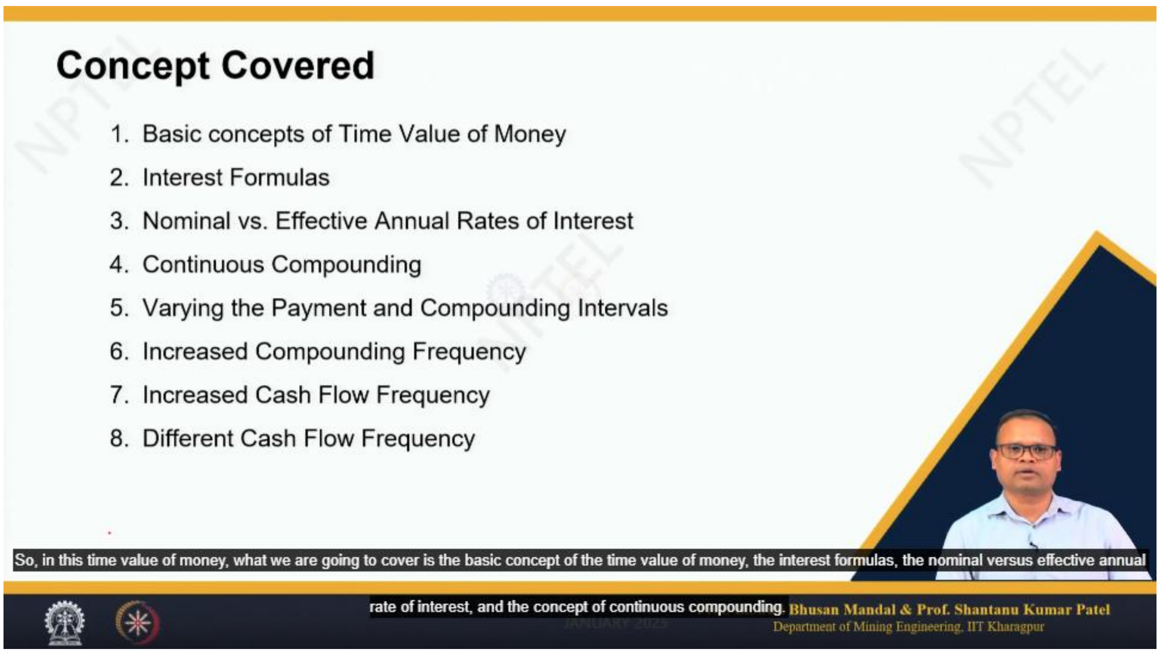
MINERAL ECONOMICS AND BUSINESS

Prof. Shantanu Kumar Patel

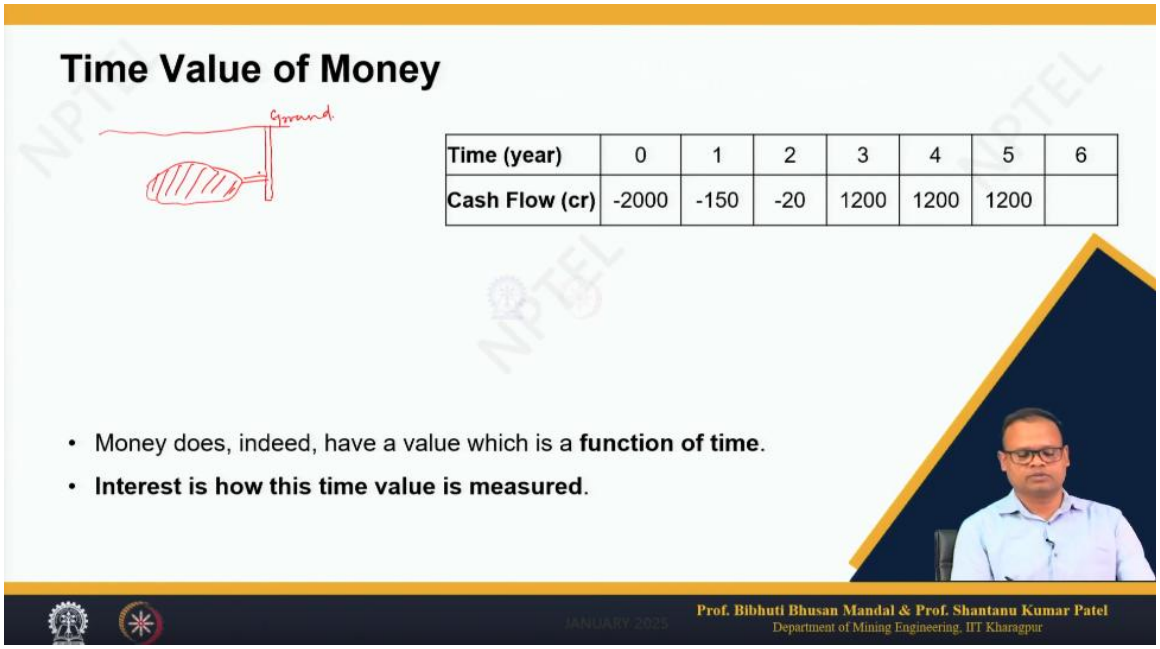
Department of Mining Engineering

IIT Kharagpur

Lecture 22: Time Value of Money - 1



Hello everyone, and welcome again to this course on Mineral Economics and Business. So, this is our lecture number 22, and this is the first part of the time value of money. So, in this time value of money, what we are going to cover is the basic concept of the time value of money, the interest formulas, the nominal versus effective annual rate of interest, and the concept of continuous compounding. And then we are going to look into different problems related to varying payment and compounding intervals, the increase of compounding frequency, the increase in cash flow frequency, and the different cash flow frequencies. So, today in this lecture, the first lecture of this time value of money, we are going to cover the first two parts here up to the interest formulas.

