MINERAL ECONOMICS AND BUSINESS

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Week 9

Lecture 43 : Cost Volume Profit analysis - II

Hello everybody, welcome again to this lecture on Costing in Mines. Today, we will be discussing the cost-volume-profit analysis part 2, which will cover the concepts of non-linear break-even analysis and certain other related topics. In the previous lecture, we talked about linear break-even analysis, other break-even points, the concepts of the break-even chart, what it actually means, and we know that all the cost functions and revenue are not necessarily linear. So, we will cover the concepts of non-linear break-even analysis here. Related topics like marginal profits, marginal cost, and average unit cost will be discussed today. In real-world scenarios, the break-even chart we discussed in the last lecture does not always follow a straight-line relationship.

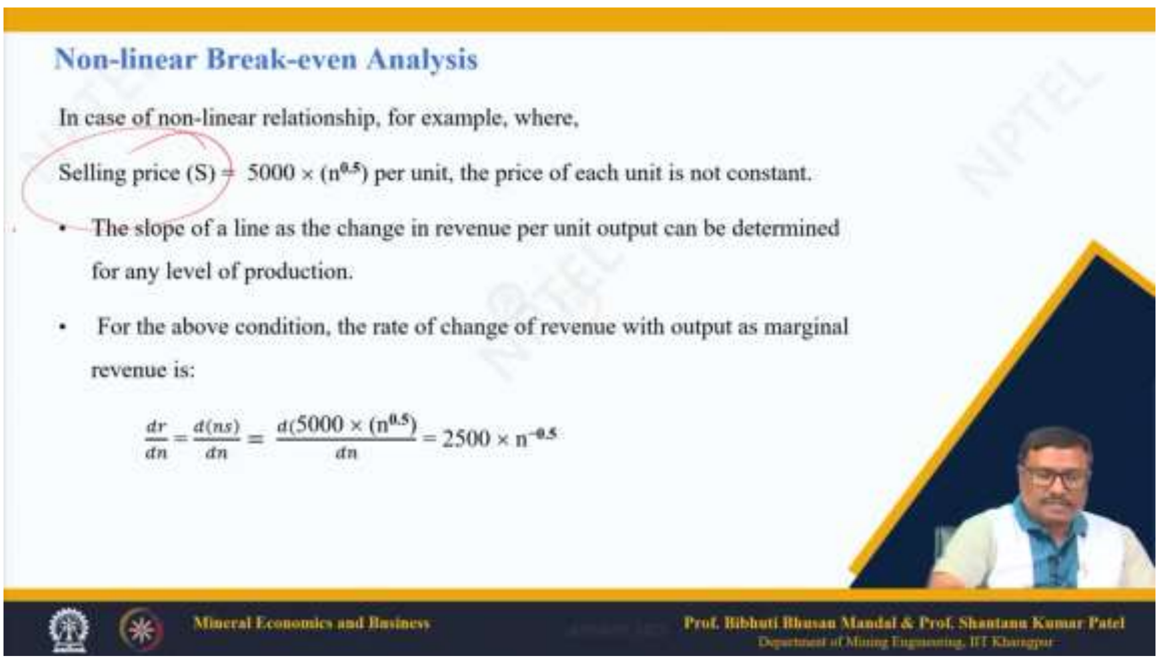
That means what we have seen in the previous lecture—that the cost is a linear function and the revenue is also a linear function—does not hold when production increases. Say you are increasing from 10,000 to 12,000, 12,000 to 50,000, or beyond that. It does not mean that the variable cost will remain the same at the same rate—that is, the linear relationship between the number of units produced and the revenue, cost, and variable cost. At the same time, some additional fixed cost may be required to increase the capacity. So, those things will now be variable.

Non-linear models are therefore required for better analysis. At the same time, the revenue we earn—the more we produce—maybe we have gone far beyond our break-even point. So, we can also reduce the price. Depending on the market demand, the price may vary. So, these relationships are not at all linear.

In a real-world scenario, that is why we say that the breakeven chart does not always follow straight-line relationships. So, by studying these non-linear relationships, what do we gain? We gain deeper insights into how revenue and cost change with production levels—when they increase or decrease, how they behave. So, unlike linear models, these non-linear functions focus on marginal values. That means

they highlight how revenue and cost shift as the output increases at a particular point in time. When we have reached a certain level, how they change depends on the stage we have already reached. So, one key concept in this approach is marginal revenue. This refers to the extra income earned from selling one additional unit at a given production level. At that point, what is the relationship between the revenue we earn and the different cost components related to the production of one unit?

And by comparing this with the price at which we sell in the market, what is the marginal value at that particular level? It will change depending on the production level—maybe up to a certain level, it becomes linear, after which it does not remain so. So, in a linear model, the marginal revenue will always remain constant, but in a non-linear system, it can fluctuate due to various factors like demand or the pricing strategies we adopt. For example, in this case where we demonstrate a non-linear relationship, if the selling price $S = 5000 \times (n^{0.5})$, then the price per unit is a function of the number of units produced and is therefore not constant at all. So, the slope of the line, as the change in revenue per unit output, can be determined for any level of production.

