MINERAL ECONOMICS AND BUSINESS

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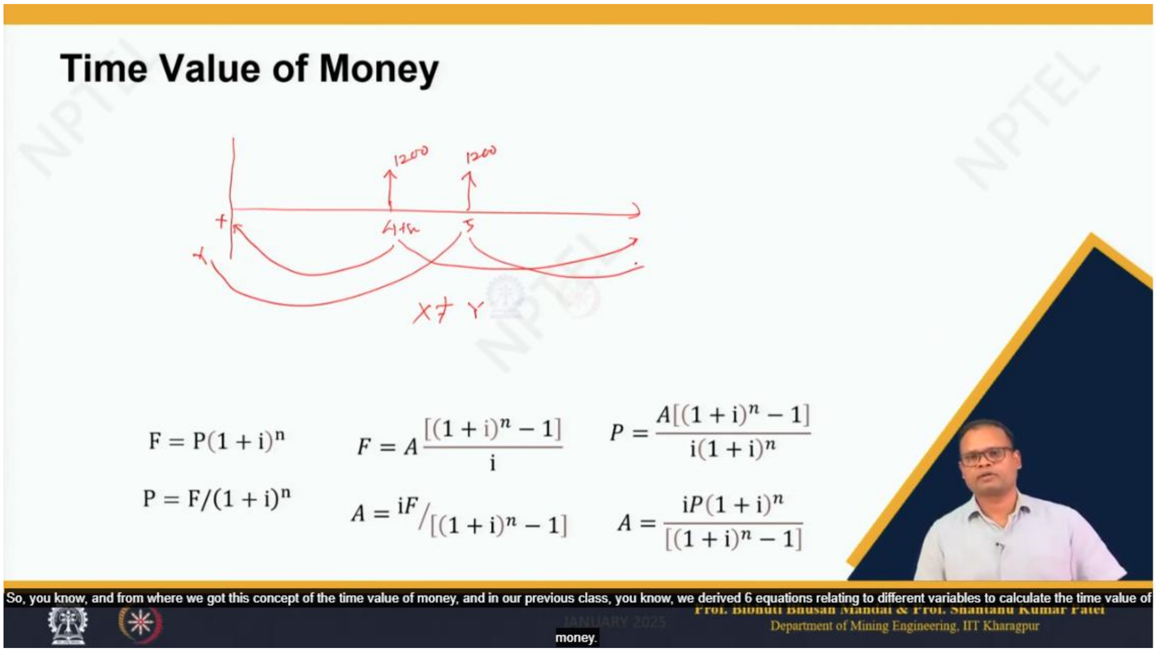
