Structural System in Architecture

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Lecture – 13
Bending Stress in Beam - III

Welcome to the NPTEL online certification course on Structural Systems in Architecture. We are going through the module number 3, the structural mechanics; and today is the last part of the bending stress on beam. Today we are going to have the lecture number 13.

So, in this lecture we will go through the following concepts:

- Advantages of Flanged Beam
- Built-up Beams
- Section Modulus of Asymmetric
- Beam Section with Composite Materials

The learning objectives of this particular the lecture will be:

- Application of Bending equation in various beam sections.
- Justification of advantages of flange sections over others.
- Outline the theory of bending conceptually in beams with composite material.

We should know what composite material in case of a beam? How this theory of bending is applicable? It is important to know because we sometimes use some of the composite material in our structural members like the concrete, RCC is one of example of composite materials.

Now, let us first discuss about the advantage of the flanged sections. But, before we go to the flanged section, let us very curiously see the stress diagram given in Figure 1. The figure shows the stress distribution diagram over the section of a beam, because of bending.

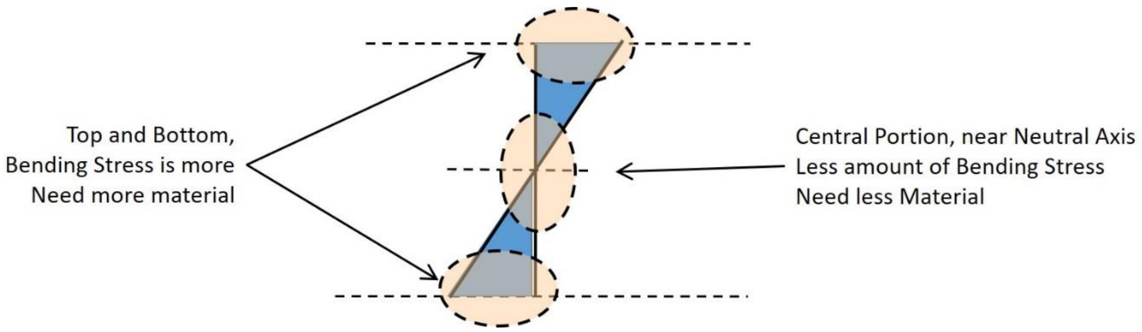


Figure 1: stress distribution diagram of a beam section due to bending

So, in the figure you can see that on top on bottom of the beam section the bending stress is more; that means, you require more material over those areas to resist the bending stress or to pull down the stress to the permissible limit. Whereas in the central portion, near to the neutral axis or near to the CG axis of the beam, the bending stress is less or minimal. Therefore, it requires less amount of material. So, the material distribution also should follow the stress distribution from top to bottom. It should be in such a way that, we can do effective use of materials to result very economic beam sections.

So, here if we consider a rectangular kind of a beam, as shown in Figure 2 (a). Here the two portions which are near to the neutral axis or the near the centre, we do not require them anymore, because we know that the central portion of the beam experiences less amount of stress.

