Mine Automation and Data Analytics

Prof. Radhakanta Koner

Department of Mining Engineering

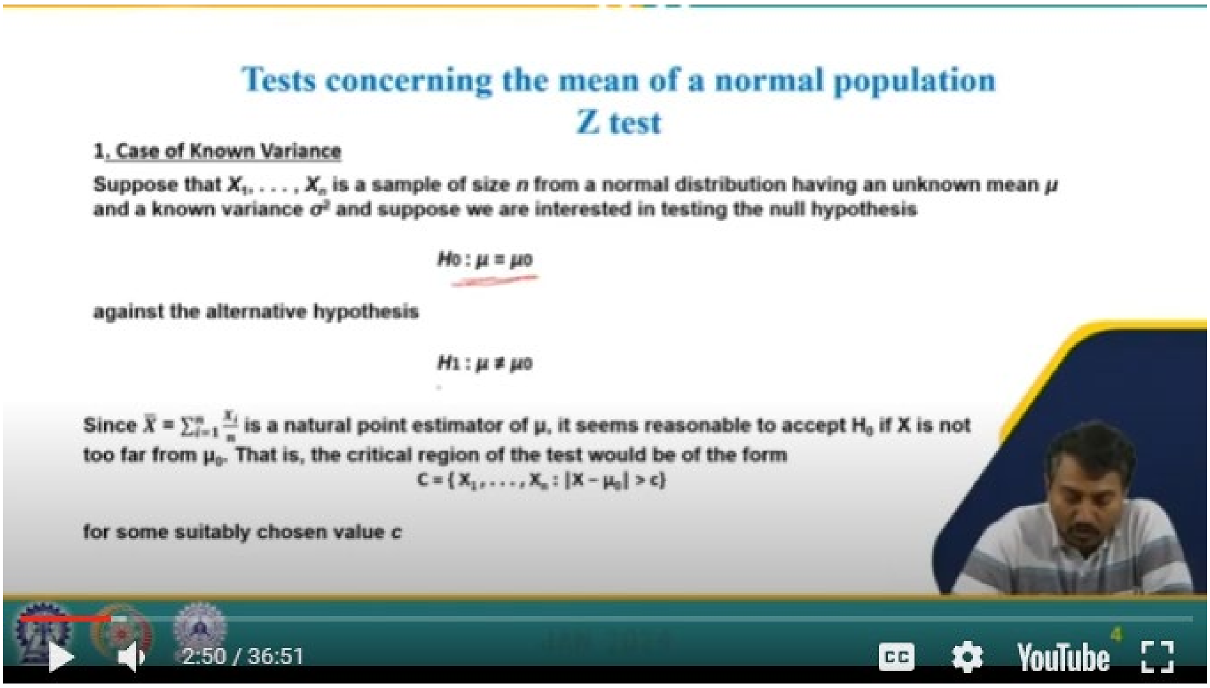
IIT (ISM) Dhanbad

Week - 9

Lecture 42: Hypothesis Testing - II

Welcome back to my course. Today, in this lesson, we will discuss hypothesis testing in continuation of what we discussed in the last class. This part focuses on the Z-test, one of the most popular hypothesis tests. So, in this lesson, we will cover the Z-test concerning the mean of a random average population, and we will discuss four examples of the Z-test and how we reach whether to accept the null hypothesis or reject the null hypothesis based on the data that prevails on the outcome of the experiment. So, the concept of the test concerning the mean of an average population is the Z-test. So for the cases of the known variance, suppose the sample comprises X1, X2 up to Xn, the sample size of n from a normal distribution having an unknown mean that is mu and a known variance sigma square, and suppose we are interested in testing the null hypothesis, H0 is equal to mu is equal to mu0.

Against the null hypothesis, the null hypothesis against the alternative hypothesis of H1, which is mu not equal to mu0. So, from the observation, the X bar is found out from the natural data point of the experiment's outcome. So X1, X2, X2 up to Xn divided by n for the value of i equals 1 to n. So, this is a natural point estimator of mu, and it seems reasonable to accept the null hypothesis, which is H0, if X is not too far from the mean, which is mu0.