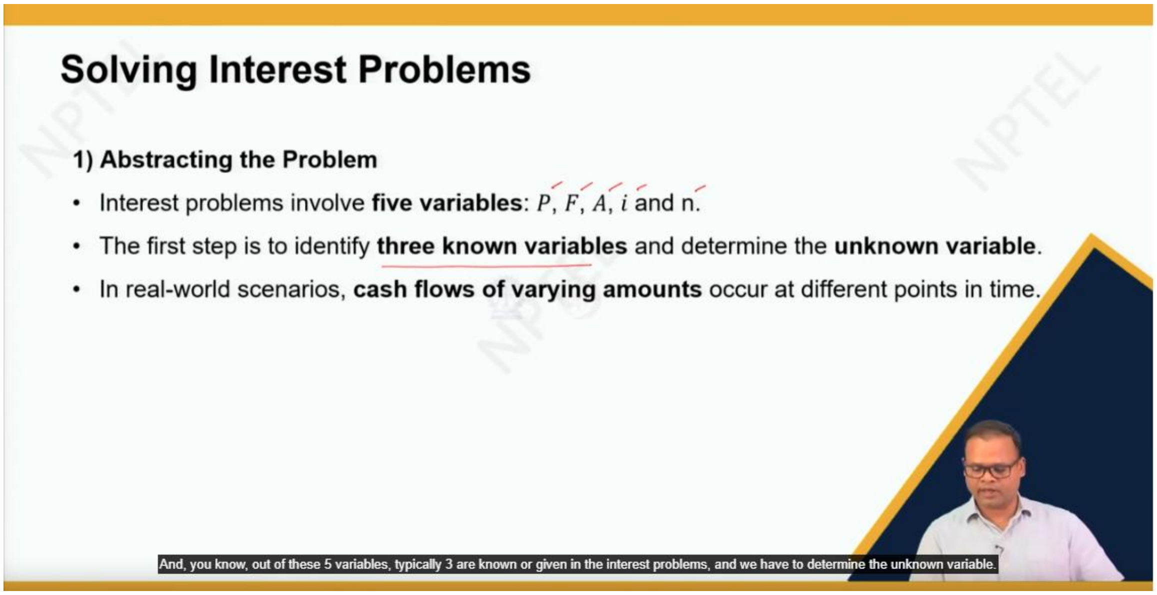
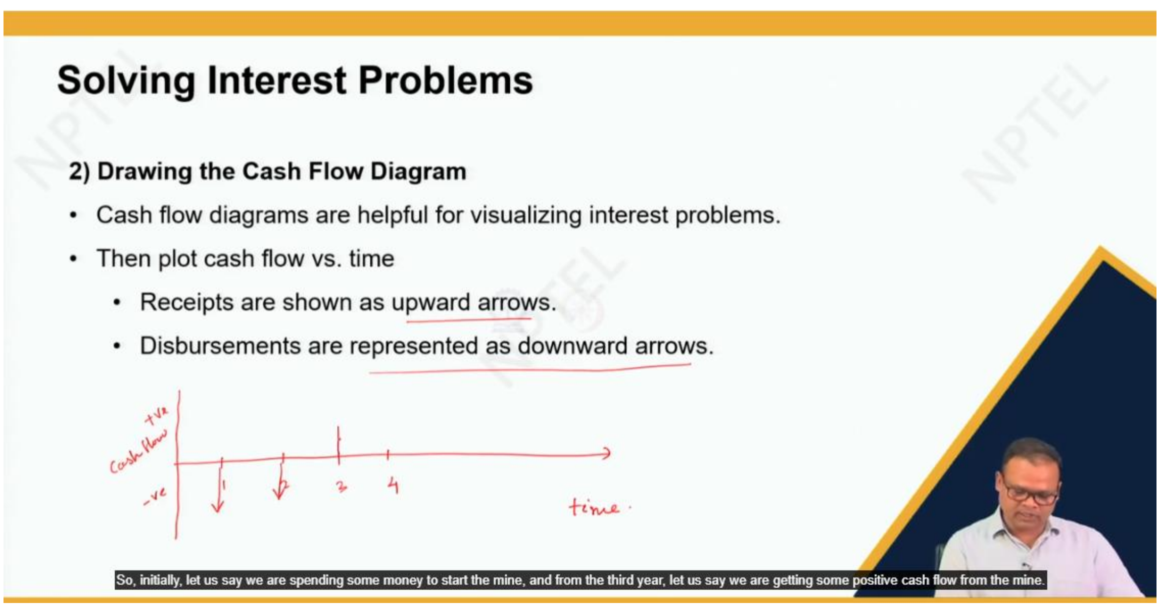
Lecture 23: Time Value of Money - 2



