MINERAL ECONOMICS AND BUSINESS

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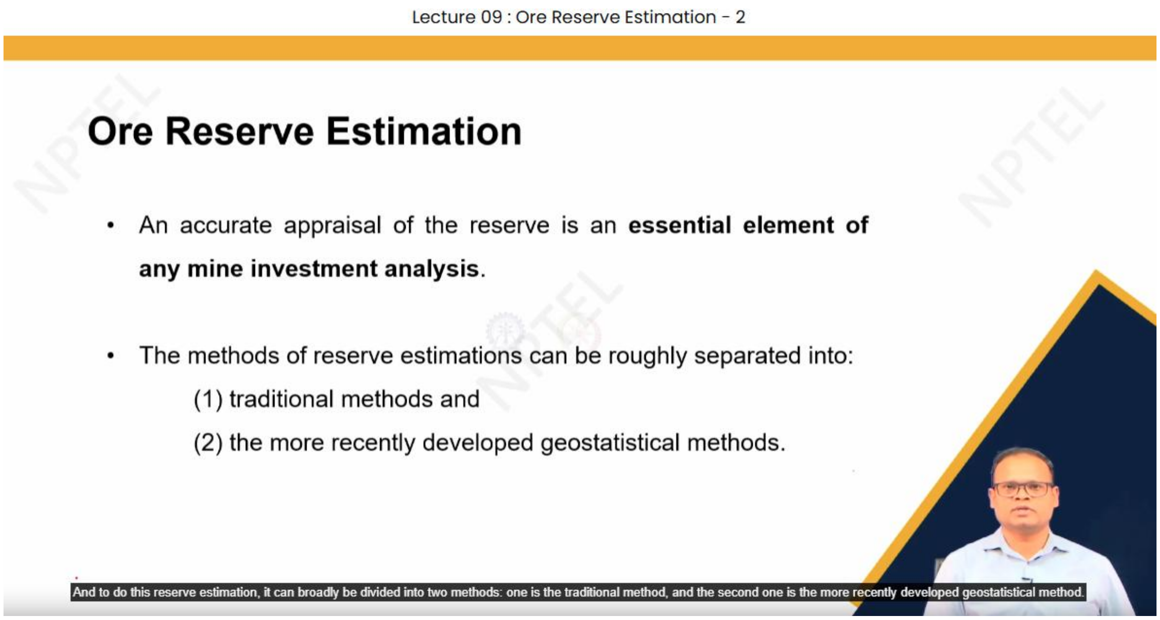
Lecture 09: Ore Reserve Estimation - 2



Hello everyone, and welcome again to this course on Mineral Economics and Business. So, this is our lecture number 9, and this is the second part of ore reserve estimation. So, in our previous lecture, we saw that an accurate appraisal of the reserve is an essential element of any mine investment analysis. And to do this reserve estimation, it can broadly be divided into two methods: one is the traditional method, and the second one is the more recently developed geostatistical method. So, in our last class, we discussed the various traditional methods, and in today's lecture, we are going to see the geostatistical method.

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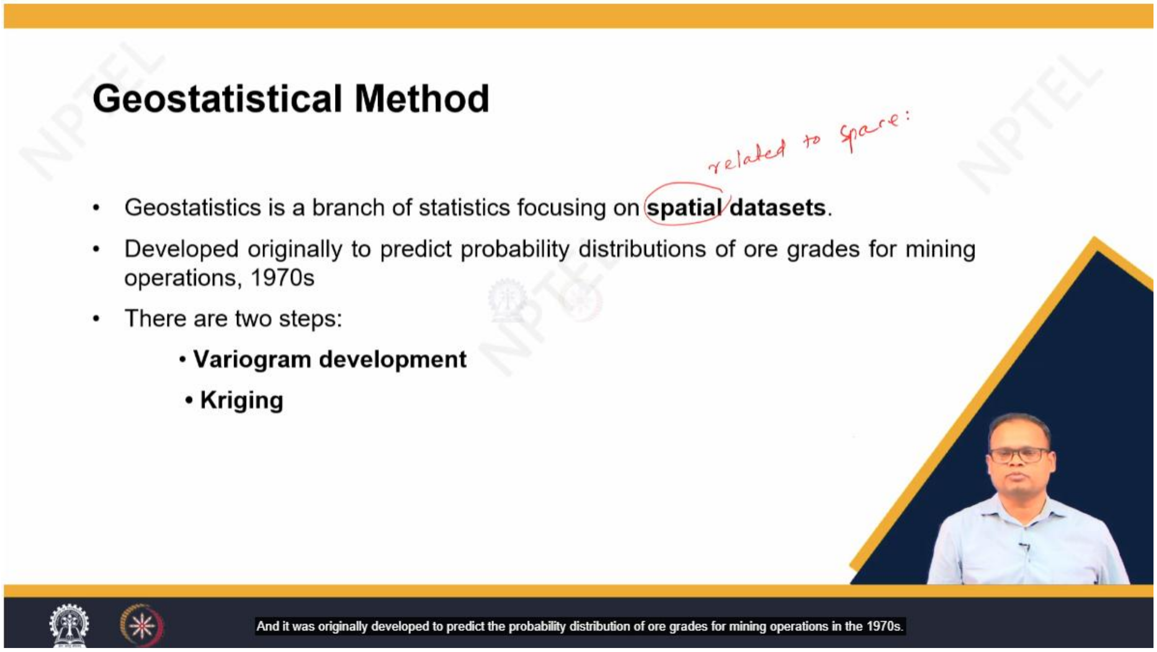

