MINERAL ECONOMICS AND BUSINESS

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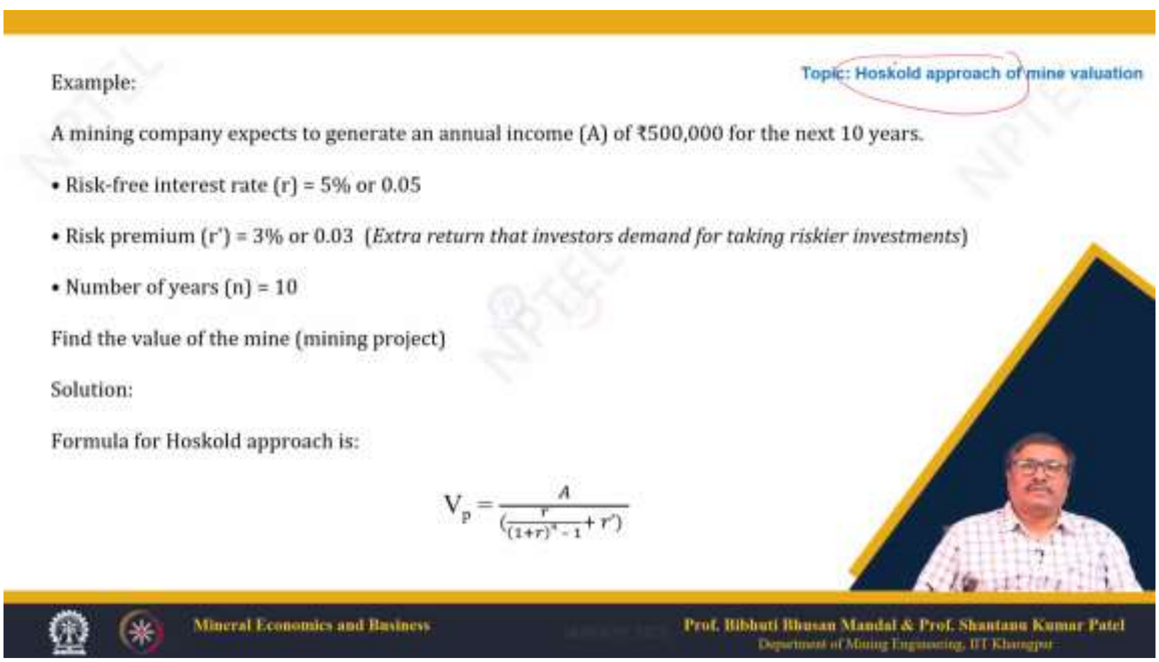
Week 7

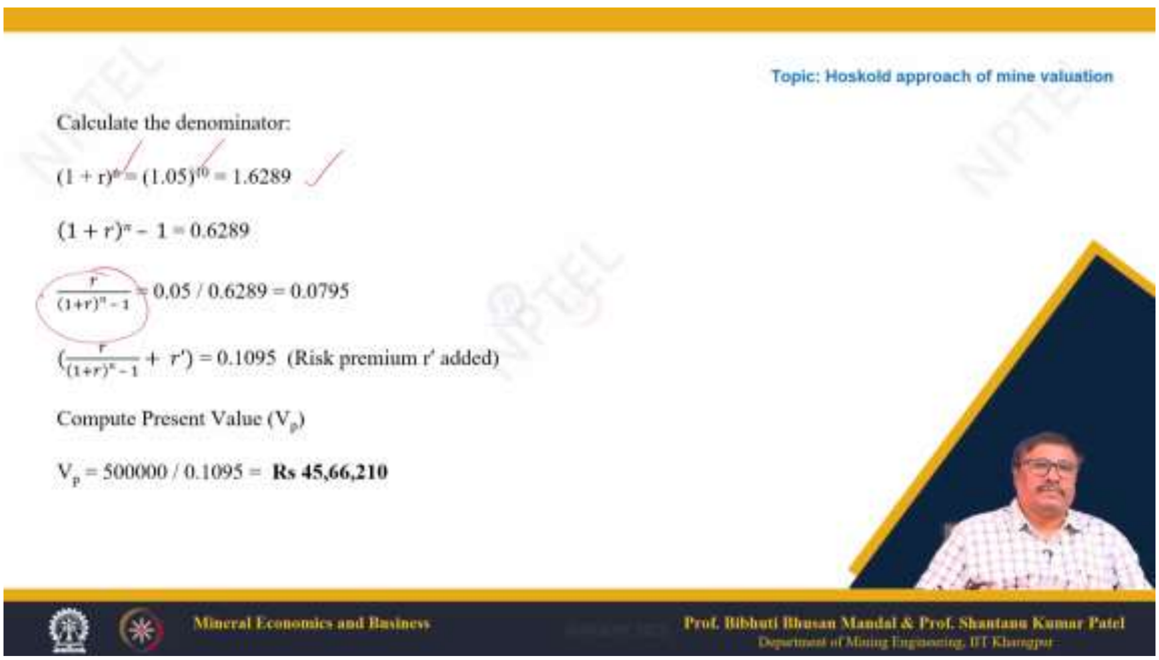
Lecture 31 : Problems and solutions - I

Hello, welcome everybody. Today in this particular lecture, we will not be learning additional concepts or topics, but we will be discussing problems and solutions related to mine valuation and the cost of capital that we studied extensively last week. So, to start with, I will ask you to remember what we said about the Hoskold approach. The Hoskold approach of mine valuation. In this, what we are talking about is that we try to determine the value of a mining project.