Hello everyone, and welcome again to this course on manual economics and business. This is our lecture number 23, which is the second part of this time value of money. So, in our previous lecture, what we were seeing was, let us say we have a mine, and let us say in the fourth year we are getting some 1200 crores from the mine, and in the fifth year also we are getting 1200 crores. But, you know, if you want to get the present value of this 1200 crore at t equal to 0, let us say this is x and this is y, and this x and y they are not the same, x not equal to y. Similarly, you know, if you see the future value of this 1200 crore at the end here, they are also not the same. So, you know, and from where we got this concept of the time value of money, and in our previous class, you know, we derived 6 equations relating to different variables to calculate the time value of money.

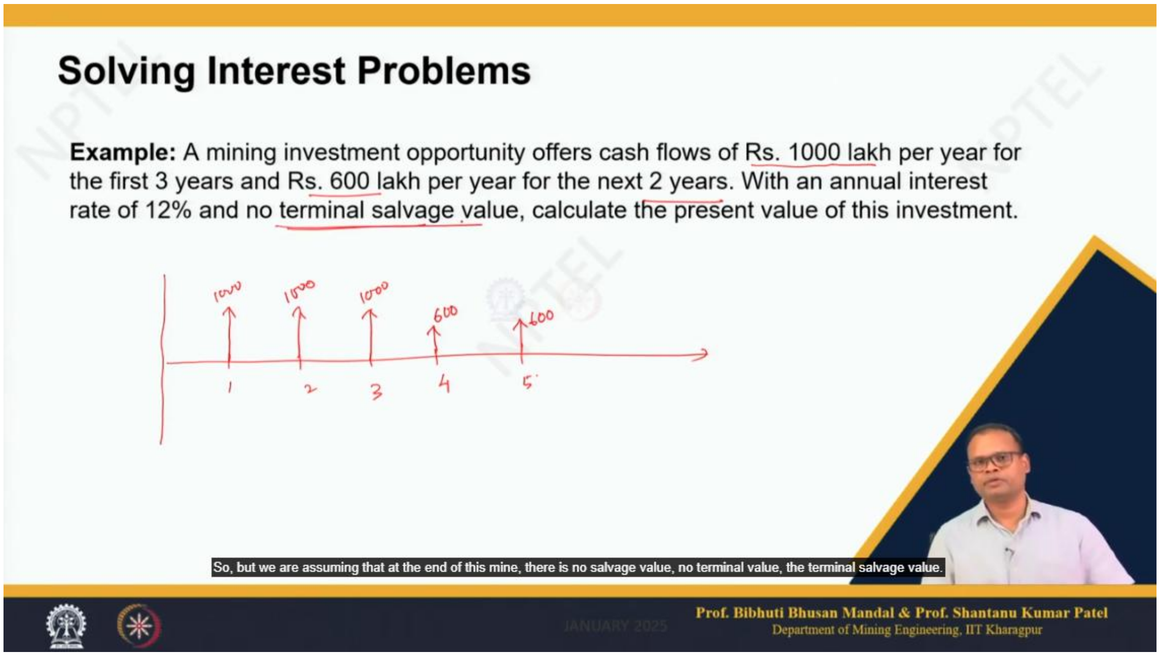


Now, coming to solving the interest problems, there are 2 steps that we need to follow. The first step is to abstract the problem, as we know from our previous class that we have 5 variables in these interest problems: first is P, F, A, i, and N. And, you know, out of these 5 variables, typically 3 are known or given in the interest problems, and we have to determine the unknown variable. But in a real case scenario, you know, the cash flow of variant amounts occurs at different points in time. So what this means is, let's say, you know, first year, second year, third year, fourth year, like that, and the amount of money that we are going to or the positive or negative cash flow that is happening to our mining business is, let's say, a1. a2, a3, a4, like that. So, this a1 to a4, they may not be the same.



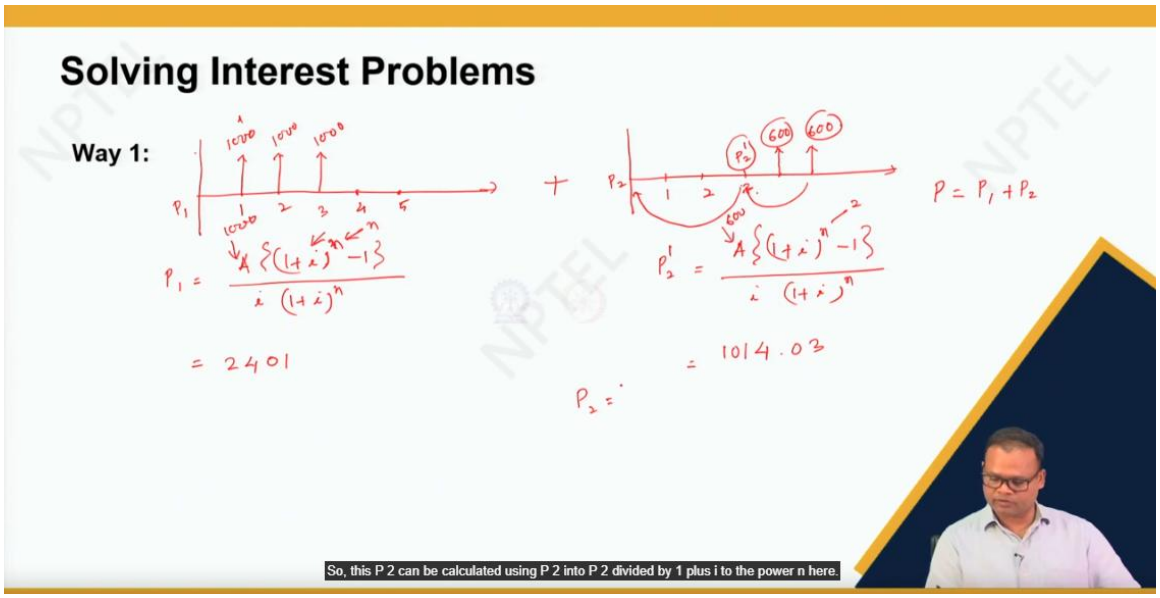
So, the second step to solve this interest problem is to draw a cash flow diagram, which we are doing, you know, from the last class. So, these cash flow diagrams are helpful for visualizing the interest problem. And, you know, this cash flow diagram is the plot of cash flow versus time, where the receipts are shown in upward arrows and disbursements are represented as downward arrows. So, what this means is, you know, in the y-axis we have cash flow. And in the x-axis we have time.

So, this can be months or years, and the positive cash flow is upward, and negative cash flow is downward. What this means is, let us say this is the first year, second year, third year, and fourth year. So, initially, let us say we are spending some money to start the mine, and from the third year, let us say we are getting some positive cash flow from the mine. So, the positive will go in the upward direction, and the negative will go in the downward direction.



