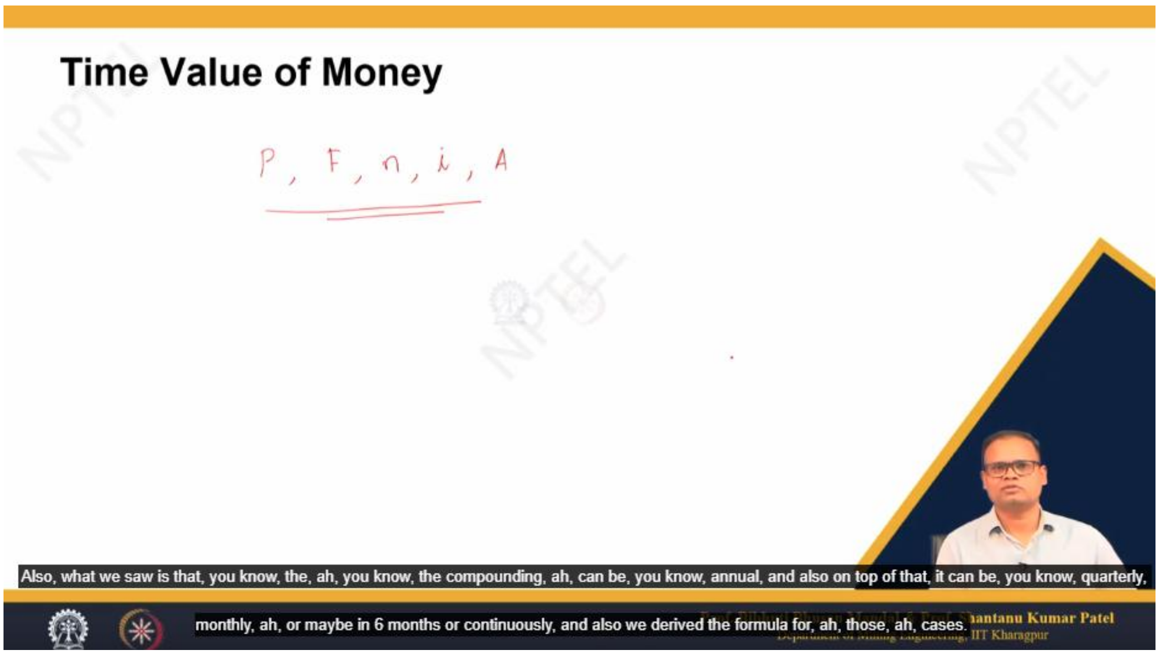
MINERAL ECONOMICS AND BUSINESS

Prof. Shantanu Kumar Patel

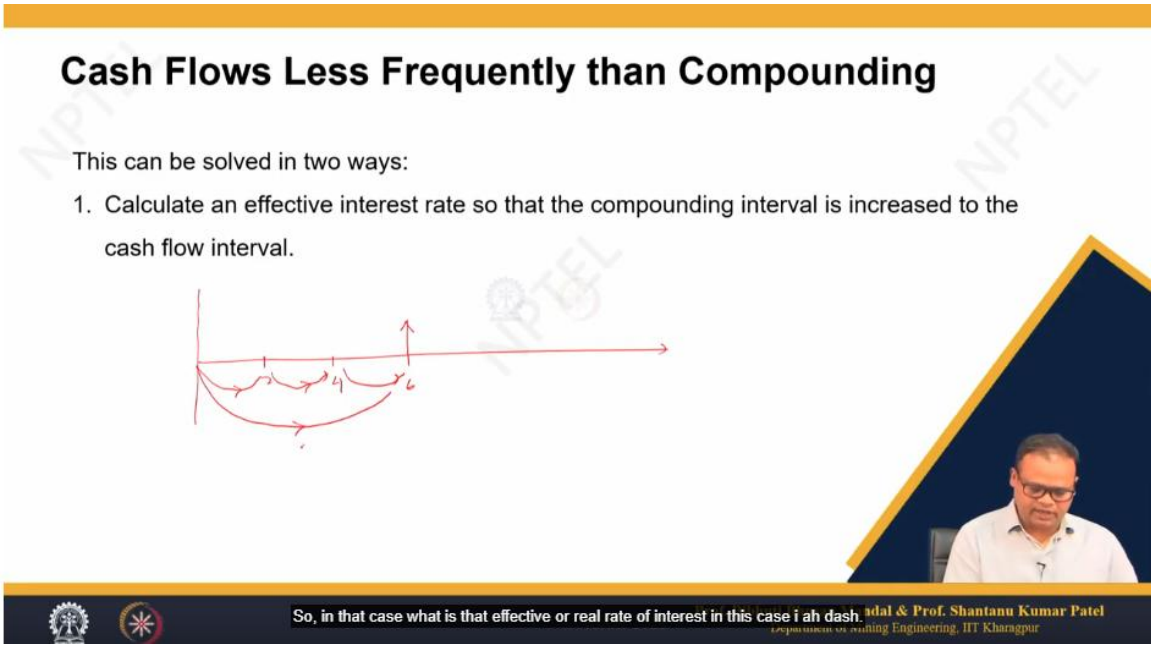
Department of Mining Engineering

IIT Kharagpur

Lecture 24: Time Value of Money – 3



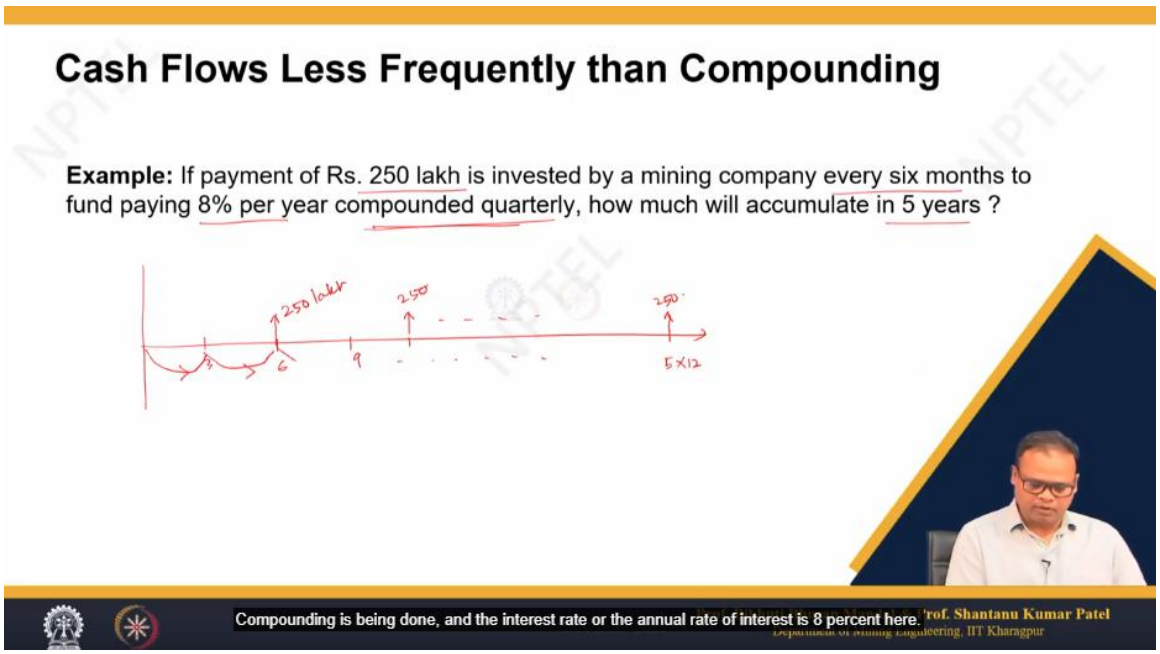
Hello everyone, and welcome again to this course on mineral economics and business. Ah, so this is our lecture number 24, and this is the third lecture on the time value of money. Ah, so, you know, in our previous two lectures, what we saw was that, you know, for the time value of money problem, ah, we have 5 variables: the initial amount P, ah, the final amount F, ah, the number of periods N, the interest rate I, and, you know, the amount of cash flow A occurring at the end of each, ah, ah, period. So, you know, typically out of these 5 variables, ah, 3 are given, and we can find out, you know, the other 2 variables, ah, using our derived formulas. Also, what we saw is that, you know, the, ah, you know, the compounding, ah, can be, you know, annual, and also on top of that, it can be, you know, quarterly, monthly, ah, or maybe in 6 months or continuously, and also we derived the formula for, ah, those, ah, cases. So, as long as the cash flow occurs at the same frequency as compounding, ah, the formula developed in the preceding sections can be applied without difficulty.