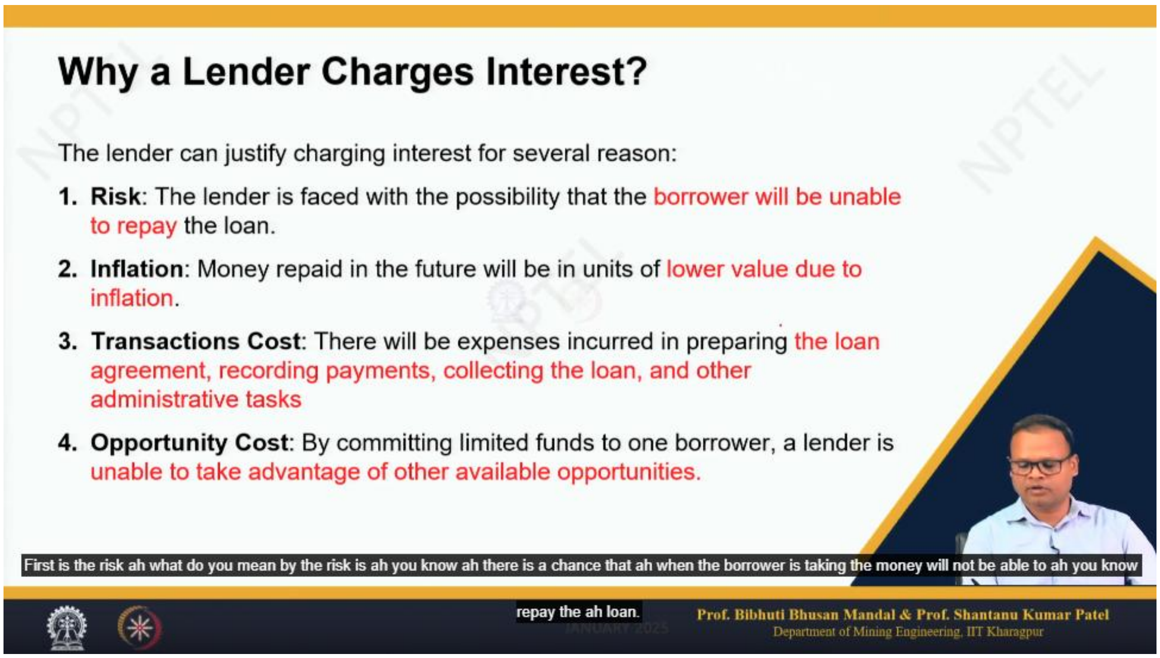


So, let us say we have a mineral deposit at a depth below the ground. So, this is our ground surface, and the deposit is here. So, in that case, to get this mineral out, either we can go for the underground method like a shaft, we can make a shaft and then enter this ore body. So, this is our underground method, or maybe what we can do is go with the open-cast or open-pit method to get our mineral from the ground.

So, in these cases, what we have is, if you want to plot the time versus the cash flow, the cash flow means it can be positive if you are getting money out of the mine, or maybe it can be negative when we are investing some amount. So, if you see, because of the development cost either for underground or open-cast, there is a huge cash flow to the project, which is why we have minus 2000 at time equal to 0 here, and the first year it will further get reduced. This is just an example, let us say, minus 150 in the second year, it becomes let us say minus

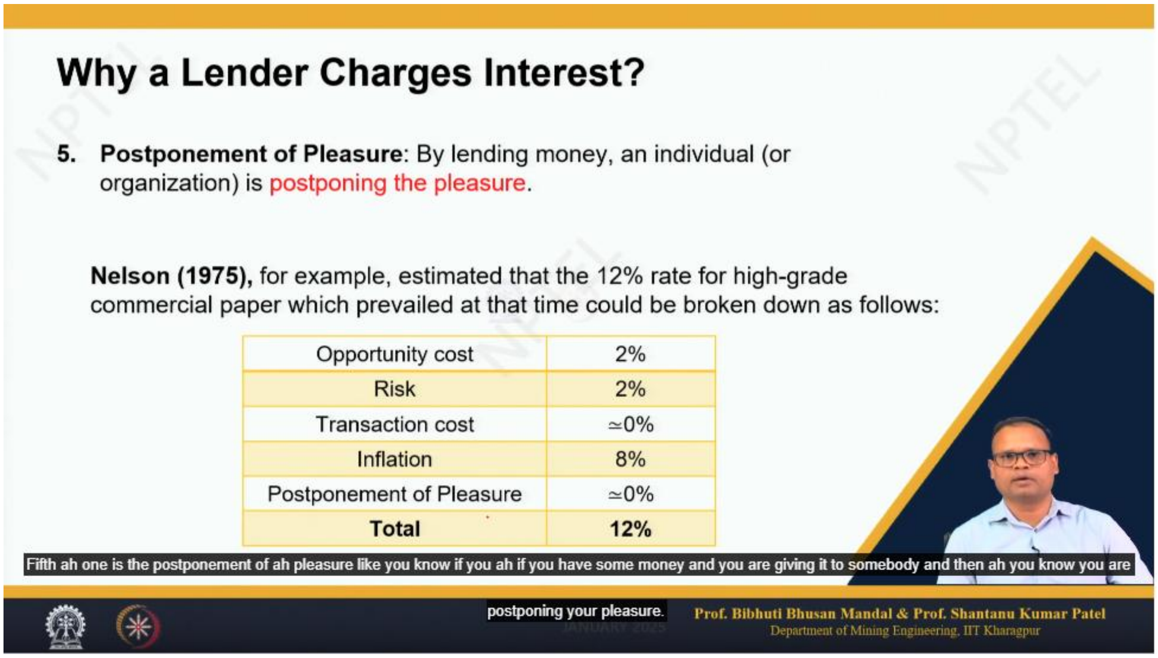
And from the third year what we are going to get is ah you know some some ah money out of this ah ah the the mine. And and fourth year ah again we are going to get money let us let us say the third year is ah 1200 crores and fourth year is ah again 1200 crores, fifth year is 1200 crores and this goes on. Now, if you see you know this the the amount of money this 1200 ah crores that we are going to get ah has a present value at t equal to 0 which can be x. And ah at the the amount of money that we are going to get in in our fourth year and let us say this present value of this money at t equal to 0 is y. So, ah this x and y ah they will not be equal Or you know we can say that this money does indeed have a value which is a function of time and the interest is how this time value is measured. So, ah now about the interest ah you know the lender can justify charging interest for several reasons ah.

