MINERAL ECONOMICS AND BUSINESS

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Week 8

Lecture 37 : Internal Rate of Return (IRR)

Hello, welcome to this lecture on the internal rate of return. This is a very important concept that we are going to discuss. In the previous lecture, we discussed the net present value, and in the context of that net present value, we will be talking about today's subject: the internal rate of return. And the problems in using IRR, the advantages and disadvantages, and then a comparison with NPV—that means a comparison between the application potential of NPV and IRR in decision-making processes. To give you a definition at the beginning, we define the internal rate of return.

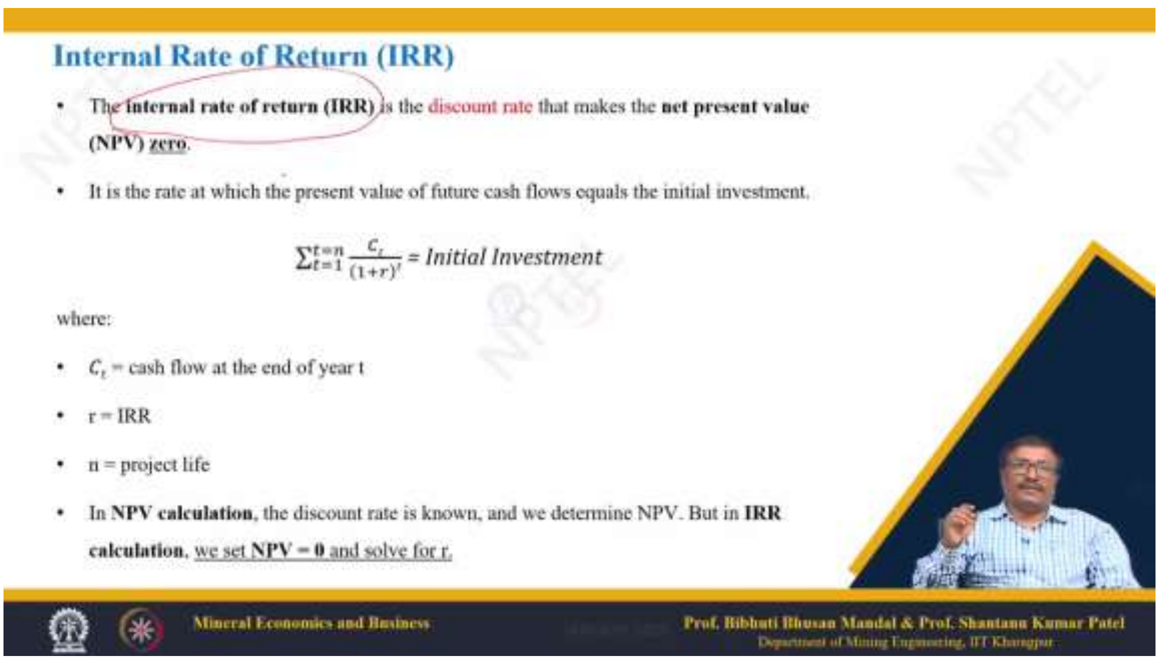
As that particular discount rate which will make the net present value zero. That means, say you think about the equation where we try to find out the net present value; you need a discount rate for that. Why do you require a discount rate? You need to transform or convert all the future cash flows to the present value.

So, you need to apply a discount rate. Now, here is a concept where the discount rate is not known—that means we are trying to find out a discount rate where the net present value, instead of being positive or negative, becomes zero. So, how to calculate that? We use the known application or formula for NPV, except that we just rearrange it.

$\sum_{t=1}^{t=n} \frac{Ct}{(1+r)t} = Initial\ Investment$

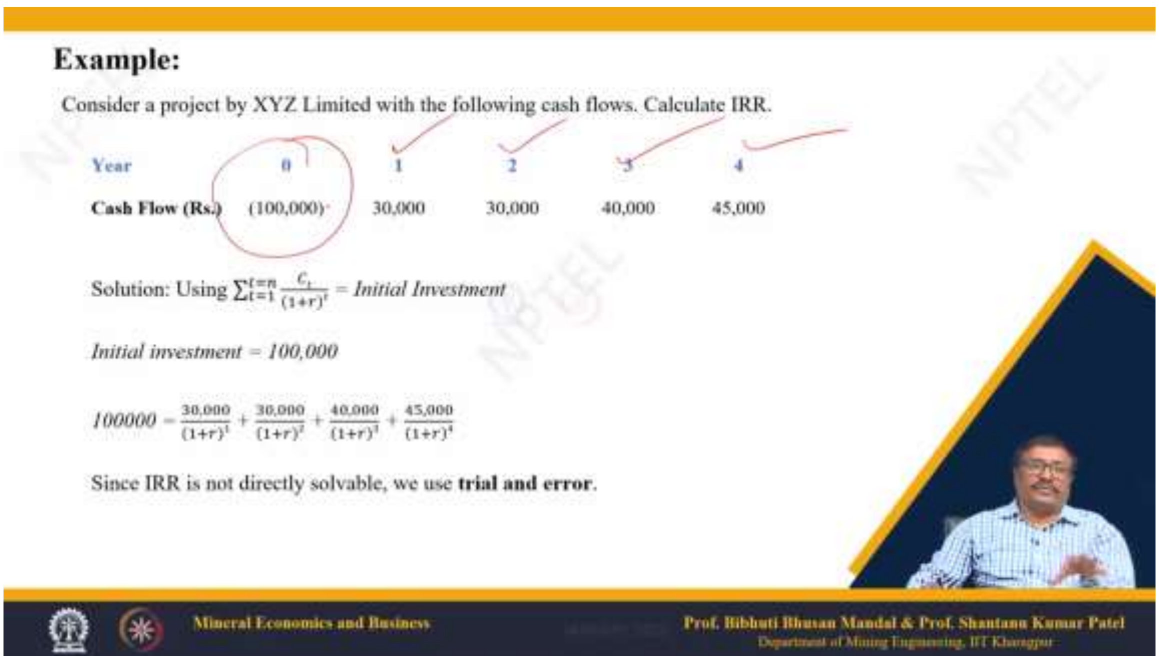
where:

- Ct = cash flow at the end of year t
- r = IRR
- n = project life



Initial investment is a cash outflow and all the cash inflows that we have summed up, then if we equate that that means, this minus this is equal to 0 that means, the net present value become 0. So, just we rewrite the value the equation for NPV and we put the value NPV equals to 0 and we rearrange the equation and we find this. Now, in this case again just to remind you that the C_T's are cash flows at the end end of year T and the R that you see here is the IRR where this relationship holds true, N is the project life and in NPV calculation the discount rate is known and we determine NPV, but here the IRR calculation in IRR calculation we set NPV equals to 0 and we find this R which is IRR.

that means, when NPV becomes 0 that means, we find out that this particular discount rate and apply the same relationship rearranging it and find out the value of R where NPV becomes 0 that is the concept. Now, we will take a very simple example and try to understand what is this, what this IRR is all about. Consider a project where we have the cash flows on the at the end of first year, second year, third year and fourth year and in the beginning we have 100,000 this is cash outflow, the bracket means it is a negative flow. Now, the solution if we just try to find out the present values of all this future cash flows, then we find out by using this 30000 by 1 plus r plus 30000 by 1 plus r square next and second year we have 40000 by 1 plus r cube.