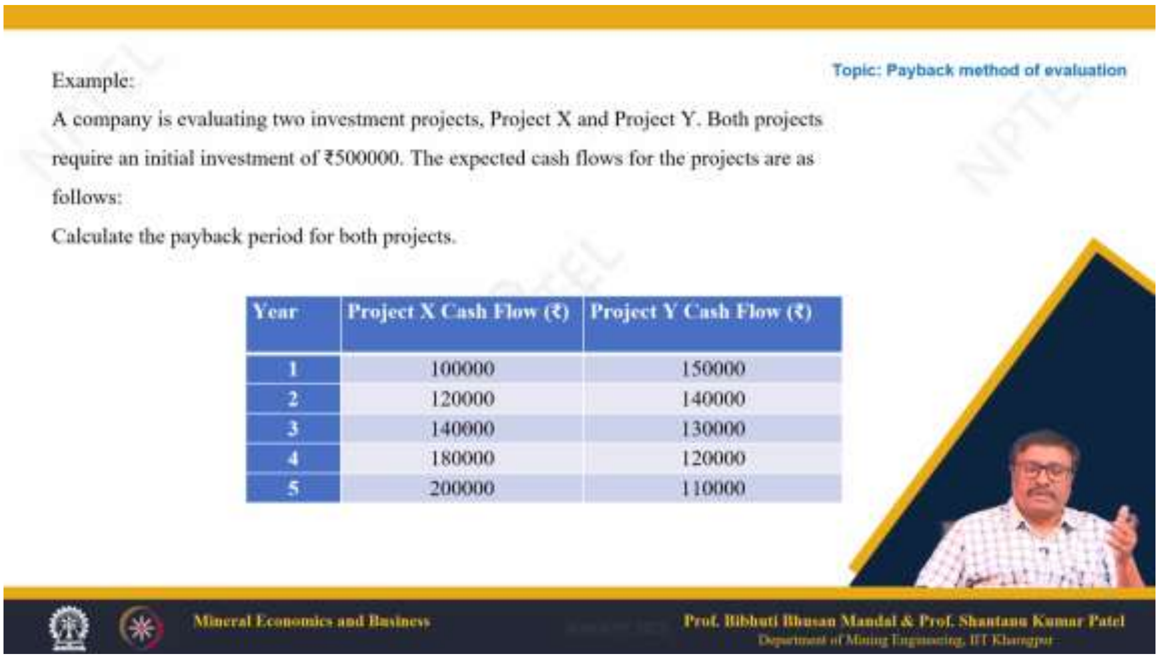
Now, this is only an example of how it is done—nothing else. Not the exact values or realistic values have been used here. So, for example, a mining company expects to generate an annual income, denoted by 'A' here, of 5 lakhs for the next 10 years. Let us assume a small mine where we are expecting to generate or earn about 5 lakhs per year for the next 10 years. And the risk-free interest rate in the market is known to be, for example, assume it is 5%. And the risk premium—extra return that the investors demand for taking the riskier investment—means they are taking more risk in the mining business.

So, the extra premium is assumed to be a risk premium of 3 percent, and the number of years, as you have seen, is 10 years here, n equal to 10. So, what is the value of the mining project at present? So, how do you do that? The Hoskold approach is a very well-known approach, and it is denoted by the formula: A divided by r divided by (1 + r)^n minus 1 plus r', where r' is the risk premium, and r is the risk-free interest rate. A is the annual income that we are expecting in the next 10 years, every year in the next 10 years to come. If you calculate the (1 + r) part and gradually all these values, and then you add them, you will find that r divided by (1 + r)^n minus 1 then becomes 0.0795.



We have taken a few more digits after the decimal to make it more precise, and now we have added. We added the value of r dash. So, the value of r dash is added to the rate of return. So, the risk premium is r dash, which is 3 percent that we have added, and now it has become 0.1095. This plus 0.03 comes to 0.1095. So now, the present value will be this annuity every year we are getting divided by the denominator, this part.

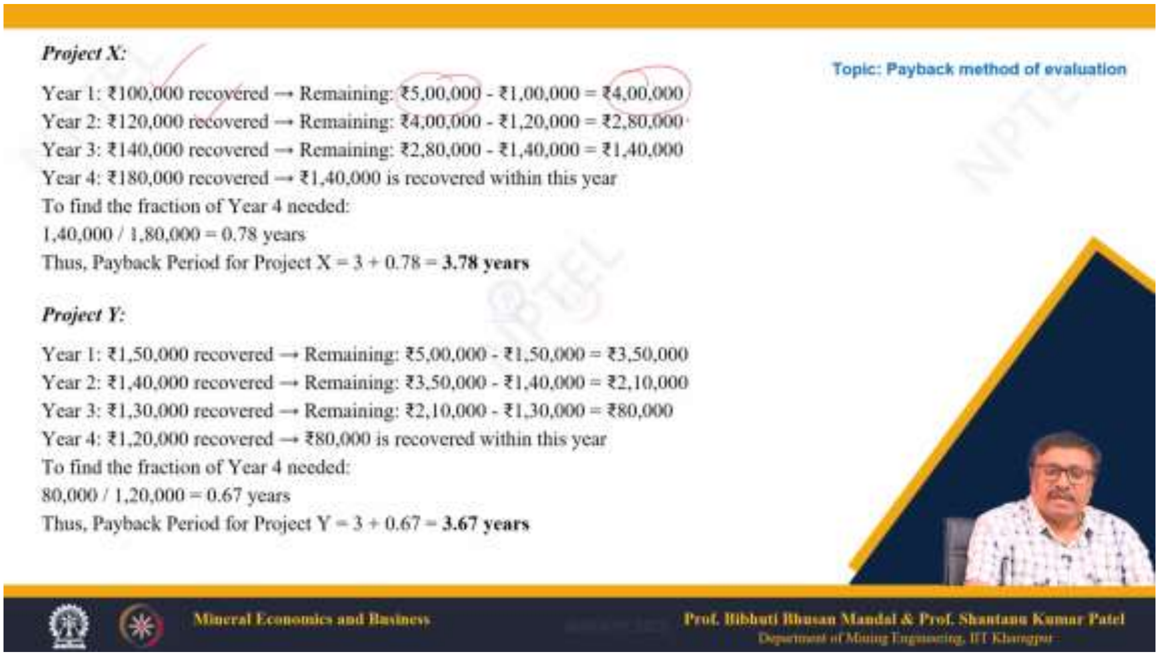
So, 0.1015 if you divide, then you get the present value as 45,66,210, roughly 45.66 lakhs. That is the present value. So, this is how, say, if it is in crores or billion dollars, whatever it is, the thing is that you can use the Hoskold approach as a rough and immediate understanding of what could be the value of a mining project that you are trying to evaluate. So, these are in the beginning we can start, and then we can go into a much more detailed evaluation process. That was all about a typical example of how to use the Hoskold approach for mine valuation. There is one more simple method where we said the mine valuation can be done by asking a simple question: in how many years I am going to get back my money? Simply, what is the payback period?



So, we call it the payback method. A rudimentary method, but it gives you an idea, and there are some people who will go by this immediately—that okay, for 5 years I can invest or 5 to 10 years I can hold to get back my money, and after that, what you get is extra or there is extra return. So, let us assume that we have two offers: Project X and Project Y, where everywhere we need 500,000 rupees for initial investment. So, the expected cash flows for the projects are tabulated here. This is for the first one, and this is for Project Y—Project X and Project Y. See that when we learn this thing, we assume for simplicity that the cash flow every year will be uniform.

Here, the cash flow is not uniform. A little more detail has been added, from which we can see that there is a constant growth in the cash flow. So, in the beginning, we have less, and then it is constantly growing. Here, it is fluctuating a little, going up and down.