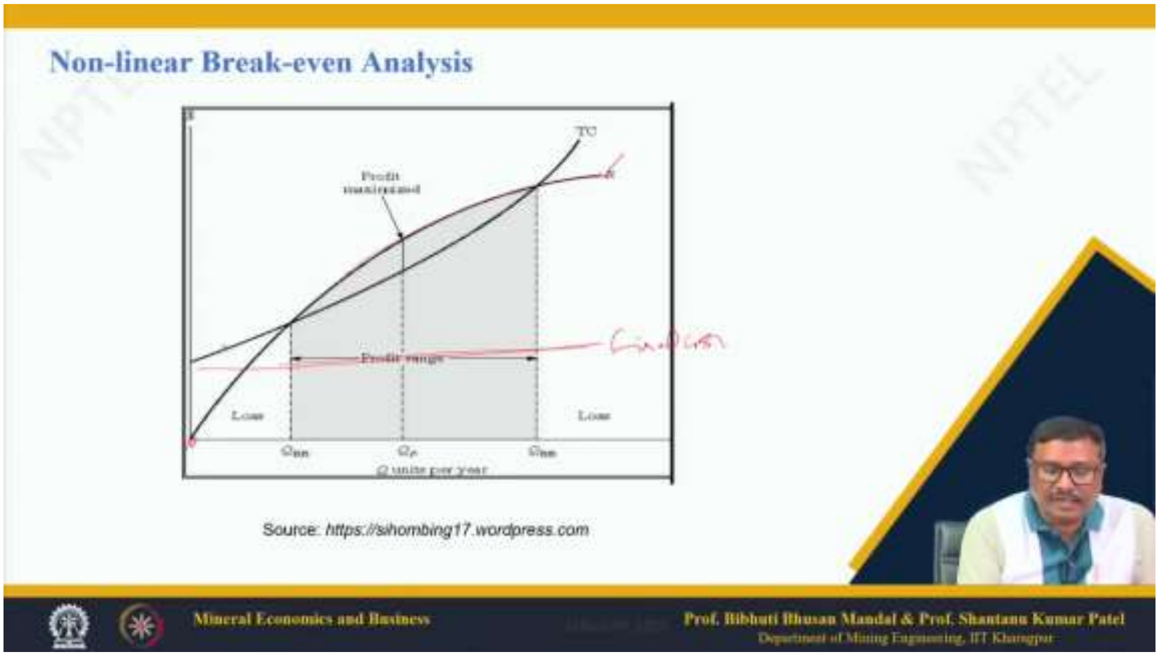


That means, at what rate it is changing revenue per unit output. So, that can be found out by differentiating. that we can write here the revenue ah with respect to ah the number of units produced dr dn equal to dns dn. The revenue is the revenue is the number of units produced multiplied by the price selling price here. So, we can find out by dns by dn.

Now here the selling price the that we have already shown on the at the top that now this is $S = 5000 \times (n^{0.5})$, So, if you take the differentiation you get:

$$\frac{dr}{dn} = \frac{d(ns)}{dn} = \frac{d(5000 \times (n^{0.5}))}{dn} = 2500 \times n^{-0.5}$$

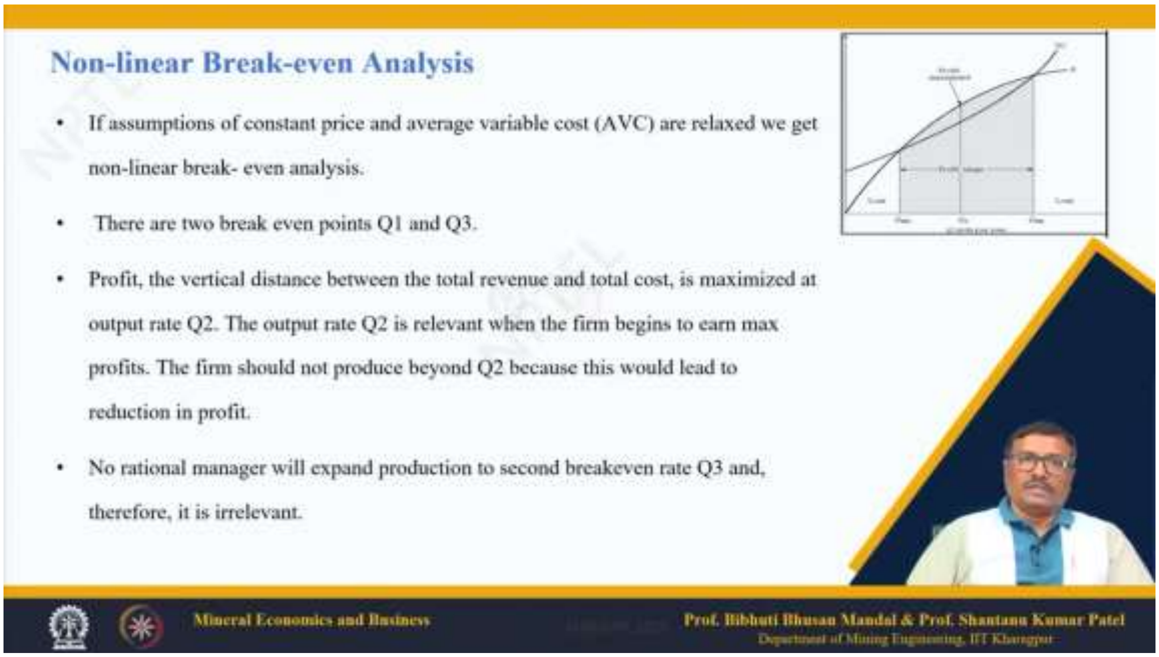
So, what happens that the in the non-linear breakeven analysis, what we see that the essential components are there, but their behaviour is changed.