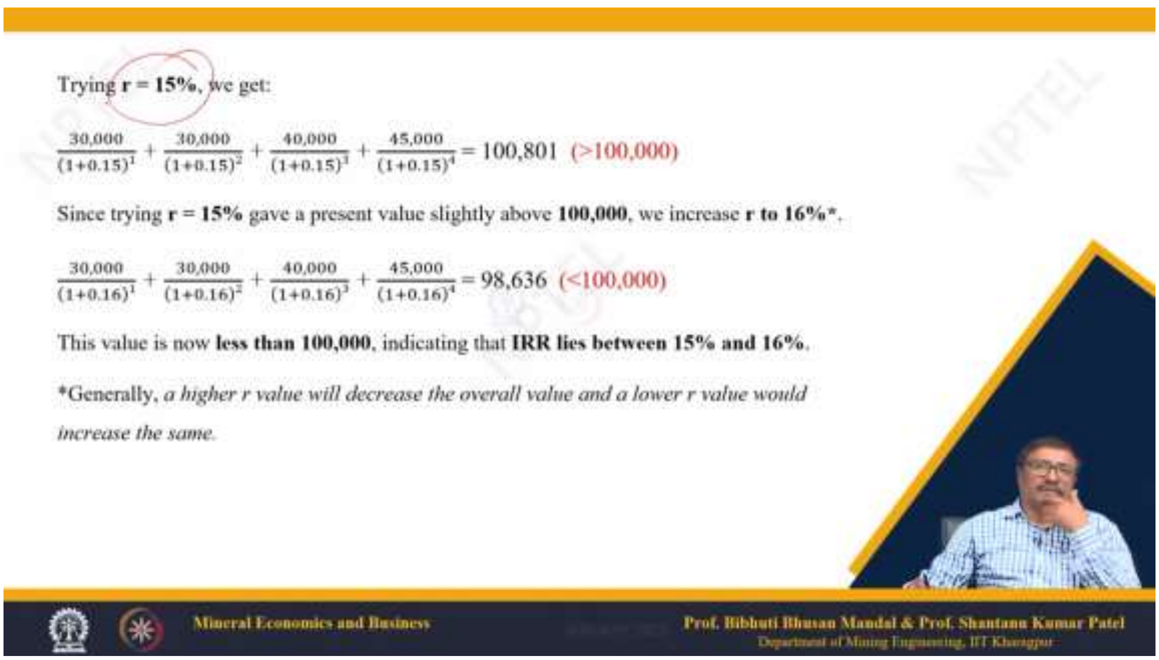


and like that and in the 45000 rupees divided by 1 plus r raise to the power 4. This is how we bring all those to the present value. Now, this net all this net present values of the future cash flows are equated with the initial investment that is 100000. By solving this we will find out the value of r. because there is only one unknown that is r, but since this is having power third power or fourth power here.

So, here the solution will not be directly available directly available. So, what we do we use a trial and error you can write a code you can use any excel sheet and easily you can find out the value of r, but if you want to do it manually for example, then we can use a trial and error method. for example, we use a value of say 15 percent for r. By putting the value of all this in all these terms in place of r we put 0.15 that is 15 percent and we get 1008001, 1,00,801. Here this is not equal to 1,00,000. it should be equal to 1 lakh, if it is equal to 1 lakh then this r value is the value that we are searching for.

So, this is above 1 lakh. That means, this is not the value. So, what we do? We increase it because since this is above 1 lakh. So, we increase this value from 15 percent above.

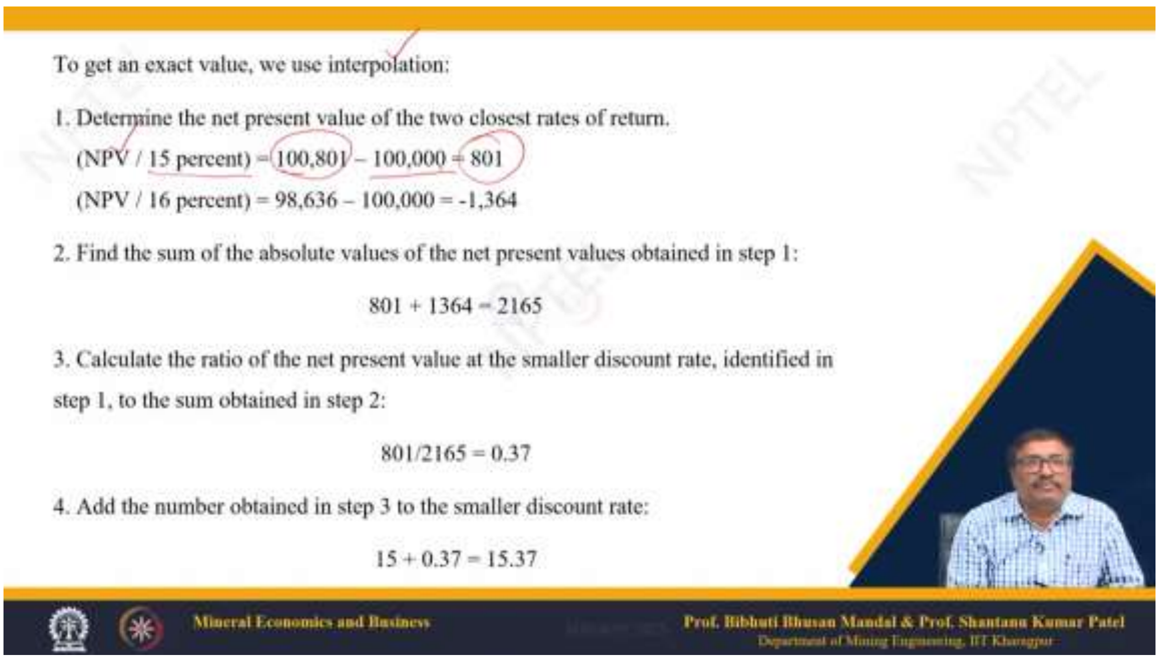
For example, we try for r equal to 16 percent because if these values are changing towards positive side from 15 to 16 or 16 to 17, then this will even this this is this value will be lesser and lesser. So, that means, a higher R value will decrease the overall ah value like the this value. and if you if you put a lower r value then it will increase the same this value. So, in this case we need to bring it down to 1 lakh.



So, what we do then instead of 15 percent we put 16 percent and take a and try it and then find that the value comes down to 98,636. So, 15 percent is less. 15 percent is less because the ah value comes to 1 lakh 801 which is above the ah above the initial investment of 1 lakh rupees. Whereas, if I put 16 percent then what happens the summation of all the present values of the future cash flows becomes 98,636 which is lower than which is below the initial investment initial investment.

Since both these things are 15 percent and 16 percent is not the correct value. The value then lies between it is lying between 15 percent and 16 percent, where it this value the net value will coming will come exactly equal to 100,000 means 1 lakh rupees. So, we have to find out that value by using a slight more calculation and approximation by interpolation that can be done. how? For example, NPV at 15 percent is 801 because this was our all the present value of future cash flows and this was our our initial investment.