Maybe it is much more developed, and this is not yet fully developed. So, it has a prospect, and it is a fluctuating one we need to understand.



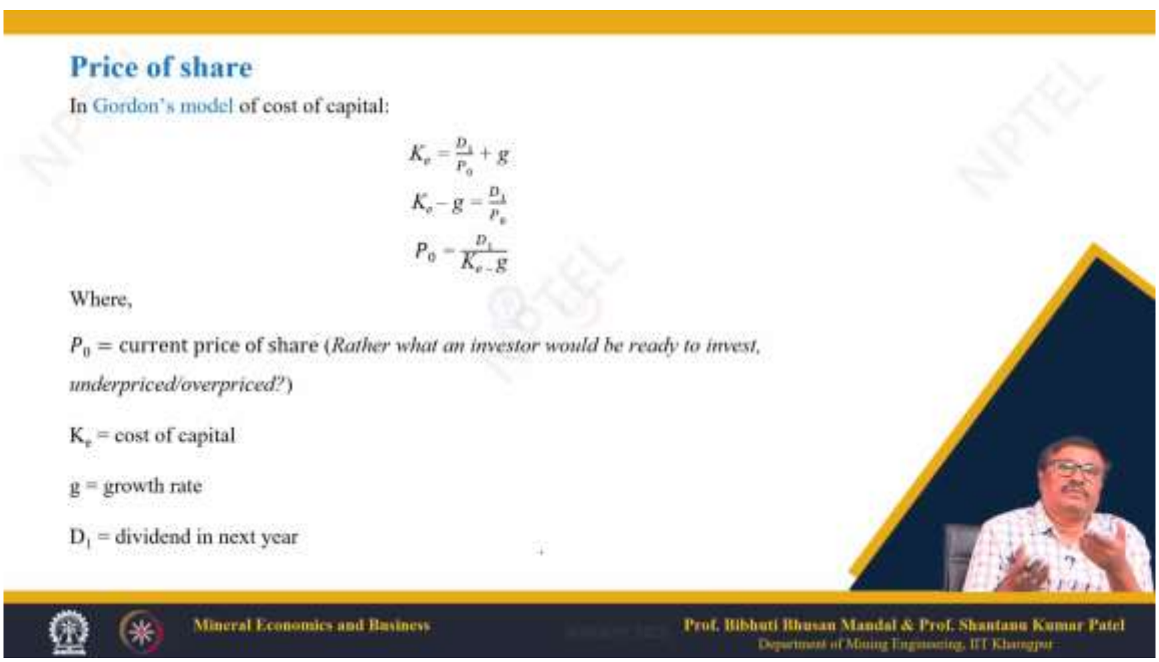
This will give us the payback period, and then, for both projects, we can evaluate the payback period and compare. So, by simple calculation, I mean every year we are recovering something. So, in the beginning, we have 500, 1000 rupees minus 1 lakh. So, the remaining is this much. So, you go on every year.

And then it is remaining 2.80 lakhs. Similarly, in year 3, you also recover; in year 4, you go on recovering. So, here, within this period, now what we have to find out is the rest amount we have recovered in the first 3 years, and then there is a part which will be remaining. So, we need to find out the fraction of that because every year we are not recovering 1,80,000 here, but what we will try to assume here is that in the next year also, we will be getting 1,80,000. So, the fraction of year 4 is needed because we recovered, say, for example, 1,80,000 in the fourth year. So, we have to only

we are supposed to recover the entire thing, but we need to recover 140,000 in the year 4. So, what we do the 140,000 divided by 180,000 is 0.78 or say or say 0.8 years roughly. So, we will add the first 3 years plus plus 0.78 that will come to 3.78 years that means, this roughly 3.8 years or so. Now, here what we see in the next case, we do the same procedure, we recover 1 lakh 50000 and then remaining is 5 lakh minus this and with this is remaining and again we go on recovering and we find out this is remaining. Again we recover 1 lakh 30000 and what is remaining.

So, in the last one we have to recover 80000 rupees. So, what we do that even though the 120,000 is recovered. So, we need to find out that how much time is required as a fraction of the year 4 was to recover the 80,000 rupees. So, 80,000 divided by 120,000 say 0.67.

So, we add these 3 years plus 0.67 and find out say this is a simple example to find out that when we have the returns which are not constant which is varying then you have to go on calculating how many years is required to recover. Here the difference is practically nothing, I mean the if you go by the years the 3.78 or 3.67 years is not there is no practical difference. what we can say in the previous example that both the projects are equally attractive equally attractive as far as payback period is important. But if the if you if you see that there is a huge difference between two two projects in terms of the payback period in that case you can take a decision that this project is more attractive to me because the payback period is less.



Now we are shifting our focus to the share equities, debt, and debentures. We have studied a lot about the cost of equities or cost of capital. Here, we are not using it directly in the first example or the first problem. Rather, we are not discussing the cost of capital directly; instead, we are starting with the price of a share. So, as we know, in the in

Gordon's model for the cost of capital, we calculated K_e as the cost of capital. Then, we can calculate the cost of capital as:

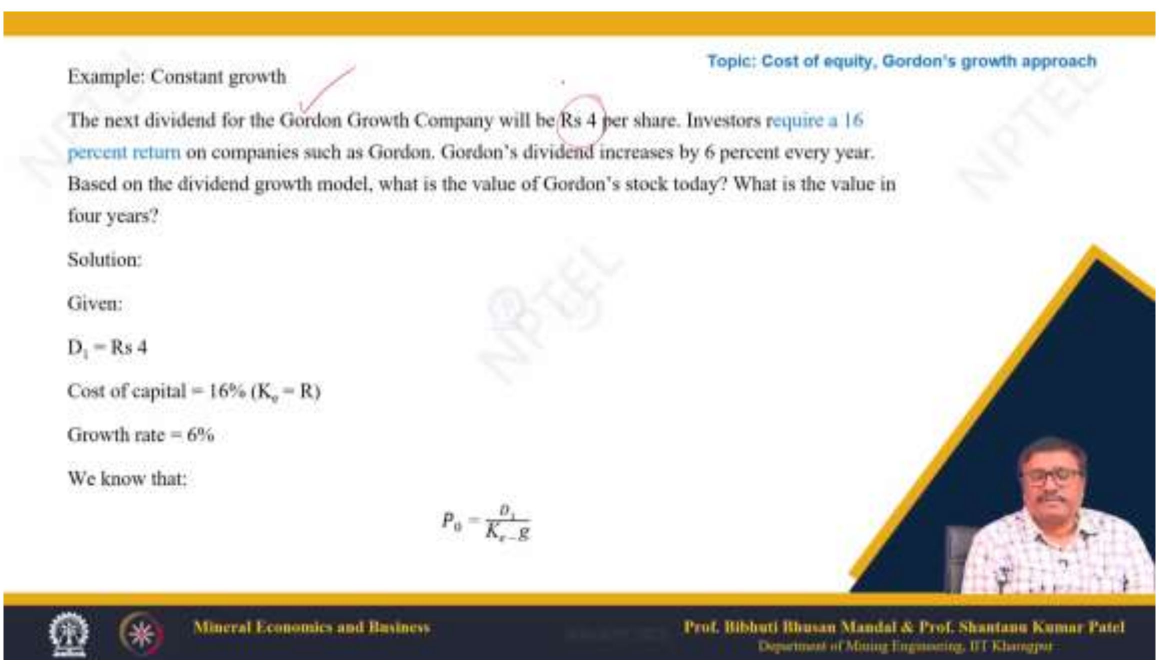
$$K_e = \frac{D1}{P0} + g$$

So, if you rearrange this, then P0, which is the current price of the share, can be calculated as:

$$K_e - g = \frac{D_1}{P_0}$$

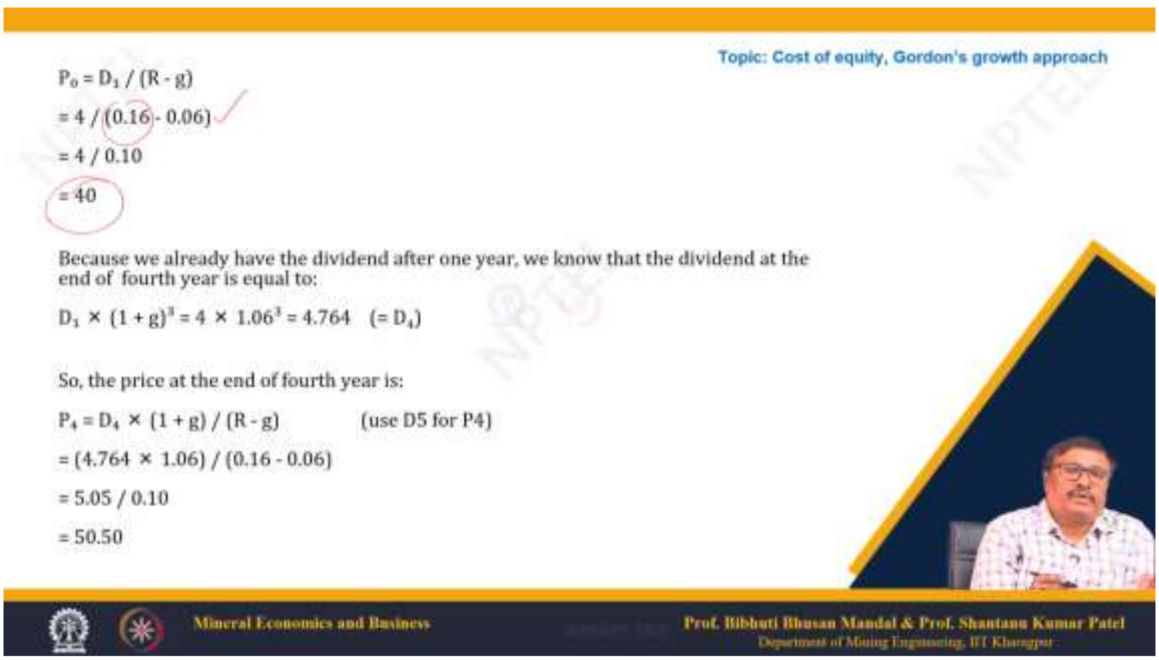
$$P_0 = \frac{D_1}{K_e - g}$$

Now, this current price of the share is what an investor would be ready to pay. That price of the share is what we are trying to indicate. It is not the current price of the share, which is already known to us. We are trying to evaluate what it should be—that means, what an investor would be ready to pay for that stock, which we are trying to calculate. So, if it is available at a cheaper rate, that means it is underpriced. If it is available in the market at a higher rate than the current price we compute from this formula, then the stock is overpriced. So, now we know that P0 is the current price in this formula. As I said once again, I repeat that K e is the cost of capital, G is the growth rate, and D1 is the dividend next year—the dividend after one year.



So, we will just use that idea in different forms in the next few examples. As you can see, in this particular case, we have intentionally used the name Gordon, even though it is actually Gordon's growth approach—who first did it—a very well-known approach. But we are given the name Gordon Growth Company incidentally. So, the next dividend for the Gordon Growth Company will be, say, rupees 4 per share. Now, the investors are expecting—or rather require—a 16 percent return on companies such as the Gordon Company. Now, this dividend is known to be increasing by 6 percent every year on average.

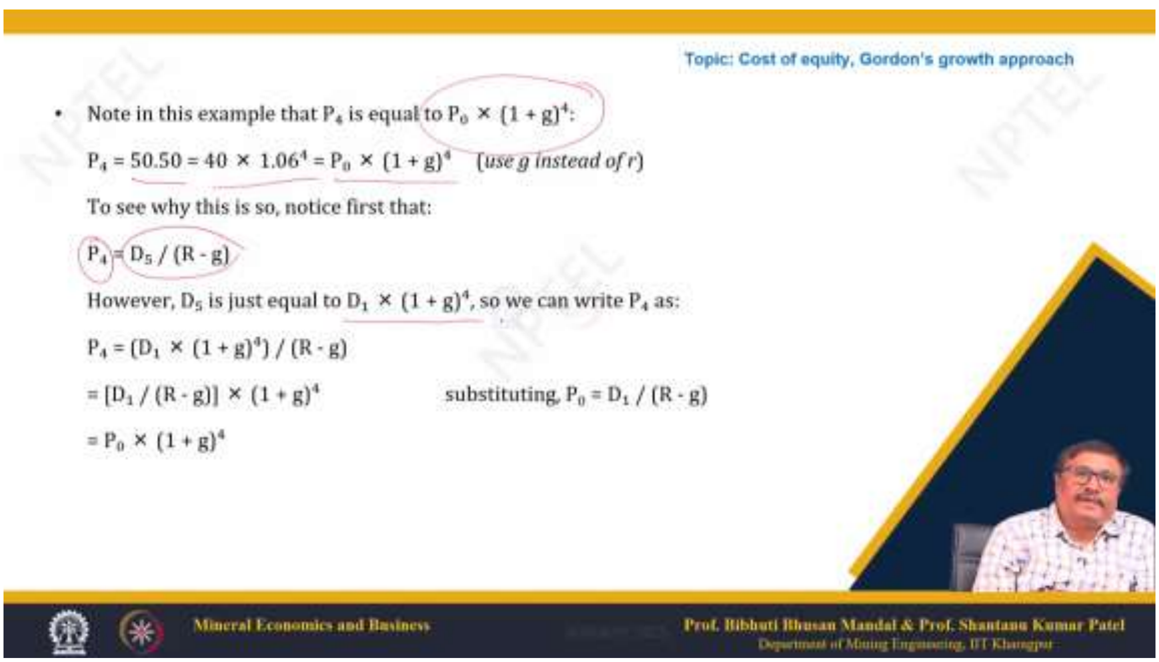
So, based on the dividend growth model that we have seen earlier, what is the value of the Gordon stock today? Gordon stock today means we are not referring to the actual market value. or the face value there, but we are trying to evaluate the stock price based on the factors or parameters we have just discussed. So, what is the value again in the next 4 years? In 4 years.



