MINERAL ECONOMICS AND BUSINESS

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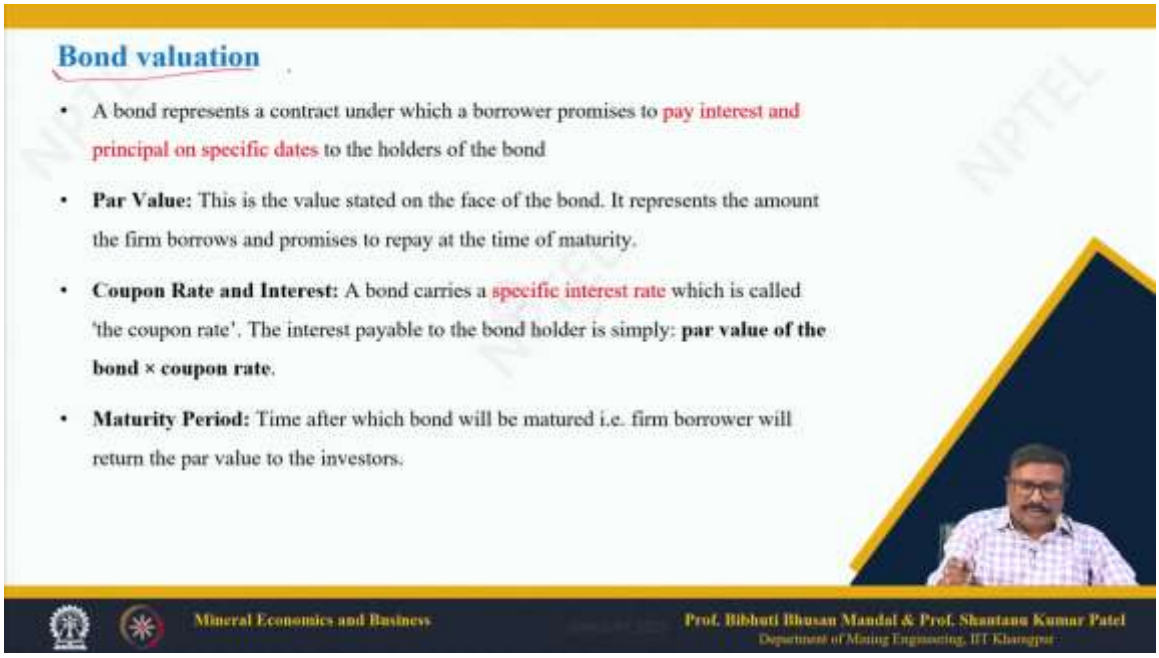
Week 7

Lecture 32 : Problems and solutions - II

Hello, welcome once again, welcome everybody to this problems and solutions part 2. In this we will be continue with the certain exercises of course, certain ideas ah related to which the problems have been formulated that I will first introduce and then ah give you some practical problems ah provide with the solutions thereafter. The concepts major 2 concepts and the related problems here are about the bond and loan amortization. Mostly we will be dealing with the bond valuation part with different forms of the ah problems.

Now, let us recapitulate rather let us try to understand once again what do we exactly mean by the bond valuation. So, bond like as we have we have talked about this ah bond debentures all these things. So, the bond represents basically a contract between the ah borrower and the company. So, under which borrower promises to pay interest and principal on specific dates to the holder of the bond.

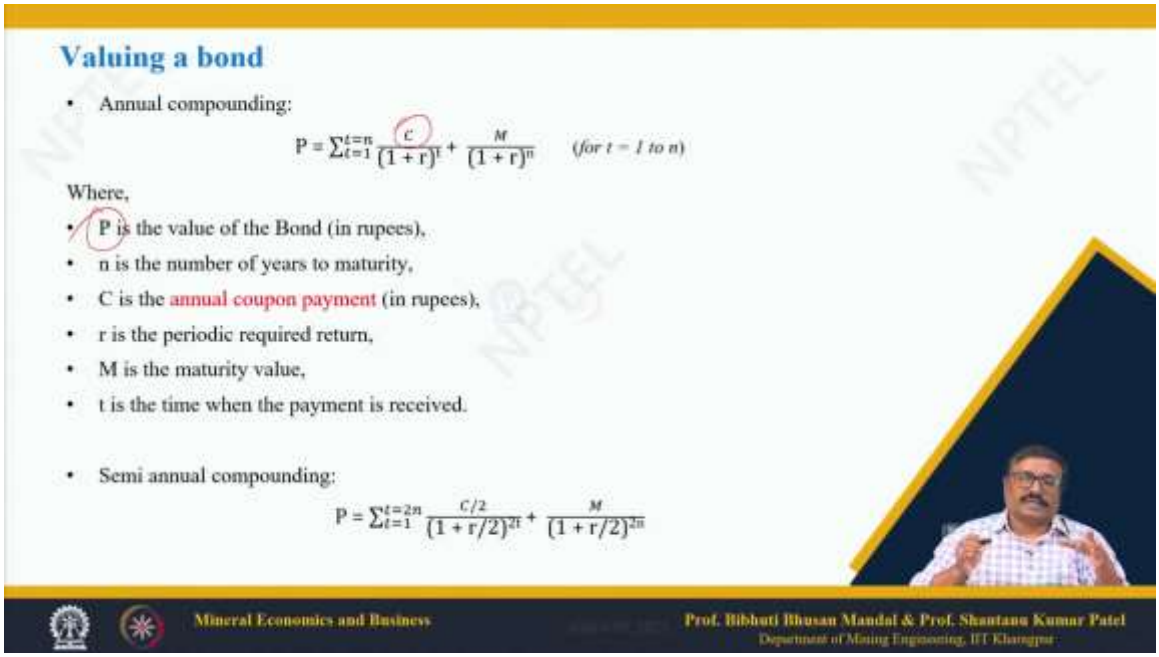


So, the company is the borrower now and the holder of the bond means somebody who has purchased the bond. So, what will happen when the bond is returned So, you have to give him the principal back at the at the end of the maturity period and in between every year or so as we decide ah within the contract that we promise to pay interest to the holder of the bond. Now, what we call the par value here to just to make a little difference ah from the ah typical phase value of the equity shares, this is the value that we state on the phase of the bond. It represents the amount the firm borrows and promises to pay repay at the time of maturity.