So, if we deduct this from the all future cash flows then we get 801. So, 801 is the NPV at 15 percent discount rate. Whereas, NPV at 16 percent discount rate will come to is coming to minus 1364 from plus 801 to minus 1364. So, what we do that



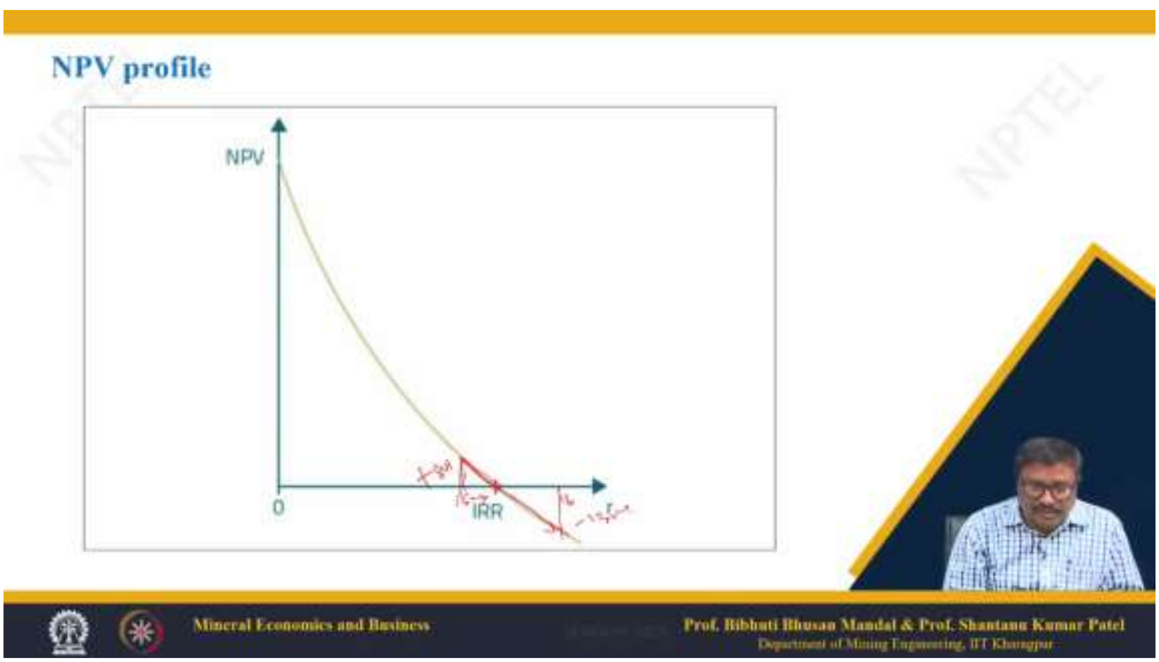
ah we can imagine that from 15 percent when we are going to 16 percent there is a change of 2165 rupees. That means, what we do? We take all the absolute values of both the both the both the NPVs like 801 and ah the absolute value of 1364. So, this is the whole the total it becomes 2165. that means from moving from 15 percent to 16 percent the change the total change absolute change is 2165.

So, now since 801 rupees is extra that means positive we have to bring down this 801 to 0 that means we have to go above 15 percent towards 16 percent proportionately. So, what we do that we find out the proportion 801 plus 1364 gives us 2165 we just try to find out. 800 on 1 divided by 2165. So, you have to move 0.37 points ahead towards 16 percent.

So, we started from 15 percent and we add this 0.37 percent or 37 percent here 37.37 and we get 15.37 as the discount rate where the NPV will be equal to 100, NPV will be 0. That means this 801 will come down and then it will become 0. Beyond that it will start showing negative values again. That means from 801 for example, here if this is say if this was 15 percent and if this is say 16 percent.

16 percent. If this is 16 percent, then say from 15 percent to 16 percent when we are coming, then there is this part was 801 and this part was about say minus 1300, 1300 or so. So, what is happening from 800 plus to minus 1300? It is we need to understand the slope, that means at what rate it is changing. So, if we can increase the 15 percent rate slowly, then we see that at one point the value will come down to 0. The NPV will become 0. We need to find out that point. So, for doing that, what have we done? We have found out the

ratio day 801 by 2165, that means you have to go 0.37 ahead of 15. So, it becomes 15.37 percent. That is at this point, it is a 15.37 percent discounting rate. So, this is the discount rate where NPV is becoming 0. So, we will call it IRR or 15, which is the value of 15.37 percent in this case. So, this is the basic concept that I have explained so far.



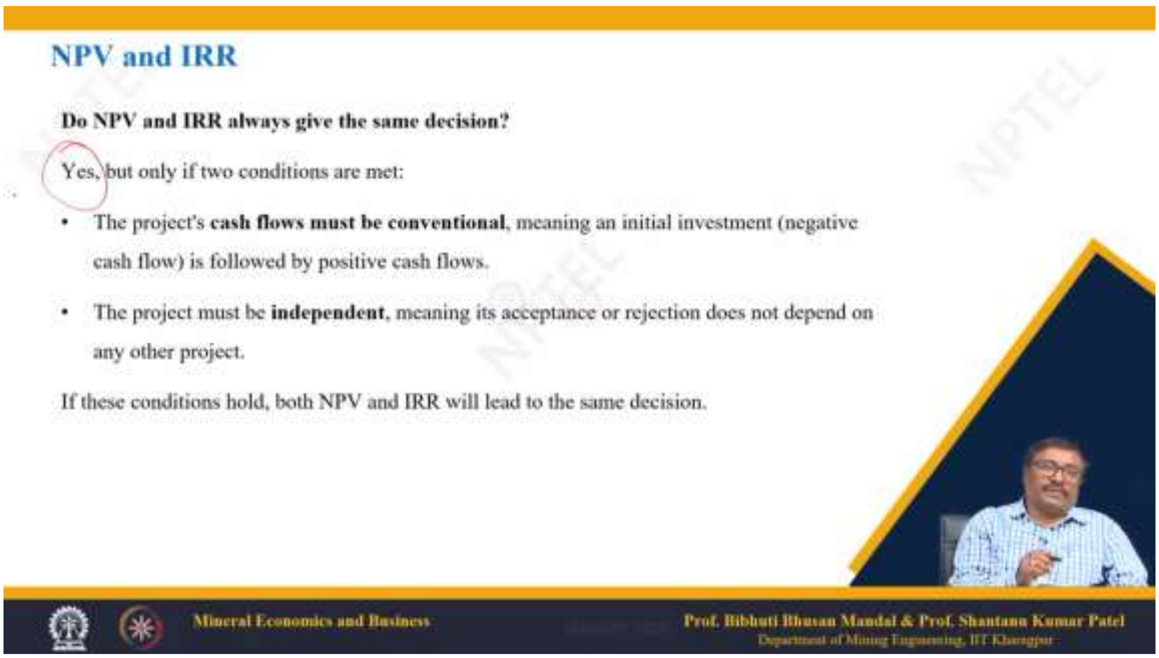
So, what you will find out is a close approximation to the true internal rate of return. It is manually done. It can be done. Now, what is the decision rule when you are using IRR as a metric? You accept the project if the IRR is greater than the cost of capital. This means the project is expected to generate a return which is higher than the minimum required, making it a profitable and viable investment. You can reject the project if the IRR is less than the cost of capital. In this case, the project is not generating enough return to justify the investment. Now, there will be a case of indifference when IRR is equal to the cost of capital.

In this situation the project neither gains or loses and the decision will depend on other qualitative things like whether the company is getting any strategic benefits like acquiring some project where you are not taking a clear decision based on IRR, but you are maybe expanding into some other areas where strategically the company will be in a beneficial position. It might be having some alignment with company goals. That means, those things qualitative factors will decide if the IRR does not give you a clear cut decision. Now, this do NPV and IRR. So, NPV also gives you a decision whether you accept a project or not.