Okay, mu0 is the original assumption, which is the null hypothesis. We are calculating from the data we found, and data is basically of i is equal to 1 to n Xi divided by n. This is an X-bar observation. So if this X bar is just a short distance from mu0, the X bar minus mu0 is the difference if I take the mod. The X bar is not too far from mu0 if this difference is minimal. We will accept the null hypothesis because the data consistently shows that it is near mu0. So, this particular difference at the level at which we will take this difference is insignificant. It is X minus mu0 mod.

So this is the critical region of the test that would be X minus mu0 mod greater than C for some suitably chosen value of C critical region. This comes from the alpha. We have already discussed the alpha type 1 and type 2 errors minus beta. So, we will primarily focus on this relation, basically the alpha. So, under these circumstances, if we desire the test to have a significant alpha level, it is our assumption. We must determine the critical value C that will make the type 1 error equal to alpha. C must be such that the probability of mu0 X bar minus mu0 mod is more significant than C is equal to alpha.

So here, p mu0, we write that the preceding probability will be computed assuming that mu equals mu0. However, when mu is equal to mu0, X will be normally distributed with mean equal to mu0 and variance of sigma square divided by n. So Z is defined by X bar minus mu0 divided by sigma divided by root over n. So this root over n will come on the top. So root over n X bar minus mu0 divided by sigma. So we will have a standard normal distribution again this Z.

$$P\{Z > z_{\alpha/2}\} = \alpha/2$$

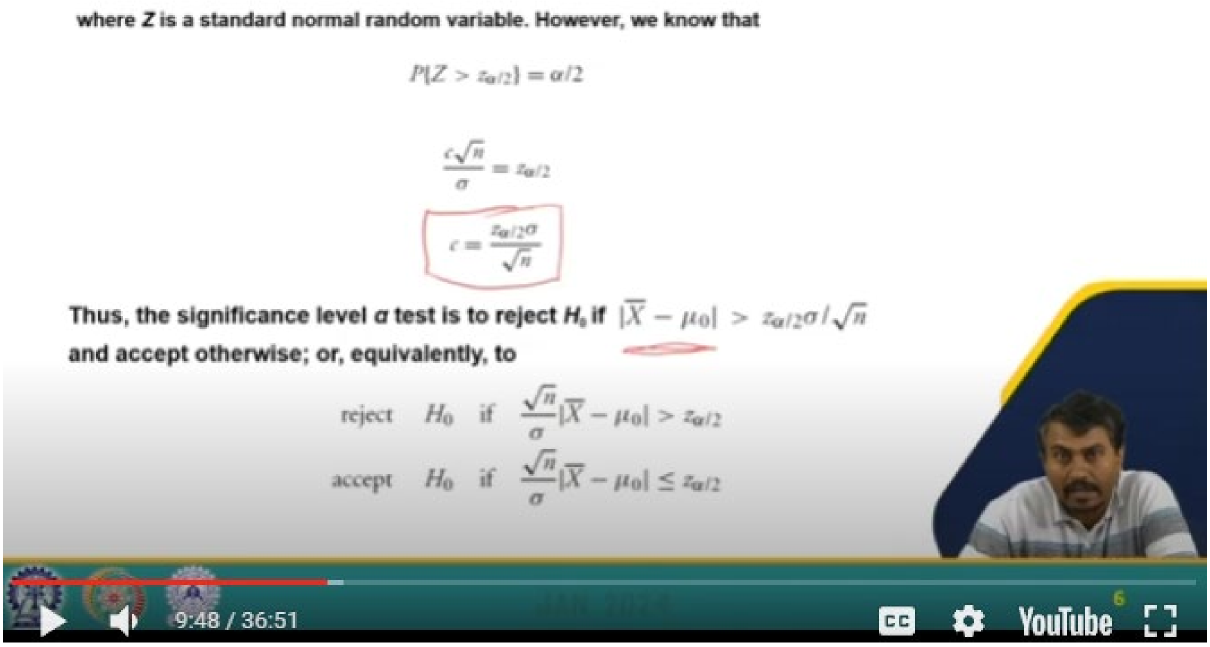
$$\frac{c\sqrt{n}}{\sigma} = z_{\alpha/2}$$