If it is written as 1000 rupees or 10000 rupees, then we pay that 10000 rupees at the time of maturity to the holder of the bond. Now, what is this coupon rate compared to the typical term 'interest'? The bond carries a specific interest rate, which is called the coupon rate. So, the interest rate payable to the bondholder is simply the par value—that means the face value—multiplied by the coupon rate, which is the interest rate. So, multiply, and you get the amount that you have to pay every year or, say, semi-annually. The maturity period, as usual, is the time after which the bond will mature for the firm or borrower.

They will return the par value to the investor. Suppose someone has taken the bond at 10,000 rupees each. So, when returning it after the maturity period, the company or the borrower will give the money back at par value. So, how do we value it? The discussion

we had now makes it simpler to understand the valuation of the bond. Now, if P is the value of the bond,



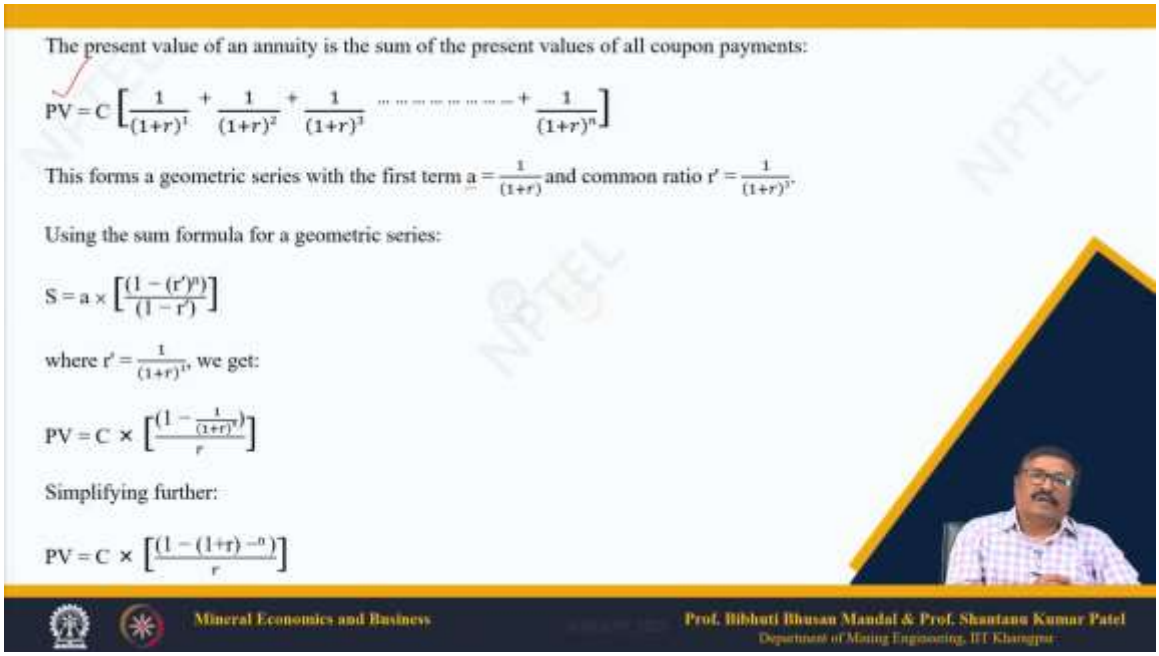
and if we know that C is the annual coupon payment in rupees—how much you are paying—then, and N is the number of years to maturity—the maturity period, rather. R is the periodic return, the required return, and M is the maturity value, and T is the time when the payment is received—say, every year, like the first year, second year, third year, and so on. So, that is what you are getting.

So, in that case we will use this simple formula:

$P = \sum(C / (1 + r)^t) + (M / (1 + r)^n)$

So, we are adding one part derived from c and the other from the derived from the value maturity value that is n. When we add this thing then we get the total value that is P. Now, this is annual compounding every year you are paying the coupon payment every year. Now, if we do the same thing for semi-annual compounding then what is the difference?

Then the coupon rate will be divided by 2 and then the time period every year you have to pay twice. So, what will happen? this period will also because since you are paying every year, every 6 months then n will now appear as twice n. So, in semi-annual compounding the formula is slightly modified, but the basic concept is same. Here we are compounding over year 1 to 2n because every year we are paying twice. So, we are just compounding it 1 to 2n.



