MINERAL ECONOMICS AND BUSINESS

Prof. Bibhuti Bhusan Mandal

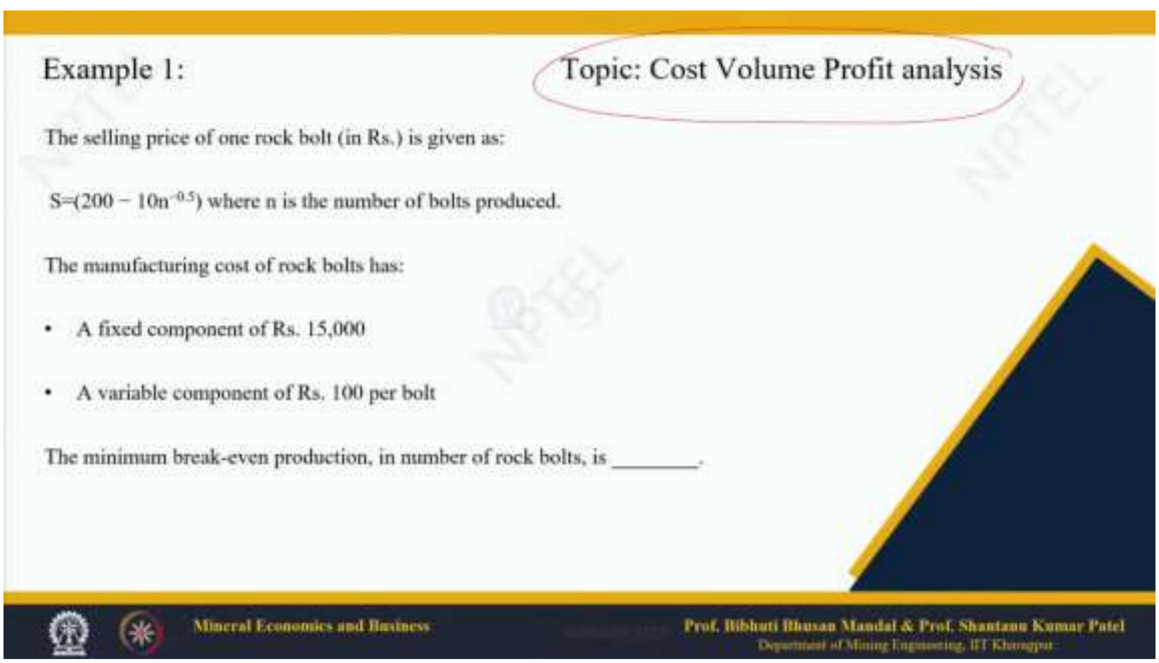
Department of Mining Engineering

IIT Kharagpur

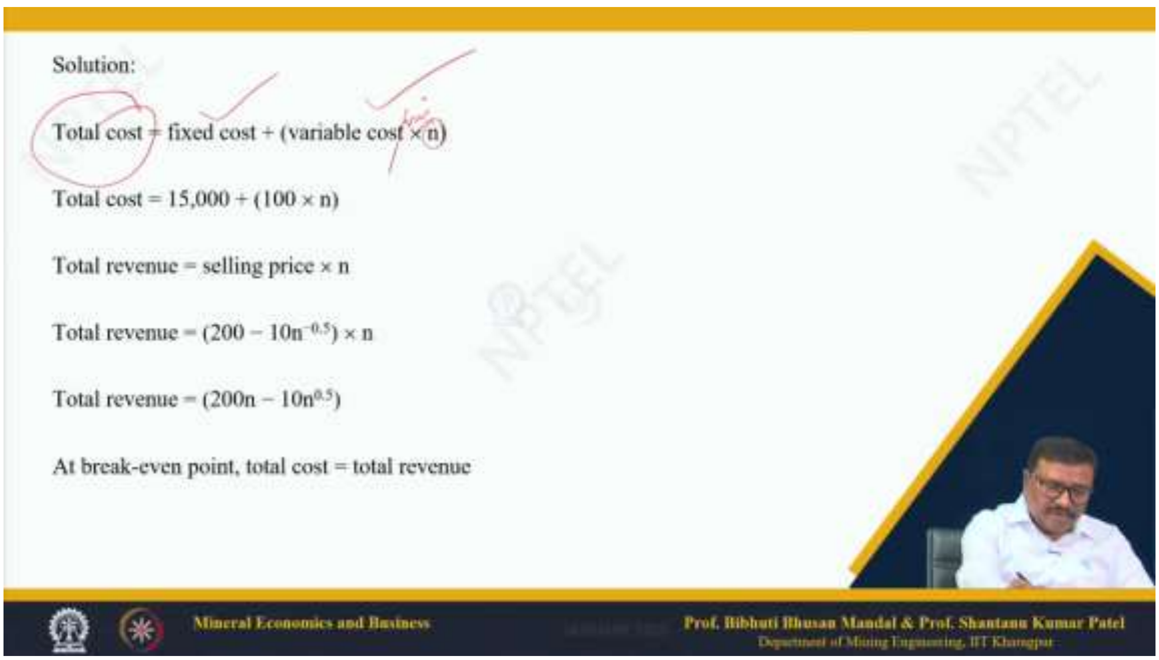
Week 10

Lecture 47 : Problems and Solutions - V

Hello everybody, today in this particular lecture, I will be having one more set of problems and solutions. Previously, we had 4 sets; now this is the 5th set, and I am repeatedly doing these things before I switch over to the tax and royalties part. And also, to the many topics related to policy matters and to conclude the entire course. Before going into that, I just want to again go through certain important problems and solutions because the students are very much interested in the numerical problems and the techniques of how they solve them—that is very, very important. Many of the students are appearing in the GATE exam.