So, we have an example of how to solve these interest problems, which says a mining investment opportunity offers a cash flow of 100 or maybe 1000 lakh per year for the first 3 years and 600 lakh per year for the next 2 years. So, with an annual rate of interest of 12 percent and no salvage value, calculate the present value of the investment. So, what it is trying to say, if you put it in the cash flow diagram, so, we have year number 1, let us say 2, 3, 4, and 5. So, in the first year, what we are going to get is 1000, this is 1000, the third year is again 1000.

and the fourth year, we are getting 600, and then 600. So, we invested some money in buying equipment and developing our infrastructure. So, but we are assuming that at the end of this mine, there is no salvage value, no terminal value, the terminal salvage value. So, we have to calculate the the present value of this investment, which is at t equal to 0.



So, there are two ways to solve this problem. The first way, you know, how we can break this problem is. So, we say that, you know, like for the first three years, we are getting 1000 here. 1000 here, and the next two years, we are not getting anything in the fourth and fifth. This can be added to, you know, like the first three years, we are not getting anything. This is 0; the cash flow is 0, and in the fourth and fifth year, we are getting 600. So, if you add these two things, it becomes like, you know, the previous cash flow diagram. So, now, you know, like in the previous case, if you see, the cash flows are not uniform.

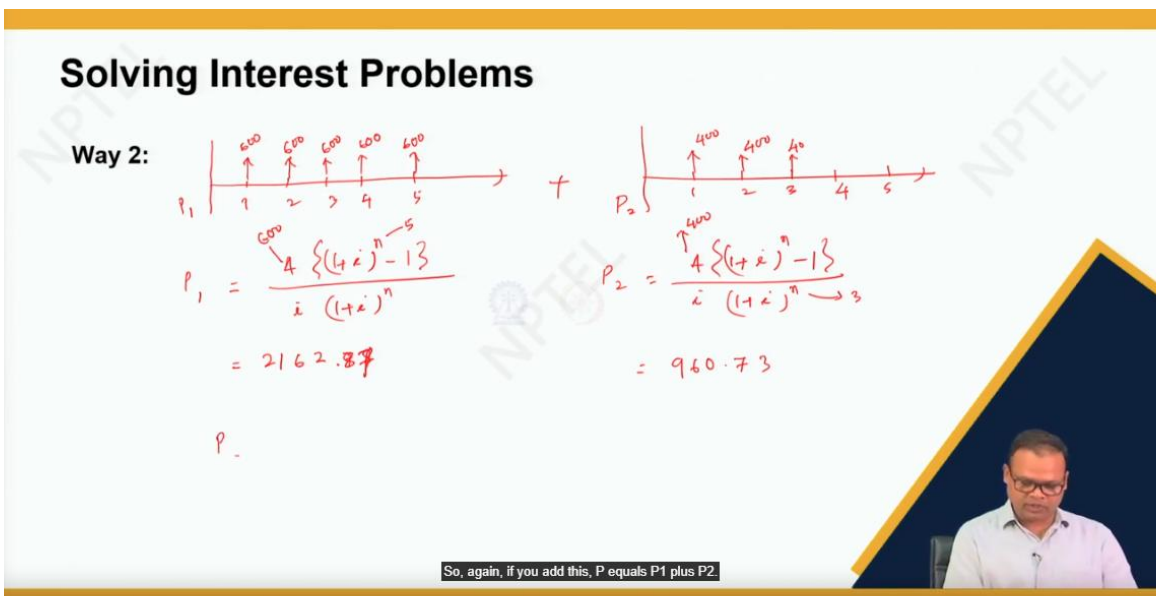
So, we could not apply the six formulas that we have, but now this problem becomes uniform, and here we can calculate for these, what is the present value P1 here. And in this case, the second part of this, we can calculate what is the you know, amount of the 600 at t equal to 3. So, let us say this is P2 dash, and this P2 dash we can bring it back to t equal to 0, which can be, you know, P2. So, if you add this problem, it becomes like our first problem. So, we can see that, you know, this P, which is the present value of the entire thing, is P1 plus P2.

Now, the question is, how do we get this P1 and P2? For P1, we know that this P1 can be calculated using A into 1 plus i to the power n minus 1, whole divided by i into 1 plus i to the power n. where we know that our interest rate is 10 percent here, and we can calculate the number of years is 3, n equal to 3 here. So, and A is, you know, our 1000.

