MINERAL ECONOMICS AND BUSINESS

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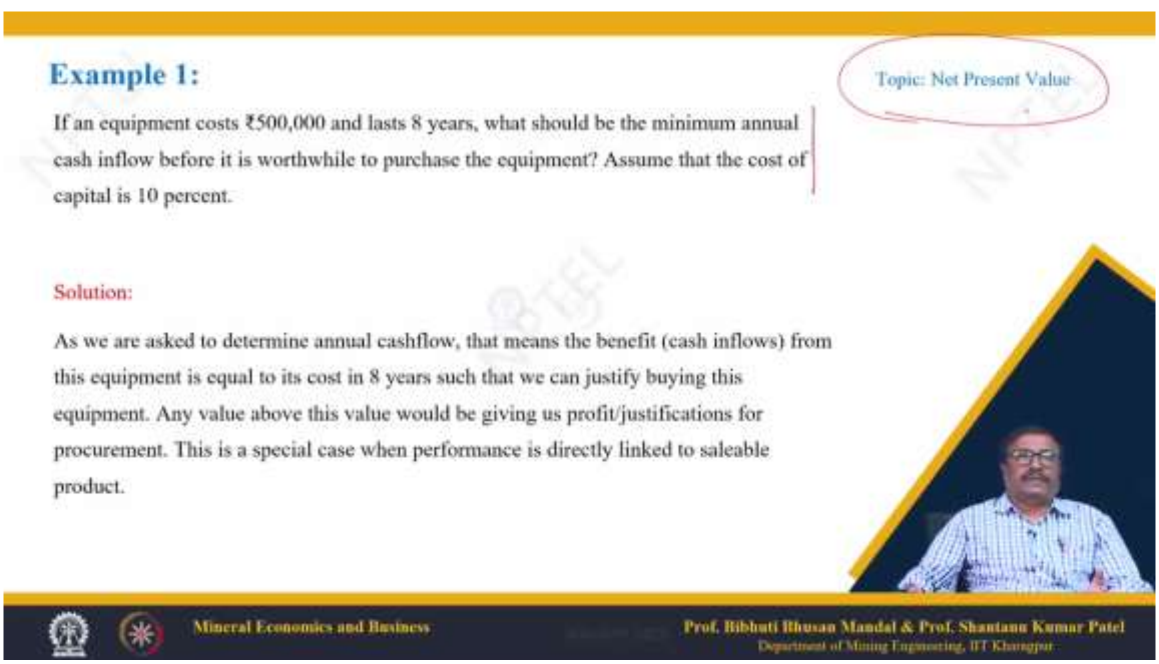
Week 8

Lecture 40 : Problems and Solutions - III

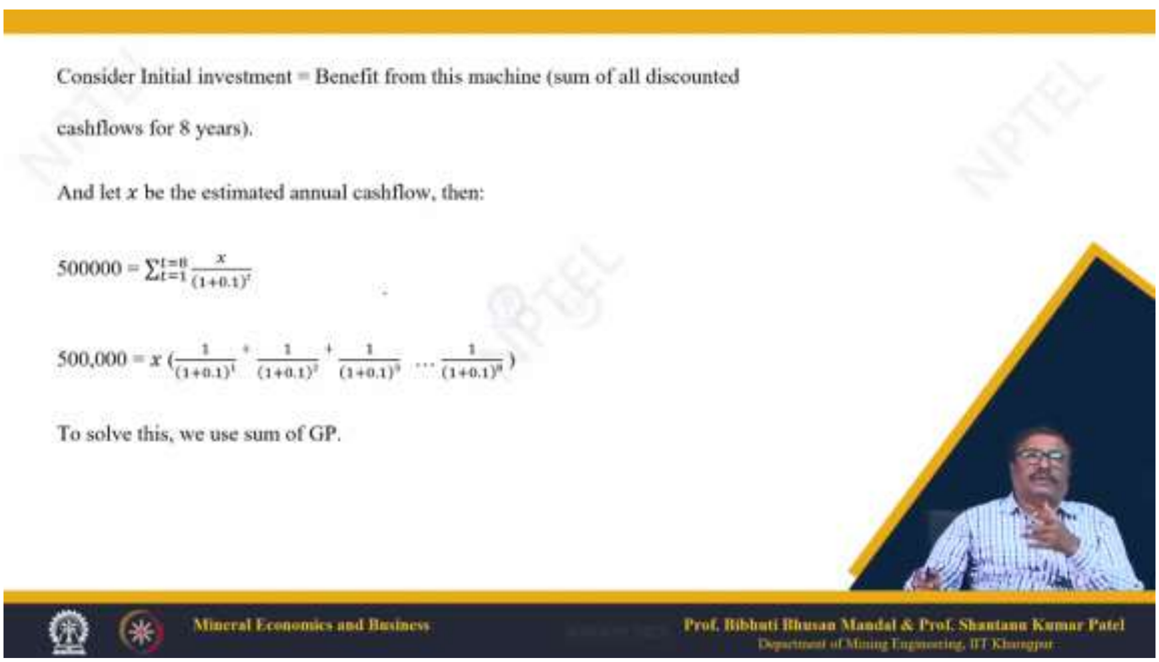
Hello viewers, welcome to Lecture 40, in which we will be having a third session on problems and solutions. Even though we are solving certain problems or doing exercises during the teaching of the theory part, we are explaining things, of course, with the help of certain examples. But then, when we are reviewing these things, we try to understand the different variations of the same problems. So, what I will do

is try to explain the same things - NPV, IRR, MIRR - these concepts which have been taught, particularly this week. In the last four lectures, I will give more examples and explain slowly the basic things and how this helps in making decisions for project evaluations. The first example, as you can see here, is centered around the net present value, or NPV - a very popular idea widely used for project evaluation. The acceptability or rejection criteria - NPV is a great instrument. So, through which you try to make a decision. Now, this problem has been framed in such a way that it is not asking you to find out the net present value; there is simply a little twist.



So, the question is: If an equipment costs, say, 500,000 or 5 lakh, and it has a life of, say, 8 years, what should be the minimum annual cash inflow before it is worthwhile to purchase the equipment? That means the decision is something like that - the life and then the equipment cost. Now, what is the return that you are getting - the annual return that you are getting? How much are you adding to the company's income? So, here the cost of capital is given—the cost of capital for the purpose of discounting is given - it is 10 percent. Now, this is a very special case, as I have written at the end of the slide.