IRR is also giving you a decision. So, are they give the same decision? Yes, in principle say we can start with the answer yes, but there are riders like 2 conditions. One is the project cash flows must be conventional, meaning that an initial investment which is a negative cash flow is followed by positive cash flow. The project must be independent.

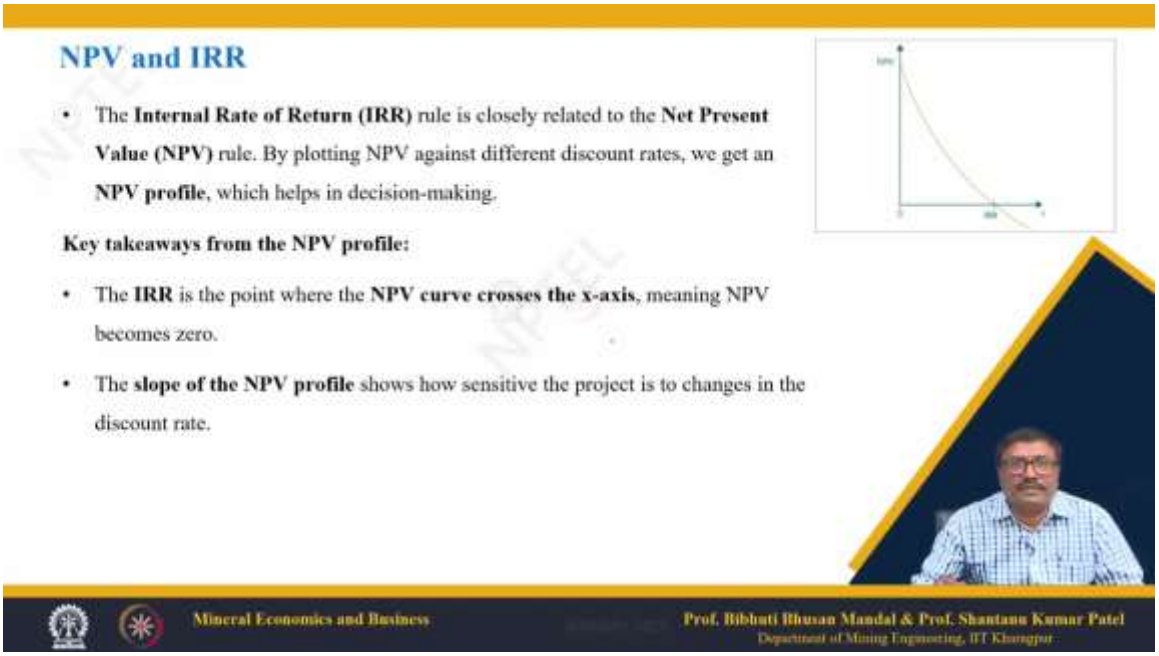
That means, meaning is acceptance or rejection will not depend on any other project. This is a clear case where it can be studied in isolation exclusively. Now, if these conditions are true, then in that case NPV and IRR will lead to the same decision. Otherwise, there will be some complications which we will be discussing later. So, what do you understand from here?

The internal rate of return or IRR rule is closely related to the NPV, as we have seen. So, by plotting the NPV against different discount rates, as we have seen through an example, we get an NPV profile. So, what you see here is an NPV profile, showing how it behaves with different discount rates. This side shows the value of NPV plotted versus different discount rates. So, what do we find from this particular profile?



The IRR is the point where the NPV crosses the x-axis, meaning the NPV becomes zero, and the slope of the NPV profile shows how sensitive the project is to changes in the discount rate. That means, if you change from, say, 15 percent to 16 percent, and it falls sharply like this, then it is highly sensitive. But if it is flat, like this, then it is not so sensitive. So, the slope of the profile will show how sensitive it is to different values of changing discount rates. So, here we clearly see that on the x-axis, we have the R or the discount rate plotted, and on the y-axis, we have the net present value.

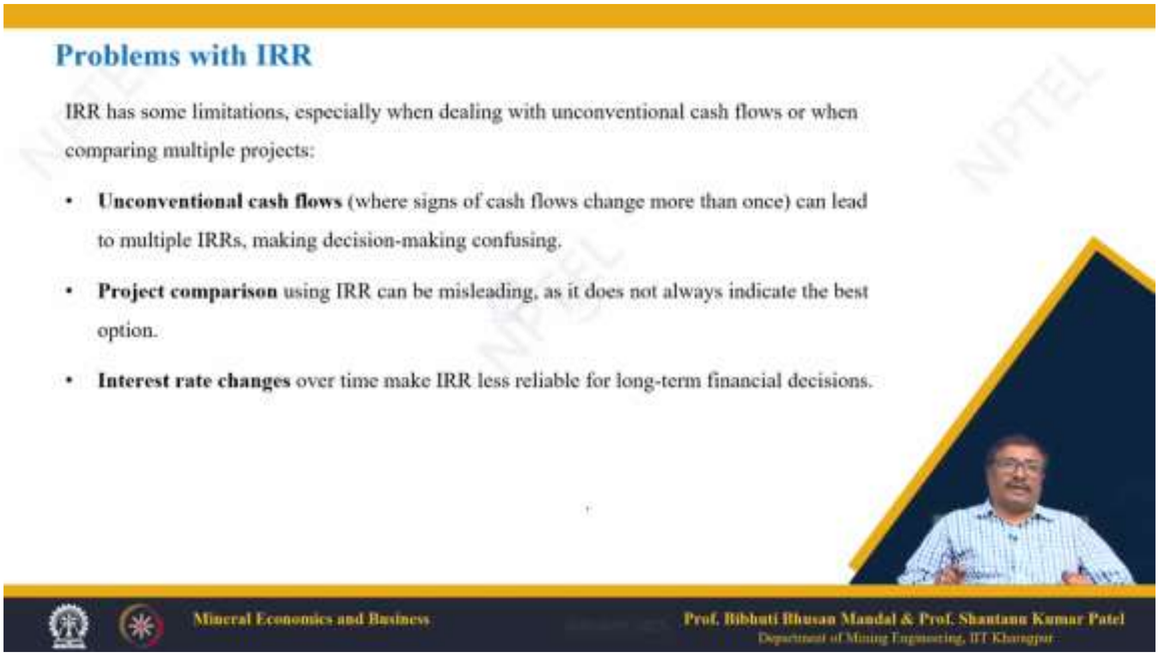
So, as we gradually move through different discount rates, there is a point where the NPV touches the x-axis. Since, as the R values increase, the NPV starts falling, the NPV drops, the NPV starts dropping. In that case, it means that with increasing R values, the NPV effectively decreases. It decreases to a point where we are interested in this point, where for a particular