So, if you put these values this becomes you know 2401 Similarly, you know if you if you calculate this P 2 ah dash, so P 2 dash equal to ah a into ah 1 plus i to the power n minus 1 whole divided by ah i into 1 plus i to the power n, where n becomes ah because this compounding is ah or maybe we are seeing it for 2 years. So, n becomes 2. So, if you put the respective value

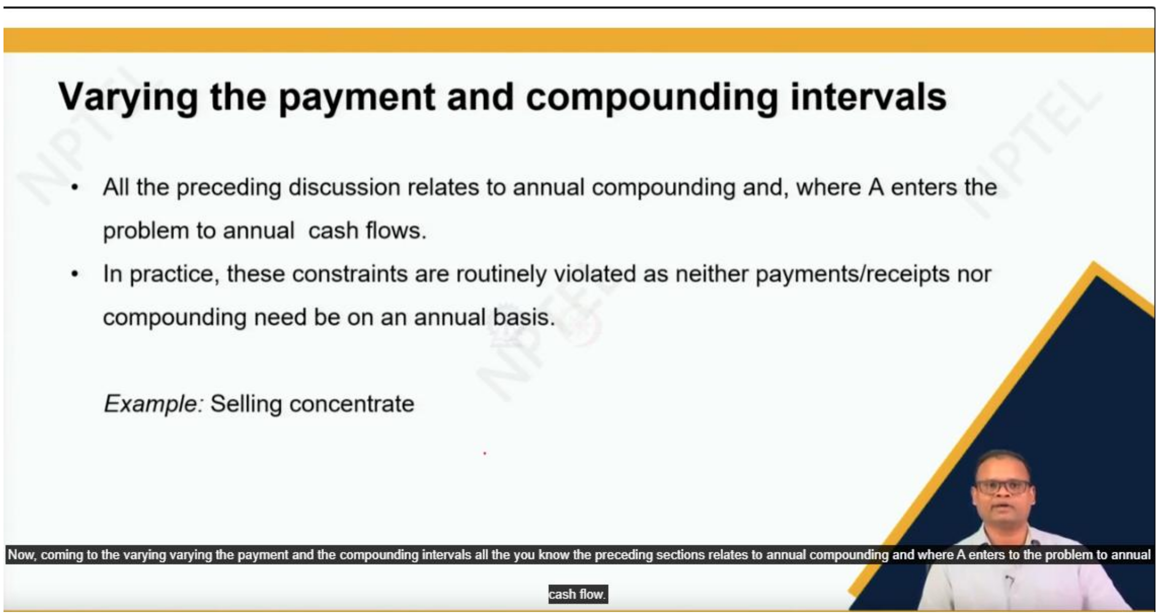
and A becomes let us say 600 and if you put these values this this becomes ah you know 1014.03. So, now, ah we know at the third value this P 2 P 2 dash equal to 1014.03. So, this P 2 can be calculated using P 2 into P 2 divided by 1 plus i to the power n here. So, or P 2 dash divided by 1 plus i to the power n here. So, n is now 3 ah.

So, you can calculate this value as 721.77. So, and ah if you add it up p equal to you know p 1 plus p 2. So, this becomes 2 2 4 0 1 plus ah 7 21 2 4 0 1 point this is 8 sorry. So, 2 point 0 1 point 8 plus 721.77 So, this becomes 3123.6 lakh rupees.