$$c = \frac{z_{\alpha/2}\sigma}{\sqrt{n}}$$

We will define the alpha for Z greater than C sigma n. This is C, this critical region C. C root over n divided by sigma is a mod of Z more excellent than C root over n divided by sigma. This probability is equal to alpha or 2 of Z, which is more significant than C root over n divided by sigma, which is equal to alpha. So here, the Z is the standard normal random variable, and this Z probability of Z greater than Z alpha by 2 equals alpha by 2. So this C of root over n divided by sigma C root over n divided by sigma is equal to Z alpha by two, or if we take C on the left side only and transfer other values on the right side, then this expression will look like the critical region C is equal to Z alpha by two sigma divided by root over n. So thus, the significant level alpha test is to reject the H0 that, is the null hypothesis if X bar minus mu not mod is more important than Z alpha by two into sigma divided by root over n and accept otherwise or equivalent to or reject null hypothesis if root over n X bar minus mu 0 divided by sigma is more significant than Z of alpha by two and accept if root over n divided by sigma mod of X bar minus mu 0 is less than equal to Z alpha by 2.

$$\text{reject} \quad H_0 \quad \text{if} \quad \frac{\sqrt{n}}{\sigma}|\overline{X} - \mu_0| > z_{\alpha/2}$$

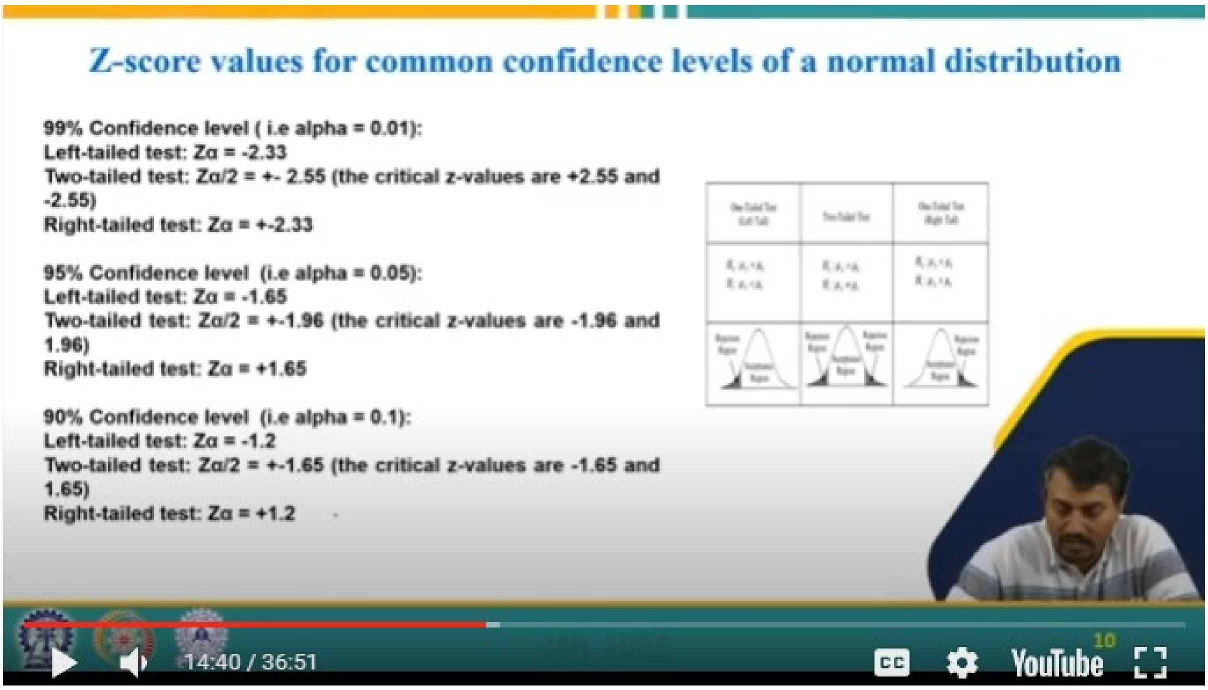
$$\text{accept} \quad H_0 \quad \text{if} \quad \frac{\sqrt{n}}{\sigma}|\overline{X} - \mu_0| \leq z_{\alpha/2}$$