First is the risk ah what do you mean by the risk is ah you know ah there is a chance that



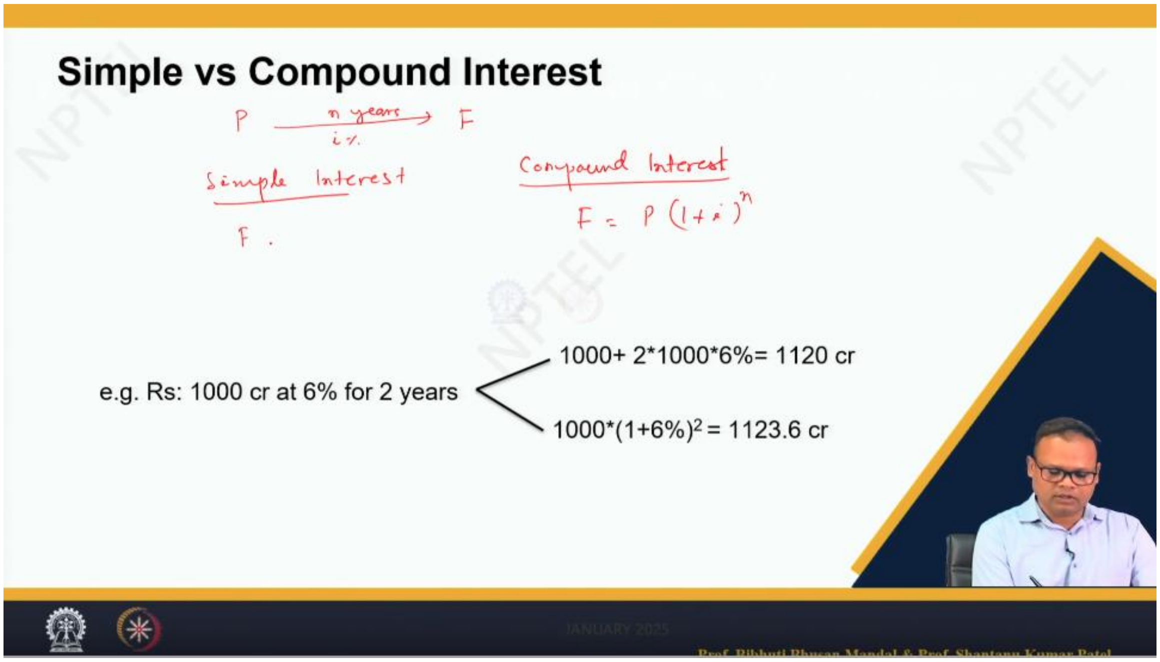
ah when the borrower is taking the money will not be able to ah you know repay the ah loan. So, there is a component of risks ah. Second thing is ah the inflation So, the money repaid in future will be in units of lower value due to ah inflation. So, he can ah let us say the inflation is ah you know 6 percent.

So, he will ah you know charge something extra more than this 6 percent ah you know so so so that ah you know he will be in benefit. Third is the transaction cost you know this they you know ah it is because of this ah different agreements ah the ah you know recording of payments ah collecting the loan and other administrative tasks ah you know the the lender put some kind of you know interest ah for his ah money he is going to lend. Fourth one is the opportunity cost by committing limited fund to one borrower a lender is unable to take the advantage of other available opportunity. So, you know if he can give the money to x he cannot give it to you know y. So, so that opportunity is lost.



Fifth ah one is the postponement of ah pleasure like you know if you ah if you have some money and you are giving it to somebody and then ah you know you are postponing your pleasure.

So, this Nelson in 1975 gave an example where he said that the the interest rate should be 12 percent for a particular case where he said the opportunity cost is around 2 percent and the risk is 2 percent. Transaction cost is almost 0, the inflation is 8 percent, and postponement of result pleasure is close to 0. So, the total interest rate becomes 12 percent. Now, for this time value of money problem, let us say we deposit some principal amount P for n years with an interest rate of i percent, and then let us say the final amount is F.