So, given here, D1 is the dividend next year, which is 4 rupees, and the cost of capital—meaning the required rate of return or expected rate of return—is 16 percent. and the growth rate is 6 percent. Now, from here, we can determine the present value of the stock. Let us see how it is calculated. So, $P0 = D1/(Ke – g)$, here, R represents the cost of capital we have already defined.

So, this comes to 16 percent, and minus G is the growth rate here. So, it comes to 40 rupees as the current price because we already have the dividend after 1 year. So, we know that now we are calculating for the fourth year. So, we know that after 1 year, the dividend is 4 rupees. So, now from here, another 3 years have to pass.

So, from here we can calculate when we know the growth rate G. then we can calculate from here and find out that 4.764 is D 4, the dividend expected in 4 year is D 4 which is 4.764. So, the price at the end of the 4th year is P 4 which is equal to D 4 into 1 plus g divided by R minus g. So, where what we are doing use d 5 for P 4 because for example, for P 0 we are doing d 1 by R minus g. So, simply by deduction you can say that for P 4 we use d 5 divided by R minus g and we know that d 5 equals to d 4 into 1 plus g. So, from there we can calculate and find out that it is 50.50 is the price in next 4 years in 4 years 4th year rather.



In this example, what we might have seen that:

$P_4 = P_0 \times (1 + g)^4$, which is usually taken in the dividend growth model dividend growth model. But we have used the same thing for the price of the share also why?

See the $P_4 = 50.50 = 40 \times 1.06^4 = P_0 \times (1 + g)^4$. is just like we are replacing r with g. So, it is showing us the same value.

So, why it is so? Let us see that $P_4 = D_5 / (R - g)$. However, $D_5 = D_1 \times (1 + g)^4$. So, we can write:

$P_4 = (D_1 \times (1 + g)^4) / (R - g)$

$= [D_1 / (R - g)] \times (1 + g)^4$

$= P_0 \times (1 + g)^4$

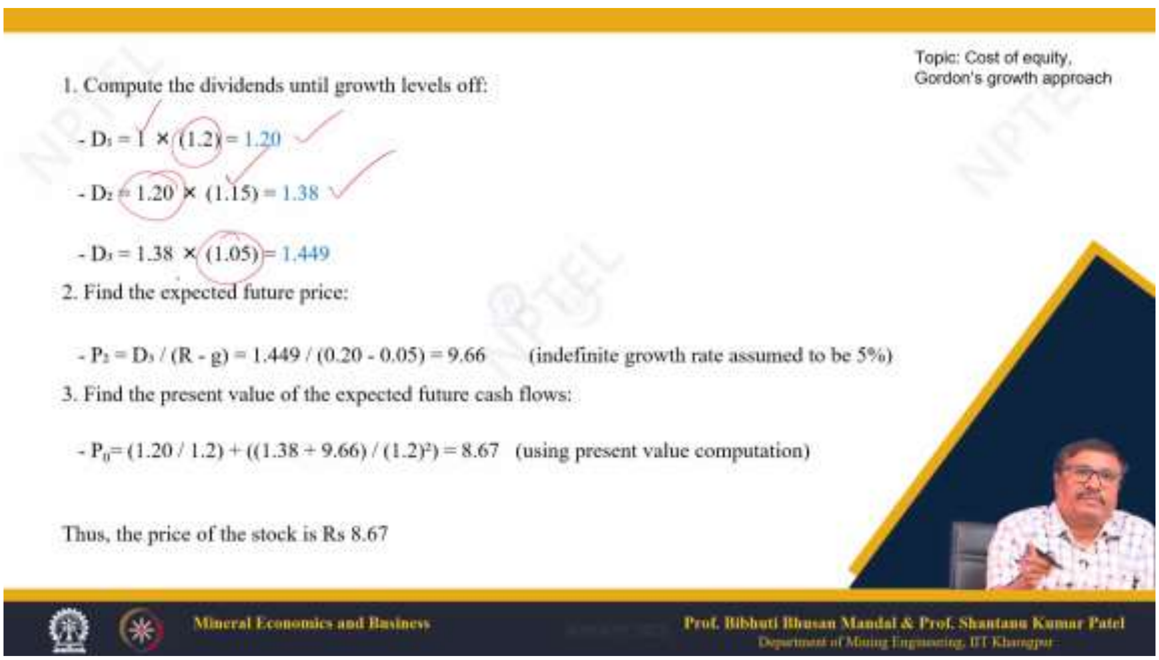
We get same result at even if it is not the direct dividend growth formula.

So, we can use the same thing in the case of the price of the shares also, similarly. Now, in this particular example, we are not talking about constant growth. So, we are now into non-constant growth. So, take for example, A firm is expected to increase dividends by 20 percent in 1 year and then by 15 percent in 2 years. That means another year added after that, the dividends will grow at a rate of 5 percent, and we are assuming that indefinitely.

So, the last dividend was 1 dollar. So, what will happen is that in 1 year there will be 20 percent, and then next year, over and above that, we get 15 percent in 2 years. After that, year after year, we expect that it will grow at the rate of 5 percent, as we have seen here.

Now, the required return is 20 percent, and what is the price of the stock? OK, the required return is assumed to be 20 percent. So, what is the price of the stock?

So, here we know that the dividend $D_n = D_{n-1}(1+g)$. So, now again, if you are trying to find out the present value, then we first try to find out the future value divided by 1 plus the discount rate to the power of 1. This is the present value formula. The present value formula, which you have learned in the time value series of lectures. So, we will use both these two concepts to solve the problem. Now, the dividends we are trying to find out. So, D 1 will be after 1 year, which is a 20 percent growth on 1 dollar, coming to 1.20 dollars, and then on 1.20 again, we are adding 15 percent.