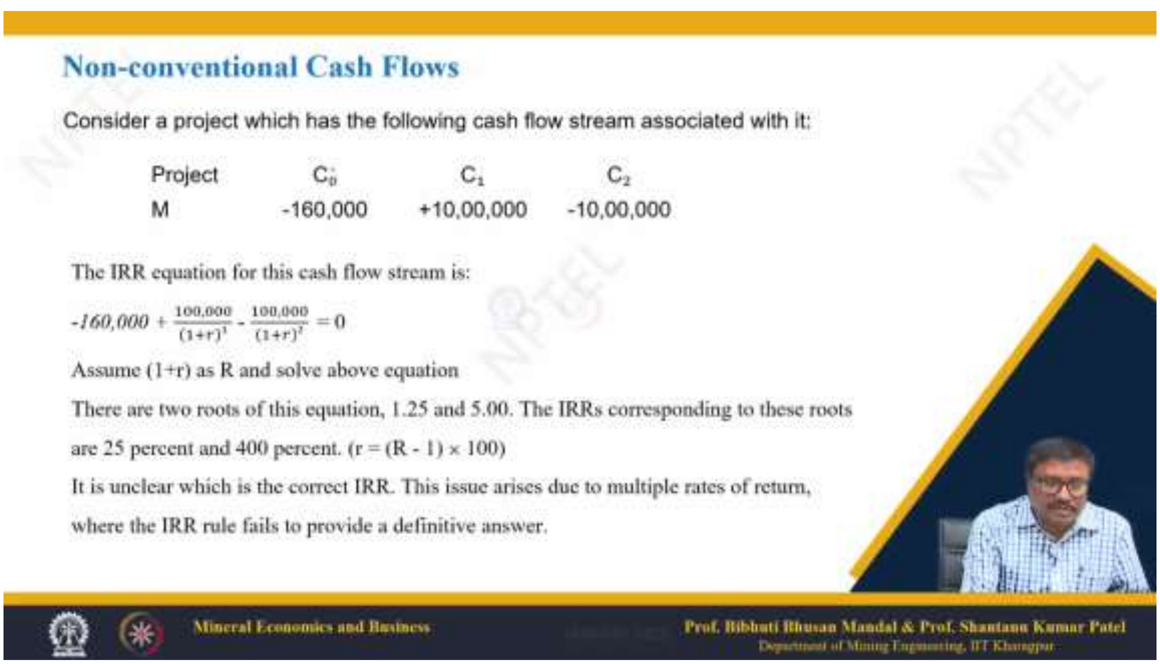
Which we now call IRR is making the NPV equal to 0. Making the NPV equal to 0. That means, if we further increase the discount rate, what will happen in that case? The NPV will become negative, as we can clearly see from the profile of the NPV. So, that is all about the basics of NPV and IRR, their relationship, and how we can use this for decisions.

More complicated examples will come soon. Now, before going further, we will also talk about the negative aspects or the problems related to IRR. What are the limitations it has? So, in cases where unconventional cash flows occur or when you compare multiple projects, then the absolute use of IRR alone will complicate the issue. For example, where the signs of cash flows—meaning unconventional cash flows—change more than once during the lifetime.

Then what happens? It can lead to multiple IRRs, making decision-making confusing. We will see examples, as I was saying. Now, also for project comparison, IRR can be misleading. We are determining IRR for project A.

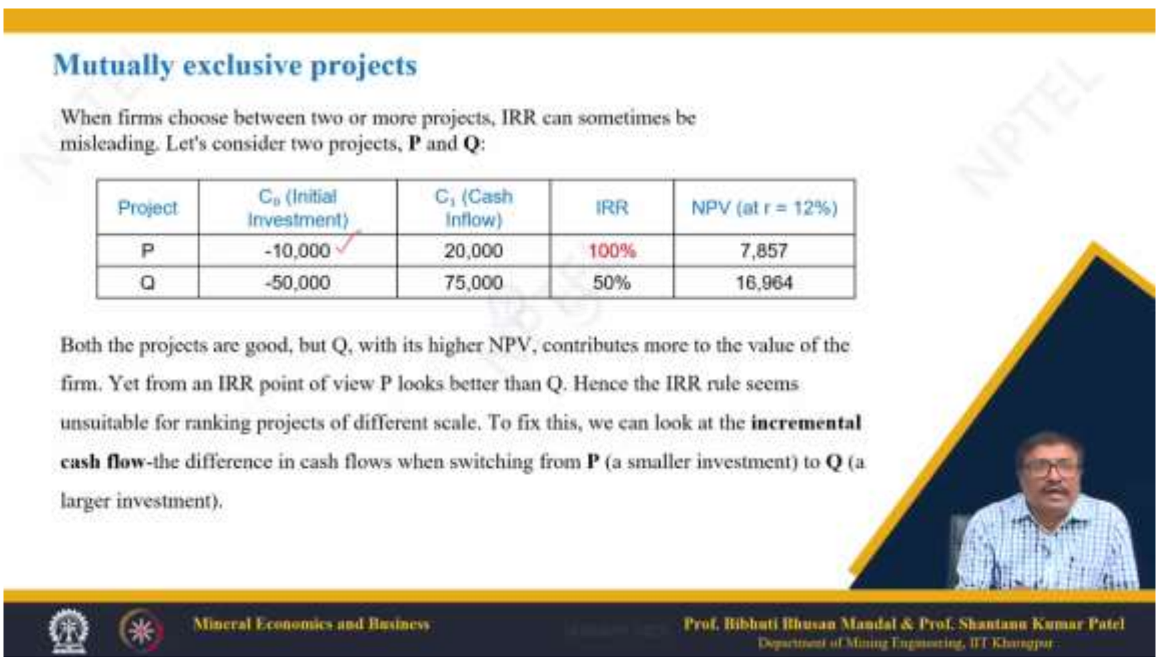


and also determining IRR for project B. So, if this IRR by seeing looking at the values of IRRs this will not always indicate the best option just by looking at IRR we will we will discuss this ah in the following examples. The interest rate changes also over the time long period that will make IRR evaluation of the IRR less reliable for long term financial decision. There we have to use some other metric other than IRR. Let us see a simple example of a non-conventional cash flow, where we have a project and the cash flows are C 0, C 1 and C 2. In the beginning, we have a negative cash flow 1,60,000, then for 2 years, first we year at the end of first year we have 10 lakhs and again at the end of second year it is 10 lakhs negative. That means, these are basically one is positive the



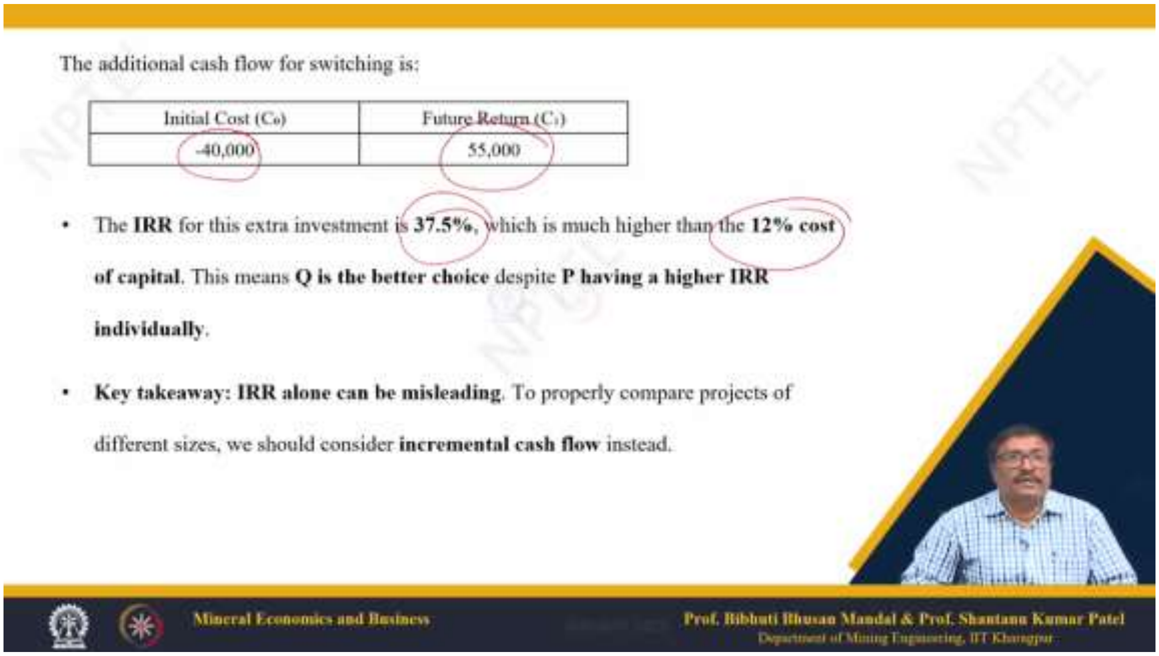
other is negative. Now, say the IRR equation for this cash flow stream is this. So, these are the 2 cash flows one is plus another is minus and then we have the in the beginning we have the cash flow of minus 1,60,000.