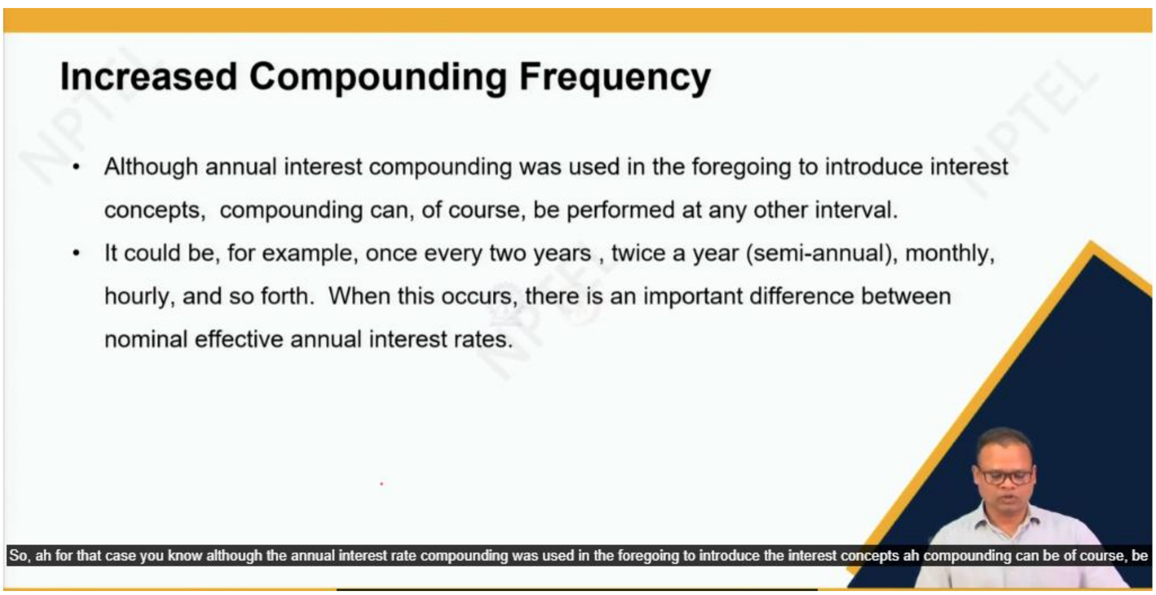


The second way to solve this problem is to break it down, you know, like what we can do is for all the way up to 5 years: 1, 2, 3, 4, and 5. So, we can say that there is a positive cash flow of 600, 600, 600, 600, and 600 here. Plus, there is a positive cash flow for the first 3 years: 1, 2, and 3, and then the 4th year and 5th year. So, for the first 3 years, we have a cash flow of 400, 400, 400, and then 4050, or we do not have any cash flow, or the cash flow is 0, we can put. And if you add these two things, it becomes like the cash flow that we saw in the previous slide. So, this can be calculated. We can use our derived formula here. So, let us say the present value is again, let us say P1, and this is P2. So, P1 is again, you know, we can calculate using A into (1 + i) to the power n minus 1

whole divided by i into (1 + i) to the power n. So, here n becomes 5, A is our 600. If you put these values and the value of i here, this becomes, you know, 2162.78, sorry, 87. And P2 becomes similarly, you know, A into (1 + i) to the power n minus 1 whole divided by i into (1 + i) to the power n. So, in this case, A is our 400. And n is 3. So, if you put the values, you know, this P2 becomes 960.73. So, again, if you add this, P equals P1 plus P2 So, this becomes again 3123.63.



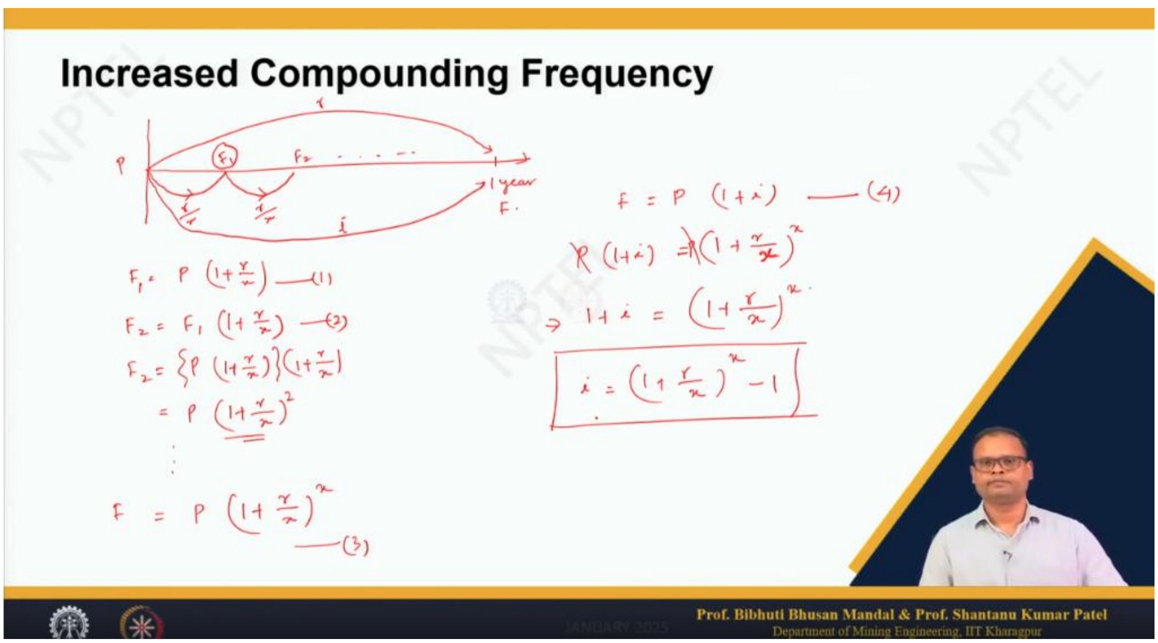
Now, coming to the varying varying the payment and the compounding intervals all the you know the preceding sections relates to annual compounding and where A enters to the problem to annual cash flow. But in practice these constraints are routinely violated as neither the payments or the receipt nor the compounding need to be on annual basis. For one of the example is let us say selling of the concentrate for a mine. So, which you know in where you know we can say that every month we are getting some concentrate And, ah like the amount that we are going to get from this selling this concentrate is let us say A dash ah and ah this is like 1 month. So, this is 1 month, this is 2 months, 3 months like that. And, ah you know this is this is going on throughout the life of the mine, but ah you know there can be you know the compounding can be let us say every 3 months. So, these kind of mismatch ah can happen ah in in real case scenario.