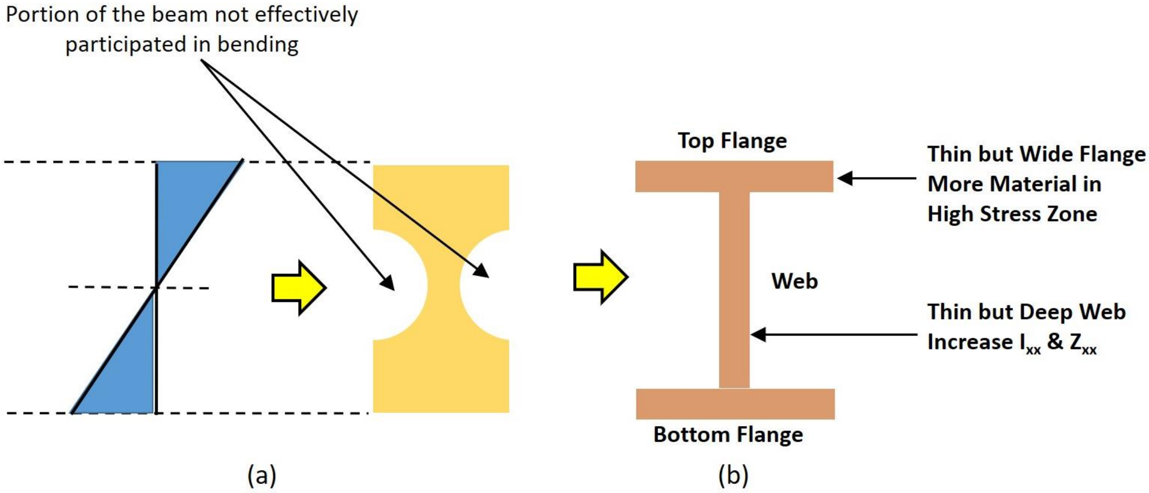


Figure 2: beam section and material used

So, why should I give same material throughout the rectangular beam section when I can economize it? Yes, in some of the cases we cannot help. Practically we cannot cast this kind of sections, for example in concrete we can't. But, in some other materials we can think of this it, we can reduce the amount of material to economize the total cost of the structure. For example, steel gives us some solutions to create such kind of beam sections.

Now, steel we can have some beam sections as shown in Figure 2 (b). This is an I beam. It is called as joist, rolled beam, or I section. Very popularly it is known as I section, because it looks like the English letter 'I'. In this section we have two flanges; top and bottom, and the central portion connecting the two flanges is called the web. As we have discussed in the earlier cases, that is stress distribution, let us see this from stress distribution and material concentration point of view. Here in case of an I section, both the flanges are thin but wide. So, this section is having more material concentration on top and in bottom, which is required in high stress zone. On the other hand, the web is deep, long and thin. In web the concentration of material is not much, here it is less. Here, the thin but deep web will increase the values of I_{XX} and Z_{XX}.

We can have series of I sections with different dimensions. In some cases, the depth remains same but the flange increases, as shown in Figure 3.

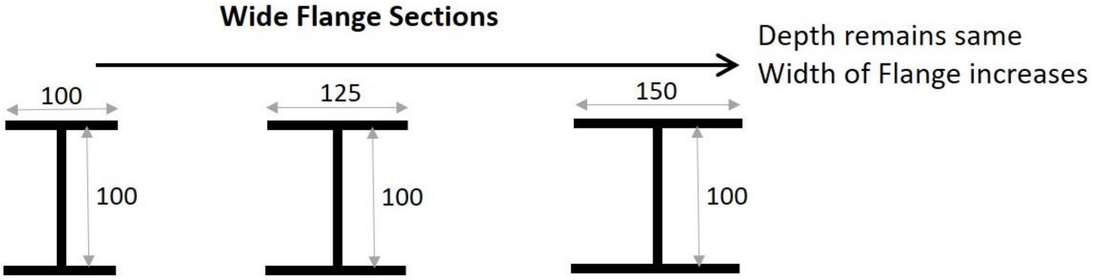


Figure 3: wide flange sections

For example, in Figure 3, the dimension of the web is 150, and flange changes as 100, 125, and 150. So, the depth remains same, but the width of the flange is increased. This is called the wide flange sections. So, I can have a series of I sections which does not change its depth; depth remain same but it is the flanges increases. Sometimes we require this kind of sections to take care of the heavy amount of bending moments.

On the contrary, we can also have some other types of sections where the flange will not be changed, width of the flange will remain same but the depth of the section, that is the web will be changed. For example, dimensions of the web are 100, 125, and may be 150; which is shown in Figure 4.

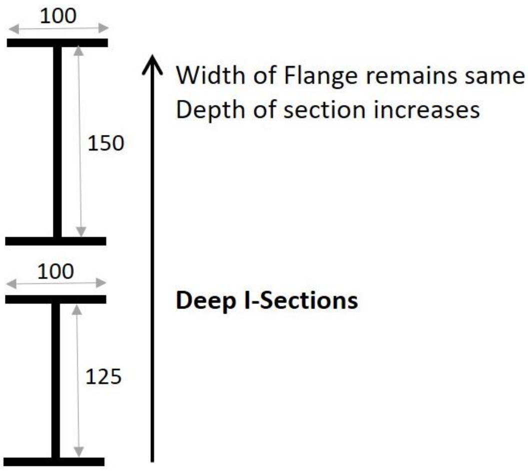


Figure 4: deep I-section

So, this is also good and this kind of the sections are also sometimes required to take care of the bending moments in beams, because with increment of the depth of the sections, you can increase the I_{XX} value and eventually, you can increase the Z_{XX} value.