So, this is a better mathematically formed statement. So this is where we superimpose the standard normal density function here, and here on the left side: Z alpha by two. Here, on the correct side, Z alpha by two is represented by this value root over n divided by sigma into X bar minus mu 0. So this is the density of the test statistic of root over n X bar minus mu 0 divided by sigma when the null hypothesis is true. So, here, we have to determine the critical value, Z alpha, by 2. So, what is the essential value of Z alpha by 2? For example, assuming a two-tail test, a 95% confidence level for alpha equals 0.05. So, a two-tail test is shown here on both sides. The tail is a standard normal distribution with 0 and 1, 0 is the mean, and variance is 1, and the critical value is minus 1.96 to plus 1.96. Outside is basically, the outside is to be rejected. So, a 95% confidence level means that 5% of the area under the curve is considered the critical region.

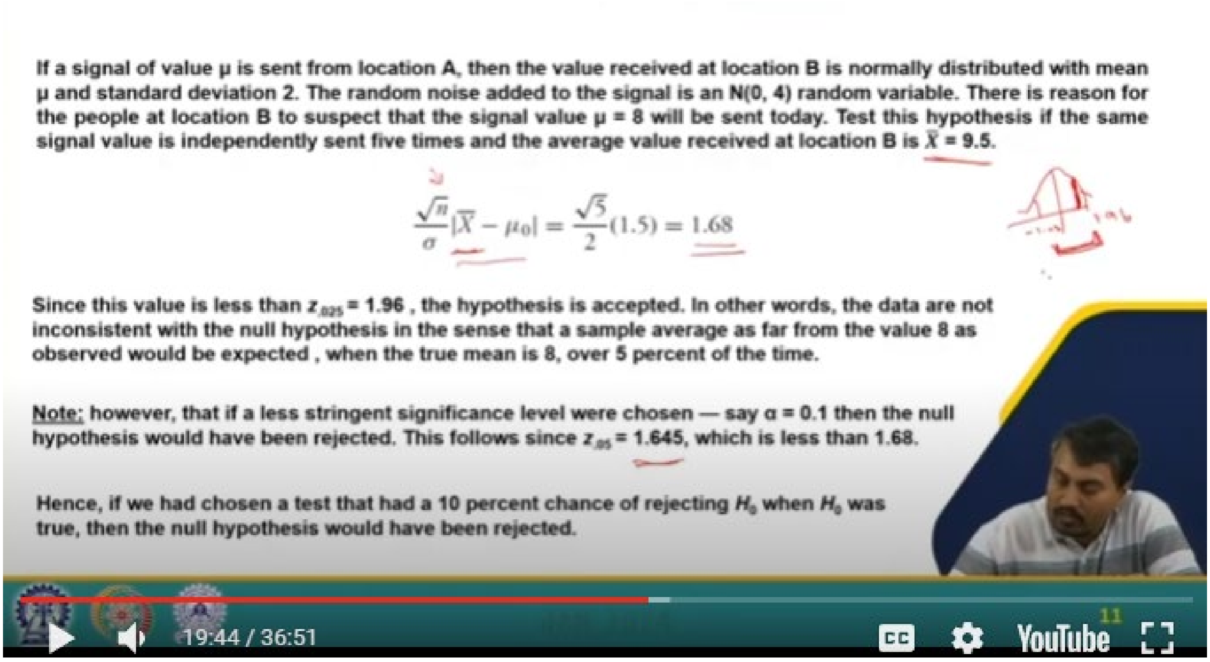
So 95% is under the curve, 5% is not under the curve that is a critical region, the rejection region, and since this is a two-tail test, half of 5%, 2.5% value would be on the left, 2.5% value on the right tail. So, looking up the Z score associated with 0.025 on the reference table, we find Z equals 1.96 plus minus—so plus 1.96 critical value for the right tail and minus 1.96 for the left. So, the crucial 95% confidence level value is Z alpha by 2, equal to plus minus 1.96. So this is again for the one-tail test left the value to be rejected, the null hypothesis to be rejected if the value is less than this area. Up to this, we will accept; after this, we will reject; up to this value, we will accept; beyond this, we will reject this side, as well as the left side for the two-tail test. For the right tail or tail, that is a one-tail test. If the value is beyond this value, we will reject it. So, this shaded region depends on the significance level we assume to be alpha. Based on the size alpha, this rejection region's size changed.