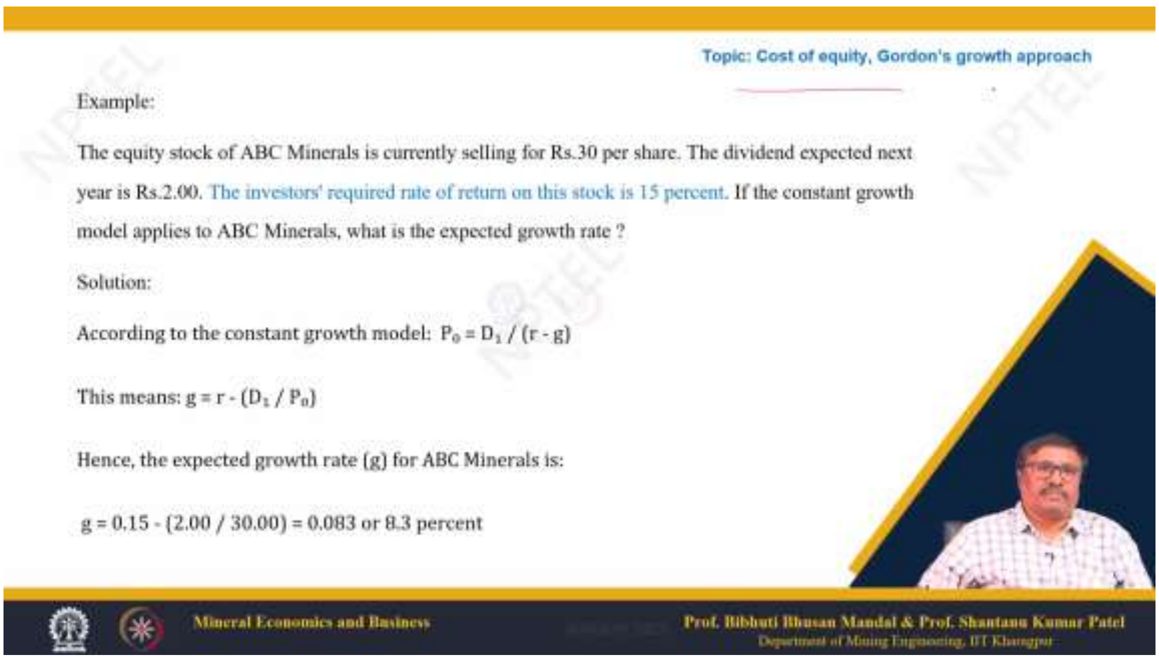


So, it is coming to 1.38. Now, again it is growing at the rate of 5 percent. So, now this is D3 becoming 1.449 dollars. Now, what is the expected future price?

That is $P_2 = D_3 / (R - g)$ So, if you substitute this 1.449 by the 20 percent return minus the growth rate, then it is coming to 9.66 because we are taking 5 percent thereafter continuously in different growth rate assumed to be 5 percent thereafter. So, now, we are just adding all those present values and bringing down to P0, the current present value net present value. Now, the present values computed together and the whole thing is coming to rupees 8.67 considering dividends growth which is not constant. In the first year, it was

20 percent, then 15 percent, and after that 5 percent. So, based on that calculation, we get that the price of the stock is 8.67. If it is available at 12 rupees, that means it is overpriced.

If you are getting it at 8 rupees, then it is underpriced, and you can go for it. You can purchase. The current price of the stock is computed to be 8.67 dollars. Sorry, the unit was dollars. Now, we are taking another case where we are talking about the cost of equity gain and using the Gordon's growth approach, one more example. So, equity stock of ABC minerals is currently selling at say 30 rupees per share. Now, the dividend expected next year is rupees 2 per share.

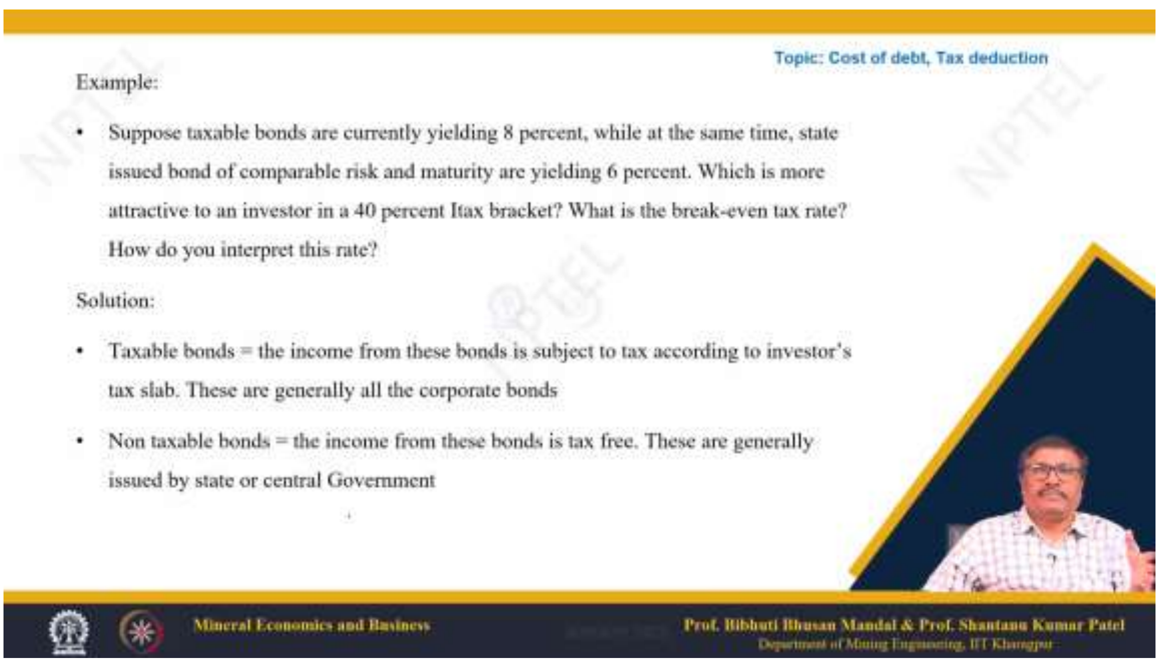


So, the investors required rate of return is 15 percent which is the we can we say that it is the ah the cost of capital here. If the from his side they are expecting 15 percent return from there. If the constant growth model applies to ABC minerals. So, what is the expected growth rate?