So, geostatistics is a branch of statistics focusing on spatial data sets. What does this spatial data set relate to? Space. It was originally developed to predict the probability distribution of ore grades for mining operations in the 1970s. Here, there are two steps: one is called variogram development, and the second one is called So, a variogram is a graphical representation of the directional interdependency of sample grades, and Kriging, named

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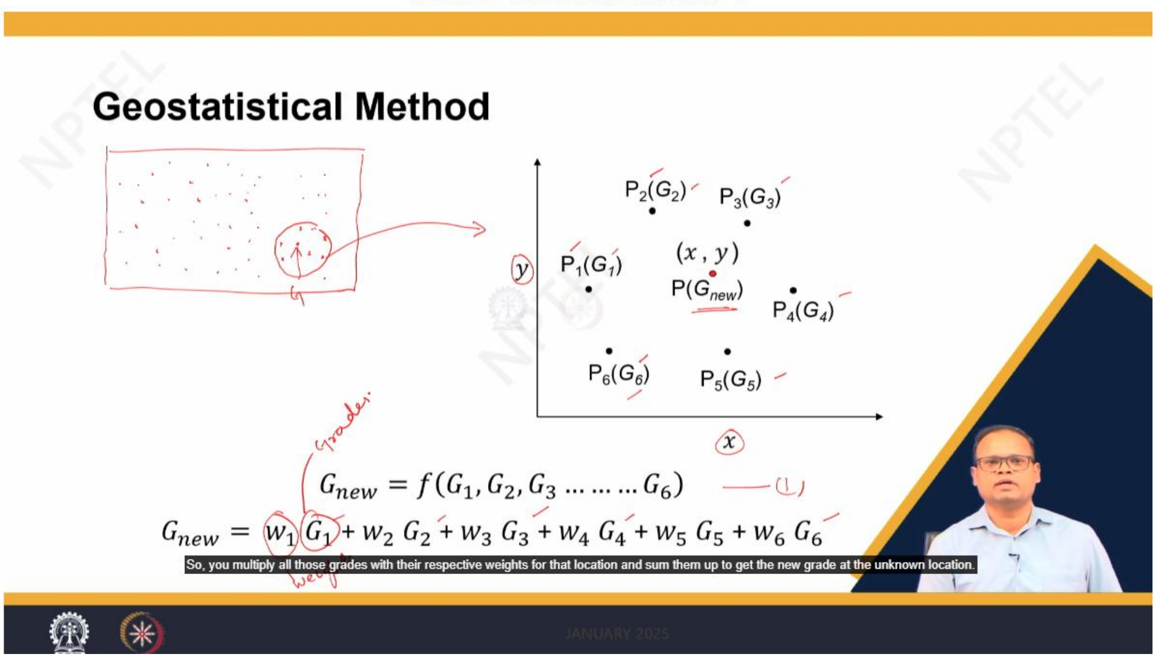


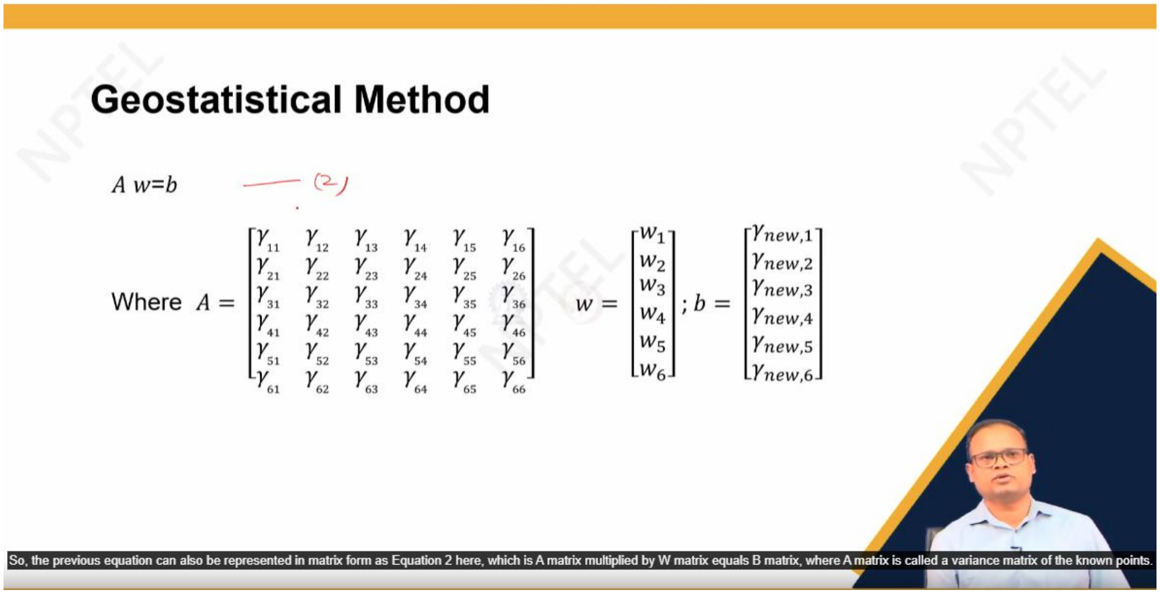
after D. G. Krige, is a mathematical procedure for assigning linear weights to samples to estimate grades at unknown locations.