So here is a straightforward example with the Z score value for an ordinary confidence level of the normal distribution. For the 99% confidence level, alpha equals 0.01, the right tail is 2.33, plus the left is minus 2.33, and for the two-tail, it is minus 2.55. The 95% confidence we have seen for the two-tail is plus minus 1.96; for the left tail test and the right tail, it is minus 1.65 and 1.65 for the alpha 0.05. For the 90% confidence level alpha 0.1, Z alpha by 2 for the two-tail test is plus minus 1.65; for the left tail test and the proper tail test, Z alpha is minus 1.2 and 1.2, respectively. So, this changes the region of rejection based on these alpha values.



So, let us deliberate on this using one example. If a signal value of alpha is sent from location A, then the value received at location B is usually distributed with a mean mu and standard deviation 2. The random noise added to the signal is an N04 random variable. So, there is a reason for the people at location B to suspect that the signal value mu equals what will be sent today.

So, the signal value is expected to be 8, and mu equals 8. So, to test this hypothesis, if the same signal value is independently sent five times and the average value received at location B is found, the x bar is 9.5. So, what is the difference between the x bar and the mu? This is 9.5 minus eight is 1.5, the sample size is five, the root is over five, the standard deviation is two, and the sigma is equal to 2. So this is found to be root over five into 1.5 divided by 2. It is calculated as 1.68. So, the value that we saw under this assumption is 1.68. So, since this value is less than 1.96 for the Z of alpha by 2.05, which is 0.025, the hypothesis is accepted because it is 1.96 minus 1.96, plus 1.96. So 1.68 is here. It is in the acceptable region. So we have to take. So, we accept the null hypothesis. So, in other words, data are not inconsistent with the null hypothesis because the sample average is as far from the value eight as observed would be expected.



So this variation from this place to this place is expected. This region is an acceptance zone. So when it is true eight over 5% over the time, it is accepted. So here we want to note. So however, if a less stringent significance level were chosen, for example, alpha equals 0.1, the null hypothesis would have been rejected. Why? For that 0.1 that is 10%, this side as a less that is 5% finally. So Z of 0.05 Z alpha by 2, it is 1.645. So 1.645, this is 1.645, this is 1.645 minus this is plus, and our value is 1.68, 1.68. So this is outside, or you can say this particular zone value is less than 1.68. So we have to reject. So, we chose a test with a 10% chance of rejecting the null hypothesis, which is H0. When H0 was true, then the null hypothesis would have been denied.

So, the correct level of significance is fundamental to choosing a specific case or a specific situation, and this depends on the circumstances, the boundary conditions, mining conditions, the field conditions, and the level of involvement of this test result with the cost effect; it also basically depends. So, there are various factors influencing the choice of alpha value. So, for instance, if rejecting a null hypothesis that is H0 would result in a significant cost that would thus be lost if H0 were true, then we might be pretty conservative and choose a significance level of 0.05 or 0.01.

And if we initially feel strongly that H0 was correct, then we would require very stringent data evidence to the contrary to us to reject H0. This is why we would set a very low significance level in this situation. So, let us explain this concept with a live example, as we promised to show you four examples. Let us start with the first example. So here, the first example is a manufacturer claiming that the mean weight of their product is 500 grams, and the mean weight of the product is 500 grams. A random sample of 36 products was selected to test this claim, and their weight was recorded. The sample mean weight was 495 grams, with a standard deviation of 10 grams. Test the manufacturer's claim by assuming the weights follow a normal distribution using a significance level 0.05. So, the null hypothesis is the technical steps we have to define and state the hypothesis. So here, the null hypothesis H0, the mean weight of the product is 500 grams, and the alternative hypothesis H1, the mean weight of the product is not 500 grams. So the H0 is mu equals 500, H1 mu0 equal to 500. So, it is a total test we have to conduct. So this is the region of acceptance beyond this value we must reject.