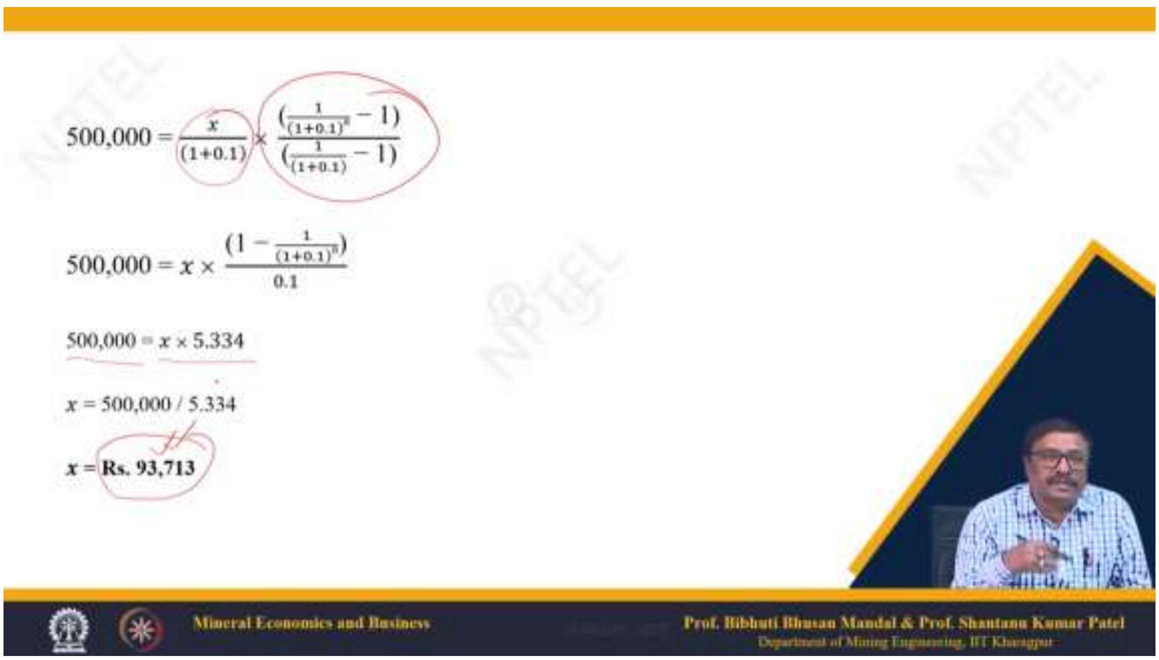
The performance of this machine is directly linked to the sellable products. That means you can convert the performance in terms of money; this is an assumption. Not all machines can be so directly used when you think about their value, cost in the market, price in the market, and the production or performance you get from that particular machine. It is not always possible to transform that in terms of money; that is not always feasible. For example, pumps - unless you have an indirect way, you have to consider these as indirect supporting machines.



Here, we are talking about a machine whose performance is directly linked to the sellable products. So, we are asked to determine the annual cash flow, which means that the benefit—the cash inflows from this equipment—is equal to its cost in 8 years. This is not just an addition problem. We can justify buying this equipment over 8 years, as it will return 500,000 rupees, but in a different form that accounts for the time value of money.

So, any value above this would give us profit or justification for procuring this machine. That means it has to perform better than what we are asking for. The calculation is quite simple once we understand this problem. Consider that the initial investment equals the benefit from this machine; these are equalized. Only then can we find out the minimum annual cash flow that satisfies this condition.

The initial investment equals to the benefit from this machine. So, benefit from this machine, sum of all discounted cash flows for 8 years. We know we know the cash flows for example, we can write the expression and we have to discount that at the rate of say 10 percent as it has been given the cost of capital. And then we find out the expression when the initial investment equals to the 8 years, cash inflows discounted at the rate of 10 percent.



Here you can see the initial value is 500,000 or 5 lakhs and here we are summing up over the life of the equipment 1 to 8 years, x is unknown which is the annual cash flow minimum. and divided by (1 + r)^t, which is a very known formula by this time. Now, if you expand this thing over 8 years and write the expression like this:

$$500,000 = x\,\left(\frac{1}{(1+0.1)1} + \frac{1}{(1+0.1)2} + \frac{1}{(1+0.1)3} \;\; \ldots \frac{1}{(1+0.1)8}\right)$$

Look at the expression as you must be familiar by this time, when you have studied the time value of money so many times in the last lectures. So, as you can see that it is a summation of geometric progression sum of GP, we can use that summation formula here.

$$500{,}000 = \frac{x}{(1+0.1)} \times \frac{\left(\frac{1}{(1+0.1)^8} - 1\right)}{\left(\frac{1}{(1+0.1)} - 1\right)}$$

$$500{,}000 = x \times \frac{\left(1 - \frac{1}{(1+0.1)^8}\right)}{0.1}$$

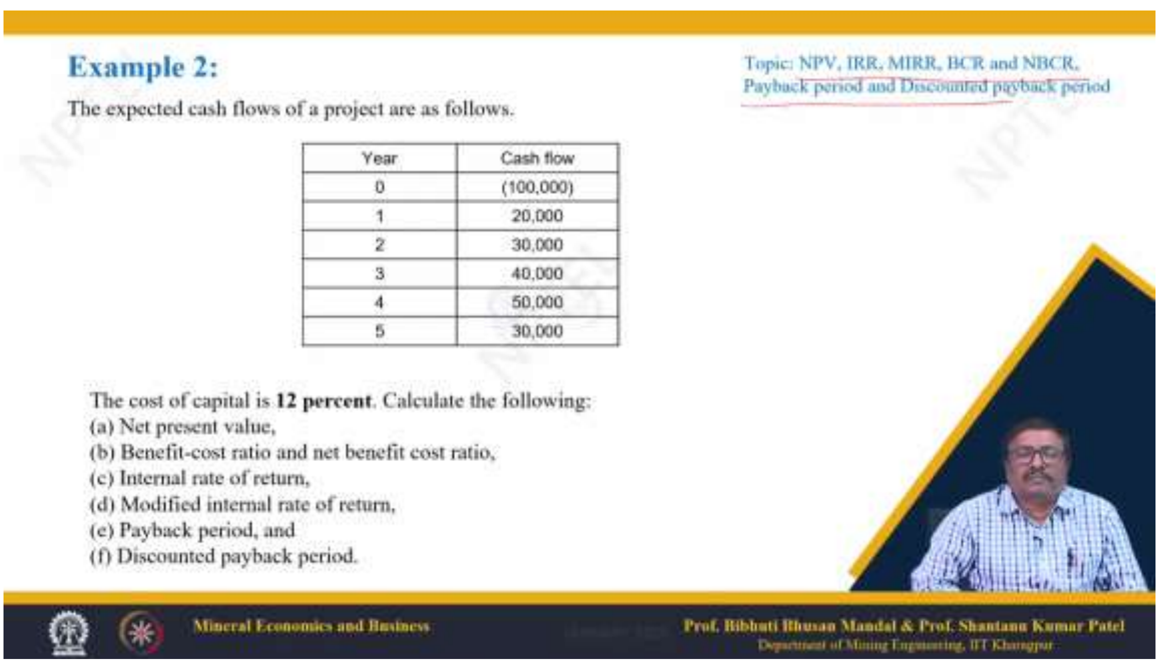
So, now we can simplify in 2 3 stages and we find that:

$500{,}000 = x \times 5.334$

$x = 500{,}000 / 5.334$

x = Rs. 93,713

So, this we require from that machine that means the output or net the cash inflow, the cash inflow should be 93,713 rupees at least to justify its purchase for its performance or as it gives us the return in 8 years.