So, now we we use the same the time value of money concept here for finding out the present value of all the C coupon that we have we are paying to the holder of the bond and also at the end also we will be adding the value of the maturity value and they will be transforming the maturity value in terms of present value. So, now here as you see:

$PV = C\ [1/(1+r)^1 + 1/(1+r)^2 + 1/(1+r)^3 + \ldots + 1/(1+r)^n]$

So, this is nothing but as you can see is nothing but a geometric series. with the first term $a = 1/(1+r)$ and the common ratio is $r' = 1/(1+r)$. So, now, it is using the sum formula for a geometric series here as you can see

$S = a \times [(1 - (r')^n) / (1 - r')]$

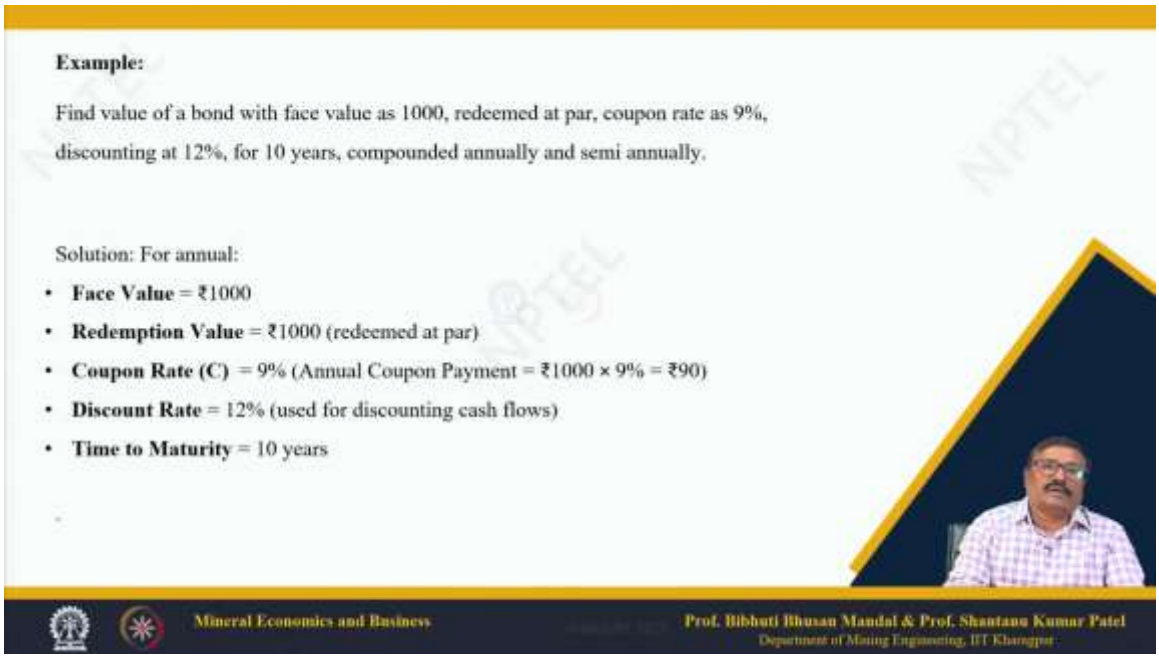
Now, so if we replace this $r' = 1/(1+r)$ in that case we get

$PV = C \times [(1 - 1/(1+r)^n) / r]$

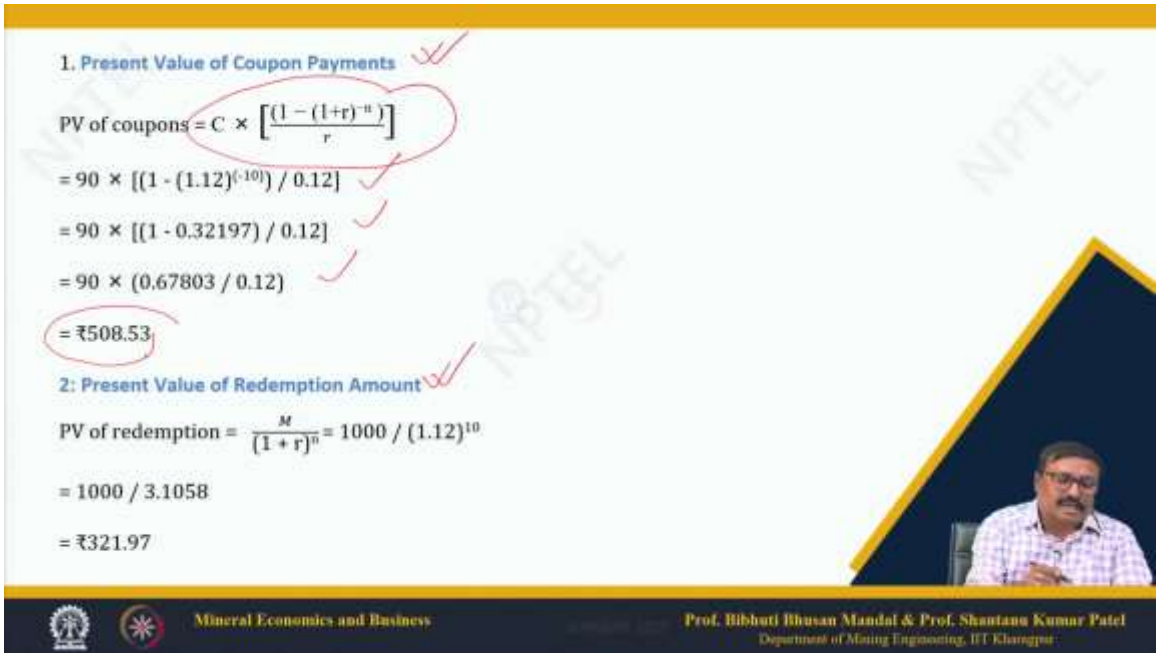
Simplifying further we will get:

$PV = C \times [(1 - (1+r)^{-n}) / r]$

So, this is the simplified formula for the present value of all the annuity and that is presented as the present value of all the coupon payments. So, we are transforming all those the coupon payments that we are making to the holder and in terms of the present value at present when we are giving the issuing the bonds to the holders. Now we will just take a simple case of the find the of the bond valuation where the value of the bond with the face value, the face value is where it is 1000 rupees here and the redemption value is 1000 because here the we are redeem redeeming the bond at par at par with the face value. And the coupon rate is here is 9 percent, annual coupon payment will be so 1000 rupees multiplied by 9 percent that will give you 90 rupees.