That means, we have need to find out the value of G. So, according to the constant growth model as we know that : $P_0 = D_1 / (r - g)$, R is the required rate of return and G is the growth rate. If you simply put these values and then you can get the G equals to 0.15 minus 2 divided by 30. So, the growth rate is 8.3 percent. if the constant growth rate model applies to ABC, it is constantly growing at 8.3 percent.

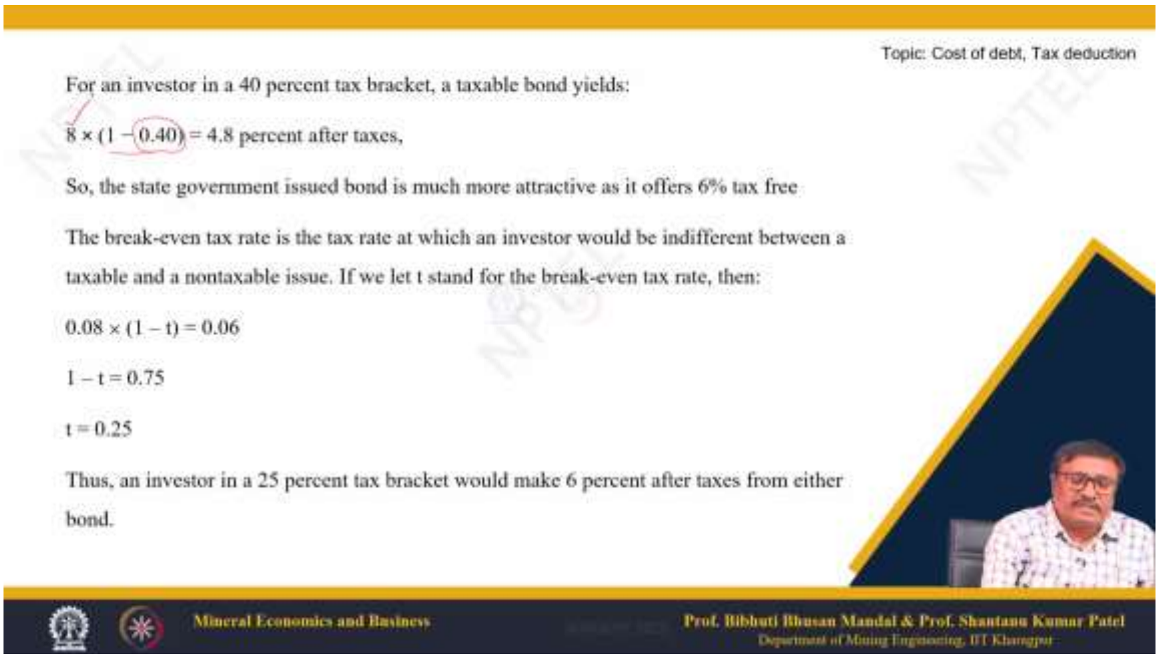
Ok with this then we are coming to the debentures of debt the cost of debt and some ah simple ah examples more examples to demonstrate what we learnt in the lectures with ah regarding the debentures and the tax deduction ah tax benefits attached to it. For example, we have suppose we have taxable bonds which are currently yielding 8 percent per year. So, while at the same time state government issued bonds bond of comparable risk and maturity are yielding 6 percent. Now, which is more attractive to an investor? So, currently taxable bonds are there which is 8 percent taxable means the the person who is earning from this is taxable.

So, that is say 8 percent yielding and the state ah bond is yielding 6 percent. So, which is more attractive to an investor in a 40 percent tax bracket for example, income tax bracket. So, what is the break even ah tax rate and how do you interpret this rate this is the So, the taxable bonds are   the income from these bonds this means that the income from these bonds is subject to income tax deductions according to investors tax slab. So, here it is 40 percent.



These are generally all the corporate bonds. So, whatever you earn from there is taxable. Now, these are non-taxable bonds. So, income from these bonds are tax free. usually in government bonds ah if it is defined so like that.

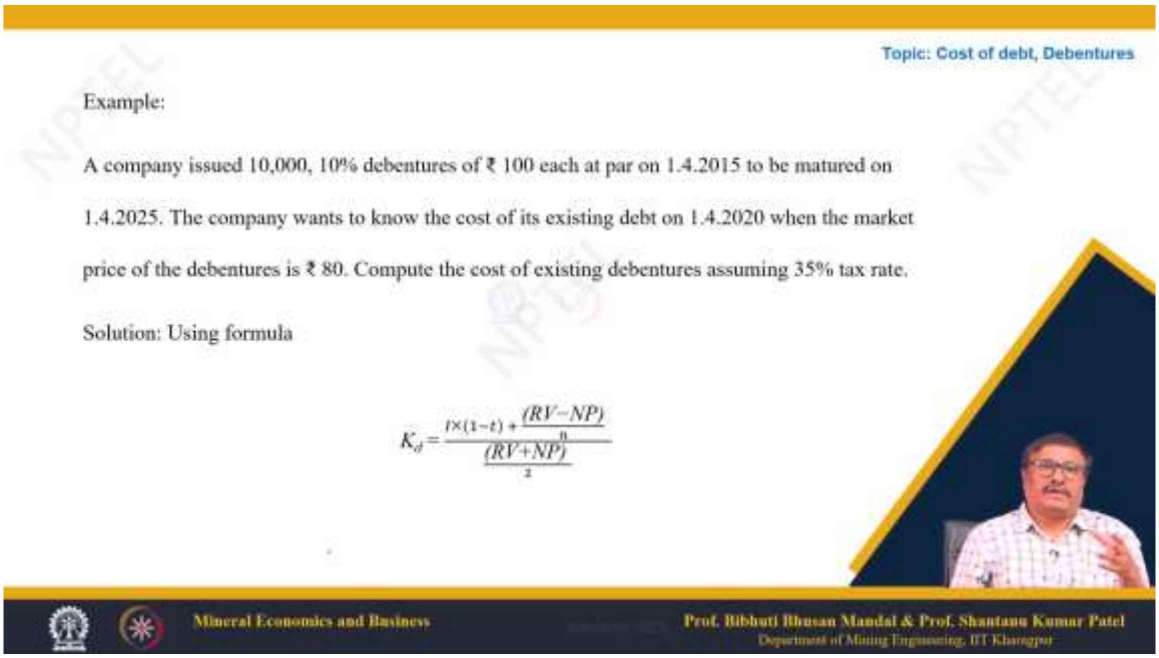
So, these are generally issued by the state or central government. So, there for example, in this case we assume that that is tax free ah that is tax free. So, for an investor in a 40 percent tax bracket, so tax bond will in ah yield actually in real this 8 into 1 minus 0.40 because there is a tax reduction here. So, basically even if even if it is yielding 8 percent, but actually what is getting 4.8 percent after taxes because he is paying tax in the 40 percent slab.