So, I have also included many questions from their previous GATE papers, and I provided certain solutions, which I will discuss when I present them. Let us go ahead now. Ah, in the first problem, which is another example from the topic of cost-volume and profit analysis, especially when the sales or the variable component are dependent on the volume itself. That means the number of units that you produce affects this cost; they are not directly proportional, as I was also giving in one of the examples in my problems and solutions series 4. Here, we are taking a simple case where we say that the selling price of 1 rock bolt is given by $S = (200 - 10n^{-0.5})$. So now, you can see that if this is the selling price, then it is dependent—this is a function of n itself, n itself. That means the number of bolts produced, if it is n, then the selling price is a function of that. The manufacturing cost—the other cost components are the fixed component—we have, for example, a small area or small factory, a small shop where we are producing this according to specifications.



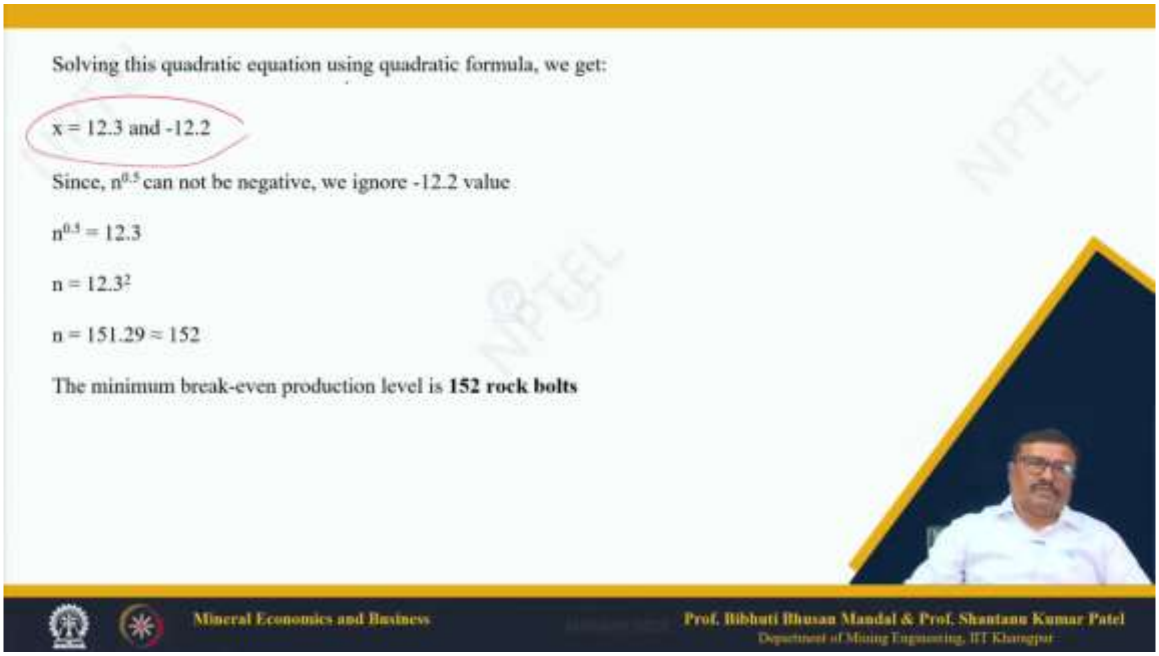
Here we have a fixed component of say rupees 15000 fixed cost you can say fixed cost component. and a variable component is rupees 100 per volt. For simplification I mean I have taken round up figures, so that it becomes easier for you to understand. So, now the question was the minimum break even production minimum that means, on the lower side. It can be multiple because it is not linear you will have 2, 3 may be anything depending on the relationship that we have here.

So, here we are expecting at least 2 results. The number of rock bolts which will give you the minimum break even that we are going to discuss today in the solution part. We know that the total cost equals to the fixed cost plus the variable cost into the number of units produced. This is variable cost per unit into the number of unit produced. So, the total cost is 15000 + 100 × N.

this is a total cost. But here the total revenue is the selling price into n, selling price per unit, selling price per unit into n. Now the selling price is a function of n, so we now put that relation here, Total revenue = $(200 - 10n^{-0.5}) \times n$, which is the number of rock boards produced. So, now we simplify these things it Total revenue = $(200n - 10n^{0.5})$. So, it becomes a little simpler 200 n minus 10 into root n that is all. as that at breakeven point again, total cost equals to total revenue, they are equal so we reach the breakeven point where this is satisfying the relation.

So, I write that part that $15{,}000 + 100n = 200n - 10n^{0.5}$. We simplify this and get $15{,}000 + 10n^{0.5} = 100n$. Further simplification gives us this equation. Now, here what we do is, for further simplification, we put $x = n^{0.5}$. If that is true, if x equals to root n, then n is definitely x squared. So, here we substitute x equal to root over n and get $100x^2 - 10x - 15000$. There you get a simplified form: $100x^2 - 10x - 15000 = 0$. So, we are simply dividing it by 10.