So, 90 rupees is the coupon rate coupon payment every year. The discount rate is here as 12 percent used for the discounting of cash flows in the time value of money equation and the time to maturity the period for which the bond has been issued is 10 years. we just replace this value into the equation that we have seen in the previous slide. So, the there are two part one is the redemption amount part and the other are summation of the present values of all coupon payments that we made.



We just remember the formula here:

PV of coupons = C × [(1 - (1 + r)$^{(-n)}$) / r]

Replace the values and calculate step by step we get the present value of the coupon payment all coupon that we paid over the period of 10 years. We get that the coupon value will be the present value of all the coupon payments comes down to 508.53. Now the present value of the redemption amount because we have issued at the per value phase value of 1000 rupees per bond. So, if you use:

PV of redemption = M / (1 + r)^n

We are getting the present value of the redemption amount of 1000 rupees after 10 years we are getting it rupees 321.97.

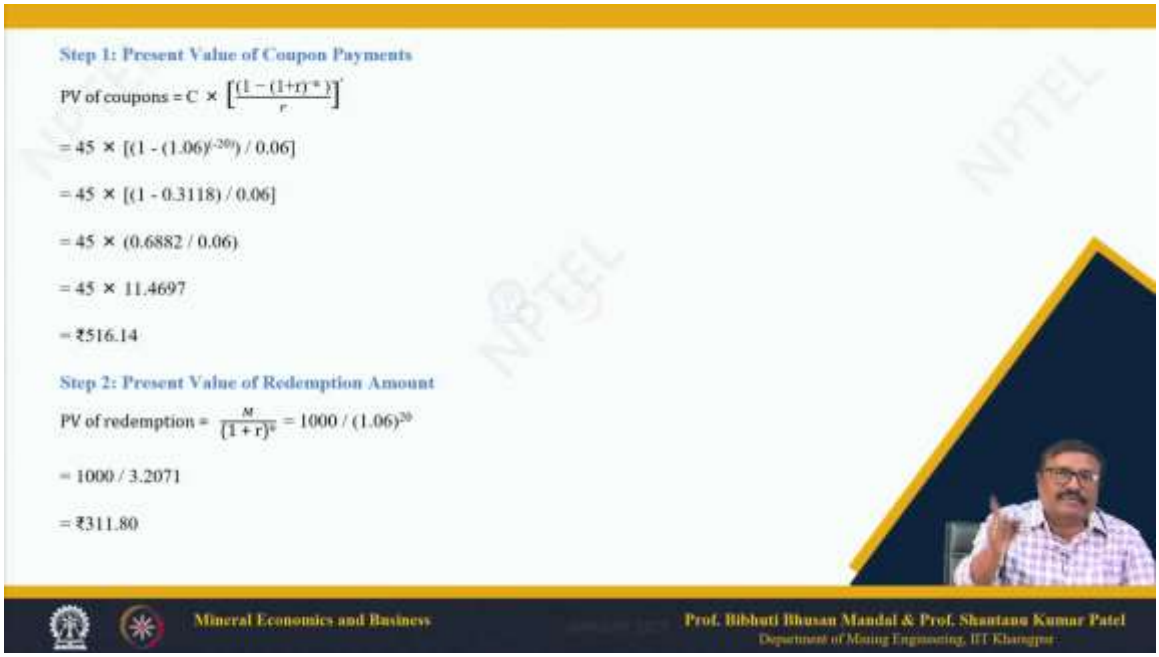
So, what is the cost to the company, the cost of this? So, ultimately, we are paying—the value of the bond will be taken in two parts: the present value of all the coupon payments plus the present value of the redemption amount. So, we now have to simply add these two values. So, the present value of the bond is the present value of the coupons and the present value of the redemption amount after maturity. We just add and get this value: 830 rupees and 55 cents.

If we just remember the formula that we derived for the semi-annual payment of the coupons, in that case, the face value remains the same, the redemption value remains the same, and the coupon rate is now divided into two parts, which is 45 rupees instead of 90 rupees now. The discount rate will be, instead of 12 percent, semi-annually discounted at 6 percent. It will be 10 years, but the total number of periods for which it will be compounded will be taken. So, using the present value of coupon payments, we replace the values which are applicable for the semi-annual payments, and it comes down to 516.14. This is based on all the coupon payments that we make to the holder of the bond.