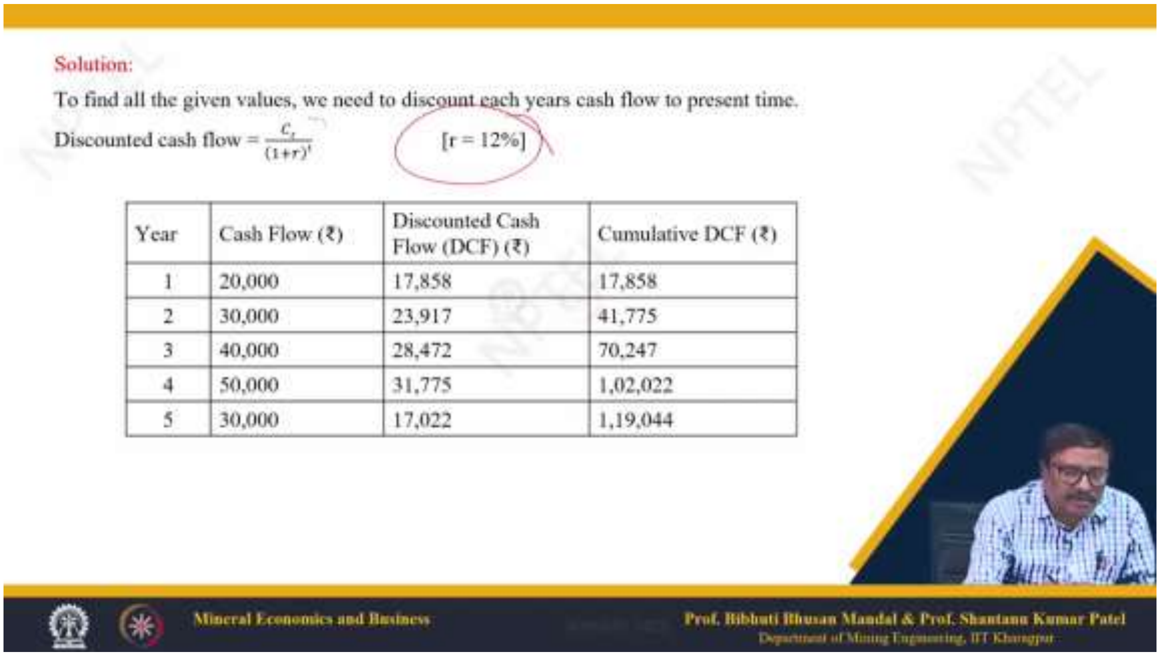
So we have to see that we have to see that the annual cash flow is above these values above these values then it is fully justified fully justified that whatever we get more than this more than the value that we have calculated then this gives us a profit. That means, above the expenses whatever whatever was the cost in the beginning we have we have actually calculated the present value of all those inflows equated them and now we find out the value of x which is the annual cash inflow. So now, this is a simple twist in the classic classical NPV problems, but it is quite interesting. Now, we will go to another example in which all these the parameters which we calculate indicators for different purpose like the NPV for project evaluation we have different indicators like NPV net present value then IRR internal rate of return.

Then there is modified internal rate of return to offset the disadvantages of IRR certain drawbacks in use of IRR. Then we have the benefit cost ratio which is also known as a profitability index and also the net benefit cost ratio which is a modification which is modification which we have learnt earlier. payback period calculation and discounted payback period calculation on the same data. On the same data, we will apply all this. In practical project evaluation, you get a huge number of cash inflows and outflows, but the basic ideas are same.

In my experience of teaching this subject in the classroom, what I found that the calculations related to this does not require a high mathematical skills. But, it definitely requires the basic understanding of the subject. Application of the subject is not difficult for the engineering students not at all, but they do not know the basic of these ideas even though they are taught. in part somewhere in the first year or second year. Here we try to make you understand the basic things more clearly and with the purpose that you will be able to use these ideas in the field of mining, engineering, mineral economics in particular.

The cost of capital in this case has been given as 12 percent. The expected cash flows are projected as this. This is projected, the expected cash flows are listed here. This is projected means when you are evaluating a project, then you make a schedule. So, the estimated returns will be this. So, this is not very difficult once you have understood the method of working and then how much production will come from the mine.

So, you can make a schedule over, say, 20 years or 30 years, as practical as possible, as you can estimate all those things and try to make a schedule which could be close to reality. In future, there would be some variation, but more or less these values will persist. Now, see, in the beginning, say, we have a huge capital investment. You can think of these figures in crores or in lakhs also. So, depending on the size of the project, the idea will remain absolutely the same. Now, in the beginning, we have put a bracket—that in statements like this, with a bracket means this is negative or cash outflow.



And all these things which are without brackets are cash inflows—these are positive values. The cost of capital being 12 percent, we have to calculate all these performance indicators, the project evaluation indicators like the net present value, benefit-cost ratio, and net benefit-cost ratio. We also have to find out the internal rate of return, the modified internal rate of return or MIRR, then the payback period and discounted payback period. We will proceed now—how to calculate one by one. To find all the given values, we need to discount a year's cash flow to present time—that means, we know that the r is 12 percent, it is given.

So, what we do is we use a simple discounted cash flow technique:

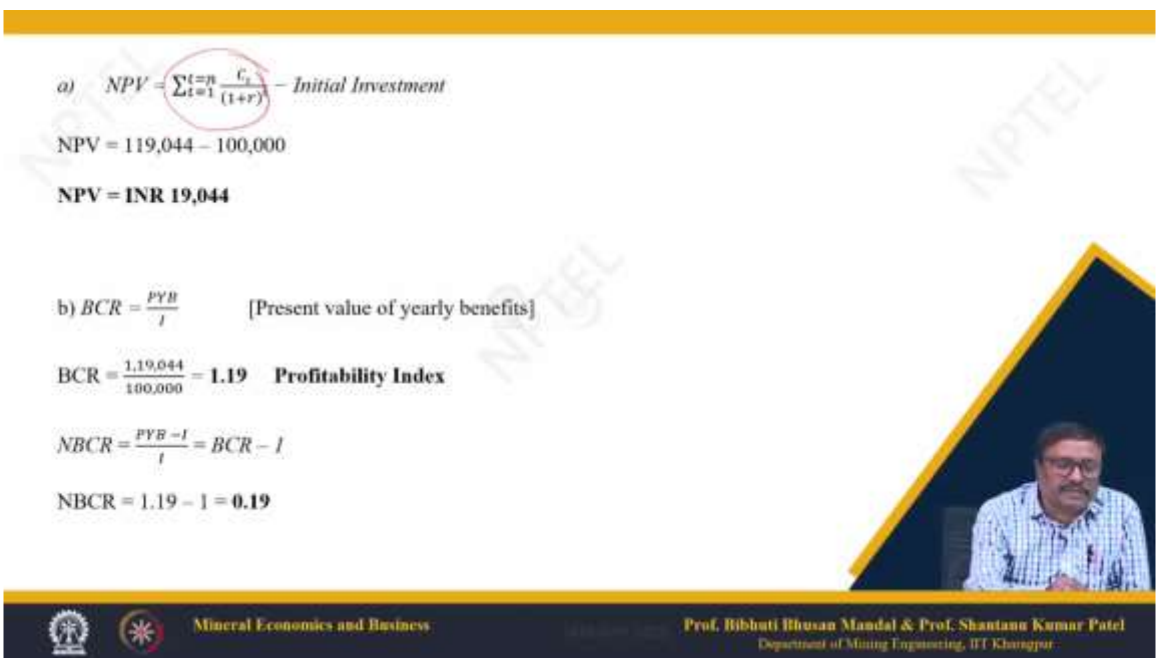
- Discounted cash flow = $\frac{C_t}{(1+r)^t}$