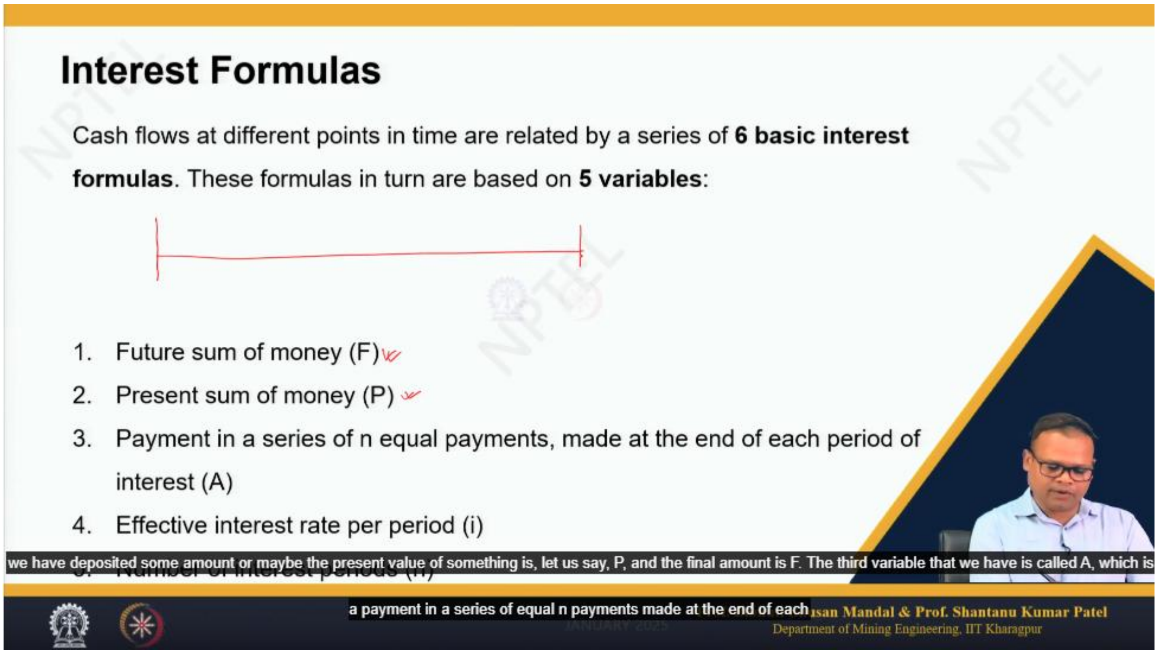


So, in that case, how much F will be depends on whether we are using simple interest or compound interest. So, from our previous knowledge, we know that if we are investing the principal amount P for n years with interest rate i, this F becomes

$$\boldsymbol{F = P(1 + \mathrm{i})^{n}}$$

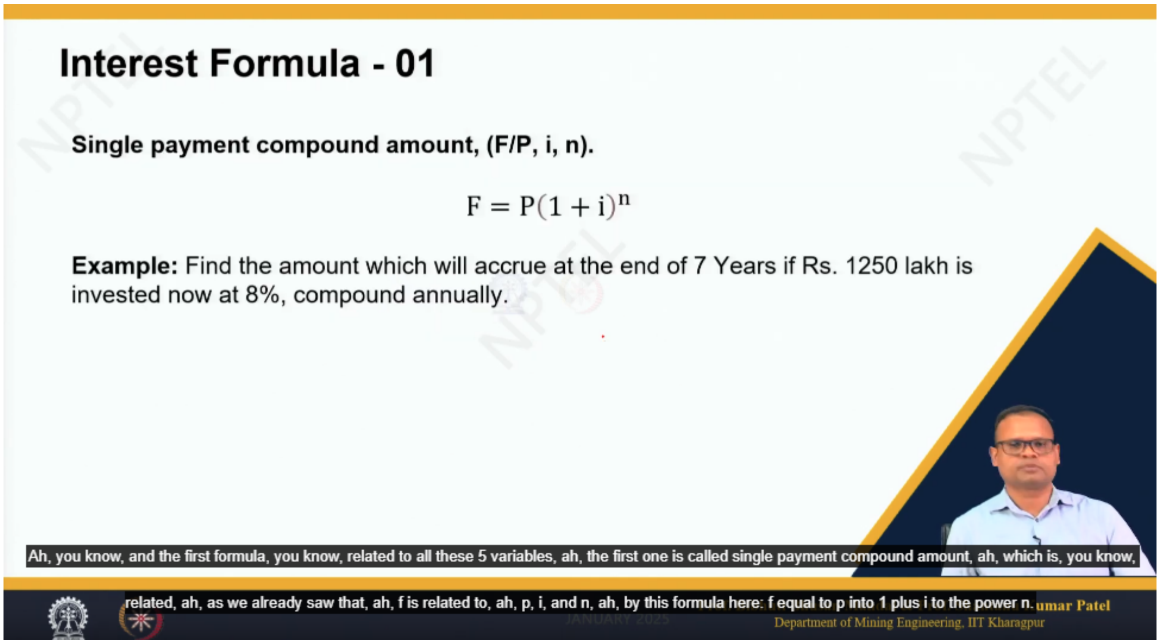
whereas for simple interest, this F becomes P plus the interest component, which is P × i (in percentage) × number of years n. We have an example here: if we deposit 1000 crores at 6 percent for 2 years, according to the simple interest it becomes the initial principal amount P plus 2 (because it is 2 years), this n becomes 2, and this is our P × i. So, this becomes 1120 crores, whereas for compound interest the 1000 crores, using the previous formula here, becomes 1123.6 crores.

So, in this course, if it is not specifically stated as simple interest, what we are going to do is assume that it is compound interest. So, the cash flow at different points in time are related by six basic interest formulas. These formulas are based on five variables, and the first variable that we have is the future sum of money.



The second variable is the present sum of money, and what we are seeing here is, let us say we have deposited some amount or maybe the present value of something is, let us say, P, and the final amount is F. The third variable that we have is called A, which is a payment in a series of equal n payments made at the end of each What this means is, you know, if we plot t equal to 0 and t equal to 1 here, let us say 2 years, 3 years, 4 years, all the way up to, let us say, the nth year, and the amount of money we are getting at the end of each year is our A. And then we have, you know, this effective rate per period, which is i. So, this, you know, the interest rate is in percentage, and then we have the number of, you know, interest periods, which is n. So, every interest problem is composed of 4 of these variables, you know, t, f, i, n, and A, and out of these variables, 3 are given. And, ah, the fourth one must be determined. So, we have some standard notation here, ah, it says, you know, for example, f slash p comma i comma n, which means, ah, find the first variable here.

So, what is this f, ah, given that, you know, p, i, n are given here. So, similarly, it can be anything like, you know, it can be, ah, you know, A is A slash p i n, let us say, in here, you know, what is the value of A given these 3 values are given. Ah, you know, and the first formula, you know, related to all these 5 variables.