So, I have calculated some of the wide flange section as shown in Table 1.

Table 1: the wide flange sections

Section	Width	Area	MI	MR	MR/Area
50x100x50	50	2000	3866667	16.1	8.06
75x100x75	75	2500	5383333	22.4	8.97
100x100x100	100	3000	6900000	28.8	9.58
125x100x125	125	3500	8416667	35.1	10.02

Here, in all the cases, the dimension of the web is constant; but the dimensions of the top and bottom flange changes. The thickness of each members of the section is kept as 10. So, in the 1st case, the 50x100x50, that means the width of the flange is 50 (for both top and bottom), and the web is 100. Similarly, for the other cases too. Using the formulas discussed earlier in the lecture, the area, MI, MR, and ratio of MR/Area is calculated, and here the $\sigma_{permissible}$ is kept as 250. From here we note that, as we increase the width of the flange, the area increase, the moment of inertia increases, the moment of resistance increases and MR/Area also increases. The MR is important in resistance point of view and Area is important in cost point of view. The higher is the value of MR/Area better is the section and more beneficial. The graph of this is plotted in Figure 5, shown in blue line.

Similarly, now if we increase the depth then, let us see what happens.

Table 2: the deep I sections

Section	Depth	Area	MI	MR	MR/Area
100x50x100	70	2500	1920833	13.7	5.49
100x75x100	95	2750	3980729	21.0	7.62
100x100x100	120	3000	6900000	28.8	9.58
100x125x100	145	3250	10756771	37.1	11.41

So, here the flange remains same as 100, with thickness 10. I have increased the depth from 50. Therefore, it goes like 70 and then to 95, 120, 145 like that. The area I have calculated which is increasing, MI, the moment of inertia increasing, MR is also increasing, MR by area is also increasing like that but considerably high amount over here, the jump is very, very high as shown in Figure 5. This jump is very high with compared to wide flange sections. So, because of the height increasing this jump is much more and at some point it is overshoot the wide flange sections.

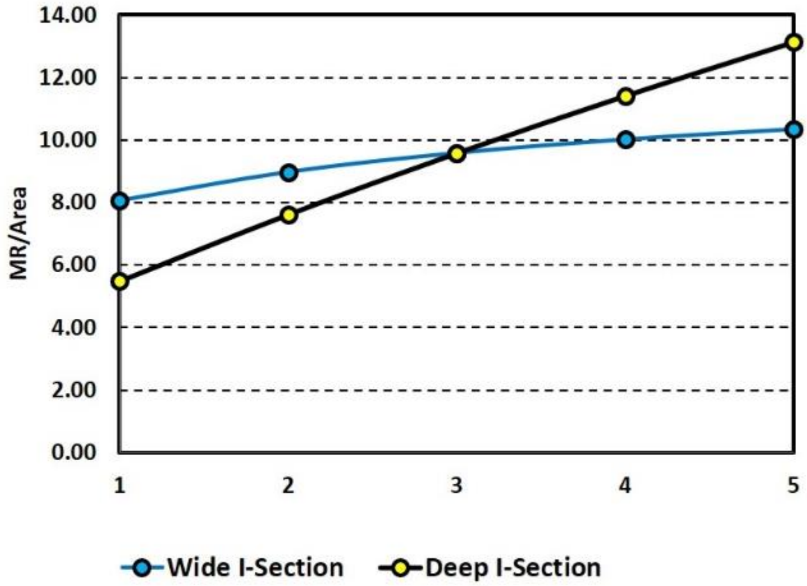


Figure 5: comparison of wide flange sections and deep I sections

So, these calculations can also be done easily in excel sheet, where you can put your values and you can find out lot of things. By putting the formulas, the moment of inertia, the Y top, Y bottom, the Z bottom, Z top, the MR, the moment of resistance of the sections, stress in the top and bottom most sections etc. By changing the values, for example the bending moment, you can also check whether the section will be stable or will it fail. If we put higher values, which is more than permissible, than it will reflect as fail. If the stress at top and bottom are higher than permissible, the top and bottom conditions will be reflected as fail. So, in this way you can actually see that all the things can calculate by virtue of excel sheet.