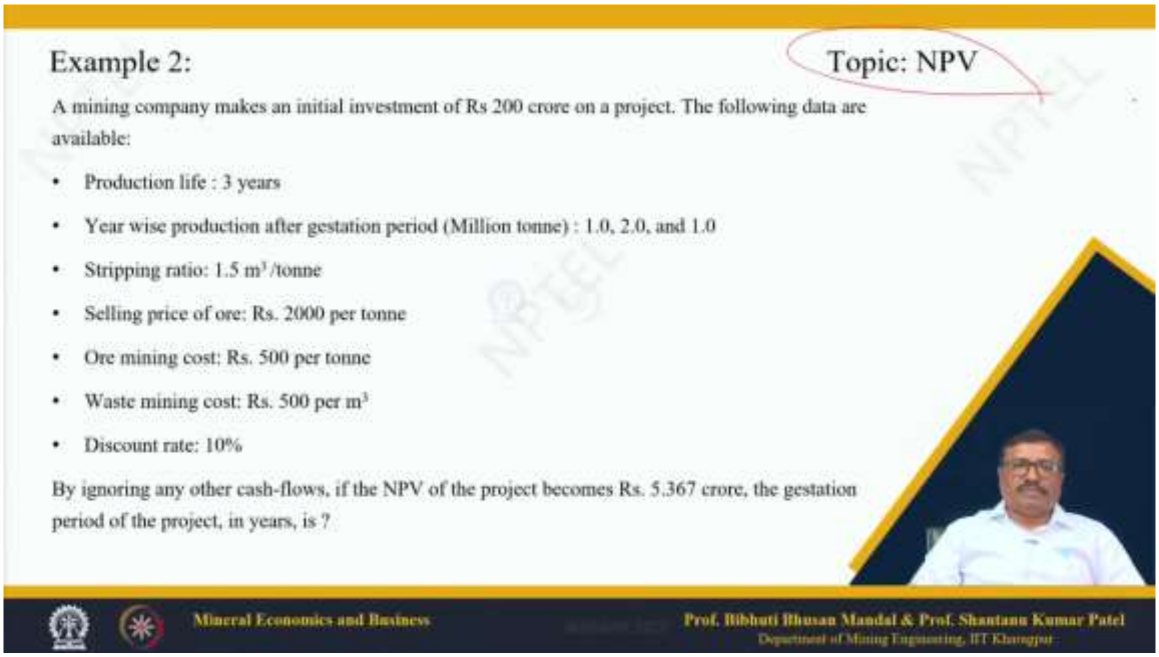
So, we get $10x^2 - x - 1500 = 0$. This is a simple quadratic equation that we try to solve. So, by solving this quadratic equation using the standard formula, the procedure, we get this, which I am not showing. You can do it yourself; it is nothing for you. So, x equals to 12.3 and then minus 12.2. So, now if n is a positive number, if n is a positive number, we have said that x equals to root over n. If n is a positive number, in that case, the root over n cannot be negative.



So, we ignore this negative value which has come out from the solution. So, we take the positive value, which is 12.3. Now, this 12.3 is root over n. So, x squared is your 152. n squared is n squared is your 12.3, n raised to the power 0.5 is 12.3. So, n is the square of 12.3, which we ultimately get as 152. 152 rounded up because 151.29 number of rock bolts.

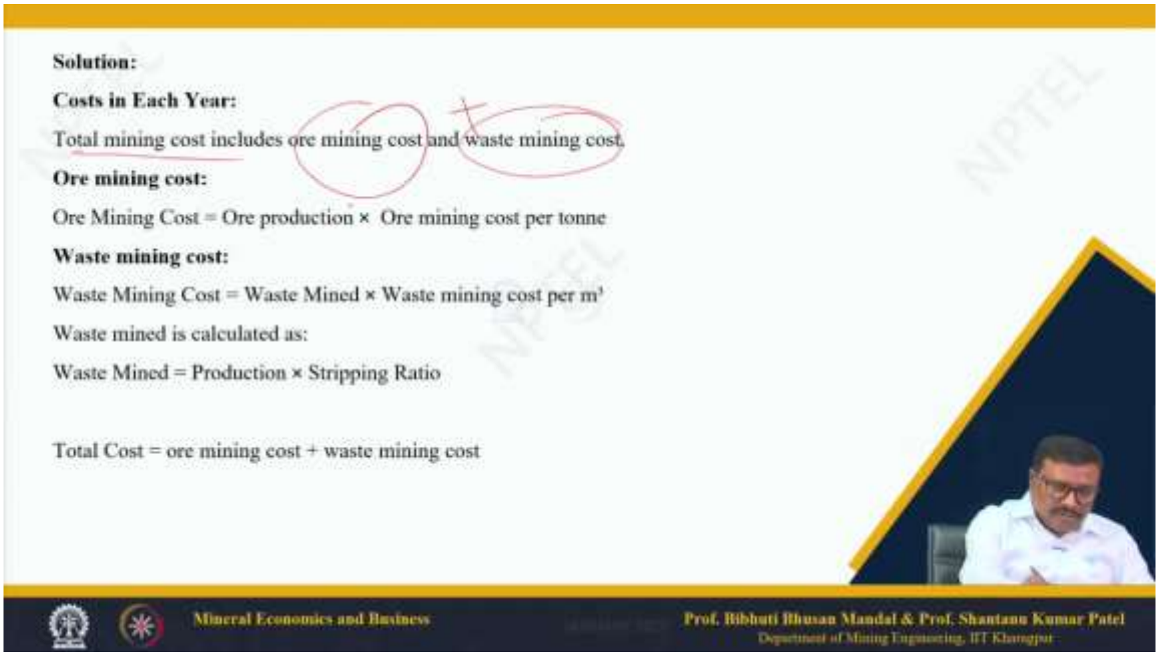
So, we have rounded up to the next whole number that is 152. So, the minimum breakeven point production level is 152 rock bolts that is the answer to this. We are not doing any other analysis further because the question was limited to the minimum breakeven point that is 152 rock bolts. We are going to a very popular concept in mineral economics or any project evaluation, the next example is from that topic the net present value. It is a combination of two concepts, but then ah I will just first read out and then you try to understand how you are going to solve this.

The mining company makes an initial investment of rupees 200 crore on a project ok. Now, the following data are available production life is 3 years and year wise production after gestation period is 1.0, 2.0 and 1.0. These are all in million tons, this is only to make you understand this is not a absolute real life problem, but it gives you enough scope to further understand the topic. And the stripping ratio is 1.5 meter cube per ton, selling price of ore is rupees 2000 per ton. ore mining cost when you are mining ore then you will be spending rupees 500 per ton and when you are mining waste then per meter cube you spend rupees 500.



We are considering a discount rate of 10 percent for the purpose of calculating the time value of money. So, we ah in the question they say that the by ignoring any other cash flow, if the NPV of the project becomes 5.367 crores. So, there is no other cash flow, the cash flow is already given. The initial investment part is there and then your 1.0, 2.0 and 1.0 these are the ah cash flows only the year wise production. In million terms that you have to convert into money that is different issue, but there are only two things one is that we have initial investment and other is that we are getting from sales revenue.