This part is the revenue starts from 0 and starts from 0 and then it goes increasing and then since it is a non-linear then it takes for example, this. This is not exactly the graph of the functions that we have shown in the previous slide, but this is only for illustration purpose illustration purpose. And the total cost for example, if this is your the fixed cost line, this shows you the fixed cost and then it will be our total cost or Tc, total cost which is fixed cost plus the variable cost. Look at this, this is also not linear here.

So, in a non-linear function, what is happening is the revenue function, the revenue line, and the graph that shows the line representing the total cost are not straight lines. Here, it is intersecting at two different points. So, here, for example, this is quantity 1, this is quantity 2, and this is quantity 3. Quantity 2 is very important, and I will explain it. At two places, at two breakeven points, that means for quantities q1 units and q3 units, we reach the breakeven points at two places.

So, now, if we go by the values, what happens if we reach the breakeven point here? Then, to reach the breakeven point, nobody will go here—not at all. Why should I produce more to reach the same breakeven point? There is no point. Where the total cost, as we learned in the previous class, equals the total earnings. So, here we reach the breakeven points here and also here. So, we will prefer this one. The question is, when we are crossing this, the profits start increasing, but if you look at this again, it falls to zero.

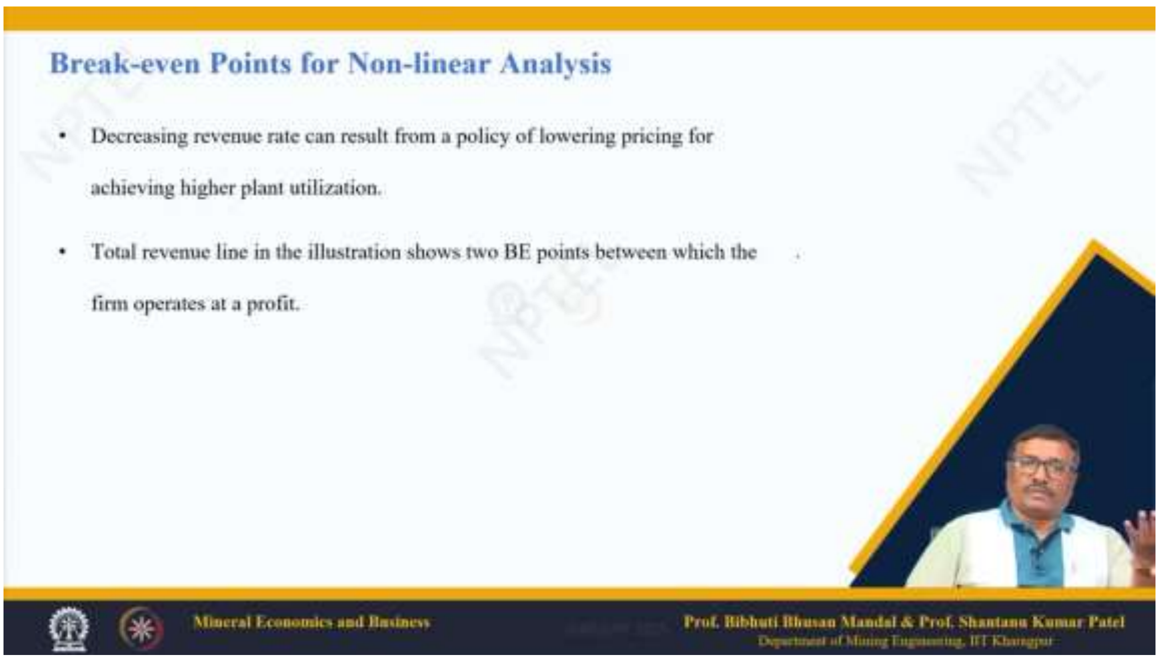


At the breakeven point, the profit equation means the profit becomes zero. So, here the profit is equal to zero, and here also, the profit is equal to zero. In between, there is a point where we have the maximum profit. And that we have to determine—between these two breakeven points, where is that point Q2, the number of units where the profit is maximized? That means it is increasing in this direction, slowly rising, and here it is maximizing. Then, it starts falling again. So, here, what we do is again reach the breakeven point. Between two breakeven points, we have the point of maximum.

Let us now again look at this curve. So, if the assumptions of the constant price and average variable cost are relaxed, that means they are not linear at all; they are not constants. Then we get the non-linear breakeven analysis. And the common example or illustration is like this. There are 2 breakeven points, as I was telling, at Q1 and Q3 here, Q3 here at 2 points.