So, all these things now the NPV the net present value total is 0 and we have to solve for r. If we solve for r for this we can use a simple technique like we can use r equal to 1 plus r or all those things . So, I find out that there are 2 roots that means r is 1.25 or say 5.00 there will be 2 roots here. will be 2 roots here. Now, the IRRs corresponding to these values will be so, since r equal to r equals to 1 plus r. So, the corresponding r values will be 0.25 and 4.00 there will be 2 values. Now, for the first part this one is 25 percent that means, r is 25 percent.



Whereas, for the value of 4, it becomes R equal to 400 percent, 400 percent. It is unclear which IRR is correct. This issue arises due to multiple rates of return, where the IRR rule fails to provide a definitive answer in this case. So, in the case of non-conventional cash flows, this can happen; this confusion can occur. Now, we are talking about cases of mutually exclusive projects.

When firms choose between two or more projects, if you take this project, then you cannot take that project. That means you have to go for only one out of two, for example, P and Q. The cash flows are listed here. Initial investment here is 10,000 minus and this one is 50,000 minus, and the cash flows are 20,000 and 75,000, for example, after 1 year.



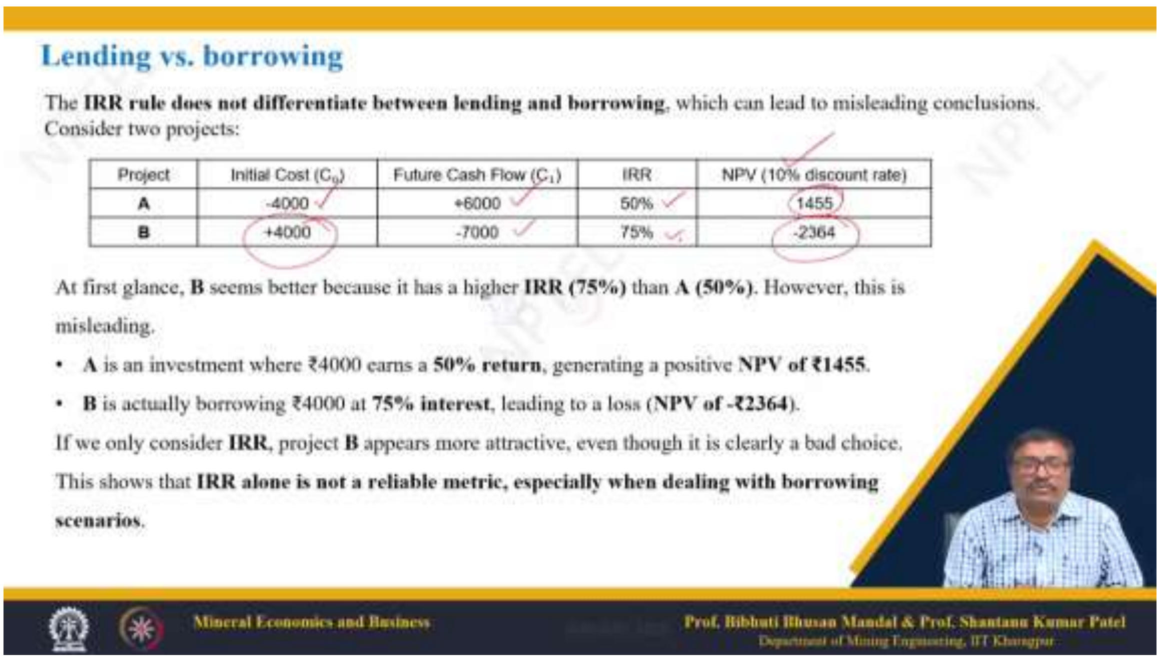
We are not going into a bigger series for the purpose of doing the same thing, but this is for giving an example.

So, if you look at this, you calculate this, then IRR is 100 percent here, and here it is at 50 percent of IRR, the net present value will become 0. Now, if we use these cash flows, then the NPV for project P is coming as 7,857, whereas for Q, it is 16,964. Clearly, the net present value is higher here. IRR for P is more attractive because it gives you a 100 percent IRR. So, both projects are good, but Q, with its higher NPV, both have net present value positive. So, both projects are good, but Q, with its higher NPV, contributes more to the company, more to the value of the firm.

So, from an IRR point of view, P looks better than Q. Hence, the IRR rule seems to be unsuitable for ranking projects of different scales. Say, here we are contributing more to the value of the firm, whereas in IRR, if you are comparing the IRR values of P and Q, you see that we have 100 percent here, whereas we have 50 percent here. That means, from an IRR point of view, this is better; from an NPV point of view, this is better. So, to fix this confusion, we can instead look at the incremental cash flow—the difference, that is, the difference in cash flows when switching from P, a smaller investment, to Q, a larger investment.

How is it taking place? We will go to the next slide and see. So, the additional cash flow from 10,000 to 50,000—say, initial cost—we are adding 40,000 more as initial cash flow, which is negative. But in future returns, it is going from 20,000 to 50,000 to 75,000. So, we are getting an additional 55,000 rupees.