Other than this, in this problem, no other cash flow is considered. We have to find out the gestation period of the project in years. That means, how many years the initial



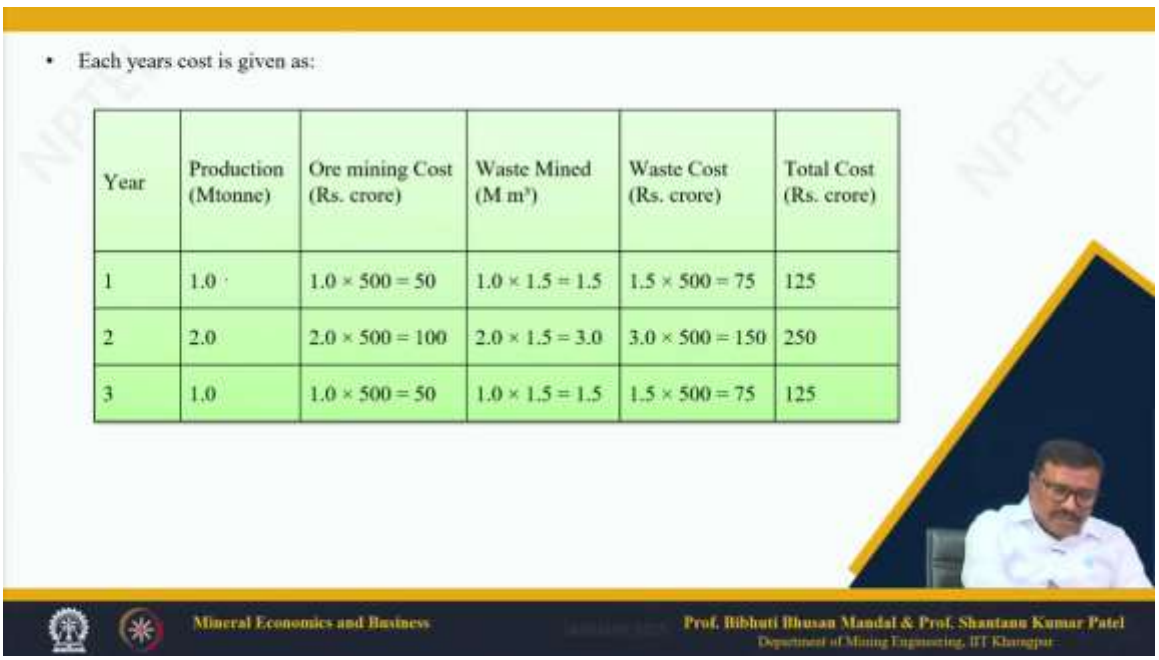
investment will be blocked, after which we will start getting the profit. That means, our gestation period—the compensation to the initial investment—is over. Whatever we are getting now will be the net cash flow, which will be profit. So, in years, I have to—we have to calculate.

Now, for calculating the cost in each year, the total mining cost—you see that—includes ore mining cost plus waste mining cost. So, ore mining cost is the Ore Mining Cost = Production × Ore Mining Cost per tonne. Similarly, the waste mining cost is the Waste Mined × Waste Mining Cost per m^3. Waste mining cost per meter cube. See, this ore production will be in tons, and the ore mining cost per ton will give us the total mining cost. Now, the ore waste mine is calculated as waste mined.

And then, production into stripping ratio, because this was the waste mining cost. So, how do I get waste—how much waste is to be mined? So, that depends on the stripping ratio. So, this production, say in tons, multiplied by the stripping ratio. So, the meter cube per ton.

So, total cost will be ore mining cost plus waste mining cost this will be our total cost. Let us see:

Year	Production (Mtonne)	Ore mining Cost (Rs. crore)	Waste Mined (M m³)	Waste Cost (Rs. crore)	Total Cost (Rs. crore)
1	1.0	1.0 × 500 = 50	1.0 × 1.5 = 1.5	1.5 × 500 = 75	125
2	2.0	2.0 × 500 = 100	2.0 × 1.5 = 3.0	3.0 × 500 = 150	250
3	1.0	1.0 × 500 = 50	1.0 × 1.5 = 1.5	1.5 × 500 = 75	125



So, for 2 we will get 100 crores and for again we will get 50 crores. This is the total amount of mining mean ore mining cost. For waste mining we have million meter cubes. So, in that case we are using this values that I have shown earlier. We are actually 1.0 million ton multiplied by 1.5 meter cube per ton. So, that that means, we will we have to take 1.5 million meter cube of waste for generating 1 ton of ore. Similarly, 2.0 × 1.5 that means, we have to take out 3 million tons or 3 million meter cube of waste for getting 2.0 million ton of ore. So, like that we have made a schedule here.

So, now, we have the ore mining cost calculated here. For waste calculation, we know that it is 1.5 million cubic meters multiplied by 500. So, we get 75 crores as expenditure here: 150 in the second year and again 75 crores in the third year. Now, the total cost we have to add is ore mining cost plus waste mining cost, and we get 125, 250, and 125. These are the total costs that we are spending. Now, we will see how much we are earning and then we will do the calculation.