So, we will look at these two terms and understand them in the next slide. But before that, let us take an example, like the one we were discussing in our previous lecture. Let us say this is an area where we are doing our borehole investigation, and let us say these dots are our boreholes that we have drilled to explore the ore body. So, all around this area. So, as we discussed in our last class, the exploration part, where we are looking at the grids at different locations, is a very tiny fraction compared to the size of the ore body.

And what we need to know from this exploration is that the dots here represent the points where we know the grade of the ore body. But from here, what we need to know is, for any unknown location within this area, what is the grade of this point in this location. So, in this case, let us take an example. Take one area here and put it in this figure on the right side. So here, what I have considered is this: the x-axis and the y-axis represent the location of different points. P_1, P_2, P_3, P_4, P_5, and P_6.

So, P_1 to P_6 are the locations where we drill the boreholes and where we know the grade of this ore body as G_1, G_2, and all the way up to G_6. So now, here the question is: we have a point P. With coordinates (x, y), the grade at this point is unknown, and that grade at the new point we have to calculate using this geostatistical method. So, for this grade at this new point P, it can be represented using Equation 1 here, which says the grade at the new point is a function of G_1, G_2, all the way up to G_6. Or we can write it like this: the grade at the new location is $G_{new} = w_1 G_1 + w_2 G_2 + w_3 G_3 + w_4 G_4 + w_5 G_5 + w_6 G_6$

What are these W's? They are called weights, and G's are the known grades. So, you multiply all those grades with their respective weights for that location and sum them up to get the new grade at the unknown location.



So, the previous equation can also be represented in matrix form as Equation 2 here, which is A matrix multiplied by W matrix equals B matrix, where A matrix is called a variance matrix of the known points. Between point 01 all the way to point 06. So, here, for example, this gamma 2-4 is the variance between the second point, let us say this is P_2, and we have point P_4. So, what is the variance between these two points? We will see how to calculate the variance between two points in the next slide.

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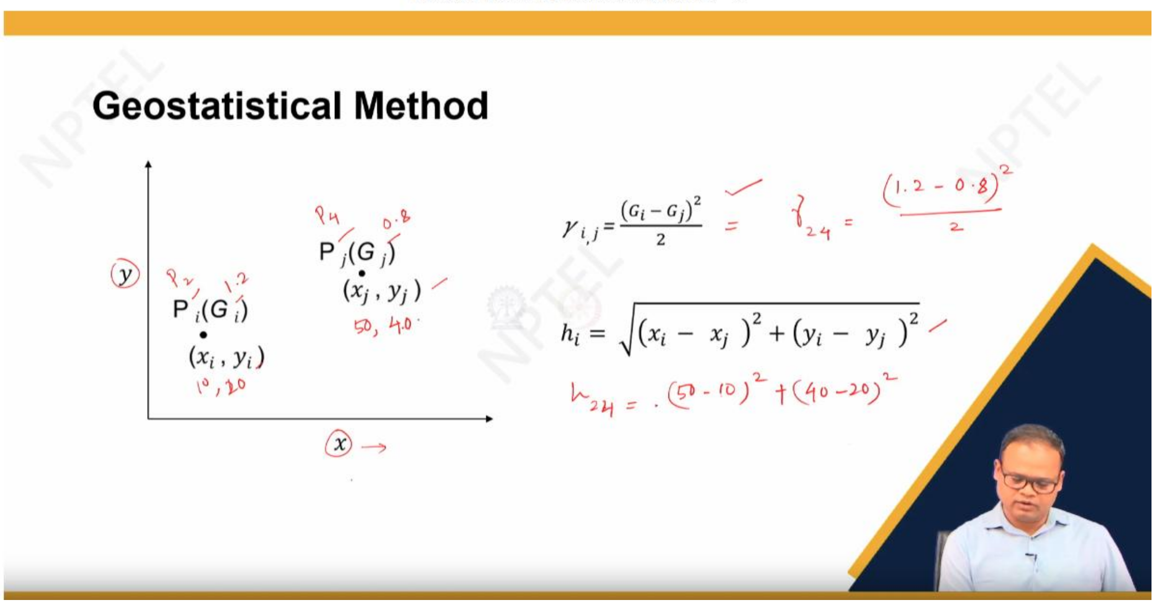


W is the weight matrix that we have to find out, like we need to know how to multiply it with the corresponding known grades to find the new grade. And B is another variance matrix or a column matrix for the new point and the unknown point. So, we will see how we calculate these matrices in the next slide. Before that, the first thing is to calculate the variance. So, to know about the variance, we have this plot here on the left side where we are plotting x and y, and we have point P_i and point P_j.

With grades g_i and g_j, the coordinates of point P_i are (x_i, y_i), and those of point P_j are (x_j, y_j). So, the variance between point 1 or P_i and P_j is represented using the equation here, which says