So, ah for that case you know although the annual interest rate compounding was used in the foregoing to introduce the interest concepts ah compounding can be of course, be performed at any other interval ah it could be for example, once in 2 years a year which is called semi annual ah then monthly hourly and so forth. ah When this occurs there is a there is an important difference between the nominal rate of ah nominal rate of interest and the effective annual rate of interest. So, ah you know like as we were saying like you know let us say if you we have the cash flow diagram. So, ah and this is let us say ah 1 year

And, you know, the nominal rate of interest is, let us say, r. And instead of this, you know, 1-year compounding, what we are doing is we are dividing this into x periods. So, first period, second period, third period, like this. So, there are, you know, x periods like this, you know, x, or maybe in 1 year, we were compounding this for x times. So, in this case, you know, if you do like that,

The real rate that we are going to get is, let us say, i. So, i is different than r, of course. I know I will be a little bit more than r. So, here we have 3 terms: i is the true or effective annual rate of interest that we saw, you know, is the thing, this compounding, multiple compounding effect; i is the nominal annual interest rate; and x is the number of compounding periods, like, you know, how many compounding we are doing within a year. So, for this case, you know, if we do like that, So, this is our cash flow diagram, and we saw that this is 1 year.



And we are dividing this, you know, interest rate into multiple or maybe x periods. So, and this nominal rate of interest is, let us say, r. So, if you divide it into x periods, So, this interest rate becomes r by x, and again, this is r by x. Like that. And if you are depositing or, you know, investing some amount P, this, let us say, after the first year, let us say this final amount is becoming F1, and this is F2, and this is going on all the way up to 1 year where we have F equal to 0.

First Compounding Period:

$$F_1 = P \times (1 + r/x)$$

Second Compounding Period:

$$F_2 = F_1 \times (1 + r/x) = P \times (1 + r/x)^2$$

After x Compounding Periods (1 year total):

$$F = P \times (1 + r/x)^x$$

Effective (Real) Rate of Interest:

$$F = P \times (1 + i)$$

Equating Both Forms:

$$P \times (1 + i) = P \times (1 + r/x)^x$$

Canceling P from both sides:

$$1 + i = (1 + r/x)^x$$

Effective Interest Rate:

$$i = (1 + r/x)^x - 1$$

So, this is the relation between the real rate of interest, which is i, r is the nominal rate of interest, and x is the number of periods per year. So, based on that, we have an example here: one borrows, you know, 100 lakh, sorry, 1000 lakh from a finance company which charges an internal rate compounded 2 percent per month.

So, what is the true or effective annual rate? So, here you know we, uh We can see that, you know, it is being compounded per month. So, in this case, you know, r becomes—because there are 12 months. So, 2 into 12 equals 24 percent or

0.24, and you know it is because there is compounding every month. So, our x becomes—our number of periods becomes 12. So, in this case, you know, we can calculate the real rate of interest, which is I, related to 1 plus r by x whole to the power x minus 1. So, if you put the value of, you know, r here and x here.

So, this becomes 0.0268 or 26.8 percent, which is a little bit more than, you know, your nominal rate of interest, which is 24 percent. So, you know, if we increase the number of periods, you know, like, let us say it is annual. So, in that case, in this table here, it compares the different compounding frequencies, number of periods, annual nominal rate, and effective annual rate. So, if the compounding is once per year, the nominal rate becomes, you know, the effective annual rate, and similarly, if it is semi-annual, quarterly, monthly, weekly, and daily.

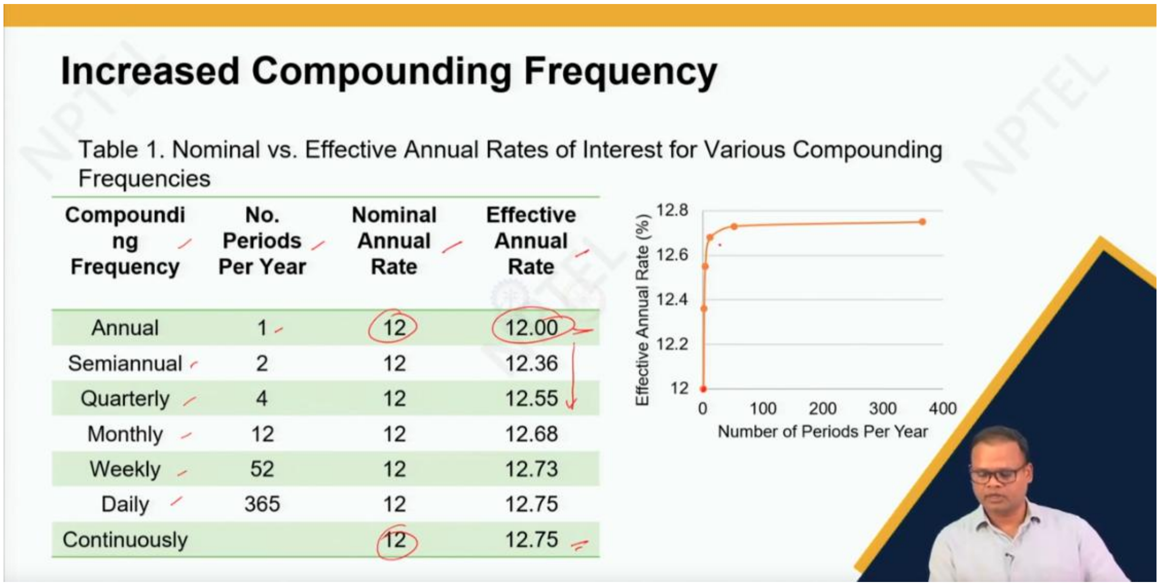
So, it becomes, you know, continuously increasing a little bit from the nominal rate if it is continuous. That means there are, you know, infinite kinds of compounding. Then, the effective interest rate is increased to 12.75 from this 12. So, this is plotted in the right-side figure here, and you can see initially it was 12 for the first case, this 12 and 12. And, you know, if you increase the number of periods per year it goes to some peak, you know, in, let us say, daily, and after that, it is not increasing much. So, the limiting case for, you know, increasingly frequent compounding is instantaneous or continuous compounding, as we discussed, you know, when x tends to infinity.