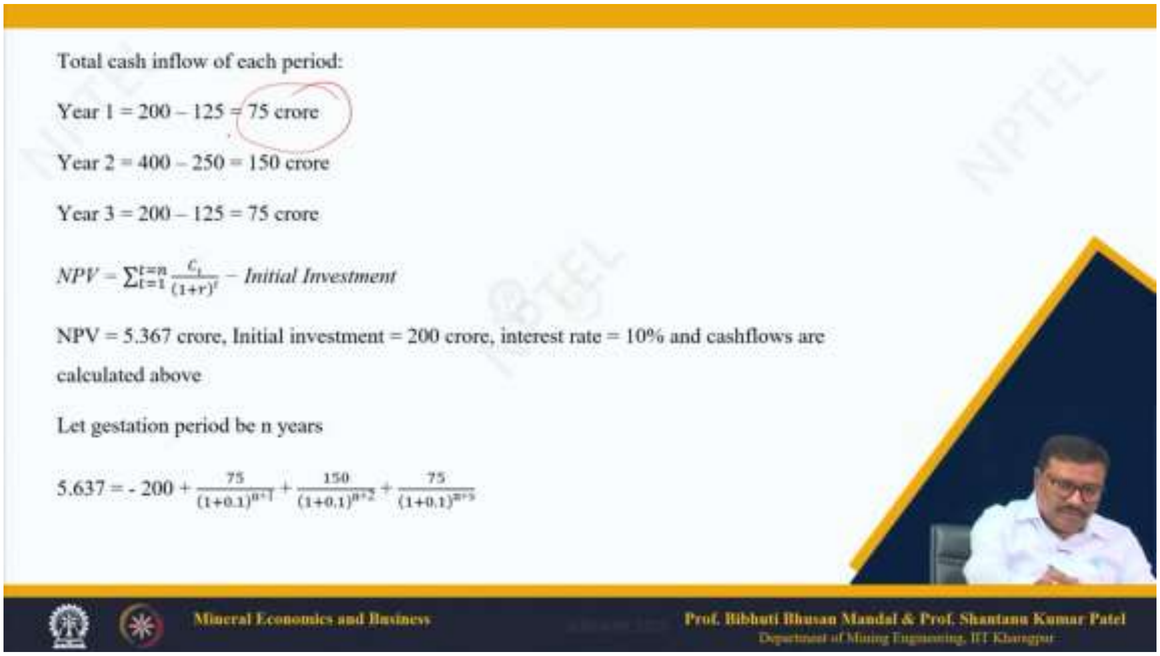
Year	Production (Million tonne)	Selling Price (Rs./tonne)	Revenue (Rs. crore)
1	1.0	2000	200 crore
2	2.0	2000	400 crore
3	1.0	2000	200 crore

What is the revenue that we are earning by doing this? We are having 1 million tons, 2 million tons, and 1 million tons in 3 different years. So, the selling price will be accordingly, rupees per ton is given here. 2000, 2000, 2000. So, we multiply and we get rupees 200 crores, 400 crores, and 200 crores.

Year	Production (Million tonne)	Selling Price (Rs./tonne)	Revenue (Rs. crore)
1	1.0	2000	200 crore
2	2.0	2000	400 crore
3	1.0	2000	200 crore

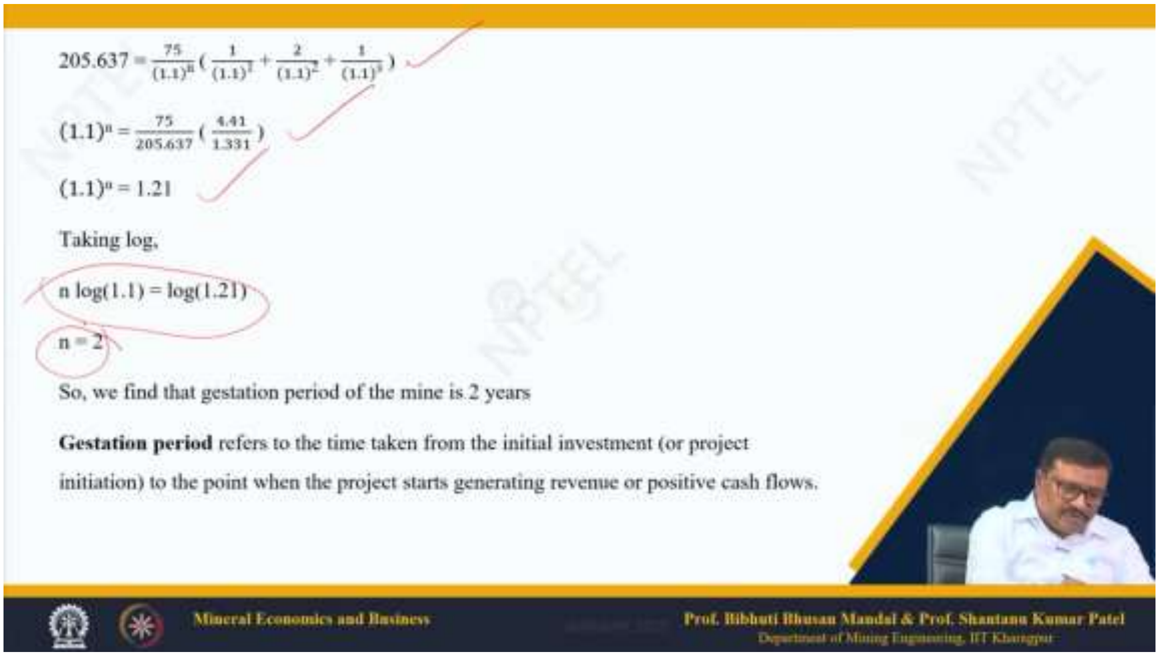
These are our sales revenues. We have also shown the total cost in the previous slide. Now, we do the usual calculation. What is this usual calculation? The total cash inflow for each period, for example, year 1, will be the cash inflow minus your cash outflow, and then you get the net cash flow of 75 crores, which is positive.

Again, in year 2, we have the cash inflow minus outflow, and we get 150 crores. of net cash flow. Again, in year 3, we take 200 minus 125 and then we get 75 crores. If you see the previous table, it is clear how much you are spending and how much you are earning. Now, NPV will be our total as the cash inflow, which is converted into present value, minus the initial investment that will give us the net present value.