What does that mean is, ah, just to repeat, ah, you know, if this is our cash flow diagram, this is 3 months, 6 months, 9 months, 12 months, like that up to, ah, you know, some N, ah, months. And, you know, if the cash flow is occurring at the end of every 3 months and the compounding is being done also in every 3 months, then we can use the formula, but there can be 2 possible variations to this. Ah, the first one is the cash flows are less frequent than compounding. So, what does this mean is, ah, you know, with an example. So, this is, ah, this is, let us say, 2 months, 4 months, 6 months, like that, and the compounding is being done, let us say, every 2 months, ah, but the cash flow is occurring, you know, ah, at every 6 months.

Or the second case, the cash flow is more frequent than compounding. So, in this case, you know, if we have 2 months, 4 months again, 2 months, 4 months again, and 6 months, then, you know, the cash flow is occurring every 2 months, but the compounding is happening every 6 months. So, ah, like, you know, for the first case, ah, you know, this can be solved in two ways. First is to calculate the effective interest rate, so that the compounding interval is increased to the cash flow interval. And what does this mean is, so we have 2 months, 4 months, and 6 months here, ah.

So, and and the compounding is done let us say every 2 months. and the cash flow is happening every 6 months. So, in that case what is that effective or real rate of interest in this case i ah dash. So, that you know this compounding frequency ah or the new compounding frequency which will be 6 months will match to the cash flow ah frequency which is 6 months. Second thing is to determine an ah reduced equivalent intra period cash flow, so that the cash flow interval is reduced to the compounding interval.



So, again coming to this the same example we have 2 months, 4 months and 6 months and this cash flow is occurring every 6 months and the compounding is being done every 2 months. So, in that case ah you know like ah what will be the A dash, A dash and A dash or maybe break this A into ah 3 A dash every 2 months, so that it is equivalent to ah the amount A. So, we have an example here to you know show this ah ah to ah ways to solve this problem. So, if a payment of 250 lakh is ah invested ah by mining company every 6 months ah to fund paying 8 percent per year compounded quarterly ah how much ah will it accumulate in 5 years.

So, what does this says ah in in cash flow diagram. So we have, you know, three months, six months, nine months, all the way up to, you know, five years. So five years into, you know, 12 year or 60 months. So in this case, what is happening is, you know, the the investment is or the cash flow is being there every six months, which is 250 lakhs. and ah you know again 12 months 250 lakhs all the way up to ah 60th month.

And here, the compounding is being done quarterly. So, it is every 3 months. Compounding is being done, and the interest rate or the annual rate of interest is 8 percent here. So, every year, this is 8 percent. So, as we saw, the first way to solve this problem is to find the real rate of interest with 6 months of compounding.

So, in that case, what we can do is Again, the cash flow diagram: this is 3 months and 6 months, and our compounding is done every 3 months. So, this is 2 percent because 12 months is 8 percent. So, every 3 months. So, 8 divided by 4 times of compounding, or every 3 months, is 2 percent.

So, in this case, what we are going to do is find this effective or real rate of interest if we compound every 6 months. So, what is this I dash? And we know that this r here is 2 percent into 2, or in 6 months, the interest rate is 8 percent divided by 2. equal to 4 percent. And from our lecture number 2 of this time value of money, we know that I dash is 1 plus R by X whole to the power X minus 1. So, in this case, X is the number of compounding, which is 2, and R equals 4 percent.

which is equal to 0.04. So, if you put the value i dash becomes your 0.0404 what does this mean is instead of you know this 3 month compounding we can say that i dash equal to ah 0.0404 or 4.04 percent. Now, if you see ah you know like ah if you can redraw this ah cash flow diagram we have 3 months 3 months 6 9 and 12 like that in every every 6 months we have a cash flow of 250 lakhs and now the compounding is being or equivalent compounding is being done every 6 months where i dash equal to 4.04.