Now, this 20,000 is our cash flow, divided by (1 + r), where r is 0.12. So, 0.12 to the power t will become like this, and it will go up to 5. 1 by 1. So, what we can do is you can make a factor which will look like this: (1 + 0.12), and you can go on multiplying this several times. So, like that, if you find out this, it will come around, say, 0.89 or so. If you multiply it with 20,000, you get this figure - this is the discounted cash flow.

In the next year, what you do is multiply 30,000 by 0.89 twice, and you get this value. In the next year, what you do is use that multiplication factor—this factor—and go on using it depending on the number of years. So, here what you do is multiply 40,000 by 0.89, then by 0.89, and then by 0.89. So, you get this value. Like that, you calculate for the fourth year and also for the fifth year.

Now, this column is making a schedule of the cumulative DCFs. This is the discounted cash flow individually calculated for each year, and on this side, what we are doing is: in the first year, the cumulative remains the same; in the second year, this plus this, This plus this will be 41,775. Similarly, you add 28,472, and you get this figure. Like that, you go on making a schedule of the cumulative discounted cash flows.

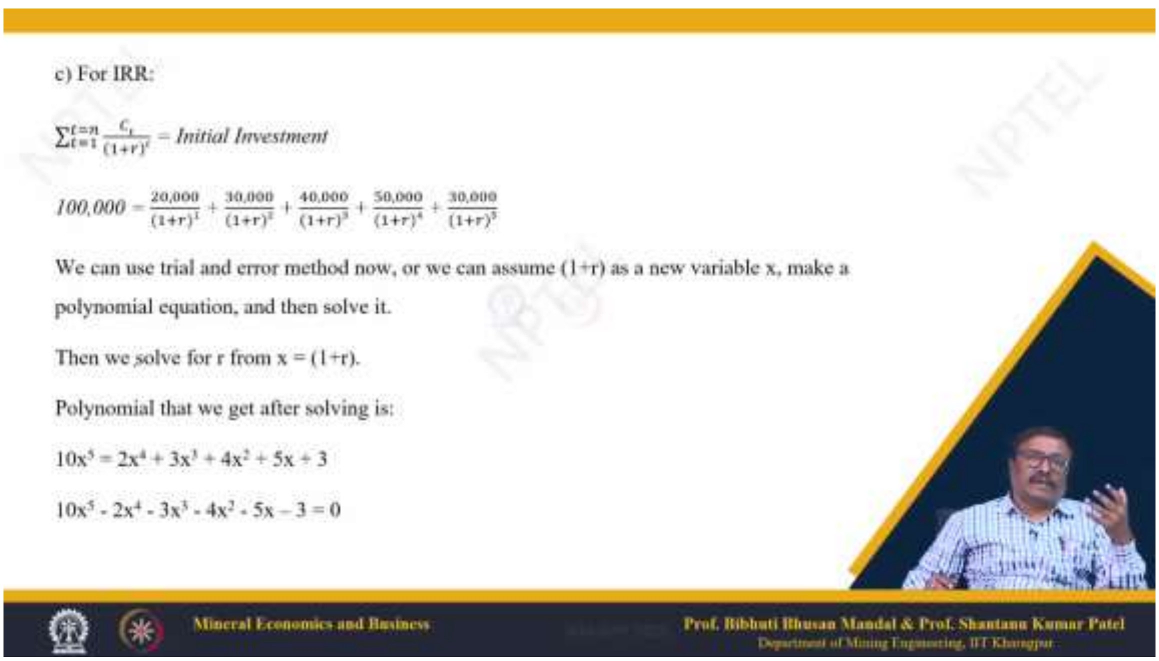
So, this will be useful for various calculations. We are just now making one table in which the basic calculations are done. Basic calculations are done. So, the net present value is—we can calculate this as all the cash inflows minus the cash outflow, which is the initial investment. So, when we add all the cash inflows, it comes to 1,19,044 minus the initial investment, which was 100,000.



So, you get the net present value as 19,044. So, NPV is positive. For example, now up to this point, it is fine. Now we are going for another indicator called the BCR, or benefit-cost ratio, benefit-cost ratio. So, this is basically the present value of all yearly benefits, which is nothing but 1,19,044, divided by the initial investment, the I. I is the initial investment. That is how much you are getting.

In return for the investment that you made, you invested 1 lakh and you are getting 1 lakh 19,044 rupees. So, here the ratio is 1.19. This is a profitability index, which is greater than 1. If they are equal, the index is 1. So, you are not going beyond what you invested - you are not getting more - and if it is less than 1, then you are getting less than what you invested. Why should you go for that, right?

Now, we are giving a slightly modified the net benefit cost ratio in which what we are doing that the extra amount the present value of all the yearly benefit minus the investment that means, the net what you get you are getting the net profit rather that you are calculating whole divided by i. So, what will happen effectively, it is nothing but the 1.19 minus 1 is 0.9. That means, you are getting at the rate of 19 percent. The gain is nothing but 19 percent of what you actually invested. So, this is giving a straight cut percentage wide gain over the investment.



And here it is profitability index that means, total the cash inflows is how many times the investment, how many times the investment that gives the profitability index. So, we have calculated NPV, we have calculated profitability index, we have calculated the net benefit cost ratio and benefit cost ratio also. We are going to IRR, in the IRR part as we know that IRR is defined as that discounting rate at which the NPV becomes 0, NPV equal to 0. It cuts the x axis, it crosses the x axis that particular point. So, here what we are doing?

That actually means that the initial investment and the all the cash inflows together the present value of all the cash inflows together are equal. So, if you deduct this from this it should be 0 same thing written in a different way. So, here what we see that 1 lakh was which was initial investment equals to all the all the cash inflows discounted to the present time they are all brought to the present value. Now, this 20000 say for first year, we know that this is the cash inflow $(1 + r)^1$. Similarly, 30000 by $(1 + r)^2$, then $(1 + r)^3$, 50000 divided by $(1 + r)^4$ and then 30000 divided by $(1 + r)^5$.