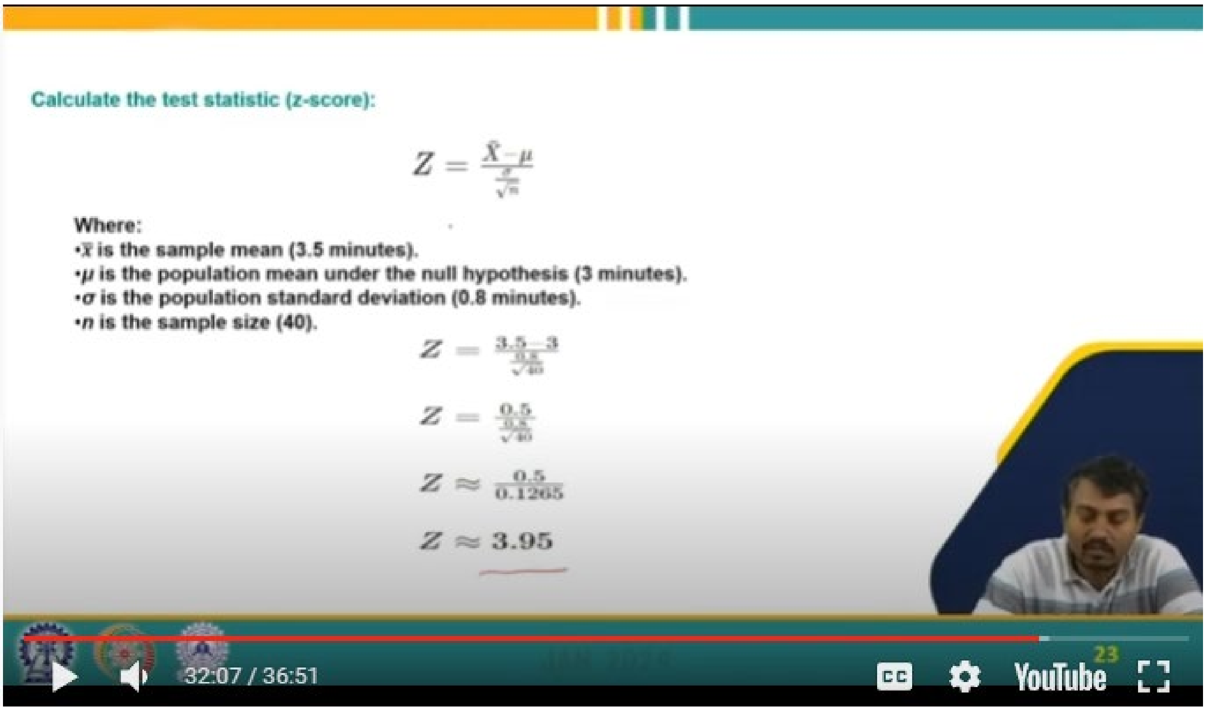
So, the level of significance alpha is 0.05. So, let's calculate the test statistic, which is the Z score. So Z equals X bar minus mu divided by sigma divided by root over n. X bar is the sample mean we calculated from the 36 observations, and mu is the population mean under the null hypothesis. The assumption is that the mean weight of the sample or the product is 500 grams, sigma is the population standard deviation, and n is the sample size.

Here, we have chosen 36. So, the X bar is 495, mu is 500, sigma is 10, and n is 36. If we calculate that X bar minus mu divided by sigma divided by root over n, it comes minus 3. Now revisit this particular table for the alpha of 0.05 double test, two-tailed test, plus minus 1.96, which is the critical zone beyond which we have to reject. So, it is seen that the technical step is now to determine the critical value. So, since it is a two-tailed test, the critical Z values are minus 1.96 and 1.96 at a significance level of 0.05. So, the decision rule in the next step is that if the absolute value of the Z score is more significant than 1.96, we reject the null hypothesis, and here it is. Now, let us make a diffusion. The calculated Z value minus three falls in the rejection region less than minus 1.96. So, we reject the null hypothesis. So, the conclusion is the last step; since we reject the null hypothesis, we have sufficient evidence to conclude that the mean weight of the product is not 500 grams. So therefore, based on the sample data, there is enough evidence to suggest that the manufacturer's claim is incorrect at a 0.05 significance level. Let us see the second example. The second example here is that the manufacturer claims that the average lifespan of their light bulbs is at least 1000 hours, and you believe that the average lifespan is less than that.