So, this ah now fits to our previously derived ah formula ah which says F equal to a into 1 plus i whole to the power n minus 1 whole divided by i. So, here a equal to our 250 lakhs, i equal to we know that 4.04 percent and n becomes n is the number of compounding or the number of periods. ah. So, which is you know every 6 months we are getting ah once ah compounding. So, 2 times a year into 5 years is 10.

So, if you put these values this F becomes 3007.1 lakh rupees. So, this is the first way to solve this problem. ah and the second way ah is to divide this 350 lakh in 3 months frequency. So, what we are going to do here is again this cash flow diagram. So, we have you know 3 months here, 6 months here.

So, we are going to divide this 250 ah lakh rupees every 6 months to some a dash and a Now, this 2 a dash becomes ah you know equivalent to 250 ah lag here. So, in this case ah you know this a dash cannot be 250 lag divided by 2 equal to 125 lag, because if we divide this a dash ah or make it A dash equal to 125 lakh, the first A dash after one compounding it will become A dash into 1 plus i ah plus the sixth ah the the next A dash will be again A dash because there is no compounding. So, this and if you put A dash equal to 125

and with any interest rate 1 plus i ah plus 125 is greater than ah your 250. So, in that case what we are going to do is break this ah you know ah A dash ah or ah 250 lakh that we are going to get ah with ah 2 compounding of A dash each which will make you know final amount of 250 lakh. So, or in other words what we can write is 250 equal to a dash into 1 plus i to the power n minus 1 whole divided by i. So, here i we know this is 2 percent and n we know that it is within because this is 2 types of compounding and that is happening to get this ah 250 lag.

So, n equal to 2 and A dash we have to find it out and F equal to ah you know we know that ah 250 lag. So, if you put all these things in the formula here, this will give us A dash equal to ah you know 123.76 lakh rupees. So, ah this ah the same ah you know ah problem can be rewritten So, we have 3 months, 6 months, 9 months all the way up to 60 months and here the A becomes or A dash become 123.67 lakh rupees every 3 months all the way up to 60th month. So now, it ah fits to a formula like ah the the cash flow is every 3 months and the compounding is also every 3 months.

So, you know, we can put this in our previously derived formula here, which says F equals A multiplied by 1 plus i whole to the power, or maybe in this case, we can write A dash multiplied by 1 plus i to the power n minus 1. Whole divided by i. So, here A dash equals 123.67 lakh rupees, and you know i equals, you know, that it is 2 percent. n is, you know, 4 times compounding a year, and there are 5 years. So, this is 20, and you know, using these three things, we can put this in the formula above, and we can calculate F equals. You know, 3007.1 lakh rupees, which is, you know, similar to what we get using our previous way of solving this kind of problem.

Case 1: Compounding Every 6 Months (Effective Interest Rate Method)

Annual interest rate (r) = 8%

Compounding is done quarterly → 4 times a year

Quarterly interest rate = r / 4 = 0.08 / 4 = 0.02 (or 2%)

Effective interest rate for 6 months:

$R = 0.02 \times 2 = 0.04$

Effective interest rate (i') for 6-months:

$i' = (1 + R/x)^x - 1 = (1 + 0.04/2)^2 - 1 = 0.0404$ or 4.04%

Using compound interest formula:

$F = A \times ((1 + i)^n - 1) / i$

Where:

A = 250 lakhs, i = 0.0404, n = 10 (2 payments/year × 5 years)

$F = 250 \times ((1 + 0.0404)^{10} - 1) / 0.0404 = 3007.1$ lakh rupees

Case 2: Dividing 6-Month Cash Flow into Quarterly Installments

Cash flow of 250 lakhs every 6 months is split into two equal payments (A') every 3 months

Let the equivalent quarterly payment be A'

$F = A' \times ((1 + i)^n - 1) / i$

Where:

F = 250 lakhs, i = 0.02 (2%), n = 2 (two quarters make one 6-month period)

Solving for A':