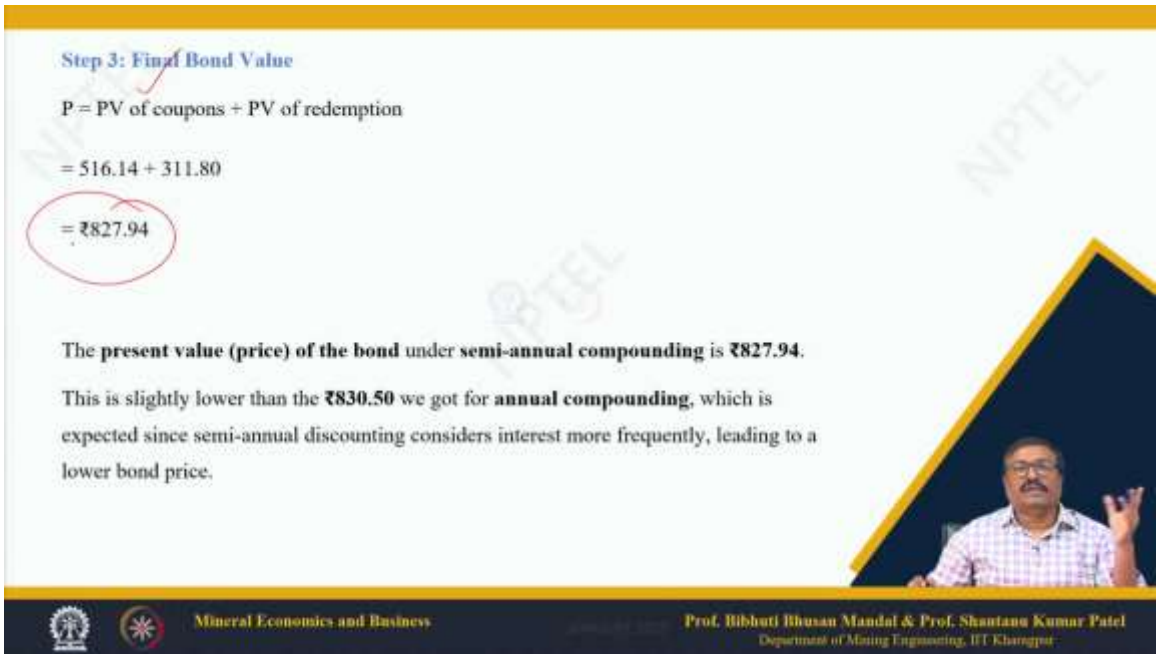
On the basis of the contract, where we are supposed to—or promised to—pay them semi-annually every 6 months. Now, the redemption amount is similarly calculated here—not compounded—we use the time value formula and transform that future value of 1000 rupees after 10 years, which means 20 different periods, and we use 50 percent of the interest rate to find out the present value. So, that comes to 311. Now, we just add again and see how the values are coming. Now, we again add this present value of the coupons and the present value of the redemption—that means, the maturity value, which is at par with the face value or par value.

So, it comes to 827.94. There is a difference between these two approaches. One is when we are doing it annually, and the other value comes when it is paid semiannually. Now, you see that the present value of the bond under semiannual compounding is 827.94, which is slightly less than the 830 rupees when we did for annual compounding just before. So, which is expected because the semi-annual discounting considers the interest

rate more frequently, leading to a lower - of course, slightly lower - bond price, as we have demonstrated through these two approaches in the bond valuation.



Now, let us see the comparison of the bond prices based on the coupon rate versus the discount rate. In which 'par' means, as I said, when you are at par, that means the coupon rate and the discount rates are the same. That means the price or bond price is equal to the face value or par value; that is why it is called par value. And in premium, what is happening is the coupon rate is higher than the discount rate, and the price is higher than the face value. Whereas, in the case of discount, the coupon rate is lesser than the discount rate, and the price is definitely lesser than the face value.

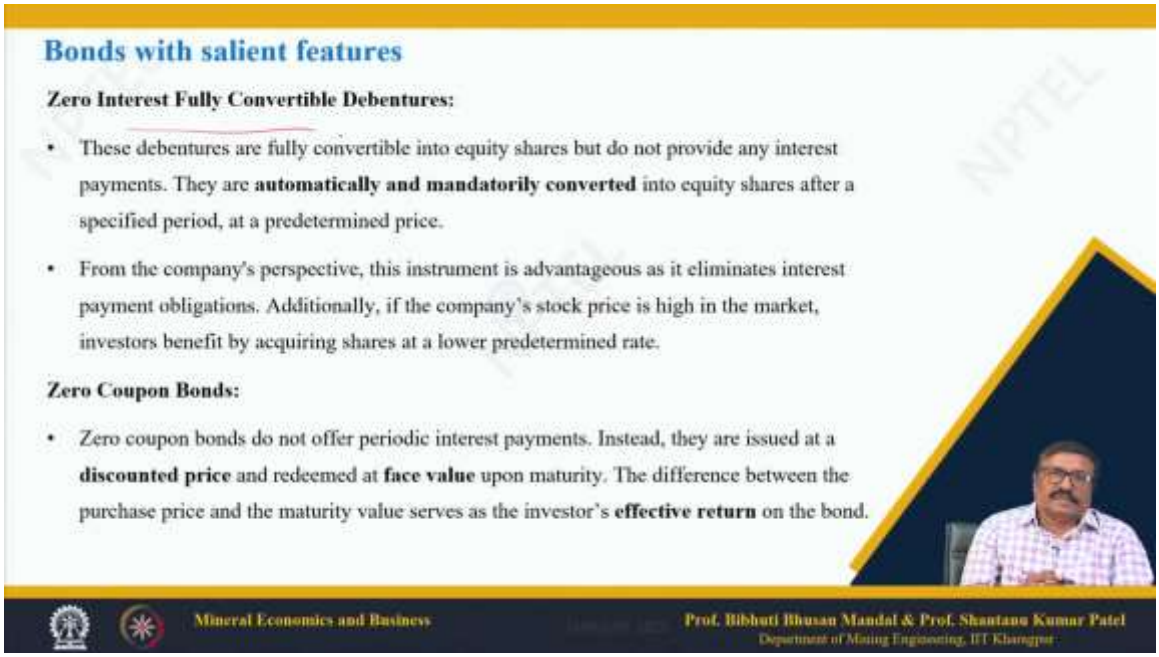


So now, if we compare again in simple language: if the coupon rate is higher than the discount rate, the bond is expensive—that means it requires a premium—and if the coupon rate is lesser than the discount rate, the bond is cheaper—that means we are offering a discount, basically. And if the coupon rate and discount rates are equal, bonds are being traded at face value—that means at par, or we say it is the par value. There are certain variations in these bonds—there are many different forms of bonds which are traded in the market and issued by the company. Now, there is something called zero-interest and fully convertible debentures. Will we just explain what this is? And there is also something called a zero-coupon bond.