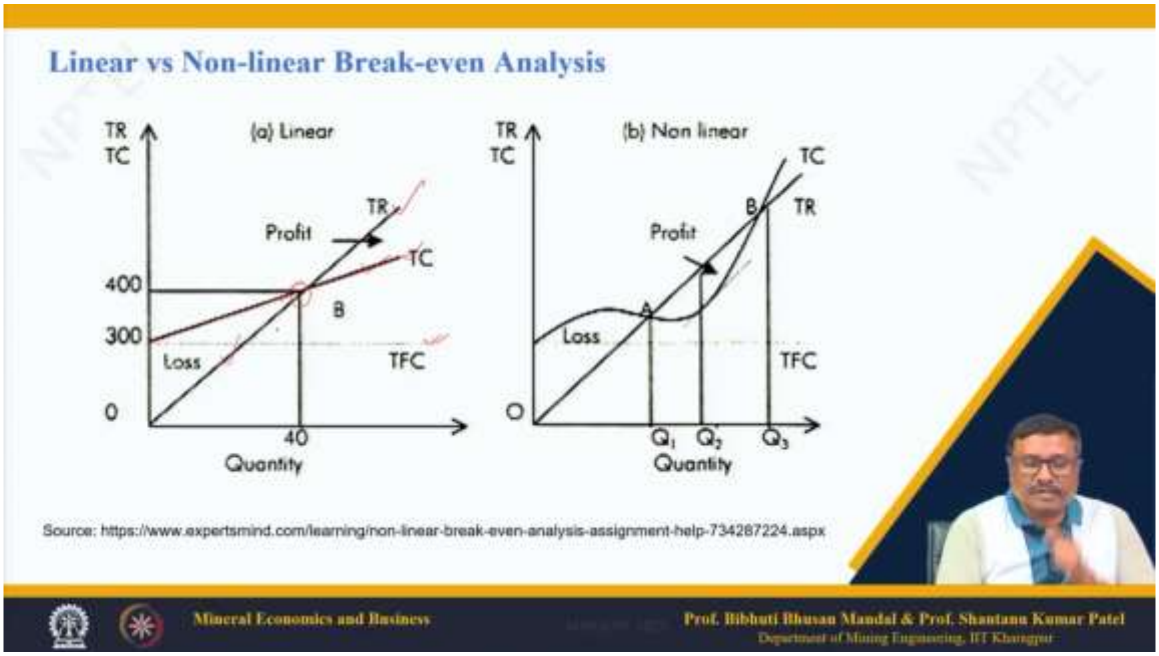
And profit, the vertical distance between the total revenue and the total cost at any point, for example, this is the profit beyond the breakeven point. This profit is the vertical distance between the revenue and the total cost. This is the revenue, and this is the total cost. How big the difference is shows you the profit.

Now, the firm should not produce beyond Q2 because this would lead to a reduction in profit again. Why? Because, as I was telling earlier, from Q1 onwards the profit is increasing, and at this point, it is maximized and again it falls. So, here we have the point of maximum profit, Q2, maximum profit. So, now the firm, that is why, should not produce beyond Q2 if this analysis, as this demonstrates or illustrates, that we should not go beyond Q2 because this would lead to a reduction in profit, further reduction in profit till it is again 0 and the so-called breakeven point that is absolutely irrational.



So, no rational manager again We will expand production to the second breakeven rate Q 3 and therefore, it is irrelevant at all the Q 3 is not relevant for us. First thing that breakeven point acceptable is the Q 1 and the point Q 2 units number of units where we reach the maximum profit. So, the decreasing revenue rate can result from a policy of lowering the price for achieving higher plant utilization. You are improving and increasing the production and then reducing the price which is possible, which is feasible also.

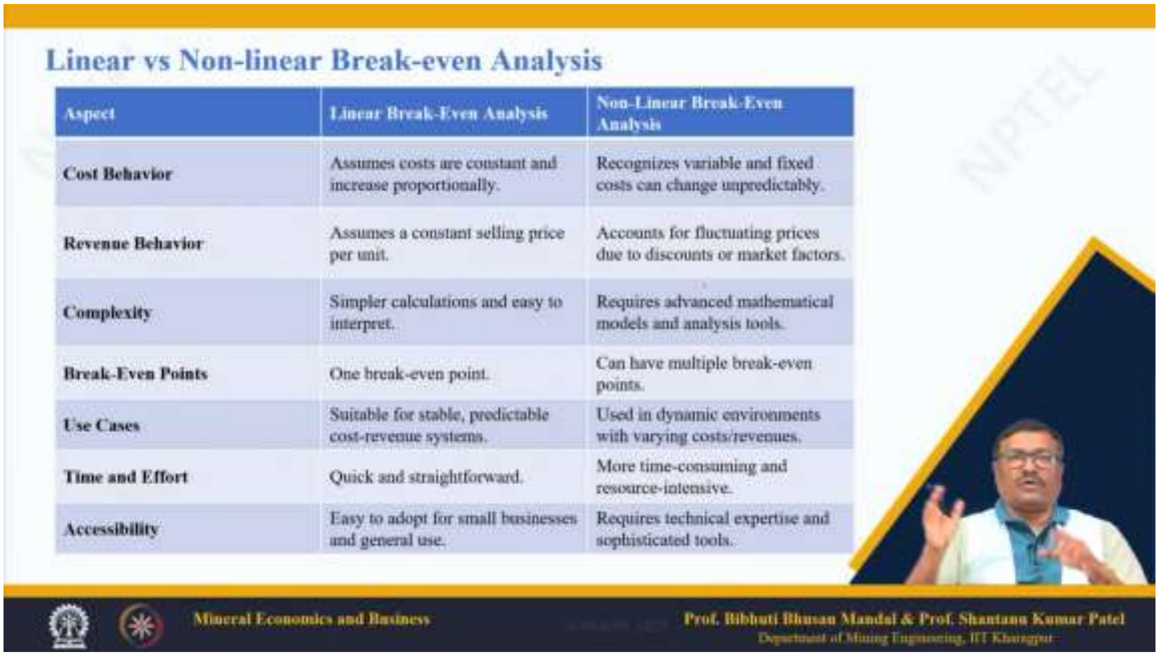
Now the total revenue line in the illustration shows that there are two breakeven points between which the firm operates at a profit. The profit point is between these two breakeven points. So, in case of linear   is very simple. This is the total revenue and this is the total cost. This is the fixed cost and this is the total cost.