What we are doing? Just for simplification as it is manageable here, we are not doing it for 20 years or 30 years, then it will be an approximation and then other techniques will be will be used. You can assume some value and see where you are and again you try with another value called trial and error. Or even nowadays you can use the excel calculation, the MS excel, Microsoft excel where this function is nowadays inbuilt, where you can find out by putting these values. But anyway, when we have these values which are manageable, we just assume that (1 + r) equal to x, that simplifies the whole issue here.

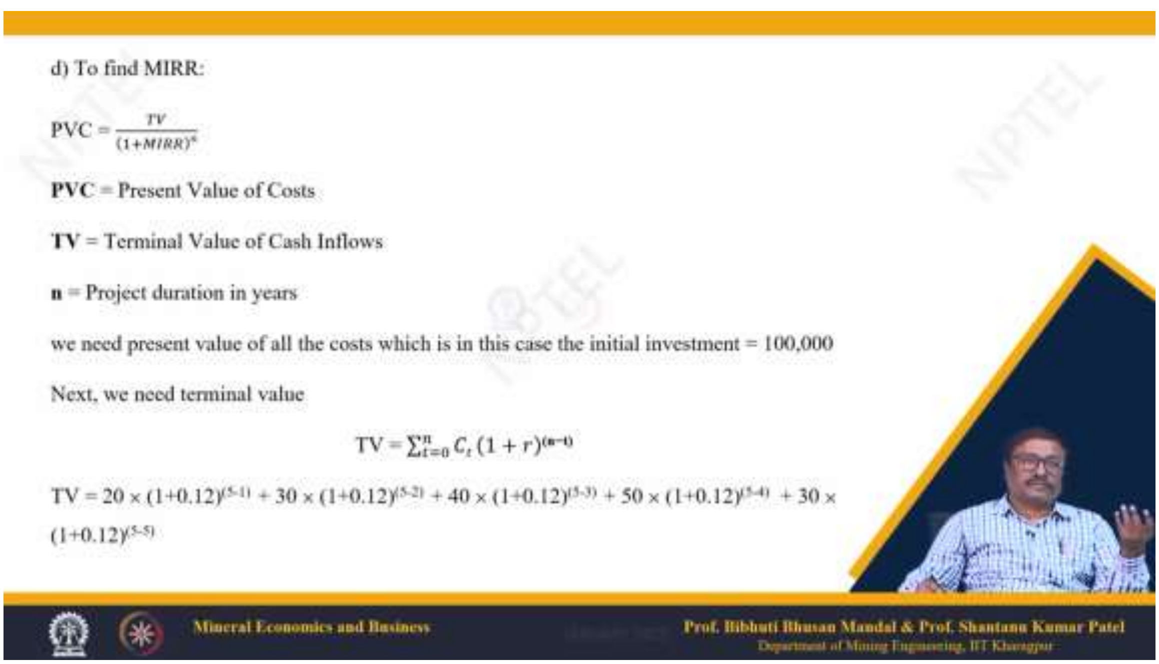
So, what it becomes 20000 divided by x plus 30000 divided by x square 40000 by x cube 50000 by x raise to the power 4 and 30000 by x raise to the power 5. If you now you transform the whole thing then it takes the form of a polynomial which looks like $10x^5 = 2x^4 + 3x^3 + 4x^2 + 5x + 3$, solve this equation. On solving this, we get X as 1.1869. Other values that we get are irrational and therefore, ignored, rejected.

we keep 1.1869, other values are irrational and therefore rejected, ignored. We assume that $x = (1 + r)$. So, $r = x - 1$ and from there we get r equals to 0.1869 or 18.69 percent. So, IRR becomes 18.69 percent. The internal rate of return from this project is projected to be 18.69 percent, remember this figure it will be useful at the end. Now, we are going for MIRR.

The MIRR is a modified form. It is more realistic; it takes the cost of capital as the discounting rate. These additional corrections have been made to make it more practical. Here, we have:

- $$\text{PVC} = \frac{TV}{(1+MIRR)^n}$$

That is the formula. Where PV is the terminal value of all cash inflows—terminal value means, at the end of the project, we are going forward with terminal values. And PVC is the present value of costs—cost of financing, rather, the present value of all costs.



N is the project life in years. So, we need the present value of all the costs, which in this case is the initial investment, equal to 100,000 to simplify. Otherwise, if other figures are there, all have to be brought to the present. Now, we need the terminal value. To compute the terminal value, we use the same formula as I have explained earlier:

TV = $\sum_{t=0}^{n} Ct\ (1+r)^{(n-t)}$

where n is the life of the mine and t is the year when the cash flow is considered. Accordingly, these are all in 1000s, so 20,000 has become 20, 30,000 is 30, and so on.

In the indices, you use the n - t and calculate the expression here. TV equals to ultimately 209.78, which is in 1000s 209.78 thousand. This is not a difficult calculation; we can easily do it with a calculator. Now, the same formula we have shown in the:

- PVC = $\frac{TV}{(1+MIRR)^5}$

Put the value of PVC - we know that the initial was 100,000, and this is 209,780, which is nothing but in 1000s (we have just multiplied by 1000)