So, the first one is the debentures which are fully convertible into equity share, but do not provide any interest payment in between nothing. So, we just sell it in the market the the holders will get the benefit of mandatorily getting the equity shares which will be converted from the debenture. The debenture will be rather converted into equity share after a specific period at a predetermined price. these are all fixed in between you are not going to get anything. It is little risky definitely, but when that it is predetermined price and which is higher than what you are purchasing at the moment.

So, people are easily attracted, but in between you do not expect any return on the investment. So, from the company's perspective The instrument is advantageous because it eliminates the obligated interest payment. So, that every year or every 6 months you

have to pay interest that part is foregone. So, the company's perspective is this, this is advantageous.



So, you do not have any obligation like that. But if the company say in addition, if the company stock price is high in the market. So, investors benefit by acquiring shares at a lower predetermined rate. If the predetermined rate is lower and the current price is high when it is being at the end of the maturity period then the investors will be easily attracted they should definitely because they are getting stock price at low price even though it is already being traded at higher higher price. Now, we coming we are coming to the 0 coupon bonds.

The 0 coupon bond they do not offer periodic interest payment ok. Instead, they are issued at a discounted price. This is a variation and an interesting variation that they are issued at a discounted price. That means, if the face value is something we are giving at a discounted price, but they are redeemed at face value upon maturity. So, the difference between the purchase price and the maturity value serves as the investors effective return on the bond.

if I am say say giving you a bond which is the face value is 1000 and you are we are getting a you are getting at a discounted price like say 800 rupees. So, the difference is

the income over the period of maturity, but because when it is being returned to the company that means, when it is being redeemed then it is redeemed at a face value, but it was purchased at a discounted price. That means the difference that you are getting is the return or effective return on investment through zero coupon bonds. There are also dip discount bonds. The deep discount bonds are a type of zero interest bond.

that are issued at a discounted price and redeemed at face value upon maturity. But this this bonds do not provide periodic interest payment and the investors receive their returns only at the end of the lock in period just the previous one it is the same thing in a different name. Then the inflation bonds the inflation bonds the inflation concepts will be taught by Professor Patil in the ah next few lectures. So, this bonds are designed to adjust the interest rate based on the inflation ensuring that investors receive returns that are protected from inflationary effects.

That means, when between the issue date and the maturity date, the inflation has to be adjusted by saying that the base interest rate is just 11 percent, and if there is considerable inflation, it is going to have a considerable effect. Then, what we do is say that in inflation bonds, the interest rate will be adjusted as per the effective inflation during that period. So, the effective return becomes 16 percent. This safeguards the investor's

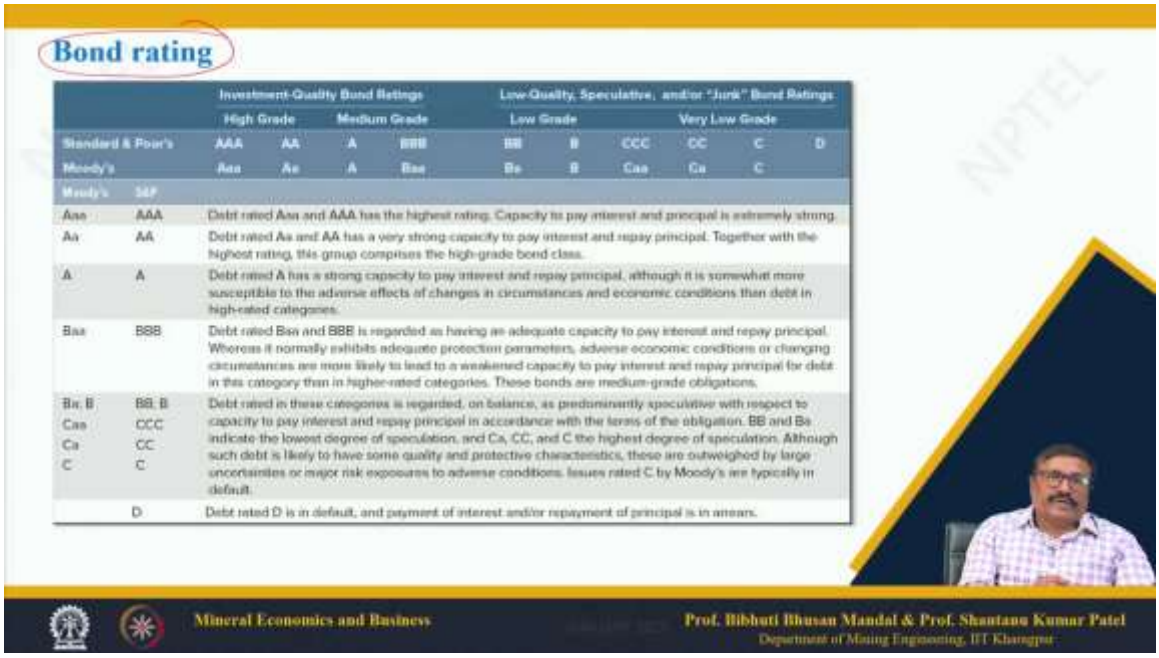
purchasing power because we are adjusting the return according to the inflation in that particular period. These are different variations of the concept of the bond.

There are also floating rate bonds. In floating rate bonds, we have seen that the coupon rate or interest rate is fixed, but here the variable interest rate makes the name of the bond 'floating rate bonds.' So, the interest rate does not remain fixed but changes depending on market conditions. These bonds are particularly useful for issuers seeking to hedge against interest rate volatility in the market.

So, they have become very popular as money market instruments. They have been successfully issued by institutions like IDBI (Industrial Development Bank of India), ICICI, and many others. Now, we come to option bonds. We have also discussed before that option bonds can be either cumulative or non-cumulative. Interest payments are made either periodically or upon maturity.