The first one is called single payment compound amount, ah, which is, you know, related, ah, as we already saw that, ah, f is related to, ah, p, i, and n, ah, by this formula here:

$$\boldsymbol{F = P(1 + i)^{n}}$$

we have an example here, you know, ah, find the amount, ah, which, ah, will accumulate at the end of 7 years, ah, if Rs, ah, or rupees, ah, 1250 lakh is invested now at, ah, 8 percent compounded annually. So, what we have here is, ah, we need to identify those 3 given variables first, ah, we can see that at the end of, ah, 7 years means, ah, n becomes 7, ah, and if 1250 lakh is, ah, invested means our p becomes, ah, 1250 lakh. And the interest rate is 8 percent. So, i becomes, you know, 8 percent, or we can write it, you know, 8 divided by 100. So, and using the previous formula here,

F=P·(1+i)n

Where:

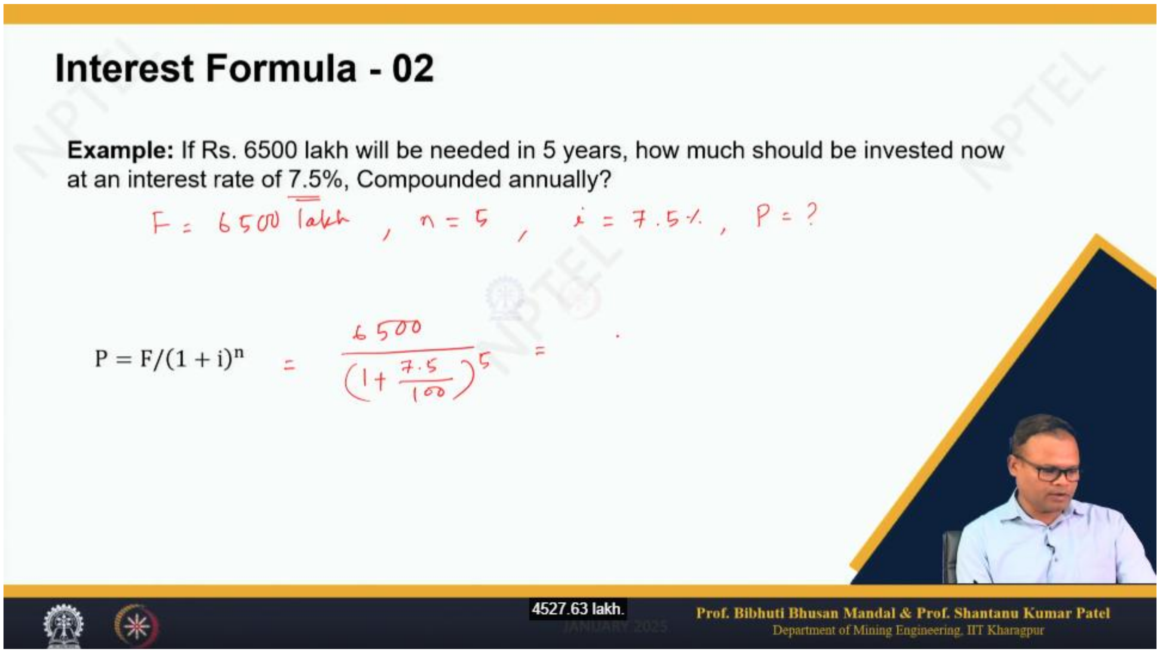
F: Future value

P=1250(in lakhs)

i=0.08 Interest rate (8%)

n=7 years

F=1250· (1+0.08)7=1250· (1.7137) ≈ 2142.28 lakhs



So, this is our first interest formula. The second interest formula is called single payment present worth. So, this is the reverse of the previous formula we saw that, like, if we have the present worth as P. And F is the final worth, and we have I as the interest rate, N as the number of periods. Then we saw that

$$\boldsymbol{P}\ (\mathbf{1}+\mathbf{i})^{\boldsymbol{n}} = \mathbf{F}$$

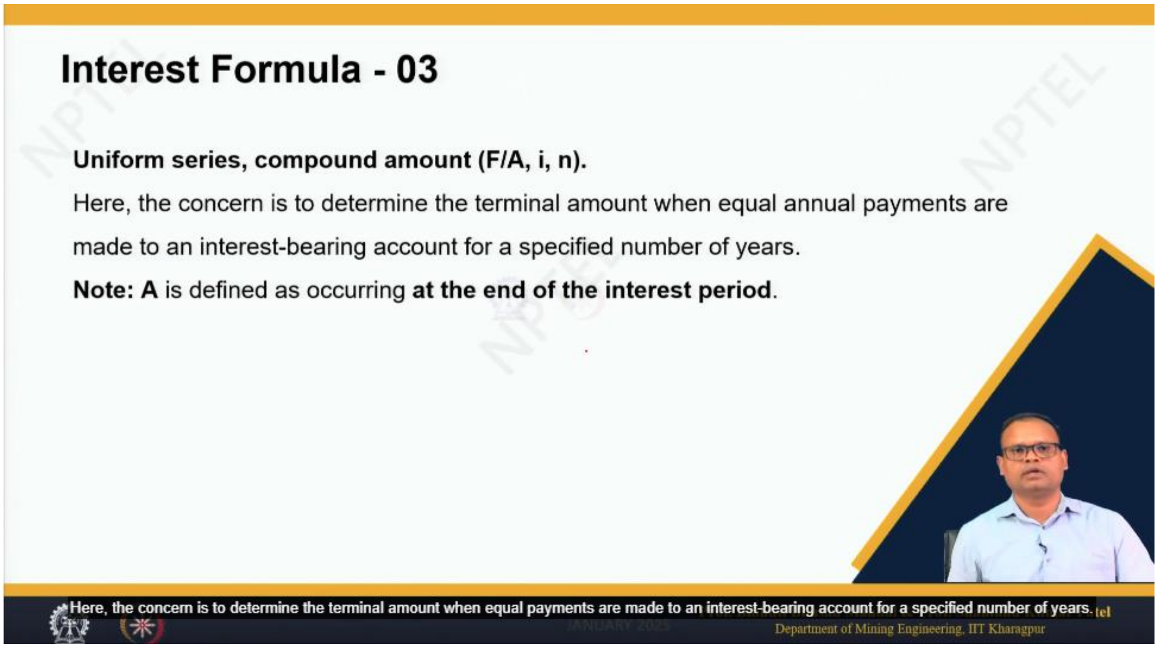
So, if you take (1 + i) to the power n to this side, it becomes

$$\boldsymbol{P} = {}^{\boldsymbol{F}}\!/_{(\mathbf{1}+\mathbf{i})^{\boldsymbol{n}}}$$

So, this is called our single payment present worth. So, based on this, we have this example here: if 6500 lakh rupees will be needed in 5 years, how much should be invested now at an interest rate of 7.5 percent compounded annually? So, what we need here, or what is given, is F equals 6500 lakh.