This is for illustration purpose only. So, the total revenue and the total cost is crossing at a point where so you find out the break even point. So, anything above this you are making profits anything less than this you make a loss that is as simple as that, but here what is happen the T c the total cost and the total revenue For example, here we have this this one is the total revenue which is straight cut here this is not not linear, but there is this part is non-linear. There can be another option which we have we have shown earlier that both the things can be non-linear.

Here we are showing one linear and another non-linear in this case also. There are two points which we can consider as breakeven points. So, here we see that at Q1 and Q3 we have breakeven, and at Q2, the distance between the total cost and the total revenue. This is the total cost, and this is the total revenue. So, this is maximum.

So, this is the maximum profit. This is the point of maximum profit, that is the difference between the linear break-even analysis and the non-linear break-even analysis. We will just go through this once again quickly, that the cost behavior will be compared between the linear and the non-linear breakeven analysis. We can say that there are certain aspects where these things differ from each other.



Aspect	Linear Break-Even Analysis	Non-Linear Break-Even Analysis
Cost Behavior	Assumes costs are constant and increase proportionally.	Recognizes variable and fixed costs can change unpredictably.
Revenue Behavior	Assumes a constant selling price per unit.	Accounts for fluctuating prices due to discounts or market factors.
Complexity	Simpler calculations and easy to interpret.	Requires advanced mathematical models and analysis tools.
Break-Even Points	One break-even point.	Can have multiple break-even points.
Use Cases	Suitable for stable, predictable cost-revenue systems.	Used in dynamic environments with varying costs/revenues.
Time and Effort	Quick and straightforward.	More time-consuming and resource-intensive.
Accessibility	Easy to adopt for small businesses and general use.	Requires technical expertise and sophisticated tools.

Let us see the cost behavior aspect. The linear breakeven assumes that the costs are constant and increase proportionately, whatever may be the figure. But the non-linear breakeven analysis recognizes that the variable and fixed costs can also change unpredictably, depending on the volume of production. And the revenue behavior assumes a constant selling price per unit. For the linear, but here we can play with the selling price depending on the discount that we can offer when somebody purchases a large quantity, or there are other market factors that influence the price fixation.

About the complexity in calculation, it is very simple to calculate and easy to interpret. But in the nonlinear case, it requires advanced mathematical models and analysis tools. We have shown something for illustration purposes. But when you, for example, in the case of a mine, there are so many parameters influencing the price and the variable cost. And also, in certain cases, the fixed cost makes the model very complex.

In that case, you need a good computational model to find out the breakeven point or the point of maximum profit. As far as the breakeven point (BEP) is concerned, we have one breakeven for the linear case, and there can be multiple breakeven points. We have to find out the relevant and practical breakeven point for the non-linear analysis. We use it for stable, predictable, and cost-revenue systems where cost and revenue are predictable. This is for linear and small-scale operations, but for large and dynamic environments with varying costs and revenue, we should use non-linear break-even analysis. Time and effect are quick and straightforward for linear analysis, but non-linear break-even analysis is more time-consuming and resource-intensive.

This is easy to adopt in terms of accessibility—meaning how quickly you can adopt it. It can be easy to adopt for small businesses where you can find the number of units needed to reach the breakeven point, especially when the number of variables is small and well-defined in small-scale businesses. But here, we require technical expertise to understand all the costing processes, summarize them in the form of revenue and cost functions, and only then can we perform a breakeven analysis. However, the number of variables will be large and non-linear. So, this is definitely more complicated compared to linear breakeven analysis. We will get one more example.

So, here n = Number of units produced and sold

V = Rs 250 per unit

F = Rs 23000

s = Rs $5000n^{-1/2}$ per unit

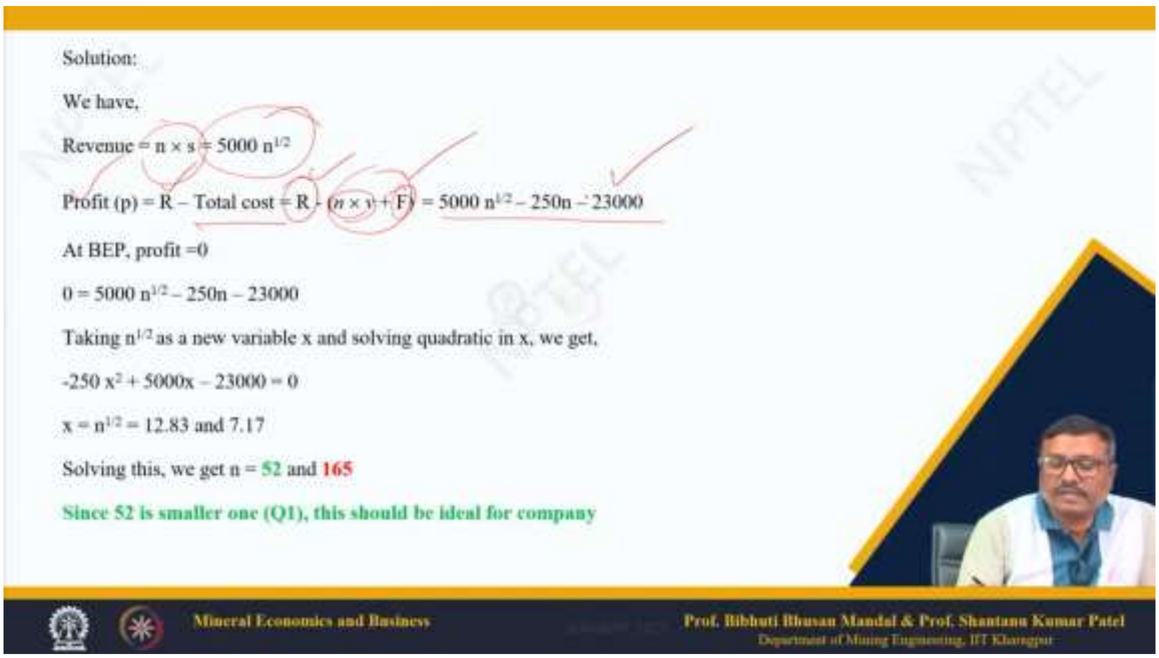
So, find n for breakeven ah points that is number of units to reach BEP and also find which of the breakeven point points should be ideal for the company which of the which one of the breakeven, but because since this is non-linear. So, there will be multiple BEP which should be relevant ah   for for the company. Now we have the revenue function as:

Revenue = n × s = 5000 $n^{1/2}$

So, the profit equation now:

Profit (p) = R – Total cost = R - ($n \times v$ + F) = 5000 $n^{1/2}$ – 250n – 23000

The total cost is the total variable cost plus fixed cost.



So, at BEP the profit equation now we can rewrite the profit equal to 0, at profit equal to 0 we find out the   break even point.

So, this this whole thing becomes 0 at break even Now we just for simplification what we do that we take x equal to root n to make it simple. So, we rewrite the equation and we find that:

0 = 5000 $n^{1/2}$ – 250n – 23000

Taking $n^{1/2}$ as a new variable x and solving quadratic in x, we get,

-250 x^2 + 5000x – 23000 = 0

$x = 12.83$ and $7.17 = n^{1/2}$

Solving this, we get n = **52** and **165**

Since 52 is smaller one (Q1), this should be ideal for company

Q1 be ideal for the company, there is no point in going to Q3 to reach the BEP, not at all. The relevance of Q1 and Q3 lies in the determination of the maximum profit, because at both these points—Q1 and Q3—we reach the profit equal to 0, which means the BEP is reached at 2 different points: 52 and 165. The company will go for 52 units here.

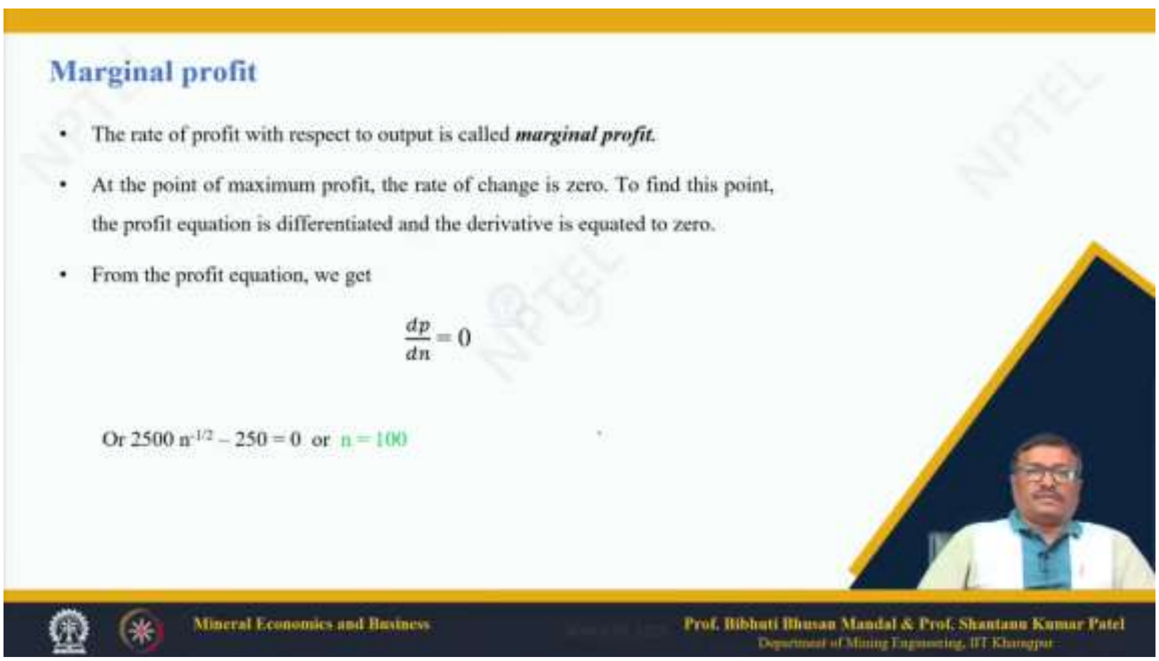
So, what is the limitation of the non-linear break-even analysis? The complexity in the cost and revenue behavior; it assumes that the cost and revenue are non-linear, leading to more intricate calculations, but it is practical. Variable costs may rise disproportionately due to inefficiencies or declining productivity at higher production levels. This may not be fully predictable.

