining that you require the help of probability.

Financial analysis there is a vast scope and these financial model are basically relying on the probability theory how the market changes how the revenue generated what is the possibility of making profit under all these conditions what is the what is the psyche of the stakeholders under these these situations and you have to address all that and finally you have to you have to run the business profitably so overall there is a vast applications of these probability theorem and statistics in the whole mining process chain and the mining project so I hope the students and you all will assess the importance of this particular theorem and the subject and you will go through and we will subsequently go through in the details of this specific part in the subsequent lectures so these are the differences so in this relation we have discussed the independent event what is independent event the multiplication theorem the total probability based theorem and also we have highlighted the applications of this probability in the mining industry thank you