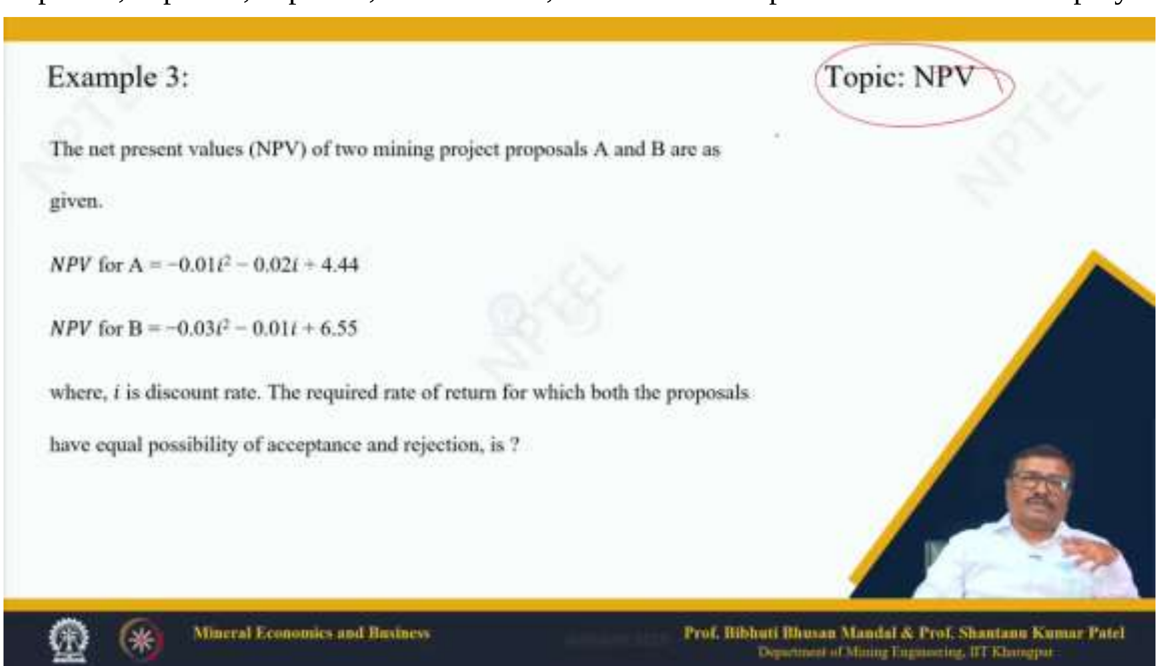
So, the present value of all the net cash flows minus the initial investment will give us the net present value. From here, you see that the NPV is already given as 5.367 crores. The initial investment is given as 200 crores, which we have to offset in the gestation period, meaning we have to earn 200 crores first and then start earning the profit. So, now the interest rate is also given as 10 percent, and cash flows are calculated above, which we have already seen. Now, what we are seeing is that this is the NPV, which is given. Then, this is the first-year cash flow, the initial investment at year 0, minus 200, and then we are finding out the present values of all future cash flows.

How are we doing this? After n years, we are trying to find out the year in which the 200 crores should be recovered. So, the recovery starts from n plus 1. So, this is $75/(1+0.1)^{n+1}$. Similarly, next is $150/(1+0.1)^{n+2}$, and then $75/(1+0.1)^{n+3}$, n denotes the number of years



for which we are trying to calculate the gestation years as n. So, this means these are the years required to offset the initial investment, and then we start making profits. So, this is how we have written this to solve for n. Now, we simplify step by step, and ultimately we take a log here on both sides and get n equals 2. So, we find that the gestation period for the mine is 2 years. This refers again to the time taken from the initial investment or project initiation to the point when the project starts generating revenue or positive cash flows.

So, that we have assumed to be n. So, the other years will be outside the gestation years: n plus 1, n plus 2, n plus 3, like that. So, this is how the problem is solved. I hope you

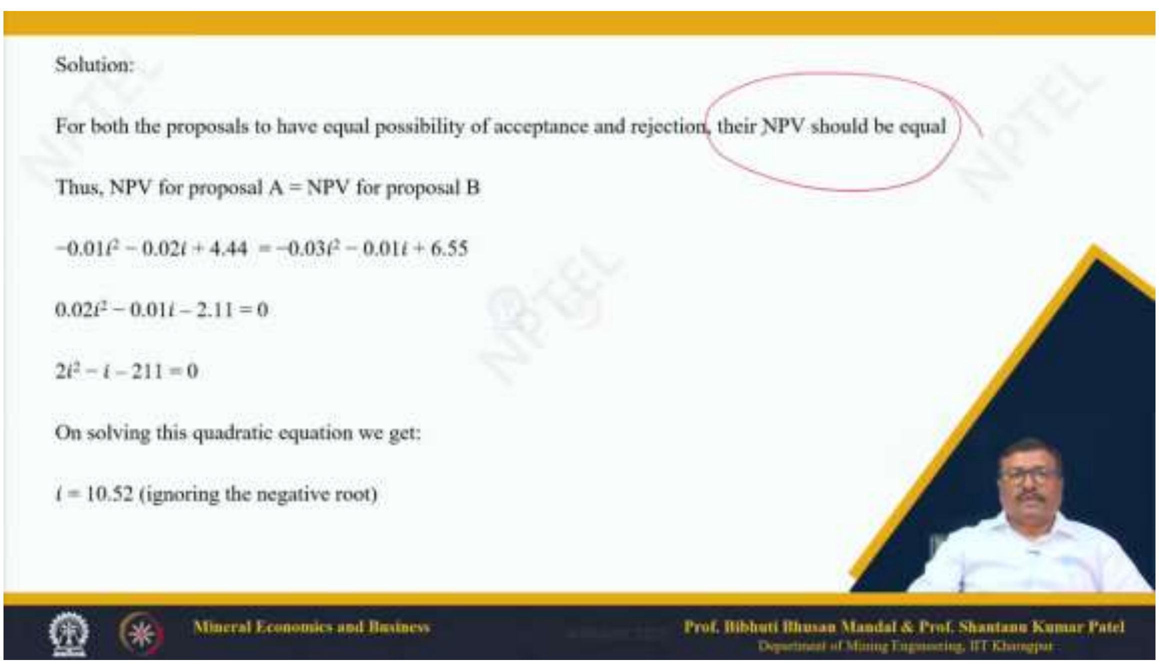


have enjoyed this particular problem. This was in GATE.