$250 = A' \times ((1 + 0.02)^2 - 1) / 0.02$

A' = 123.67 lakhs

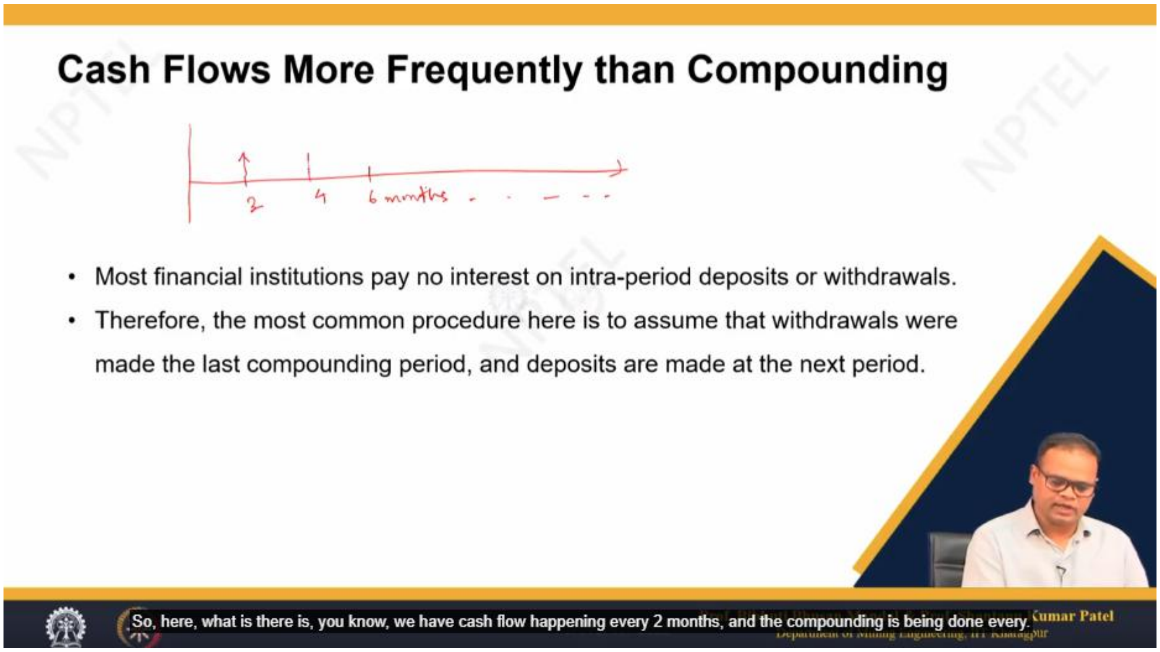
Now the full cash flow becomes 123.67 lakhs every 3 months for 60 months (20 periods)

Using the formula again:

$F = A' \times ((1 + i)^n - 1) / i$

Where: A' = 123.67, i = 0.02, n = 20

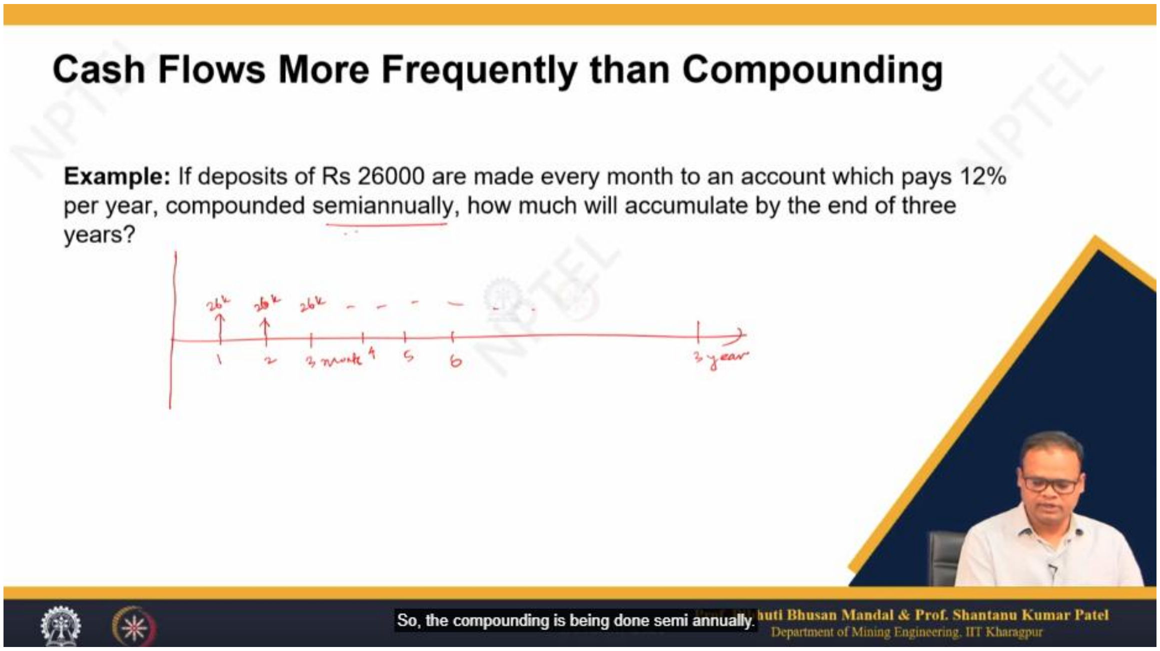
$$F = 123.67 \times ((1 + 0.02)^{20} - 1) / 0.02 = 3007.1 \text{ lakh rupees}$$