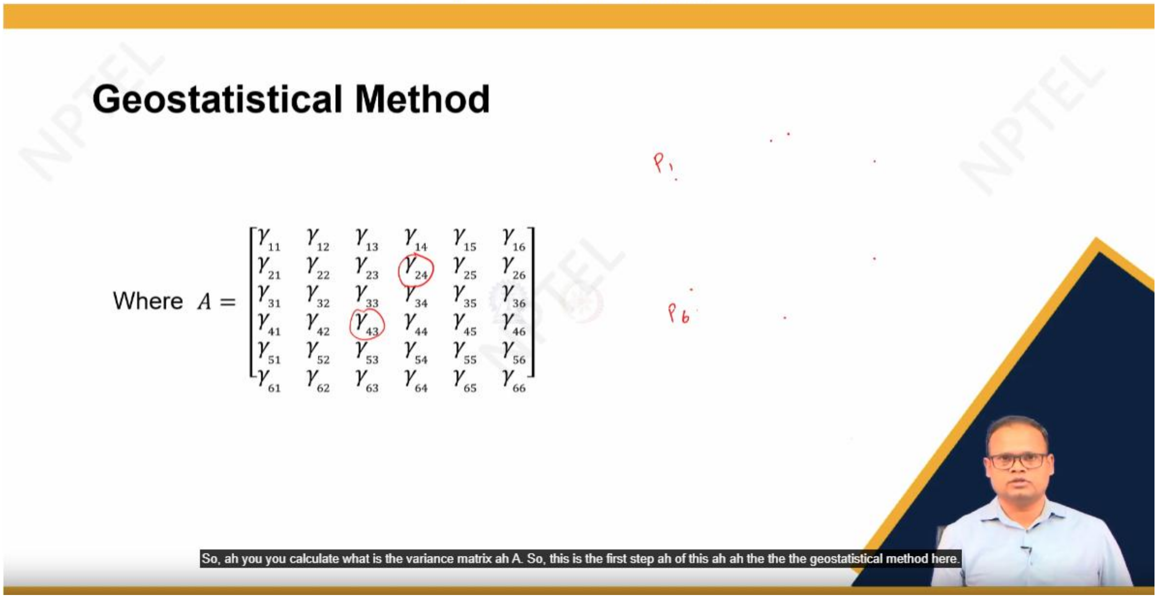
$$\gamma_{i,j} = \frac{(G_i - G_j)^2}{2}$$

And the distance between two points, as we know, is

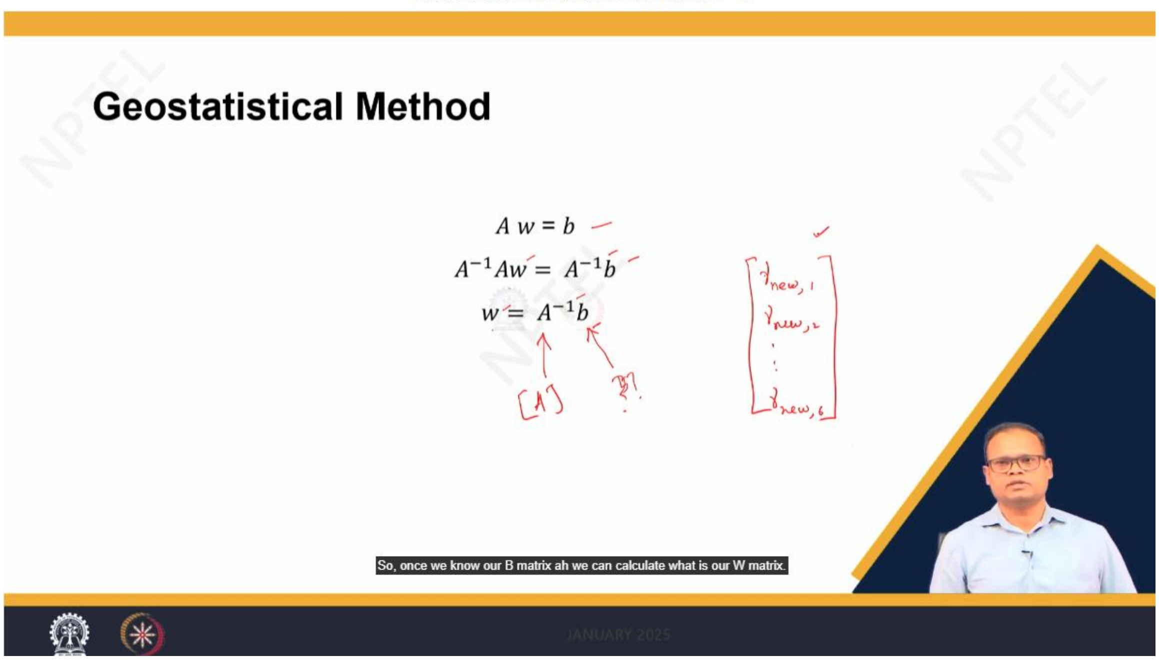
$$h_i = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$$

For example, let us say we talk about P_2 and P_4, and let us say the grade here is 1.2 and the grade here is 0.8. This coordinate is, let us say, (10, 20), and Pj's coordinate (or P_4's coordinate) is, let us say, (50, 40).

So, the variance γ_{24} becomes $(1.2 - 0.8)^2 / 2$. Here, 1.2 is the grade at point number 2, and 0.8 is the grade at point number 4. So, the whole square is divided by 2. grade at point number 4. So, whole square divided by 2.