You can get this every year or so, or every 6 months, or you can get the entire interest payment at the end of maturity. The options are given: it can be either cumulative or non-cumulative. To attract investors, these bonds may also offer a redemption premium at the time of repayment. That means, if I have received 1000 rupees against the issue of one bond from the holder. Then, when the company redeems the bond - going back from the

holder to the borrower - the borrower will pay a premium. That is the attraction to the investor because we are promising to return a value which is more than the par value or the face value. This is to immediately attract the investor to invest in the company.



Depending on so many variations—the performance of the companies, the rating of the companies, the performance of the bonds—the debts are given a bond rating. So, these are investment-quality bond ratings, ranging from high grade to medium grade, then low grade, and very low grade. So, the quality is falling in this direction. So, if we start with the rating triple A by Standard and Poor's or say AA by Moody's—these are mostly followed in the US. But we also have similar ratings in India.

So, when you are going to purchase a bond, you can go through the rating of the bond by well-known companies or firms which rate it depending on the performance. For example, triple-A-rated bonds have the highest rating; the capacity to pay interest and principal is extremely strong. That means the company is highly capable and highly recommended. Because they pay interest, they have the capacity to pay interest at regular intervals and the principal. So, this is extremely strong.

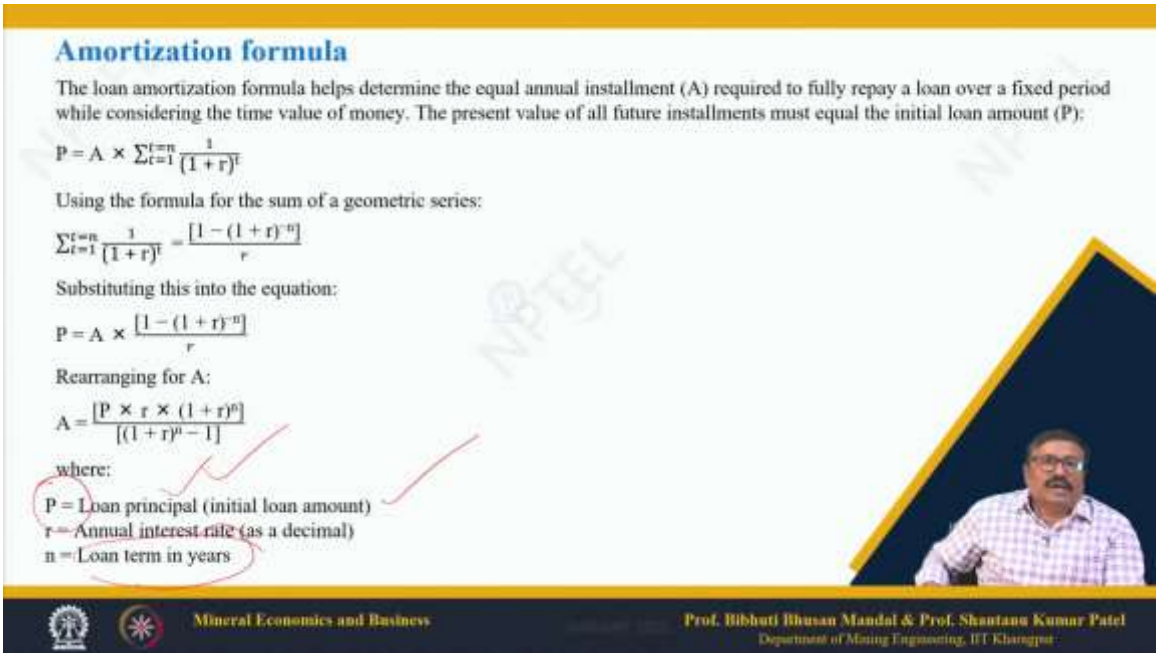
So, we give a triple A rating to the bond; it has been given a triple A rating. So, you can easily invest there. Of course, there is risk because today we give it a triple A rating, but

that does not mean it will remain triple A tomorrow, six months later, one year later, or even two years later. But seeing the trend and the general performance of the company in terms of its capacity to pay interest and the principal value when redeemed, if something is given a triple A rating, then you can easily invest, or investors are easily attracted. Bond rating is important, especially when people are searching for investment opportunities in debt instruments.

So, this can be referred to by investors to decide which bonds are preferred for their investment portfolio. Now, let's move to another topic I mentioned earlier: loan amortization. This is important because it allows borrowers to repay loans in structured, manageable installments over time. So, instead of paying the entire loan amount at once, borrowers can make fixed periodic payments that cover both principal and interest. We will show how it looks.

So, if you take a house-building loan or a vehicle loan, you will see this happening. The system helps businesses and individuals plan their finances efficiently while ensuring the lender recovers the loan amount along with interest. So, what are the key benefits of loan amortization? We know how much we have to pay. So, borrowers know their fixed periodic payments, aiding budgeting - knowing how much to pay for the loan they have taken.

So, we can go for budgeting and interest management means reduces total interest paid over time as principal reduces. Because, if you are starting paying principle, then the effective principle remaining residual part will be less. So, you will be paying less interest instead of paying the whole interest at a time. Now, the financial planning aspect of this, it helps companies manage cash flows effectively, cash flows effectively because this figures. Now, for loan amortization formula, if the P is the principle.



loan principle initial loan amount and r is the annual interest rate as a decimal and n is the loan term in years. So, the loan amortization formula that we are going to show here. It helps determine the equal amount annual ah installment that we can find out by calculating that is required to fully repay a loan over a fixed period of time while considering the time value of money is of course, very important here the same thing that we have been discussing in this lecture. The present value of all the future instalments must be equal to the initial loan amount that is the basic idea behind it.