Now, next again we have a simple problem about the NPV, which is less about NPV but requires understanding of the concept itself. We are given the net present values of two mining project proposals, A and B. We have two proposals. So, we can draw their graphs also, but here everything is given. Without drawing any graph, you can easily solve this, like the NPV for A = $-0.01i^2 - 0.02i + 4.44$ and for B = $-0.03i^2 - 0.01i + 6.55$, where i is the discount rate not known to us. Now, the required rate of return for which

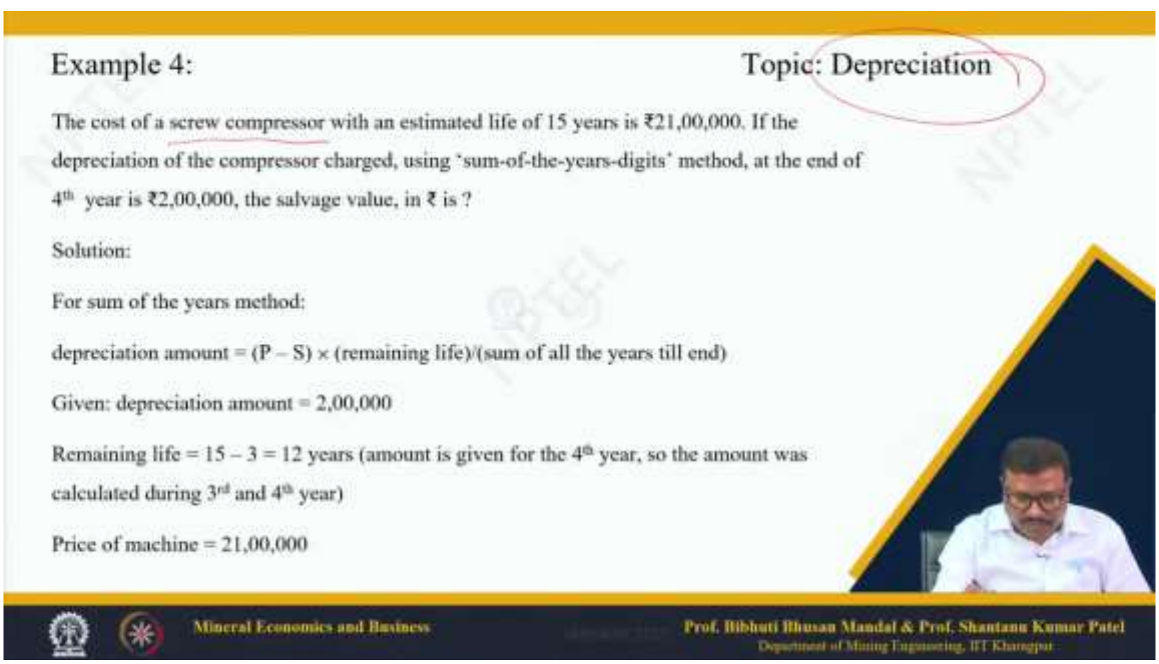
Both the proposals have an equal possibility of acceptance and rejection, which is what I mean. We have to find out the required rate of return when both the projects are equally acceptable or qualified/disqualified. We have to find that particular value. Now, for both proposals, we have an equal possibility of acceptance and rejection. So, then the NPV should be equal. We have separate expressions for the NPV for the two projects. So, now we are considering them as equal.

So, now the NPV of proposal A equals the NPV for proposal B. Writing both these expressions on both sides, we are equating both the expressions. We ultimately reduce to a simple quadratic equation: $2i^2 - i - 211 = 0$. You will get this equation when you ignore the negative root. You will get 10.52%. So, this is the required rate at which the NPVs will be equal. So, here



On the basis of this, if you calculate the NPV, then you will see whether it is acceptable or rejected. Both projects will give you the same NPV. That was a simple proposition in the question. If you understand the basic concepts of NPV, then you can easily solve this. Now, we are going to depreciation. The problem is that the cost of a screw compressor, with an estimated life of 15 years, is 21 lakhs. Now, if the depreciation of the compressor is charged using the sum-of-the-years'-digits method, then at the end of the fourth year, if it is calculated as 2 lakhs, then what is the salvage value?

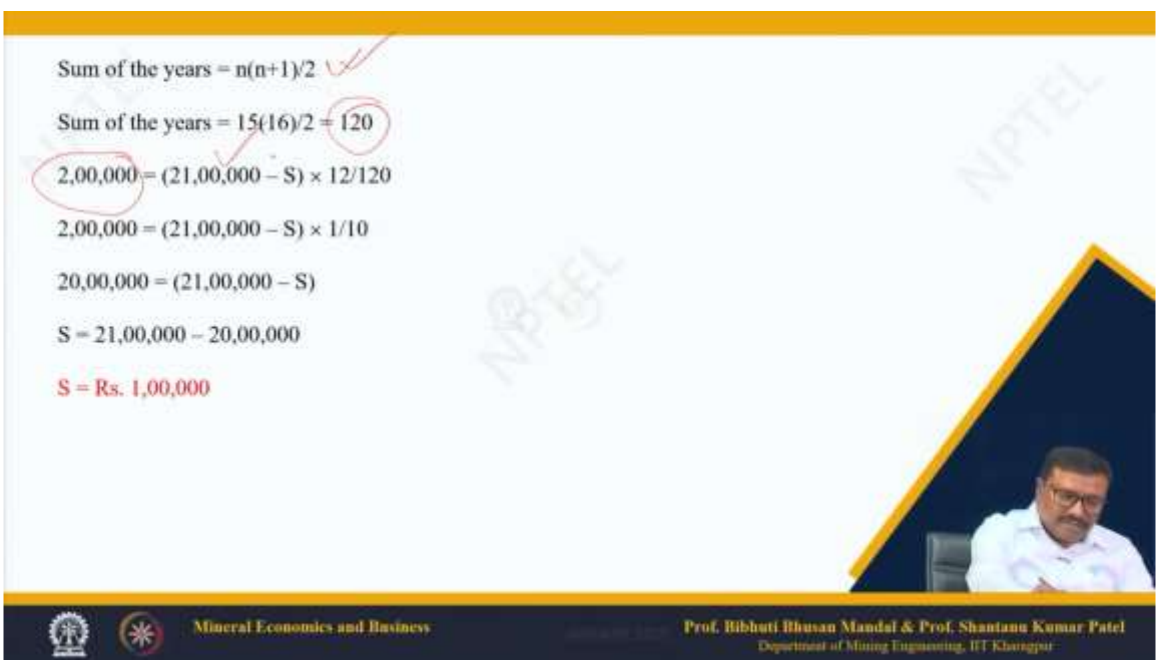
It is simply reorienting the problem. Sometimes we would ask for the depreciation chart and then find depreciation amount, then your book values. So, we are just reorienting and trying to see whether the student understands the whole concept or not. we know that the in the sum of the years method we consider the salvage value S as an essential component in the calculation. So, the P is the price which is 21 lakhs - S is the salvage value which we are supposed to calculate into the remaining life divided by sum of the digits means sum of all the years till end. Now, the depreciation amount is given 200,000 or 2 lakhs here. Now, the remaining life is 15 - 3 at the end of fourth year. So, it is remaining life is 12 years amount is given for the fourth year.