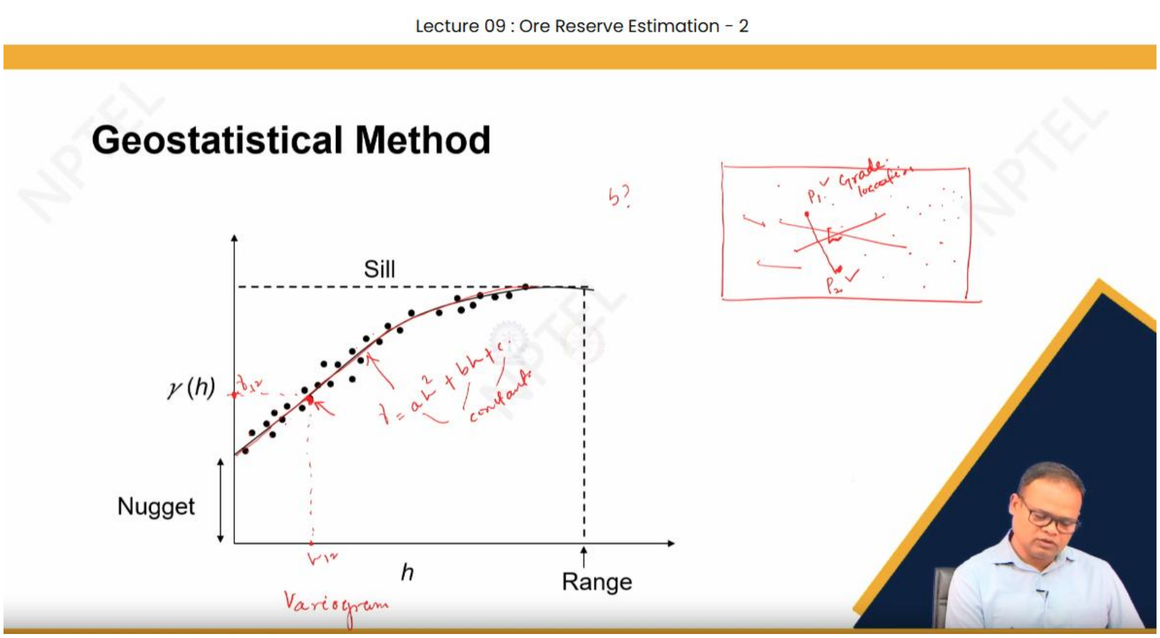


Similarly, we can calculate what is the distance between or h between 2 4 as you know 50 minus 10 square plus 40 minus 20 square whole square root. Now, ah once we know how to calculate the variance, we can calculate different variance ah ah between these 2 points ah you know or or maybe ah all these 6 points ah. So, ah for example, as we saw like you know the the variance of 2 4. Similarly, you can calculate all these variances between ah the different ah the 6 point that we ah consider in the example here like let us say you know p_1 ah all the way up to p_6.



So, you calculate what is the variance matrix A. So, this is the first step of this the geostatistical method here. And this equation you know once we know that A matrix into W matrix is our B matrix. So, here what we can do we can multiply A inverse in both the sides. So, A inverse into A ah becomes you know A inverse into ah A inverse into A ah into W matrix becomes ah A inverse into B matrix B matrix yes. So, or we can write ah you know W matrix equal to A inverse into

B matrix. Now, ah we we know how to calculate A matrix So, we can calculate ah what can be the like what will be our A inverse matrix ah the only the question is what will be our B matrix. So, this is our you know the question how how do you find it out which is ah you know we we saw that the the B matrix is gamma new comma 1 gamma new comma 2 all the way up to gamma new comma 6. So, this is our ah you know the the ah the the B matrix. So, once we know our B matrix ah we can calculate what is our W matrix. So, ah because the we we do not know the you know ah grade at the new location ah we need ah we need something called a variogram. to know our B matrix, what is this B matrix is. So, before that actually you know like to explain what this variogram is, let us let us take the example of you know our exploration data ah.



So, we we did borehole at different points and these dots here are representing the location of the boreholes. and we know ah you know what what are the grades at these ah you know borehole locations ah. For example, you know this is a point let us say this is p_1 and this is let us say point p_2. So, in here ah you know we know the coordinate of p_1 ah and know the coordinate of p_2. Similarly, we know know the grade at p_1 and we know the grade at p_2.

So, once you know the grade and location, you can calculate what is the distance between those 2 points which we call and the variance between the point P_1 and P_2 using the previous equation that we have seen which is you know g_1 minus g_2 whole square divided by 2. So, once you know you know what is my distance let us say the distance h between P_1 and P_2. So, you can find the distance h and maybe the corresponding you know gamma value

So, the gamma is here, and this is for p_1 and p_2. The gamma is this; this is the value. So, we can write it as gamma 1 2 here, and this is h 1 2 here. So, once you know this thing, you can get the corresponding point or put this point here. So, this becomes our gamma versus h plot for p_1 and p_2. So, when we do our investigation or exploration, and we know the grade and the corresponding distance between the different points, what we can do is take different combinations of h and the corresponding gamma h between different points with different distances.