This amount we need after 5 years. So, N becomes 5, and I, the interest rate given, is 7.5 percent here. So, the three variables are given, and we are going to calculate how much money we need to invest at the present time, T equals 0. So, what the question asks is: what is P? So, we have this formula: P equals F divided by (1 + I) to the power N. So, if you put these values here, F is 6500 divided by (1 + I), where I is 7.5 percent, or 7.5 divided by 100, all to the power N, where N is 5.

So, this gives us 4527.63 lakh. So, this is our interest formula number 2.



The third interest formula is called uniform series compound amount. Here, the concern is to determine the terminal amount when equal payments are made to an interest-bearing account for a specified number of years. Here, we have to note that A is defined as occurring at the end of the interest period.

What does this mean? You know, in here We have 'i' and 'n' given, and we have to calculate the value of ''. So, in this, you know, let us say if you draw the cash flow diagram, what we have here is t = 0 and t = 1 year, 2 years, 3 years, all the way up to the nth year. So, here the variable 'a', as we discussed before, is going to be put at the end of the year, not in the middle of the year. So, all the way up to the first year, second year, third year, up to the nth year. So, in this case, the question is, what will be the final amount that we are going to get at the end of n years?

So, for this case, let us say if n = 1, or in this case, the cash flow diagram maybe becomes So, this is t = 0 and t = 1, and we are depositing 'a' at the end of the first year. So, in this case, 'f' becomes equal to 'a' because it is deposited and not being compounded in this case. So, for n = 2 Let us say t = 0 here, t = 1, and t = 2.

So, what we are doing here is at the end of the first year, we are depositing 'A', and at the end of the second year, we are also depositing 'A'. So, in this case, the amount we are depositing at the end of the first year will be compounded here by 1 year. So, the value of this 'a' at the first year, at the end of the second year, it will be 'a' into (1 + i) because it is compounding only 1 year. So, the total 'f' for n = 2 becomes this 'a' here, the first one, and then the value of 'a' at the end of the first year. So, it is 'a' plus 'a' into (1 +i)

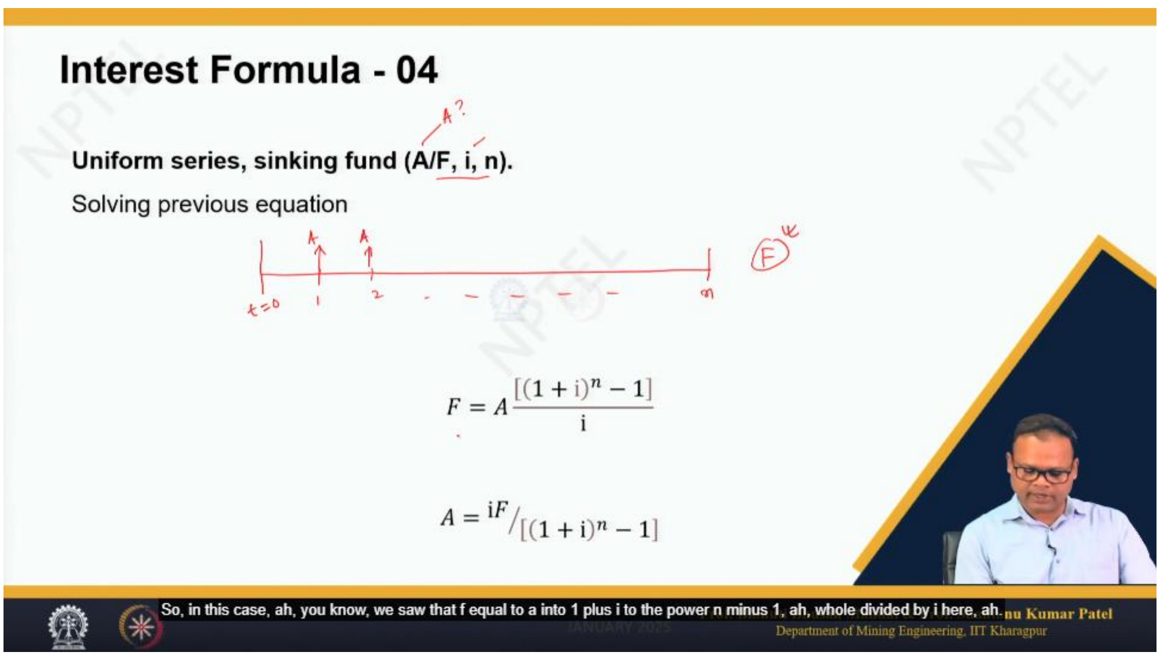
If n equals 3, similarly, you know we have 1, 2, and 3, and this is t equal to 0. Here, we are depositing A, A, and A for 3 years. You know, the first year A will be compounded, the second year A will be compounded, and F will become A plus A into 1 plus i. Plus A into 1 plus i square. So, similarly, if you do it for n years, n equal to n, then, you know, in the cash flow diagram, it will look like 1, 2, 3, all the way up to the nth year, we are depositing an amount A. A at the end of each year, all the way up to the nth year. So, in this case, this F will become A plus A into 1 plus i plus A into 1 plus i square, all the way up to A into 1 plus i n minus 1. So, let us say this is our equation number 1. So, you know, once you know that the value of F, you know, this, you can multiply equation number 1 with 1 plus i. So, this becomes F into 1 plus i equal to A into 1 plus i plus A into 1 plus i square plus A into 1 plus i cube. All the way up to A into 1 plus i whole to the power n. Now, let us say this is our equation number 2. So, if we subtract equation number 1 from equation number 2, then it becomes F into 1 plus i minus F equal to, you know, all the values in the middle will be canceled. So, it will become A into 1 plus i to the power n minus A, or we can write, you know, if we expand both sides, F plus i into F minus F equal to A, taking A common, 1 plus i to the power n minus 1. Or you know, F got cancelled. So, i into F equal to A into 1 plus i to the power n minus 1. So, F becomes A into 1 plus I to the power N minus 1, whole divided by I. So, this is our third interest Ah, we have an example here related to this formula.

$$F = A\frac{[(1+\mathrm{i})^n - 1]}{\mathrm{i}}$$

So, if it says if a payment of 725 lakhs is made at the end of each year for 12 years to an account which pays an interest rate at 9 percent per year, ah, what will be the terminal amount? So, you know what we have here is, you know, if you draw this cash flow diagram. So, this is T equal to 0, T equal to 1, 2, like that all the way up to 12 years. So, ah, and every end of every year, we are depositing 725 lakh, 725, and all the way up to 725 lakh. So, and with an interest rate I equal to 9 percent.