So, the amount was calculated during third and fourth year. So, the remaining life will be 12 years. So, now, the price of the machine is 21 lakhs this we know already. we use the formula standard formula sum of the years is n × (n + 1)/2. So, this comes to 120.

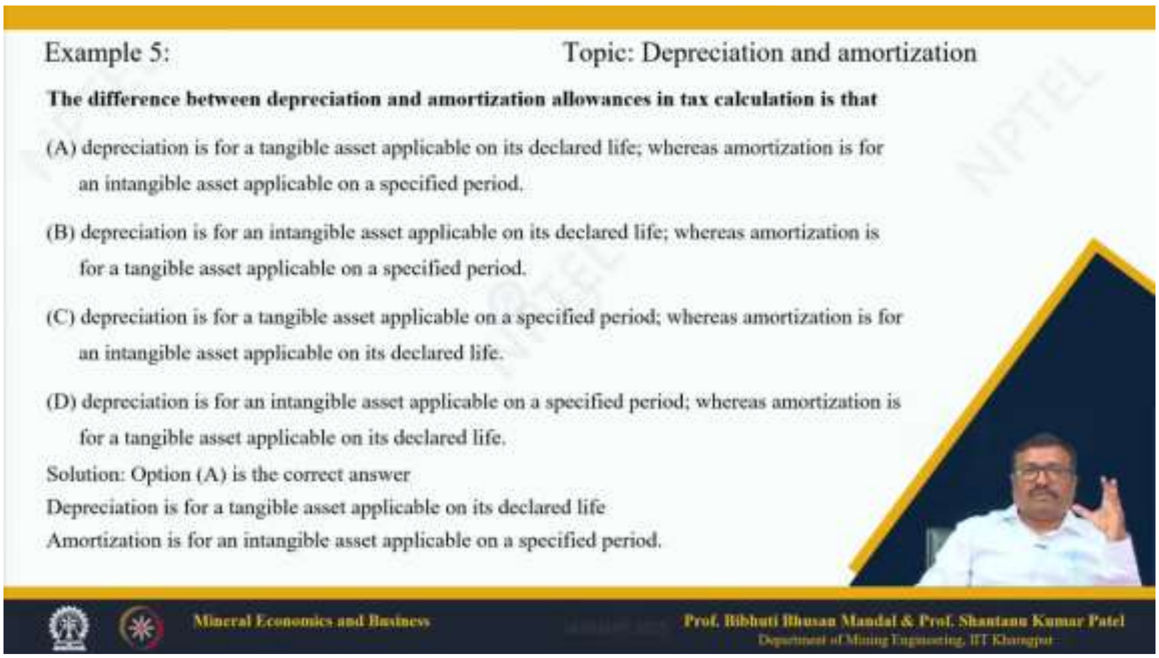
So, this 200,000 is your depreciation and it is equal to (P – S) into 12 divided by 120. This is the remaining life. So, now if you simplify this, solve for S, then you find out that the S, the salvage value, is 100,000 or 1 lakh. So, this is only a test of comprehension, whether you understand the concept or not. So, the basic question was about the sum of the years' digits method, and whether you understand the concepts of how to calculate and how to find out the salvage value by reorienting the problem, nothing else.

Ah, the topic depreciation and amortization, here we are just asking you a descriptive or, say, multiple choice. There are, there is, there will be a choice, ah, A, B, C, D, this kind of question where you have choices, you have to find out which is the correct answer. So, we had a numerical problem. So, now, we have, let us have a simple question where we have multiple choice or one could be the answer only here. So, depreciation is for it, we are, we are having statements about depreciation and amortization. And through that, we are trying to understand whether you have clear ideas about depreciation and amortization or not.



So, the depreciation is for a tangible asset applicable on its declared life, whereas amortization is for an intangible asset applicable on a specified period. Now, again, we are saying depreciation is for an intangible asset applicable on its declared life, whereas amortization is for a tangible asset. So, you can understand this is not true. Then we are writing, depreciation is for a tangible asset applicable on a specific period, whereas amortization is for an intangible asset applicable on its declared life. Depreciation is for an intangible asset again.

So, understand how much you have to proceed. So, after this, we find out that depreciation is for a tangible asset, which is applicable based on its declared life—first



thing. And amortization is for an intangible asset. So, first, you have to select which is applicable for tangible and which is for intangible. So, you reduce the problem by half—first half of the problem is done.

So, this intangible asset is applicable for a specified period—that is amortization. This is only to test whether you remember the basic concept of depreciation—where it is applicable and where amortization is applicable. These topics, like depreciation and amortization, we have discussed at length in the theoretical part. This is only a test to see whether you remember these things or not. In the GATE exam, you can expect questions like this, apart from numerical problems, where you will be required to clearly remember the concept behind the topics.

So, always try to fully understand—go through the definition several times, verify when solving problems, and discuss further so that you do not make any mistakes. Now, you can also solve the 10 to 15 years' questions of GATE. So that you can do well—nowadays, a GATE score is very much required for getting a job in the public sector, and GATE is essential for postgraduate studies also. So, I would definitely advise my students listening to me here in my class that you should prepare the questions and answers from previous years' GATE papers. So that you become very thorough and expert in solving both numerical and multiple-choice questions—I must wish you all the best.

Thank you.