So, let us next go to built-up beams. Sometimes you may need more amount of moment to be controlled or moment to be actually support. So, with I sections we can do that as we can go with lot of variety. We can go with plate, introducing some plates on top and bottom of the I section, we can use some kind of a channel section, we can also go with some kind of an angle sections and the plate, or some 2 channels can be put together and make a box sections; as shown in Figure 6. So, like that there are ample of ways to go for a geometrical construction of those different types of sections and different kind of built-up beams. These kinds of heavy sections are used in industrial structures, railway stations and maybe stadiums as well as for high-rise buildings.

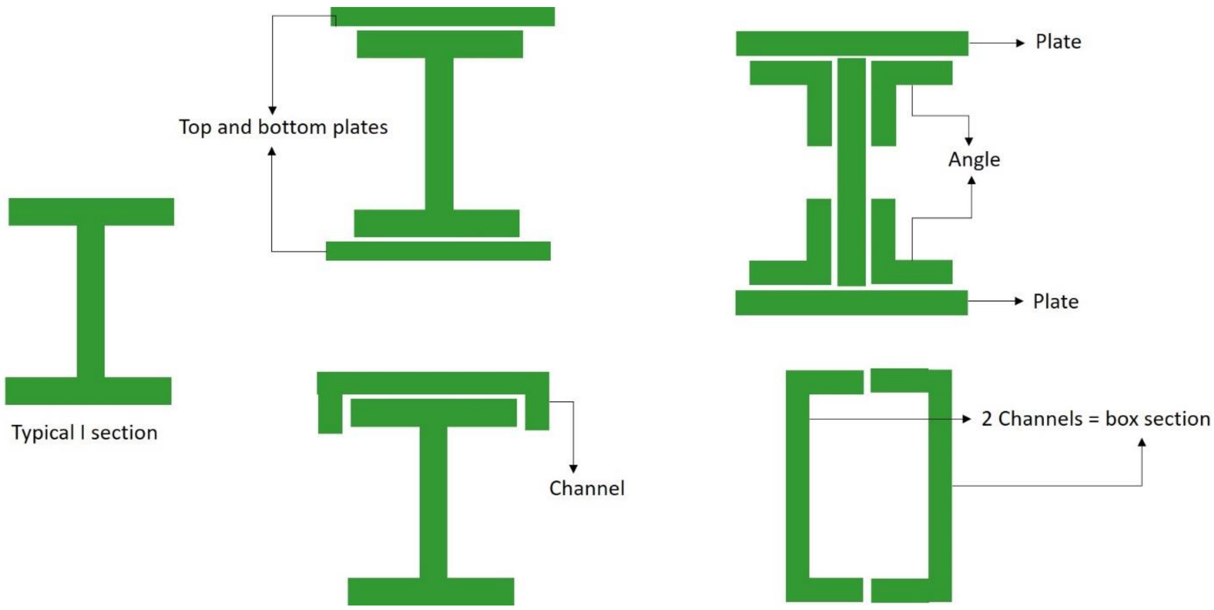
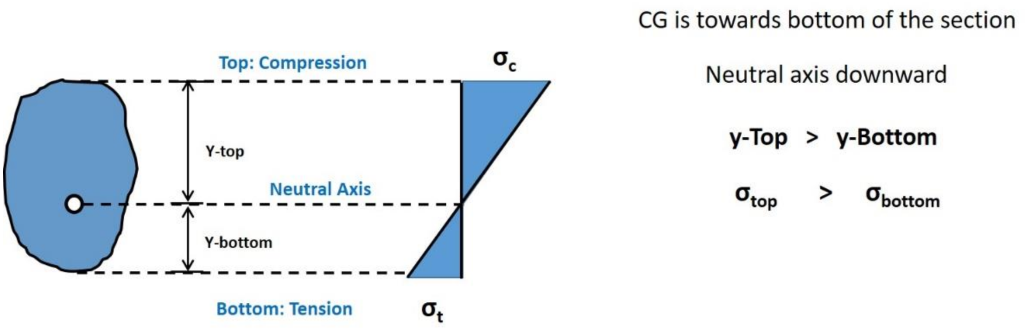


Figure 6: built-up beams

Now, let us go to another criteria that is section modulus of asymmetric sections. In previous lectures we have discussed about the sectional modulus for the symmetrical sections where the Z top and the Z bottom we calculated based on the ‘y’ top and the ‘y’ bottom; ‘y’ is the distance from the neutral axis in the top and bottom respectively; and for the symmetrical section it is equal both the way, the top to bottom. But in some cases, it is not so, we may have some asymmetrical sections too. Hence, we will have the different values of the y top and the y bottom. However, the pivoting case will be there the neutral axis.