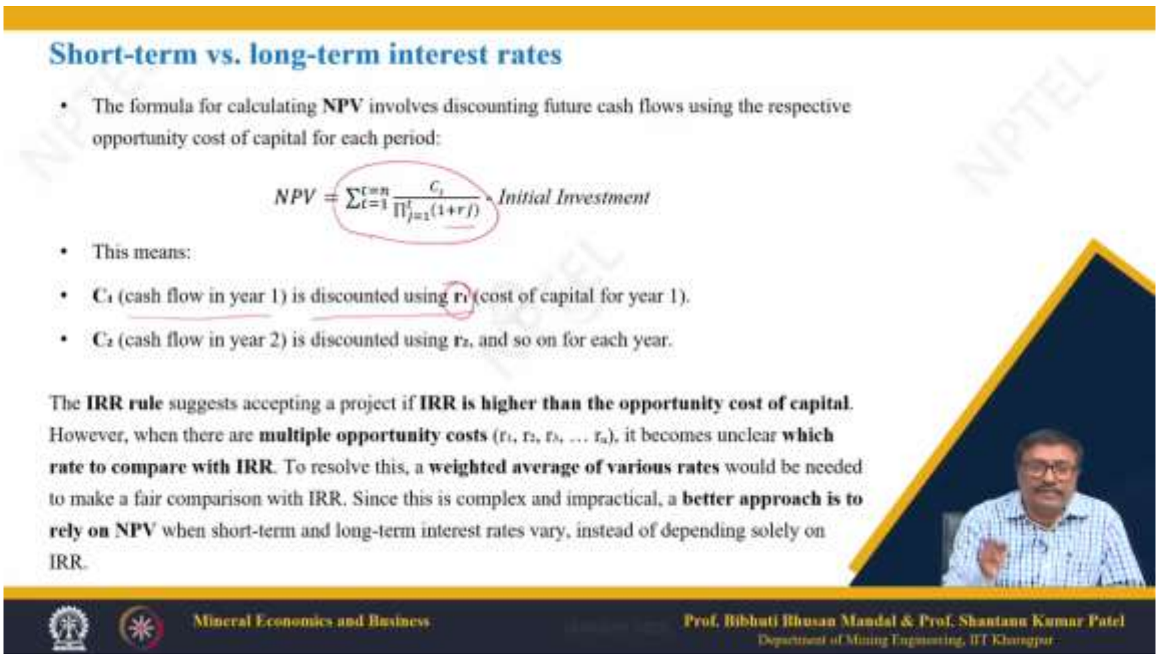
That means, with an initial investment of 40,000 additional, we are getting an additional future return of 55,000. The IRR for this extra investment, if it is separately calculated, is 37.5 percent, which is much higher than the 12 percent cost of capital. This means Q is the better choice, despite P having the higher IRR individually. The higher P was having 100 percent, whereas Q was having only 50 percent IRR. So, this is an illustration where we have to go by the incremental cash flow and delve into much more detail to understand which is the better choice.



So, what we understand from this is that IRR alone can be very misleading. So, to properly compare projects of different sizes, we should consider incremental cash flow instead of relying solely on IRR for a decision rule. Now, there is a case where we have lending and borrowing. So, we see the IRR rule will not take into account whether it is a case of lending or borrowing initially. So, the initial cost here is, say, minus 4000, which is the cash outflow, whereas in this case, we are taking a loan.

So, if you calculate these cash flows considering these future cash flows, we have 50 percent and 75 percent IRR. Whereas, the net present values at a discount rate of 10 percent are 1455 and minus 2364. So, if you just go by IRR in this column, then you see that B seems better because it has a higher IRR of 75 percent compared to A, which is 50 percent. This is misleading. Why? A is an investment where 4000 rupees earns a 50 percent return, generating a positive NPV of, say, 1455 rupees, whereas B is actually borrowing 4000 rupees. Borrowing 4000 at 75 percent interest effectively leads to a loss of 2364.

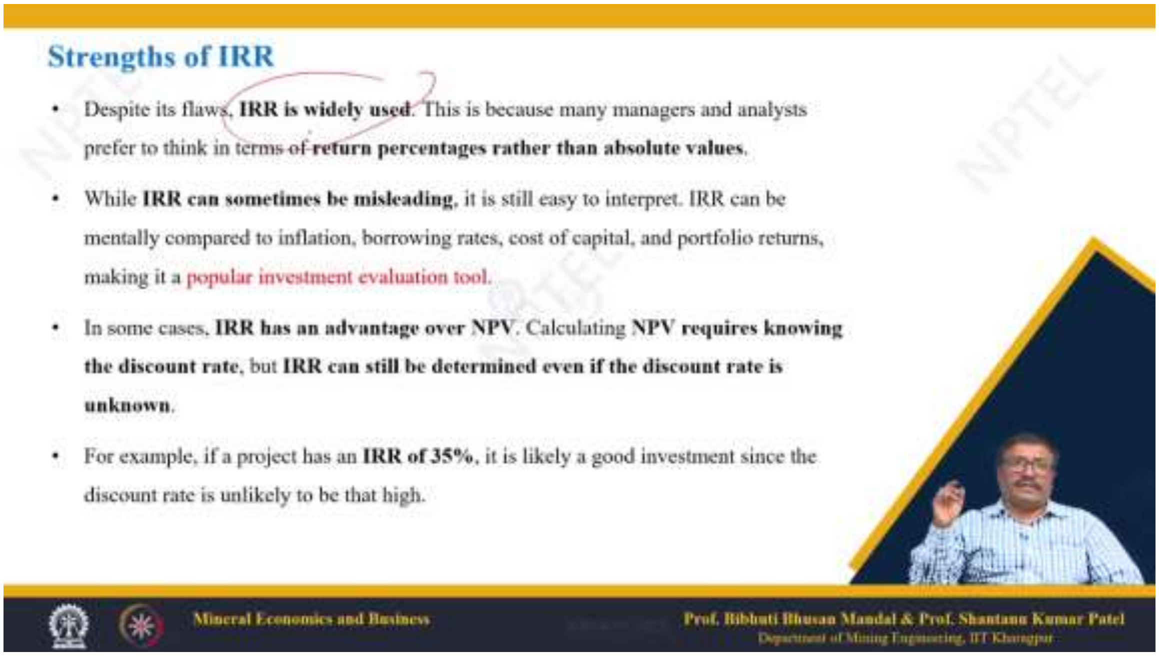
So, if you only consider IRR, project B appears more attractive. That means, by just the mathematics related to the calculation of IRR, it does not always give you the right decision. So, if you consider only IRR in this case, B appears to be more attractive even though it is clearly a bad choice. That means, again, we are saying that IRR alone should not be taken as a reliable metric, especially when dealing with borrowing or lending scenarios. Now, we are talking about short-term versus long-term interest rates. What is happening? The interest rate is changing; the interest rate is changing here.



$$NPV = \sum_{t=1}^{t=n} \frac{Ct}{\prod_{j=1}^{t}(1+rj)} - \textit{Initial Investment}$$

- This means:
- **C_1** (cash flow in year 1) is discounted using **r_1** (cost of capital for year 1).
- **C_2** (cash flow in year 2) is discounted using **r_2**, and so on for each year.

So, the IRR rule suggests accepting a project if the IRR is higher than the opportunity cost of capital. But when there are multiple opportunity costs like R1, R2, R3, or Rn over a period of time, then whom do we compare it with? Which rate is to be taken for comparison with IRR? So, it becomes unclear which rate to compare with IRR. To