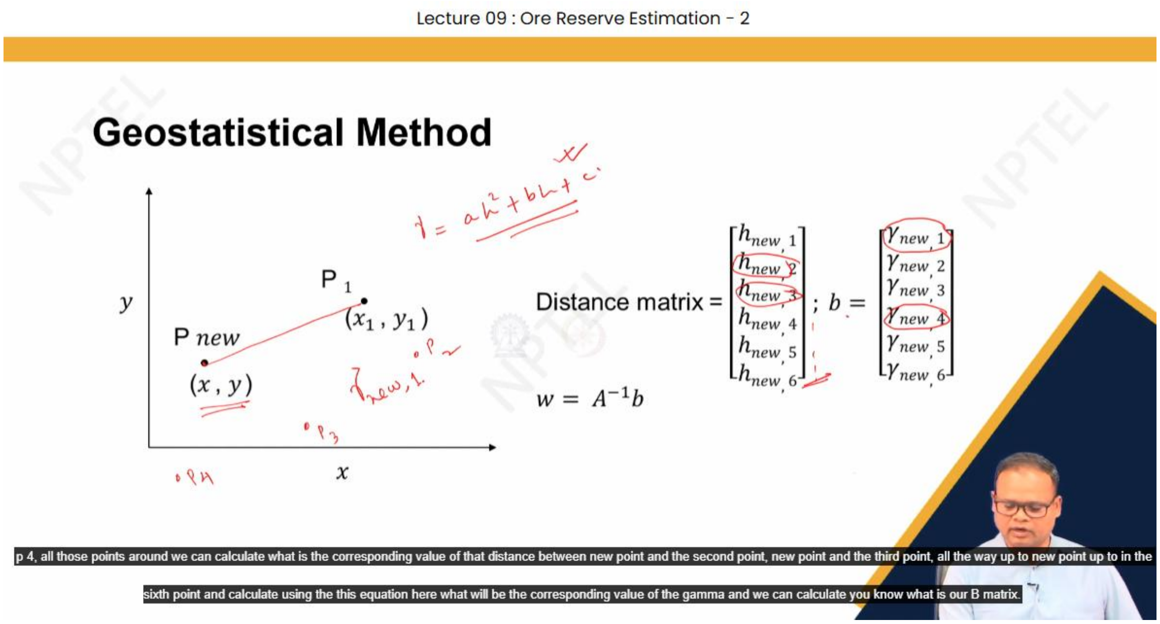
So, then put all those points, like we have put in black dots here, and try to fit them with a curve. So, this curve can be a polygonal curve, which is a function of h. So, this can be, let us say,

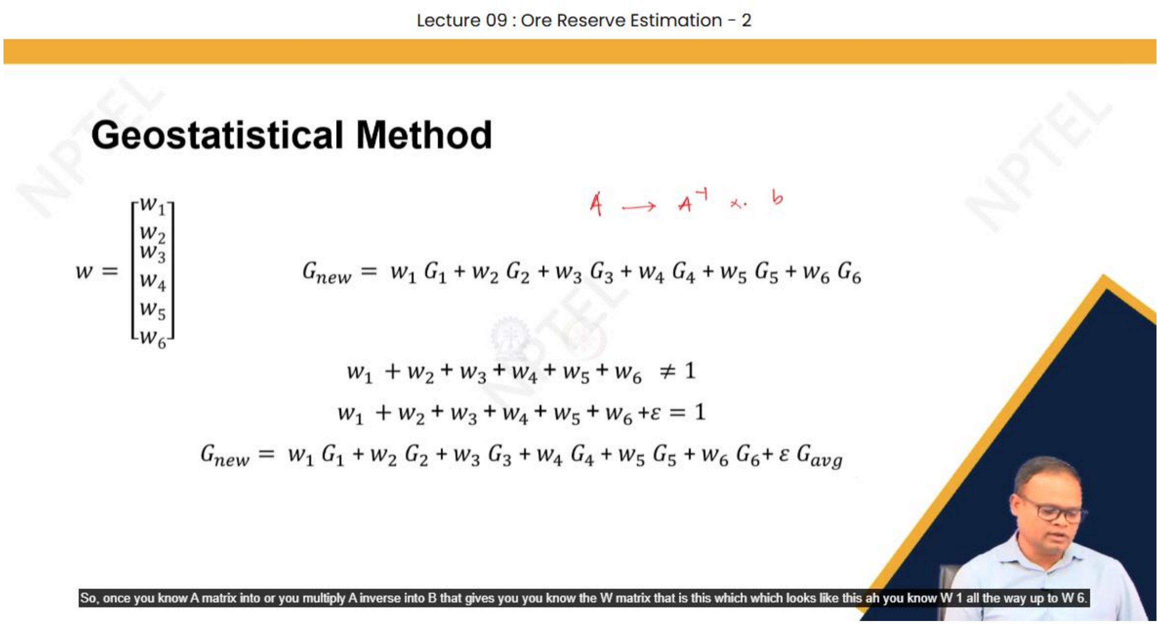
$$\gamma=ah^2+bh+c$$

where a, b, c are the constants. So, these are the constants. Constants, and the gamma is related to this.

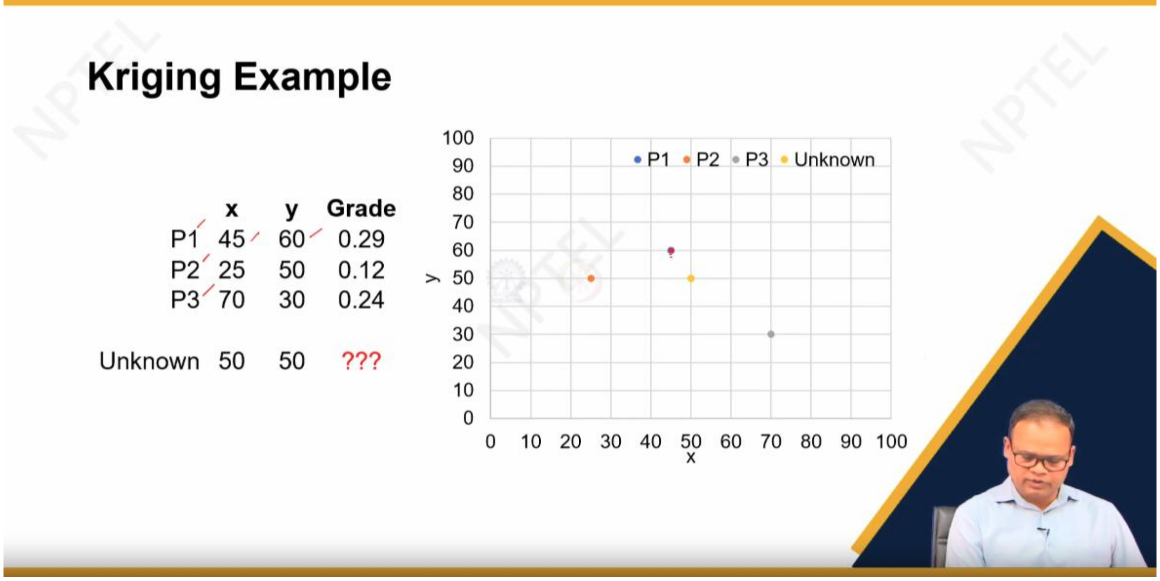
So, once we know there is a relation between gamma and h, what we can do is... We can take our unknown point or the new point, which has coordinates x, y, and if you want to calculate gamma between, let us say, the new point and the first point, what you can do is calculate the distance between them. And using the previous equation, which we saw, gamma equals a h squared plus b h plus c, or some other kind of equation fitting those data points, what we can do is... Get the distance between the new point and the p1 point, which is known to us, and put it in this equation here to calculate what our gamma new versus 1 will be. So, similarly, we have different points, let us say p_2, p_3.