To test this claim, you collect a sample of 50 light bulbs and find that the average lifespan is 980 hours, with a standard division of 40 hours. Now, you want to test whether the average lifespan is significantly less than 1000 hours at a 5% significance level. So again, the same step, so it is one tail left tail test because the value is less than the particular value that we are assuming or we are stating as a null hypothesis. So, the null hypothesis is that the average lifespan of the light bulb is at least 1000; that is, H0 mu is equal to 1000, and the alternative is that the average lifespan of H1 mu is less than 1000, and the X bar is 980. We have found that the mu is 1000, the sample standard division is 40, the sample size is 50, and the significance level alpha is 0.05. So, if you determine the Z status Z score, it will be Z, which is nearly equal to minus 3.54. So it is beyond that range. Since it is a left tail test and the significance level of 0.05, we find the critical value Z from the standard normal distribution Z at the alpha of 0.05 is 1. minus 1.645, and our value is minus 3.54, which is very much beyond this particular area. So, it is lying in the rejection region. So make a decision.

So, we reject the null hypothesis since the calculated value Z minus 3.54 is less than minus 1.645, the critical value for a 5% significance level. So, the conclusion is that there is enough evidence to suggest that the average lifespan of light bulbs is significantly less than 1000 hours. Example 3. Example 3 is a company claiming that the average response time for their customer service hotline is more than 3 minutes. It is a real case problem; many of you are in contact with customer care so that we can check it.

So, do you believe the average response time is longer than that of the company claiming 3 minutes? So, to test this claim, you collect a sample of 40 calls to the hotline and find the average response time is 3.5 minutes with a standard deviation of 0.8 minutes, and you want to test whether the average response time is significantly greater than 3 minutes at a 5% significance level. So, the null hypothesis is H0, which is mu, equal to R3. The alternative is the average lifespan response time for the customer service hotline is greater than 3 minutes, is H0, H1 is mu greater than three and let us choose a significance level of 0.05 5% level, and it is a proper tail test one tail test of the right because the value is significantly you are going to check whether the value is substantially greater than 3 minutes. So Z equals X bar minus mu divided by Sigma divided by root over n, which is found to be 3.95. X bar is 3.5, the claim is 3 minutes mu, the standard deviation is 0.8, and the number of samples we collected is 40. So, Z is equal to 3.95. And 3.95 is well beyond the critical region 3.5. Here, it is basically beyond the acceptance region, and it is false under the rejection region.



So, since it is a right-tail test with a significance level of 0.05, we find the critical Z value from the standard normal distribution table at alpha is equal to 0.05, which is 1.645. It is beyond that. So, since the calculated value Z 3.95 is more significant than 1.645, we reject the null hypothesis. So, there is enough evidence to suggest that the average response time for the customer service hotline is significantly greater than 3 minutes, which is the last example of today's lesson. An Educational Institute claims that the average score of its student on a standardized test is 75. A random sample of 50 students is selected, and their average scores are recorded.

The mean score is 72, with a sample standard deviation of 8. Let us test the institute's claim at a significance level of 0.01. So the null hypothesis H naught is mu is equal to 75, and the average score of the institute student is 75. Alternative hypothesis H 1: the average score of the institute student is not 75.

So, H1 mu is not equal to 75. An expert is given 72, mu is given 75, a sample standard deviation of eight is given, the size is 50, the significance level is 0.01, and Z is found to be minus 2.65. 0.01 here, it is two minus 2.65. This is 2.55 in the region, or 2.57 or 58.

So, since it is a two-tailed test alpha at 0.01, the critical value is plus minus 2.58 from the Z table, and the decision rule is that if the absolute value of the Z score is more significant than 2.58, we reject the null hypothesis. So, our value is 2.65, so the absolute value of the Z is false in the rejection region greater than 2.58, so we reject the null hypothesis. So, the conclusion is that since we reject the null hypothesis, we have sufficient evidence to conclude that the average score of the institute student is not 75, which is a significance level of 0.01.

Therefore, based on the sample data, enough evidence suggests that the institute's claim of an average score of 75 is not supported. So, you have seen the hypothesis test using the Z test and the examples we have illustrated. We hope to deal with more examples for the other hypothesis testing, which will clarify the significance of this hypothesis testing in engineering science and applications. These are the references, so let me conclude in one sentence that we have discussed the Z test hypothesis testing with detailed examples. Thank you.