So, if we have this, ah, you know, the question is what is the value of F. So, we have this, ah, the we derived this formula here: F equal to A into 1 plus I to the power N minus 1, whole divided by I. So, if we put all these values here, ah, this becomes 725 into 1 plus I equal to, you know, your 9 percent. So, this is 0.09, you can write to the power N. So, whole divided by I, I is, you know, 0.09.

So, you need to be careful, you know, that you do not put, ah, this is i equal to 9 percent. So, you do not put this 9. So, it becomes 9 divided by, ah, 100 or 0.09. So, this total value becomes, you know, if you calculate this, this is 14602.02 lakh. So, this is our third interest formula.



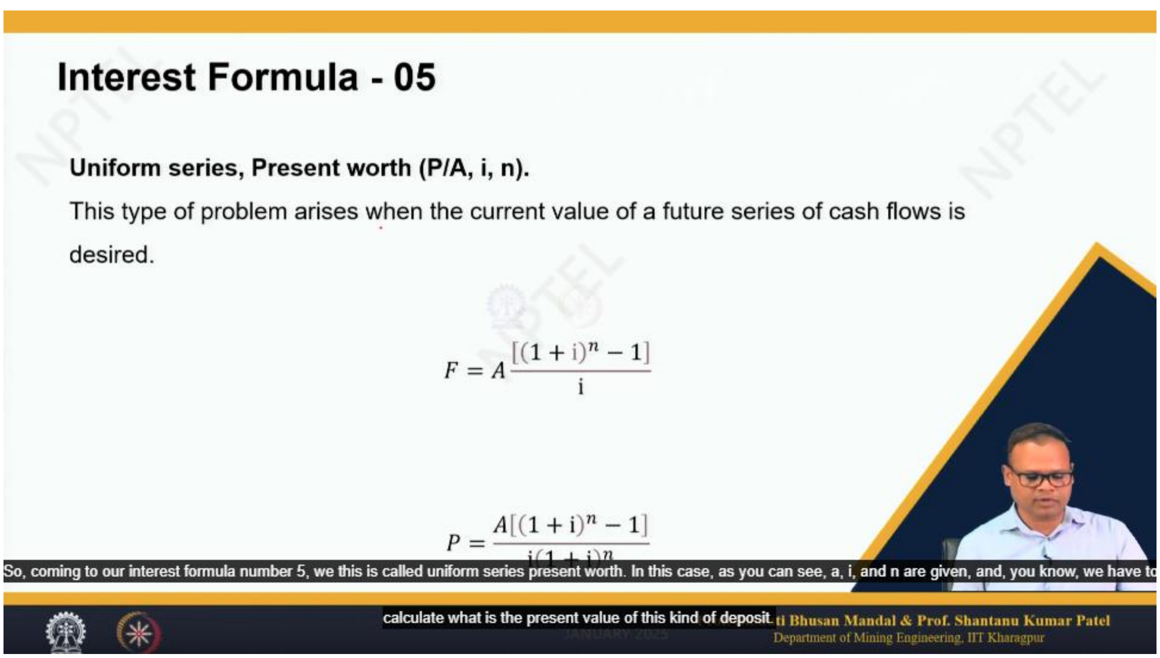
The fourth interest formula is called uniform series thinking form, where f, i, and n are given, and the question is, you know, we have to calculate what is a. So, ah, you know, or maybe in, ah, you know, the diagram, you know, you can put like t equal to 0 here, t equal to 1, 2, all the way up to, let us say, n. And what we know is, you know, ah, we know the value of f. So, what is the amount in each year that we are going to deposit, ah, so that, ah, it becomes f at the end of the nth year with some interest rate i percent. So, in this case, ah, you know, we saw that f equal to a into 1 plus i to the power n minus 1, ah, whole divided by i here, ah. So, if you just take, ah, you know, this entire thing on the right side, ah, to the left side. So, it becomes a becomes, you know, i into f whole divided by 1 plus i to the power n minus 1

So, this is our fourth interest, ah, formula, and based on this, ah, we have, ah, an example here, ah, which says, with an interest rate of 6 percent, how much, ah, must be deposited at the end of each year to, ah, yield a final amount of 2825 lakh in 7 years. So, what is given here is, ah, you know, ah, F equal to, ah, 2825 lakh. So, in here, n equal to 7 years. Ah,

$$A = {}^{iF}/_{[(1+i)^n - 1]}$$

and, ah, you know, ah, i equal to 6 percent or 6 by 100 equal to 0.06. So, if you just put it in, ah, the formula that we, ah, saw in the previous slide, ah, a equal to i into f whole divided by 1 plus i to the power n minus 1.

So, this becomes. i is 0.06 into the final amount 2825 whole divided by 1 plus i, i is 0.06 whole to the power n, n is our 7 years minus 1. So, this gives you a value of, you know, 336.556. So, every year we have to, you know, deposit or maybe invest an amount of 336.566 lakhs to yield a final amount of 2825 lakh in 7 years.



So, coming to our interest formula number 5, we this is called uniform series present worth. In this case, as you can see, a, i, and n are given, and, you know, we have to calculate what is the present value of this kind of deposit.