p_4, all those points around we can calculate what is the corresponding value of that distance between new point and the second point, new point and the third point, all the way up to new point up to in the sixth point and calculate using the this equation here what will be the corresponding value of the gamma and we can calculate you know what is our B matrix. So once we know our A inverse, or A A matrix then we can calculate our A inverse matrix and and from the previous slide we can we saw that how how to calculate this B matrix. So, once you know A matrix into or you multiply A inverse into B that gives you you know the W matrix that is this which which looks like this ah you know W 1 all the way up to W 6. So, this is up to w 6 because we have we have considered 6 point in the example.





So, ah now we know like you know what ah in in the previous case we saw that you know the g new the grade at the new ah location is ah w_1 into g_1 plus all the way up to w_6 into g_6. But you know using this geostatistical method like you know if you calculate this w matrix ah ideally this should be equal to 1, but you know then you change that it is not equal to 1. So, what we do we add or error term here to this all the summation of this w ah plus some error term is become 1. So, instead of you know using this ah let us say ah this equation here ah which is let us say 3 ah we use another equation 4 ah where we add our term error into ah g average where this g average equal to g_1 plus g_2 plus all the way up to g_6 whole divided by 6. So, the thing is you know your A matrix. you can calculate your A inverse matrix, you can calculate what is your B matrix, you multiply A inverse and B ah you get your W matrix and using the equation 4 here ah which is ah the in the bottom like you can calculate ah the grade at new location.



So, you know we have an example here instead of 6 points what I have considered here is the 3 points p_1, p_2, p_3. So, with the corresponding x and y coordinate here.

So, x is 45, 60 here. So, the first point p_1 is, you know, here. With 45 comma 60, the second point p_2 is here, which is 25, 50, and the third point is 70, 30. So, this is our p_3. So, these are the known points.

Or where we drilled our holes, and we know the coordinates and the corresponding grade. So, the grade is written here: for the first point, the grade is 0.29; the second one is 0.12, and the third one is 0.24. So, if we know these things, let us say this p unknown point is 50 comma 50, which is p new or p unknown. We can write it here, which is at this location 50, 50. So, the question is: what will be the grade new at this location, 50 comma 50? So, for this, you know, what we can do is first calculate a matrix, which is called a distance matrix, a 3 by 3 matrix.

And, you know, which says the distance between different points. So, the first term here is h 0 comma 0. So, the distance between 0 and 0, which becomes 0. So, the second term is the distance between the first point and the second point. So, which we can calculate as 22.4 from the previous coordinates.

Similarly, you can calculate h_{12} and h_{21} and all those points. So, this is the first step of this kriging we can say. And once you know all these points, what we can do is calculate the A matrix. And how do you calculate the A matrix? It is using the equation we get it from the So, let us say one of the variogram equations is gamma h equal to

$$\gamma(h) = -5\times10^{-6}\cdot h^{2} + 2.2\times10^{-3}\cdot h + 0.04$$

So, in this case, if the variogram equation is this, we can put the different h values here and get the corresponding value of the variance. So, once we put, let us say in this equation to calculate the value 0.13, what we have done here is put the 49.2 values in this equation. So, if you put it, it becomes 0.13. So, you put different values of h and you get the A matrix.

So, once you know your A matrix, what you can do is the second step: calculate the B matrix. So, for the B matrix, what we do is calculate the distance matrix, the second distance matrix, which says the first term here is we have 3 known points. So, p_1, p_2 and p_3 and the middle point is, let us say, an unknown point which is 50, 50, So, here you calculate what is the distance h_1. So, this h_1 is here.

So, we know all the coordinates between for point t1 and the unknown point let us say t, then you can calculate what is h1 and what is h2 and what is h3 using the formula that we saw before the distance And, once you know again you can use the previous equation you know which is a gamma is a function of h that that that equation and we put that equation here and get the value of the variance between ah the new point and let us say this gamma new and ah you know first point and similarly gamma new and second point gamma new and the third point. So, we can calculate what is the B matrix here. So, once you know ah you know how ah you calculate this B matrix ah using the distance and the the the um the function between gamma h ah gamma and this ah h ah this the the B matrix, what you can do is to calculate first thing is to from A matrix we can get the inverse of the matrix, where you know if we if we do the inverse of A matrix will come this matrix which is here.