The second, you know, issue that we saw is that the cash flow can be more frequent than compounding, which, you know, can be put in the cash flow diagram as we were seeing, like this is 3, 6, or 2, 4, 6 months. All the way up to some n months. So, here, what is there is, you know, we have cash flow happening every 2 months, and the compounding is being done every. 6 months. So, in these cases, you know, again, we cannot apply the previously derived formula, and for these cases, actually, most financial institutions pay no interest rate on intra-period deposits or withdrawals.

Therefore, the most common procedure here is to assume that the withdrawals were. Made at the last compounding period, and the deposits are made at the next period. What this means is, in the first case where we have, you know, deposits at 2, 4, 6, let us say this is, you know, 100,000 or, you know, 100, whatever it is. So, in these cases, these two will, you know, go to the deposit will go to the next period, and you know, it will be assumed that the total amount was deposited at 6 months. And if you have some kind of withdrawal, like, let us say this is again 2, 4, 6, the compounding here is being done every 6 months.

So, in this case if there is withdrawal, so these withdrawals will go to the you know the previous ah compounding ah period like that in this case up to t equal to ah 0.



So, related to this we have a we have an example here which says if a deposit of ah you know 26000s are made every month to an account which pays ah 12 ah which which pays 12 percent per year compounded semi annually, how much will it accumulate by the end of 3 years. So, here what we are doing is you know if you again draw the cash flow diagram what we have. So, we are depositing 26000 rupees every month or there is a positive cash flow of 26000 thousand twenty six thousand twenty six thousand all the way up to you know end of three years.

So, this is three years and this is one month two month three months all the way up to you know four five and six months. So, the compounding is being done semi annually. So, the compounding is done here So, in this case what we are going to do is you know this 26000 26000 and 26000 it will come all these 26000 and will come here and you know we will consider all those things to be you know the cash flow to be done at 6 months. So, in this case you know the amount that we are going to get or in this case the equivalent amount A will be 6 times 26000, which is you know your 156000.

So, in this case you know ah our ah it it now fits to our ah the previously derived formula we have 6 months, 12 months, 18 months all the way up to 36th month which is 3 years and every year we have 156,000 156,000. 156,000 of cash flow all the way up to 36 months.

and this compounding is being done every ah every 6 months. So, in this case we can again put this ah into a formula F equal to a into ah 1 plus i to the power n minus 1 whole divided by i. So, this becomes a equal to 156000 into 1 plus i, i is our ah the interest rate every 6 months.

So, this is 12 percent divided by, you know, 2, equal to 6 percent, and n becomes, ah. So, this is 0.06 whole to the power n. So, n equals every 6 months for 3 times or 3 years compounding. So, it becomes 6. So, to the power 6 minus 1. Whole divided by I is again 0.06.

So, this becomes Rs 10,88,150 rupees. So, this, ah, ends our lecture on, ah, the time value of money.

Monthly deposit: ₹26,000

Duration: 3 years = 36 deposits

Compounding frequency: Semi-annually (every 6 months)

Group monthly deposits into 6-month blocks:

$A=6\times₹26,000=₹156,$

Annual interest rate $r=12\%$

Semi-annual rate $i=12/2=6\%=0.$

Number of compounding periods $n=3\text{ years}\times2= 6$

$F=A\times((1+i)\ n -1)/i$

$F=₹156,000\times(1+0.06)6-1/0.06 = ₹10,88,150$

So, the accumulated amount after 3 years is ₹10,88,150.