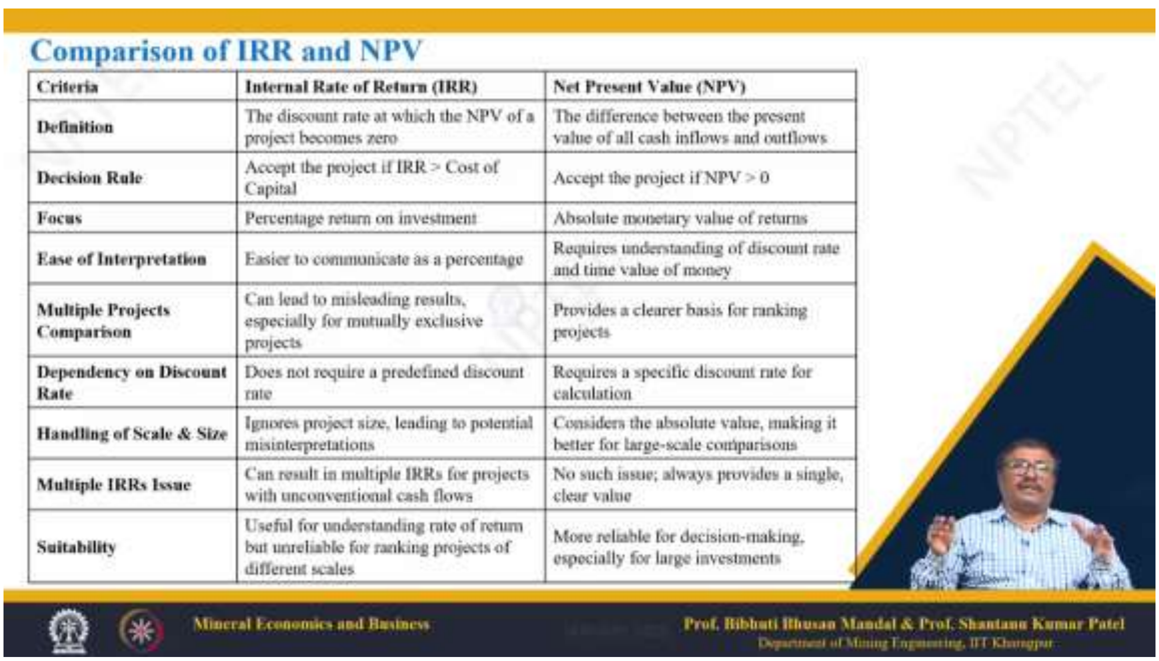
resolve this, a weighted average of the various rates would be needed to make a fair comparison with the IRR.

Since this is complex and impractical, a better approach is to rely on NPV. That means, with IRR, we do not get a clear idea. So, we now rely on NPV because NPV will take care of all those fluctuating rates and interest rates, giving you a net present value for long-term interest instead of depending solely on IRR in this case. NPV will give a result because it accounts for all these varying interest rates and provides a net present value, which helps in decision-making for choosing different projects. Even then, as I understand from this discussion so far, despite all these flaws, IRR is still widely used.

This is because many managers and analysts prefer to think in terms of return percentage rather than absolute value—not how much, but at what rate you are getting the return. While IRR can sometimes be misleading, of course, it is still easy to interpret. IRR can be mentally compared to inflation, borrowing rates, or cost of capital—meaning the rate of return can be compared with different rates, cost of capital, borrowing rate, or portfolio return. So, you can make a decision. Thus, it has become a very popular investment evaluation tool.

In some cases, IRR has an advantage over NPV also. Calculating NPV requires knowing the discount rate, but IRR can still be determined even if the discount rate is not known. For example, if the project during evaluation yields an IRR of 35 percent. 35 percent. So, it is likely to be a good investment by all probability because this discount rate cannot be as high as 35 percent, why?

So, in this, it is impractical. So, if it is 35 percent IRR, you can go for it. Go for it without knowing the actual discount rate that is used, that is coming in the future, because we can easily understand that whatever the discount rate may be in the future, it may be fluctuating also, but it will never come close to 35 percent. So, this IRR has got this strength; these are the facilities or rather the advantages of using IRR as a metric. Let us summarize the strengths and limitations of IRR in this table.


Comparison of IRR and NPV

Criteria	Internal Rate of Return (IRR)	Net Present Value (NPV)
Definition	The discount rate at which the NPV of a project becomes zero	The difference between the present value of all cash inflows and outflows
Decision Rule	Accept the project if IRR > Cost of Capital	Accept the project if NPV > 0
Focus	Percentage return on investment	Absolute monetary value of returns
Ease of Interpretation	Easier to communicate as a percentage	Requires understanding of discount rate and time value of money
Multiple Projects Comparison	Can lead to misleading results, especially for mutually exclusive projects	Provides a clearer basis for ranking projects
Dependency on Discount Rate	Does not require a predefined discount rate	Requires a specific discount rate for calculation
Handling of Scale & Size	Ignores project size, leading to potential misinterpretations	Considers the absolute value, making it better for large-scale comparisons
Multiple IRRs Issue	Can result in multiple IRRs for projects with unconventional cash flows	No such issue; always provides a single, clear value
Suitability	Useful for understanding rate of return but unreliable for ranking projects of different scales	More reliable for decision-making, especially for large investments

I have just jotted down; I will just show you what these are, whether we can compare IRR and NPV in a tabular form. Definition-wise, as we know, this is nothing but a discount rate at which the NPV becomes 0 for a project, and net present value is the difference between the present value of all cash inflows and outflows; this we already know. As a decision rule, accept the project if IRR is greater than the cost of capital. Whereas, in NPV, what you say is accept the project if NPV is greater than 0, depending on how much greater it is compared to 0; that depends on the decision-makers. Now, a focus is here on percentage return on investment, but in NPV, it is in absolute monetary terms; that means here it is a value, here it is a percentage return.

Now, ease of interpretation it is easier to communicate in terms of because it is a percentage for example, it is 22 percent IRR it is easy to communicate whereas, if here it requires understanding of the discount rate and time value of money both because if I say this is the net present value this does not say that how much we invested what was the cash flow all these things we are giving only are the net present value. So, here it is rather easier to interpret by knowing the IRR. In case of multiple projects comparison, it can lead to misleading results IRR as we have demonstrated with examples specially for mutually exclusive projects. But here for NPV, it provides a clearer basis for ranking project. We have different NPV values.

So, we can say that the net present value of this project is higher than the other project. So, it gives a clearer basis. But, here as we have demonstrated that it can be misleading if we when we compare with two different IRRs of two different project, even if the IRR is higher the project may not be attractive by looking at the other matrix. Now, the dependency on discount rate IRR does not require a discount rate, it does not require a predefined discount rate. So, you can find out the IRR first, then you find out the discount rate and other rate for comparison.

But for finding out the NPV, you must know the discount rate and then only find out the NPV of the project. Now about the scale and size, this ignores the project size leading to potential misinterpretations as we have seen, but this considers the absolute value that means, the magnitude of the of the investment and the returns are well documented or well taken care of in the calculation of NPV. Now multiple IRR issue. As we have seen that there are cases where there are positive and negative cash flows alternating. So, this can relate this can result into multiple IRRs and there we will have confusion as to which IRR is to be taken.

For NPV, it gives a clear value. When you provide the discount rates, it gives a clear value. No multiple NPV—nothing called multiple net present value has ever existed. So, here it does not have the issue of multiple NPV, but multiple IRRs exist. Regarding suitability, IRR is useful for understanding the rate of return but unreliable for ranking projects solely based on IRR.

Whereas NPV is more reliable for decision-making, especially for large investments, where we know the net present value of a particular project. It also depends on the investment and the different cash flows. So, overall, we can conclude that for decision-making related to large investments, NPV is better. And IRR has its advantages, and NPV has its own advantages. Rather, both IRR and NPV can be considered together. Whenever you are uncertain, you can switch from one to the other for decision-making.

So, we have discussed the net present value and IRR—the basics, their applications, how they are used, the challenges that can arise, and where each is more suitable: where NPV fits and where IRR fits. You can refer to the book Financial Management by PC Chandra and also Mineral Project Valuation for further study. We will explore more applications and discussions related to NPV and IRR in future lectures this week. Thank you very much.