It can be simply found out by if the annuity every year how much you are paying and using the formula for the sum of a geometric series and simplifying it this coming to:

P = A × [(1 - 1 / (1 + r)^n) / r]

Rearranging the whole thing. we are getting:

$A = [P \times r \times (1 + r)^n] / [(1 + r)^n - 1]$

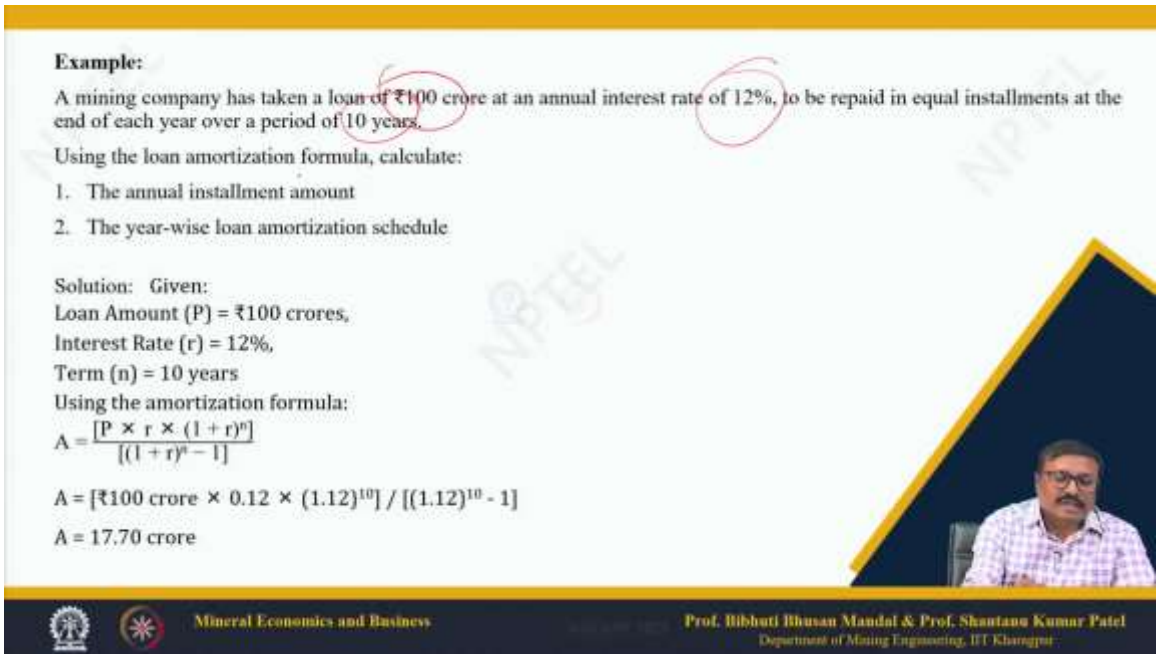
where:

P = Loan principal (initial loan amount)

r = Annual interest rate (as a decimal)

n = Loan term in years

We will be just just try to demonstrate how it looks if I put the values in that. So, in a mining company for example, takes a loan of 100 crores at an initial annual interest rate of 12 percent. to be repaid in equal installments at end of each year over a period of say 10 years.



So, using the loan amortization formula, we have to find out the annual installment amount—how much I have to pay. And the year-wise loan amortization schedule that we have to prepare is for capital budgeting, showing every year how much I have to pay, how much we are paying in interest, what principal is remaining, and how much principal is paid toward the end to make it zero. So, that the loan will be fully amortized. Now, putting the value of the loan amount (100 crores), interest rate R, and term N into the

amortization formula we saw earlier, it comes to 17.70 crores as the amount we have to pay every year. Let us make a schedule for this.

So, here you see that in the beginning, we have 100 crores, and the installment in crores is coming every year as 17.70. If you add these things, it will be more than 170 crores here. So, the interest part is 12, the principal repaid is 5.70, and the ending balance is 94.30. That means you are paying only 5.70 in principal at the beginning, and as you go toward the end, you will see that the principal amount is increasing.

But in the beginning, most of it goes to interest, and slowly, toward the end, the interest reduces while the principal increases. So, now we have made a schedule showing every year's ending balance, which you can see is gradually decreasing until, at the end, the loan is fully amortized and becomes 0.00. In the beginning, we pay more interest—that is the strategy—and at the end, you pay more principal. It increases, and the amount of interest accrued on that principal goes down and down. The interest is calculated as the beginning balance multiplied by 12 percent, as we said the interest rate is like that, and the principal repaid is the installment minus the interest. So, the difference is the principal that we have repaid.

And the ending balance will be for every period will be the beginning balance of that year and the principal that we have repaid will be the ending balance that we have. So, this you see from the beginning this one 100 say in the first year 100 we paid 5.70 and then we get 94.30 is the balance. interest is additionally paid. So, it is a typical the loan amortization schedule the table shows. So, accordingly you can use this for next 10 years or 20 years for which the loan has been taken and in the capital budgeting this can be used as a cash flow cash rather outflow or inflow whoever is calculating this thing depending on that.

some problems related to the bonds and the ah different forms of the bond, some theoretical aspects we tried to recapitulate and we solve certain problem and also we introduce the loan amortization ideas and how the loan amortization schedule is made. If you want to learn on this and solve many more problems. It can concert mine mineral project evaluation by ah this Jones and all. Mine investment analysis is another old book, but the concepts are same and it is a very good book ah Gentry and O'Neill.

And of course, I highly recommend the website www.icai.org for better learning and more understanding. course, we will be providing these things in the in the transcript of the of the lectures along with the other all the formulas which you will be able to understand with more clarity. Thank you very much for attending the class.