So, effectively, it is a growth of 4.8 percent after taxes. Now, the state government issues bonds which are much more attractive because they offer 6 percent tax-free. So, we can go by this because effectively we are getting 4.8 percent from the corporate bond, which has a tax rate of 40 percent as far as that investor is concerned. But here, the state government is issuing much more attractive 6 percent tax-free bonds. So, in this case, the 6 percent tax-free bonds are more attractive.

Now, what is the breakeven tax rate? Breakeven tax rate means what is the tax rate at which both the opportunities, or rather both the investment proposals, are equally attractive. So, instead of using that 40 we are keeping that t as a variable which we need to find out. So, the 0.08 or 8 percent into 1 minus t is 0.06 because 0.06 is the 6 percent tax-free bond.

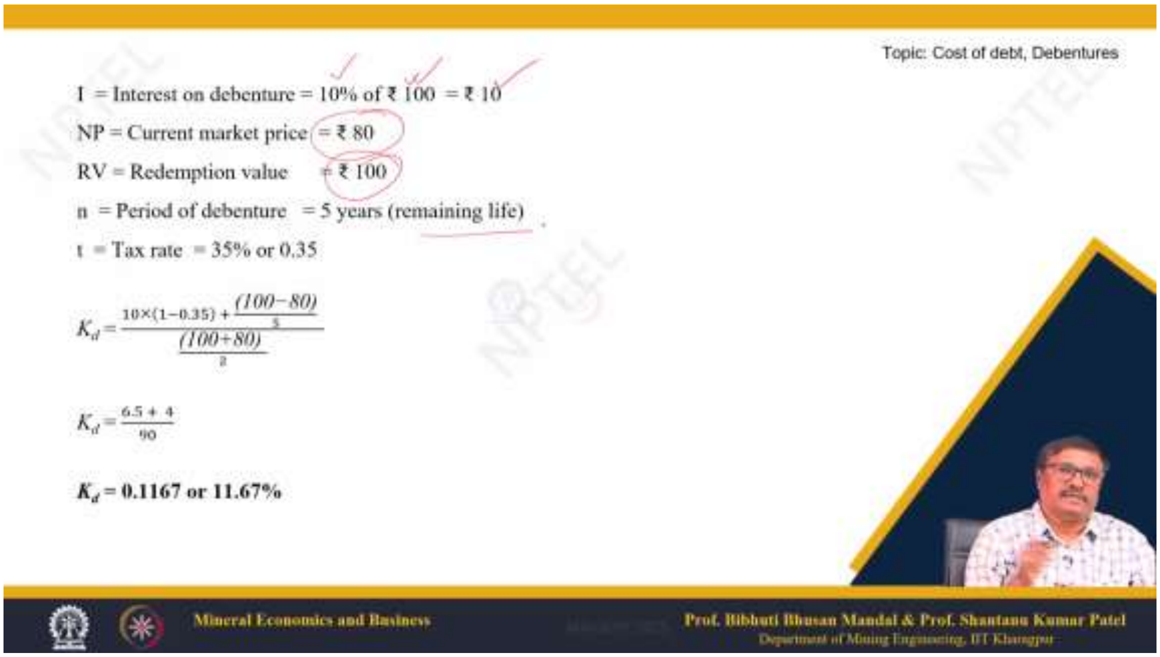
So, now, both sides are equal. So, what we are doing from here is finding out that 1 minus t is 0.75 and t is 25 percent. So, at a 25 percent tax slab, both these investments are similarly attractive. That means an investor in a 25 percent tax bracket would make 6 percent after taxes from either bond. That means the same thing: it will have 6 percent from both offers. You can choose either, and the benefit will remain the same.



Now, this is a simple use of the formula which we have learned in the lecture classes about debentures and their related calculations. And, the debentures enjoy certain things; the company enjoys certain things when they issue debentures in the market, like the tax benefits given to the company when they pay interest to the debenture holders or debt holders. Now, let us take a very simple example: the company issued 10,000 debentures with a 10 percent yield and a face value of 100 at par on 1 April 2015. It means 'at par'—we are not giving a discount, nor are we asking for a premium. These are to mature on 1 April 2025.

Now, the company wants to know the cost of its existing debt in 2020—that means after 5 years, when the market price of the debenture is ₹80, even though it was issued at ₹100; the current price in the market is ₹80. The cost of the existing debentures, assuming a 35 percent tax rate—you get the benefit of deducting the interest you pay from your income at the rate of 35 percent. So, you use the well-known formula where the cost of capital is the cost of debentures. It is: I × (1 − t), where t is the tax rate, and then the redemption value (₹100, because that was the face value) - Net Proceeds, divided by (RV + NP)/2, that is the standard formula used for calculating the cost of debt. Now, from here, you see that the interest on the debenture is 10 percent of ₹100 every year.

- $$K_d = \frac{I \times (1-t) + \frac{(RV-NP)}{n}}{\frac{(RV+NP)}{2}}$$



So, you have to pay ₹10, but the current market price is ₹80. The redemption value is ₹100, as I said—this is the face value. So, N is the period of the debenture, which is 5 years of remaining life because we have already reached 2020. So, from there, another 5 years remain. The tax rate is 0.35, or 35 percent.

Using replace, I mean substituting this value in the formula. If you find that the Kd, the cost of capital—here, the cost of capital is the cost of debentures issued by the company—is 11.6 percent, 11.6 percent. A similar problem was also solved when we took the lectures, where a similar problem was addressed. So, some of the problems which are very standard and not with so many complications—that we need not understand at this level—are explained to you. So that you have the basic idea of what exactly you mean by the cost of capital. Because there are so many financial instruments through which we raise capital from the market.

So, what is the cost of all those things under different conditions—existing interest rates, expected rates of return, growth rates? And the market price variations, the redemption values—all these things we include the basic things, and we try to understand the implication of all these things on the cost of capital. So, when you add up all those things—this cost of capital—then you must have already learned how to find out the weighted average cost of capital, which is a very important parameter in the valuation of projects. And especially in costing, that is very, very important. So, thank you very much for joining this class.

You can take these references from the ICAI website also. The Mineral Project Valuation book is a very good book from where you can take references. And, of course, the Mine and Mineral Economics—the first course approach—also, you can read some parts from here. And also, you can take examples and do more studies through many books and internet resources if you are interested in learning in detail. Thank you very much once again.