100,000 = $\frac{209,780}{(1+MIRR)5}$

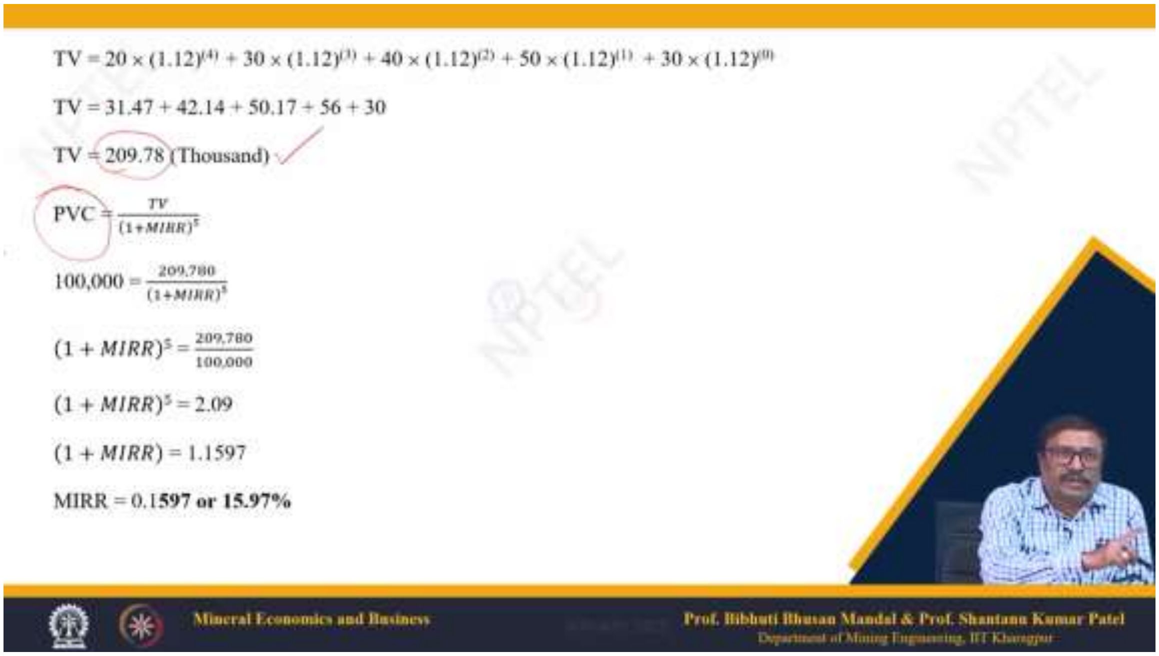
$(1+MIRR)5$ = $\frac{209,780}{100,000}$

$(1+MIRR)5$ = 2.09

$(1+MIRR)$ = 1.1597

MIRR = 0.1**597 or 15.97%**

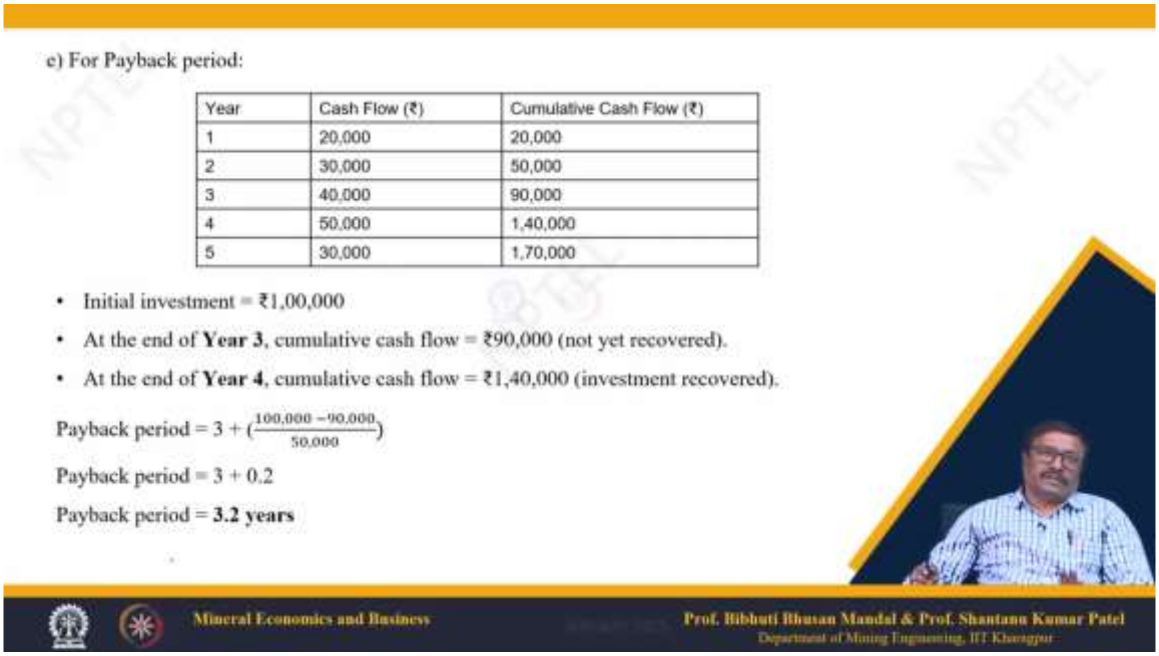
Like that, you calculate and get 1.1597, okay.



So, MIRR becomes 1.1597 - 1, which is 15.97 percent. MIRR is 15.97 percent, and IRR is 18.69 percent. MIRR is more realistic; we will go by that also. Now, for the payback period, there are some investors who are very much interested in how many years they will take to get back their money. So, the payback period from this calculation, we can easily find out these things.

The initial investment is 1. And the cash flows are given here, and the cumulative cash flows—remember that this is not discounted. These are simple cumulations. So, cumulative calculation is 20,000, and then you add 30,000 to get 50,000; 50,000 plus 40,000 is 90,000; 90,000 plus 50,000 is 140,000. You make a schedule like this. So, in total, you are getting 170,000 in 5 years, right?

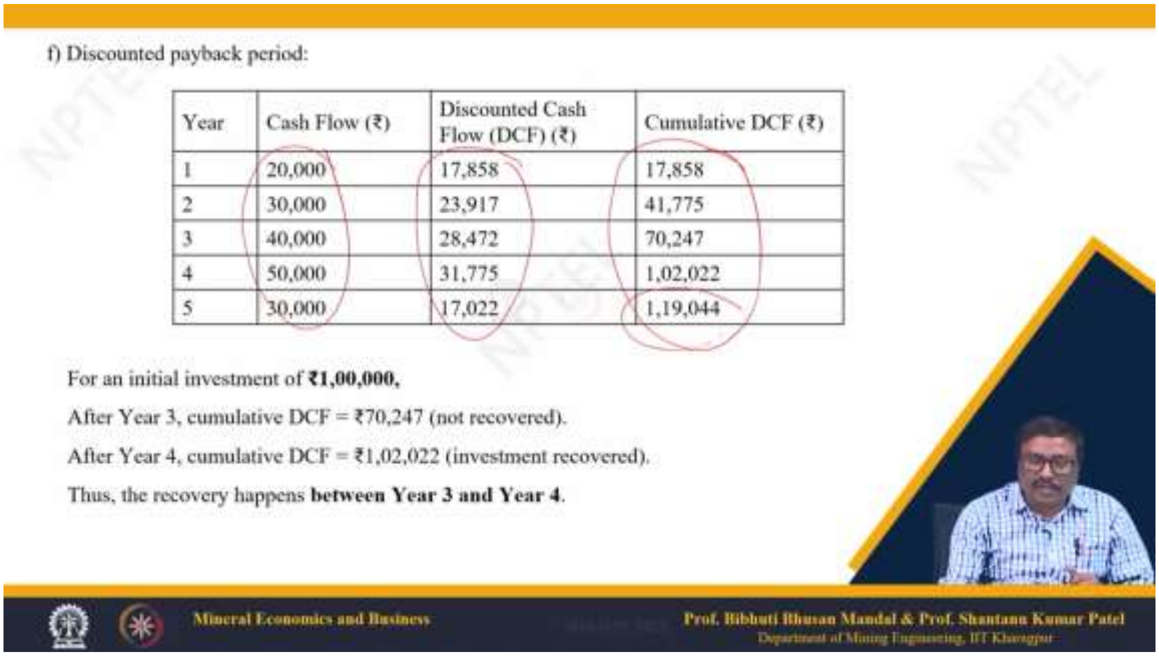
The initial investment is 100,000. So, at the end of year 3—by observation, you can see that it is 90,000. That means you still require 10,000 rupees to add, and then it will become equal to the initial investment—that it will become 1 lakh, then it will equal 1 lakh. But if you add another 50,000, as the cash flow for the next year, it will exceed and go to 1 lakh 40,000, which is above 1 lakh.