So, we know that i equals 1 plus r divided by x, whole to the power x minus 1. So, for the continuous, you know, when x tends to infinity, this becomes i equals the limit to infinity of 1 plus r divided by x, whole to the power x minus 1. So, this we can write as the limit as x tends to infinity of 1 plus r divided by x, whole to the power x divided by r, and just multiply with r minus 1.

So, this can be rewritten like this. This here. So, in this case, you know, we know that the limit as x tends to infinity of this term, 1 plus r divided by x, whole to the power x divided by r, is e. So, this becomes e to the power r minus 1, or we can write i equals e to the power you know, r minus 1. So, for continuous compounding or when x tends to infinity, the effective rate of interest becomes e to the power r minus 1.

In this case, you know, you can write that. Let us say this is our cash flow diagram, and we have P here, and we have a kind of infinite number of compounding for 1 year. So, this, you know, the real rate of interest here we are seeing is i equal to e to the power r minus 1. So, in that case, we can write the final amount as F equal to P into 1 plus i, which is equal to, you know, P into i is e to the power r minus 1 or plus 1.

So, it becomes P into e to the power r. So, similarly, you know, if it is for n years, if you see F equal to P into, or maybe you know, doing all these compounding up to the nth year. P becomes, sorry, F becomes P into 1 plus i to the power n. So, you can write it like, you know, P into 1 plus e to the power r minus 1. So, this becomes whole to the power n. So, P into e to the power r n. So, like this is the relation between, you know, F and P with the nominal rate of interest, n is the number of years.



Compounding Frequency	No. Periods Per Year	Nominal Annual Rate	Effective Annual Rate
Annual	1	12	12.00
Semiannual	2	12	12.36
Quarterly	4	12	12.55
Monthly	12	12	12.68
Weekly	52	12	12.73
Daily	365	12	12.75
Continuously		12	12.75

So, similarly, we can derive other formulas also for, you know, six different formulas for this continuous compounding. So, related to this, we have an example here: what is the worth of the present promissory note which will yield Rs. 12 lakh in 5 years if the interest is 10 percent compounded quarterly and continuously? So, we have two things here: one is the promissory note, So, this is a note, you know, or maybe a document that, you know, different banks or the government give to us. It is like, you know, all these things if you deposit or maybe give. If the present value of this, I will be, you know, this much, and n will be this much. So, this, all the information will be written. It is like, you know, a small document that they give.

So, ah the thing is you know which will yield you know 12 lakh in 5 years and the interest rate is 10 percent. So, in this case we know that this F equal to 12 lakh. ah n equal to we can say 5 years and compounded ah ah quarterly and continuously ah with ah interest rate i equal to 10 percent. So, for the first quarterly thing we can write it down like you know we know that p equal to ah f

divided by 1 plus i to the power n, but in this case because it is quarterly this i in here becomes 10 divided by 4 percent or ah you know and n becomes 5 years. So, 5 into 4 times of compounding is ah 20. So, if you put the value of all these values here this you know the p becomes you know or this p becomes 7.32 lag. So, if the compounding is done quarterly, but if it is continuously we can write down this p equation as you know f divided by ah e to the power r n from the previous slide that ah we saw ah and ah like this this becomes you know f is again 12 lakhs divided by e to the power r, r is our 10 percent.

So, n is 0.1 into n ah in in this case ah you know is ah 5 years that we are doing. So, if we put these values you know this becomes 7.28 lakh. So, ah this ah ends ah this class or the second part of this time value of money. So, in in our next class we will be continuing this ah ah lecture on time value of money.