That means this may not be fully converted into mathematical equations, which is true. But with more machine learning or other processes where the input parameters are known and the outputs are also known, then we can develop models where we can develop equations involving all the parameters influencing our cost and revenue both. Yes, there are multiple break-even points—that is true in this non-linear case. So, look at the equation: you have multiple break-even points. So, there could be confusion, but if you are looking rationally at the values you get, then you can find out the right break-even point. So, fluctuating marginal values—marginal cost and revenues—are not constant.

So, making it harder to predict is fine, and the impact of production changes on profitability. So, there will be complexity, but it is a part of life. Having said all that, now let us talk about some key concepts like marginal profit. The marginal profit is something which can be defined as the rate of profit or the rate of change of profit with respect to output. The rate of profit with respect to output is called the marginal profit.

At the point of maximum profit, the rate of change is 0, which means it is at maximum profit. The rate of change is 0, and we can show it like this. So, the rate of change is 0. To find this point, the profit equation is differentiated, and the derivative is equated to 0, as we have seen from the earlier illustrations. So, from the profit equation, we get that:

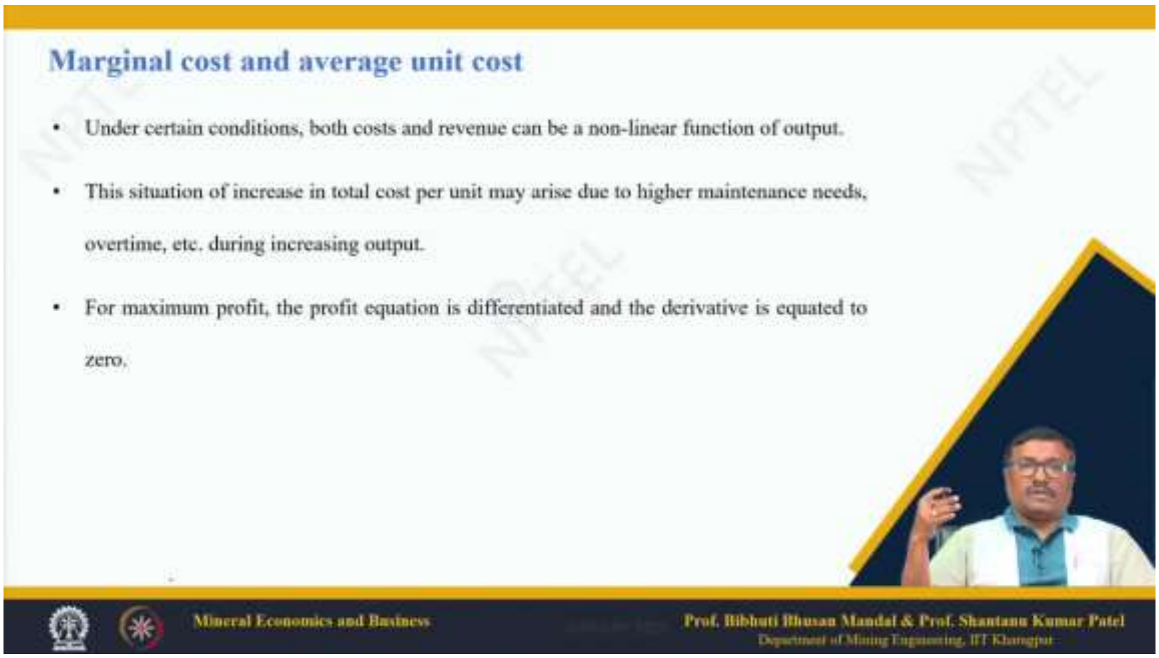
$$\frac{dp}{dn} = 0$$



Or $2500\, n^{-1/2} - 250 = 0$ or $n = 100$

So, this N equals 100 is the point where we are making the maximum profit. It is not higher than the lower breakeven and not lower than the higher breakeven. It lies somewhere in between where the curve is changing, the slope is changing, and that point is N equal to 100. That means, at quantity N equal to 100, we make the maximum profit.

The marginal profit is therefore, the rate of profit with respect to the output at a particular point at a particular point. Under certain conditions both the cost and revenue can be non-linear function of output that is definitely true. This situation of increase in the total cost per unit this may arise due to higher maintenance needs over time. That means, when you are increasing the number of units then you may incur more maintenance because the machines are running for more hours and we need overtime to be given to the workers who may be working beyond their usual shift times.



So, this is due to our plan to increase our output. So, for maximum in a profit the profit equation is differentiated and the derivative is equal to is equated to 0 this is how we find. We consider all these points we include in the equation and the profit equation is once again differentiated and the derivative we get is equated to 0 and from there we find out the number of units as we have seen in the previous slide. So, the marginal cost and average unit cost, this marginal cost and average unit cost as that the:

$$\frac{dp}{dn} = \frac{d(ns - nv - F)}{dn} = 0$$

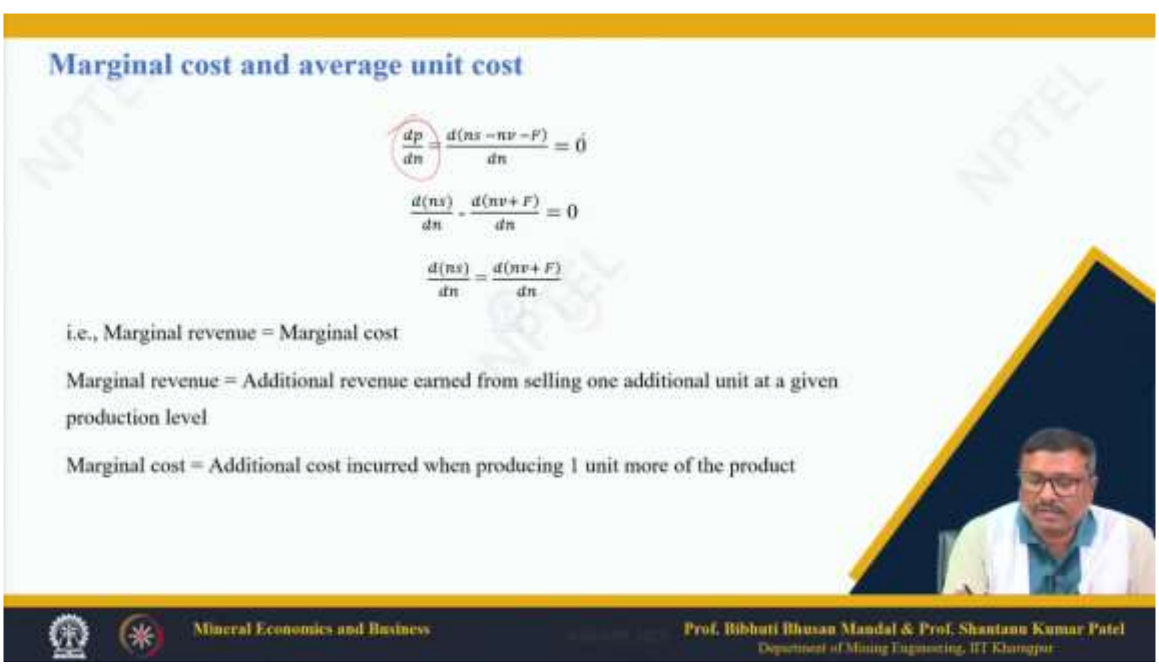
$$\frac{d(ns)}{dn} = \frac{d(nv + F)}{dn} = 0$$

$$\frac{d(ns)}{dn} = \frac{d(nv + F)}{dn}$$

Basically what is what we are telling that this is the marginal revenue and the marginal cost at a particular given point at a particular given production level when we try to understand the revenue earned from selling one additional unit that is given by:

$$\frac{d(ns)}{dn}$$

The rate the rate of change that means how much you are getting from selling one additional unit where may be the market condition and our decision to change the price for different reasons has changed and it has become non-linear but at a particular point at that point if I if I sell one more unit then what is the change in revenue at the same time at that point at that point depending on the situation what is the change per unit of production that is the change in cost per unit of production at a given at a given point. This is the marginal cost side and this is the marginal revenue So, the marginal revenue here equal to marginal cost in this in this in this case then at the point when we make the maximum profit.



So, once I just try to repeat this: the marginal revenue concept is that it is the additional revenue you earn from selling one additional unit at a given production level. And marginal cost is the additional cost incurred when producing one more unit of the product at that production level because it is non-linear. So, it does not hold true that it will be the same for all production levels; then it becomes a linear function. Can we summarize like this? The non-linear breakeven analysis extends the traditional models by accounting for the complexities of real-world cost and revenue behaviors, as we have seen and also illustrated through an example.

So, why is it costly? Because of the fluctuating variable costs and selling prices depending on the number of units produced—say, consumables, different raw material inputs—which are not necessarily linearly or directly proportional to the number of units. It introduces multiple breakeven points and emphasizes the importance of marginal profit. That means additional profit per unit and marginal cost, which means additional cost per unit at a particular level. Profit maximization occurs when marginal cost equals marginal revenue. This we have explained in one of the slides.

When the profit is equated to 0, then if we see that the marginal cost equals the marginal revenue at a particular production level. Remember that this is not linear. So, this will never remain constant. So, what happens is that at a particular level, at a particular level—say, when we are talking about this point—then the change in cost for producing one additional unit is the marginal cost, and the earning by selling one additional unit in that particular situation is the marginal revenue.

When marginal cost and marginal revenue are equal, at the maximum profit situation, marginal cost equals marginal revenue. So, how does it help? Because this offers insights into optimizing the production level. That means, when the parameters influencing the cost and the parameters influencing our price-fixing criteria are included as much as possible into the equations we form, we solve them to find the maximum profit. This gives us the optimized production level that maximizes profit. Even though it is more precise, this approach demands advanced tools and expertise, making it ideal for dynamic and complex operational environments. So, when things are very complex, the relations are not linear and are dynamic, changing with the number of units we produce and sell in the market.

So, this is more precise. It is complicated, but it gives you a more realistic analysis. So far, what we have learned in this lecture is how non-linear break-even analysis differs from linear analysis and why it is different. What causes it to differ from linear analysis? And is it more realistic? The answer is yes. The answer is yes. We are adopting a more realistic and by moving toward more realistic models, we are making it more complicated. That is true, definitely true.

And even though it is complicated, it requires more advanced mathematical skills to solve the equations and find realistic figures for achieving profit. or the break-even point, but it is worth it because it is more realistic in a dynamic operational environment. These are the references you have: Financial Management by P.C. Chandra, which I repeatedly use, and many websites, including this one, which will provide further studies for better understanding of the subject. Thank you very much.