So, this is the payback period is some something between year 3 and year 4. So, at the end of year 3 the cumulative cash flow is 90000 which is not reached which has not gone up to 1 lakh, but at the end of year 4 the cumulative cash flow is 140000 investment already recovered and you have exceeded. So, what we do what we require is 10000 1 lakh minus 90000. here and then the capacity of cash flow for the fourth year is 50000, but we need only 10000. So, what is the ratio that 10000 divided by the 10000 whole divided by 50000 which is 1 by 5 which is nothing but 20 percent or 0.2.

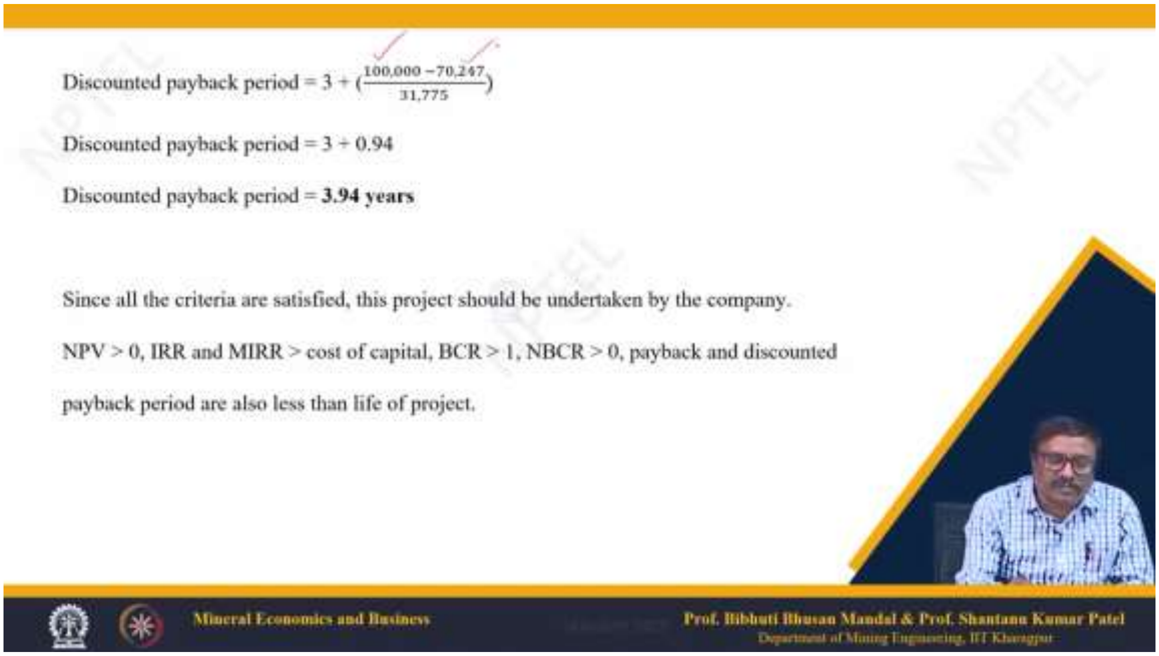
So, we need only 3.2 years to recover even though we have a schedule for 5 years, but in 3.2 years the money is already recovered. This is a simple calculation for payback period without discounting the cash flow. We will slightly modify this and see how it affects the the payback period. In the discounted payback period what we do? We do the same discounted cash flow schedule

based on the cash flow as we have seen earlier. So, the cumulative DCF will appear like this as you remember that the total cumulative discounted cash flow was 1,19,044 rupees as we have used this earlier ah for calculating the profitability index and other things. So, for an initial investment of say 1,00,000 after 3 year you can observe that we have received 70247 where we have not recovered 1 lakh. So, we are going to the next year and to see to observe what is the figure it is 1 lakh 2000 that means, we have already exceeded that means, the figure that we are searching is between year 3 and year 4.



So, as you see that the recovery happens between year 3 and year 4 that you can see from the schedule here. what we do how much we require. So, it is again the same style the 1,00,000 minus 70,247 this is the difference that was remaining and this is at the rate this is the total capacity of the next year as a discounted cash flow. So, here the discounted payback period is 3.3 plus 0.94 this is the additional part required not the entire it is almost 4 years even though in payback period it was being shown as 3.2 years very close to 3 years.

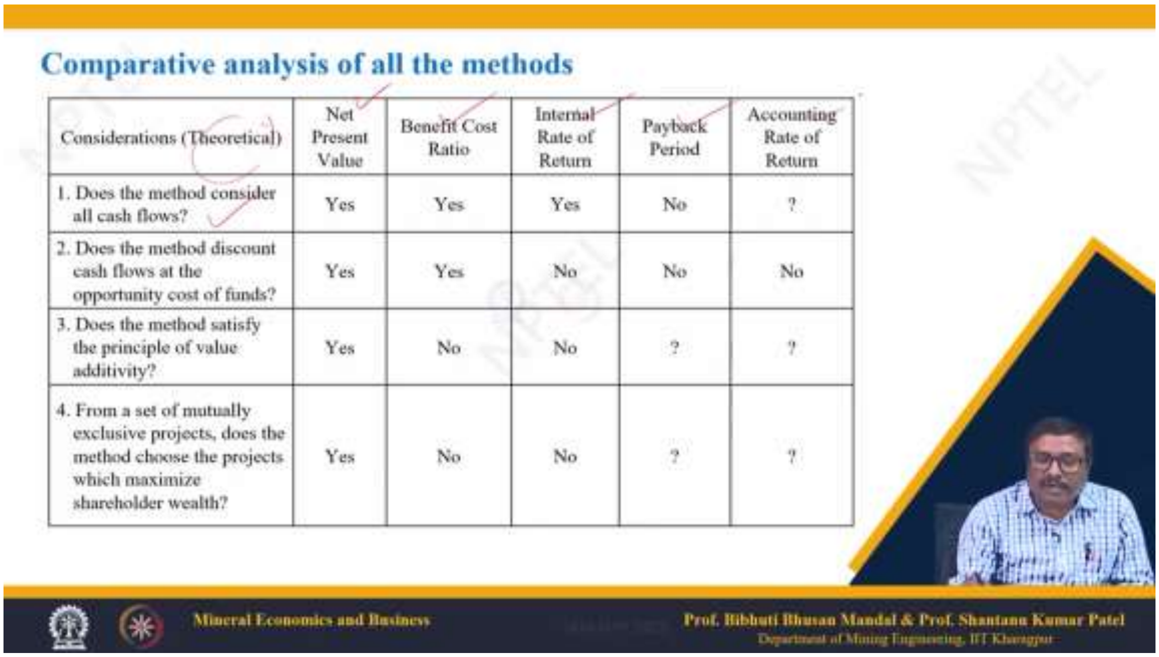
But if you are using the discounted cash flow that means, you are realistically calculating using the discounted cash flow technique then it is becoming 4 years practically 4 years. So, there is a difference between the payback period and the discounted payback period and discounted payback period is more realistic more practical. So, we can see from all the ah results that we have calculated so far, the project should be undertaken by the company that means, the project proposal is acceptable why? First thing, the NPV is greater than 0 marginally, but that depends on what scale you are using. IRR and MIRR is greater than the cost of capital in both the cases as we have shown.