So, once we know the A inverse, then what we do is you know as we saw you know the B matrix, we calculate as we explained in the previous slide. So, we multiply this A matrix or A inverse matrix with our B matrix. So, if you ah A inverse into B becomes our W matrix. So, if you just multiply this becomes you know this matrix here 0.62 ah and 0.11 and 0.25. So, this 0.62 is our W1 and this is our W2 and 0.25 our W3.

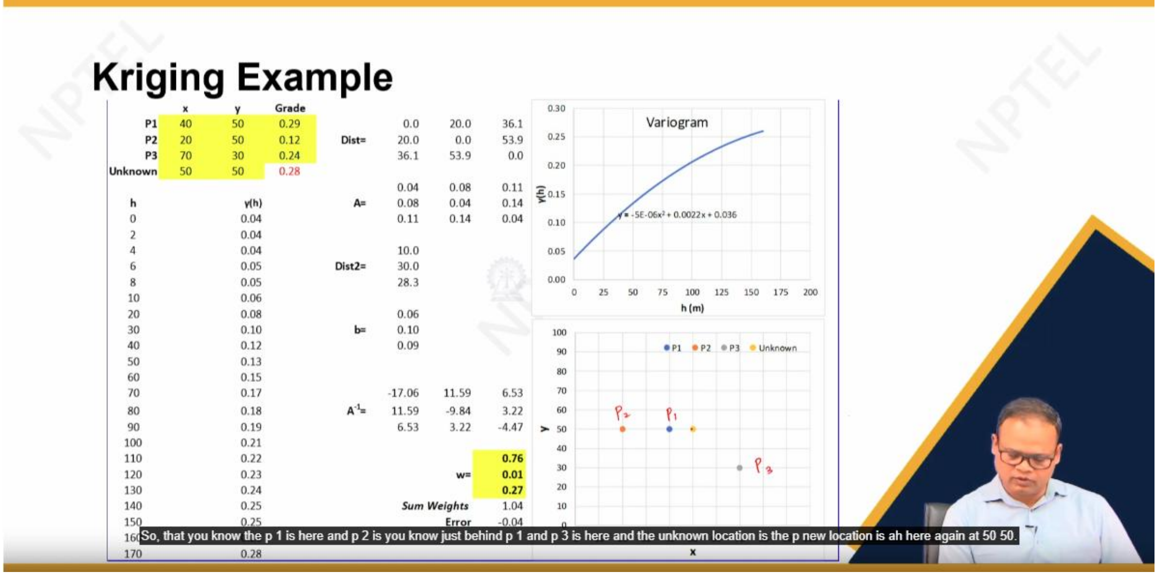
So, we know the corresponding value of our grade ah G1 and G2 and G3. So, you multiply ah you know this w1 into G1, w2 into G2 and w3 into G3 and add it ah. So, or or before that actually you know like if you add it all all these weights w 1 plus w2 plus w3 it becomes 0.98 and it is say you know as we have seen w1 plus w2 plus w3 ideally should be equal to 1, but this is not the 1 here and the error terms ah becomes 0 .02 here and if you add this error it becomes 1 and using the equation that we have seen

$$G_{new} = w_1 G_1 \cdot + w_2 \cdot G_2 + w_3 \cdot G_3 + \varepsilon \cdot (G_1+G_2+G_3)/3$$

So, if you do that you know this this whole thing is explained here. So, the unknown location the grade becomes 0.26. So, here what we have what I have done is you know ah from our borehole data first what you can do you can you know get the coordinates ah for the variogram construction.

So, in h x axis we have h and y axis we have you know gamma. So, with for different value of h from our borehole ah exploration we can calculate what is the corresponding ah you know the gamma values ah. distance you just plot it here ah in in in this variogram plot and get the average equation So, once you get the average equation the equation is written here. So, once you know this ah equation you can ah multiply this equation with the distance matrix as we saw in our previous slide ah to get first A matrix, then ah first then B matrix get the A inverse matrix multiply with B matrix and get the corresponding weights.

And you can see you know like because in in this point ah in in this figure here point p1 is very close to the unknown location. So, this is ah our unknown location p let us say this is p3 and this is p2. So, because ah P1 is very close to p the corresponding weight ah we can calculate we can see that it is it is giving highest weight and you know because this p2 is little bit far away and you know p1 is little bit hiding, p2 we have less grade here and p3 grade is 0.25 here. ah in the second example what I have shown is you know ah we we have I have placed this p1, p2, p3 almost equally ah around this ah unknown point p here or p new here.



So, in this case you can see the weights are almost the same. In the third example ah you know what I have put here like the the coordinates change the coordinates. So, that you know the p1 is here and p2 is you know just behind p1 and p3 is here and the unknown location is the p new location is ah here again at 50 50. So, in this case you know p1 is completely blocking you can say in the the point p2 .

So, you can see the and it is very close to to the unknown location. So, the weight that we are getting for p1 is the highest and we are not almost getting 0 weight for p2 because you know this is behind the point p1 and we are getting a weight of you know 0.27 for p3. So, and once you know all these weights ah as we saw that you can multiply with the corresponding grades here and get the you know the the ah value of the ah grade at unknown location. So, this is just an example with 3 points ah. So, with some kind of computer programming ah you know we can take many points ah and ah you know solve this geostatistical problem.

So, this ends our ah lecture on this ah you know over reserve estimation.