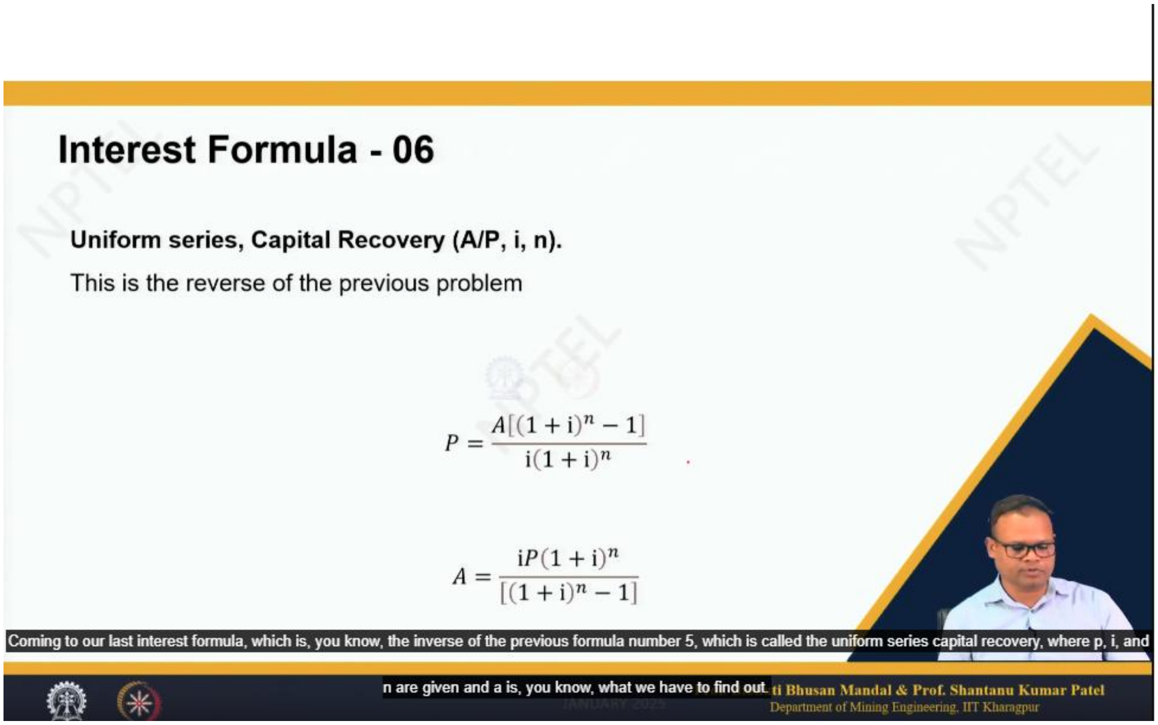
So, this P equal to what. So, or in other, you know, way. We have t equal to 0 here, 1, 2, 3, all the way up to n years. So, in this case, you know, we have we are depositing, let us say, a amount. Or there is a cash flow, a positive cash flow, a amount all the way up to, you know, nth year.

So, the question is, what is the, you know, all these cash flow values at the present time. So, what is our p. So, you know, in the previous formula, we saw that f equal to a into 1 plus i to the power n minus 1 whole divided by i, and from our first, you know, the interest formula, we saw that f equal to p into 1 plus i to the power n. So, from these two equations, you know, the equation here and the equation here. If we compare this f in both sides. So, this becomes p into 1 plus i to the power n equal to a into 1 plus i to the power n minus 1 whole divided by i. So, if you rearrange, p becomes

$$P = \frac{A[(1+i)^n - 1]}{i(1+i)^n}$$

So, the fifth interest formula based on this we have an example here. You know, an investment will yield 610 lakh at the end of each year for 15 years, and if the interest rate is 10 percent, what is the maximum purchase price or the present worth of this investment? So, what we have is A equal to 610 lakh. n equal to 15, i equal to 10 percent or 10 divided by 100 equal to 0.1. So, in this case, you know, we have to calculate what is the value of p. So, the p, as we saw in the previous slide, p equal to this.

And if you put the corresponding values, it becomes 610 into 1 plus i is 0.1 whole to the power n is 15 minus 1. Whole divided by i, i is again 0.1 into 1 plus i is 0.1 whole to the power n is 15. So, this becomes, you know, 4639.71.



Coming to our last interest formula, which is, you know, the inverse of the previous formula number 5, which is called the uniform series capital recovery, where p, i, and n are given and a is, you know, what we have to find out. So, you know, and in what does this mean is,

you know, we are investing an amount, let us say p, at the beginning of, you know, let us say starting a mine at t equal to 0, and we have time equal to 1, 2, all the way up to let us say n. And with this deposit, you know, what will be, what should be our at the end of each year. To justify this, you know, initial investment for this case. So, we saw that, you know, p equal to a into 1 plus i to the power n minus 1 whole divided by i into 1 plus i to the power n in our fifth interest formula. So, if you just take, you know, this part to the left side, A becomes

$$A = \frac{iP(1+i)^n}{[(1+i)^n - 1]}$$

i into p 1 plus i to the power n whole divided by 1 plus i to the power n minus 1. So, based on this, we have the example here, like where it says if the investment opportunities offered now to Rs 3500 lakh, how much must it yield at the end of every year for 6 years to justify the investment if the interest rate.

12 percent. So, what is given is P equal to 3500 ah lakh rupees ah n equal to 6 years and i equal to 12 percent ah ah which is 12 divided by 100 equal to 0.12. So, if you just put it in ah the formula here ah it becomes i ah 0.12 into p, p is 3500 into 1 plus i is 0.12 whole to the power n, n is 6 whole divided by 1 plus i is 0.12 whole to the power 6 minus 1.

So, if you do this, this becomes your 851.67 lag. So, what does this mean is if we ah if we have an initial investment of 3500 lakh we we need ah you know 851.67 ah rupees at the end of each year for ah for 6 years ah at at a interest rate of 12 percent to justify our investment. So, this ends ah our ah this lecture ah here ah and we will see the next part of this ah time value of money in in the subsequent lectures.