In one case, it was I think 18 percent, 15 percent. In other case, it was 18 percent or so. They are all greater than the cost of capital. Now, the BCR benefit cost rate ratio is greater than 1 and NBCR is greater than 0. Payback and discounted payback period are 3 and 4 years which is less than the life of the project.

That means before the project is over. You are recovering the amount that we invested. That means whatever you are getting after that will be additional income from that project. So, after 4 years, whatever you gain is additional. By checking—I mean going through all possible indicators—this project is viable and should be undertaken because we have calculated almost all of them. This is useful for understanding how we evaluate the project through project valuation techniques.

There are different valuation procedures, and we have also discussed in earlier classes when these specific valuation techniques are applicable. With these basic ideas, you can apply the same thing in large schedules, which are usually in mining projects. The schedules are too big—very big—and you need to have practical and realistic prices from the market, as well as the cost of equipment and manpower. You can realistically make a schedule, apply this formula, and proceed with project evaluation. If you now go by the comparative analysis of all these methods, then you can see the considerations—the theoretical considerations here. Does the method consider all cash flows, like NPV, benefit-cost ratio, IRR, payback period, and accounting rate of return? So, does the method consider all the cash flows in net present value? Yes.



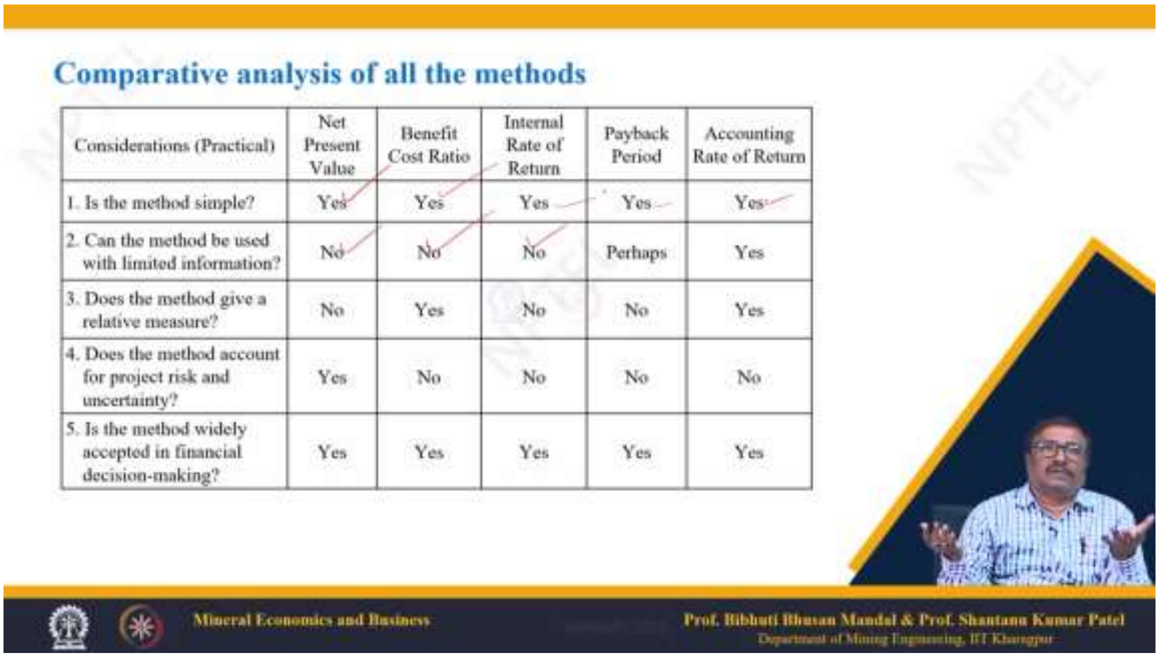
Considerations (Theoretical)	Net Present Value	Benefit Cost Ratio	Internal Rate of Return	Payback Period	Accounting Rate of Return
1. Does the method consider all cash flows?	Yes	Yes	Yes	No	?
2. Does the method discount cash flows at the opportunity cost of funds?	Yes	Yes	No	No	No
3. Does the method satisfy the principle of value additivity?	Yes	No	No	?	?
4. From a set of mutually exclusive projects, does the method choose the projects which maximize shareholder wealth?	Yes	No	No	?	?

In benefit-cost ratio? Yes. IRR? It is yes. Payback period? No. It does not; it only goes up to that period when your money is recovered. Now, does the method discount cash flows at the opportunity cost of funds?

In NPV yes, benefit cost yes, but IRR it does not, payback period does not, accounting rate of return no. Does the method satisfy the principle of value So, here only NPV talks about the value addition others are not there in terms of ratios of return or profitability. So, from a set of mutually exclusive projects for example, does the method choose the projects which maximize the shareholders weight in in case of the net present value it is yes, because it is the value addition a calculation others are not. here the here these are depends on what what context you are and these are doubtful I mean perhaps this could be indicated otherwise not.

Now other questions are also the practical things. Is the method simple? In NPV yes, benefit cost yes, yes, yes, yes these are all simple method whatever we have done. But MIRR will be definitely not not so simple. So, can the method be used with

information, these information are all required , detailed information is required for all this, but for payback period no. Your, the calculation could be simple, if you are not so critical, if you want to know Fine, in how many years I am going to get back the money? Every year I am getting 4 crore, I invested 100 crore. So, I will be requiring 25 years to that kind of calculation is there in payback period.



Does the method give a relative measure? NPV does not give a relative measure, but the benefit-cost ratio does. But the internal rate of return is only a percentage. So, the payback period is indicative of that particular project. So, it does not give a relative measure.

Does the method account for project risk and uncertainty? Yes, for NPV. Benefit-cost ratio? It does not account for project risk, and the internal rate of return does not either. This is specific to that project only, and the payback period also does not account for project risk and uncertainty—how much you are investing versus how much you are getting, like that. Is the method widely accepted in financial decision-making?

All these are accepted for financial decision-making, as they look at the problem from different perspectives. If you have doubts from one angle, take another angle and view the problem, then make a decision based on the calculations to find out the parameters listed above, as we have explained in brief as much as possible in Example 2. So, this will give you an idea of how you can do project evaluation. Using indicators like the net present value, IRR, the modified internal rate of return, the payback period, or the discounted payback period. So, with this, we come to the end of the theoretical ideas or fundamentals of project evaluation and the examples with problems and solutions.

Hope you have enjoyed this lecture. Thank you very much.