So, at the neutral axis the value of sigma will be equal to 0. So as the y top is higher than the y bottom, then this σ top will be higher than that of σ bottom, refer Figure 7. The asymmetrical beam sections will have two section modulus.



Asymmetrical beam section will have Two Section Modulus:

$$Z_{top} = \frac{I}{y_{top}}$$

$$Z_{bottom} = \frac{I}{y_{bottom}}$$

Figure 7: section modulus of an asymmetrical section

So, each will have as a Z top and the Z bottom, because as you know the Z is nothing but –

$$Z_{xx} = \frac{I_{xx}}{y_{\max}}$$

In asymmetrical sections, I cannot have a single y_{max}, I have a bottom y_{max} from the neutral axis; which will give me the tensile stress and I have a top y_{max} which will give me the compressive stress. Therefore, I have to calculate two such Z bottom and the Z top, refer Figure 7.

Now, we will see examples of some asymmetrical sections, where it would not give you the CG exactly at the central depth, as shown in Figure 8. So, the y top and the y bottom will be of different values.

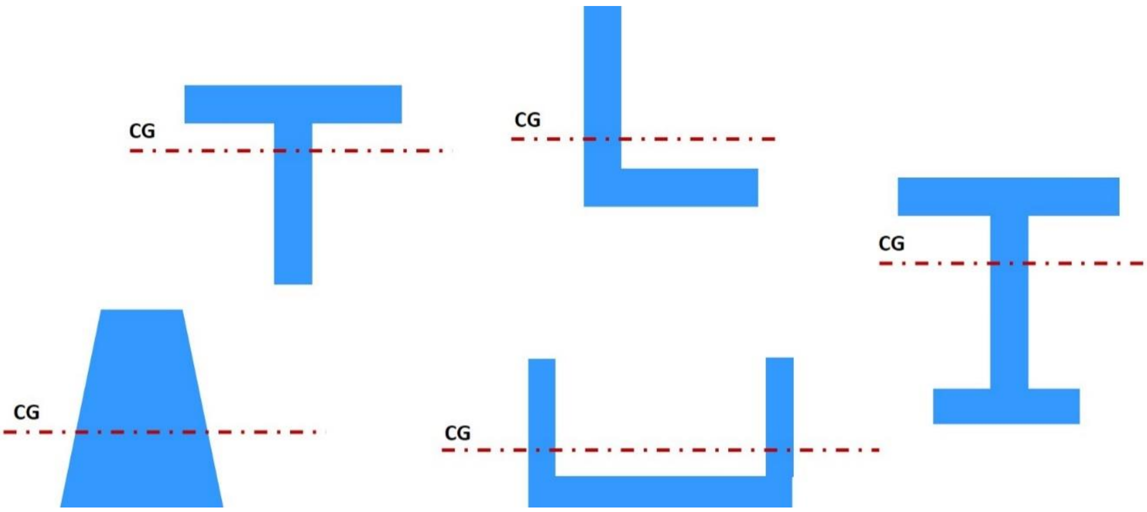


Figure 8: examples of some asymmetrical sections

So, now let us try to solve a small problem. We will consider a T section and we will try to find out the stresses in top and bottom layers of the T- beam Section. The Bending Moment on the section is 20KN-m

So, at first, I have calculated the position of the CG, and it will be at 107 from the bottom. So, as it is 107 from the bottom, the top fibre is 53 from the neutral axis. So, these two are not equal and it won't be. When we see the stress distribution diagram, then it will be like as shown in Figure 9. The σ_c will be smaller here and the σ_t will be higher; because the neutral axis is above, or closer to the top.

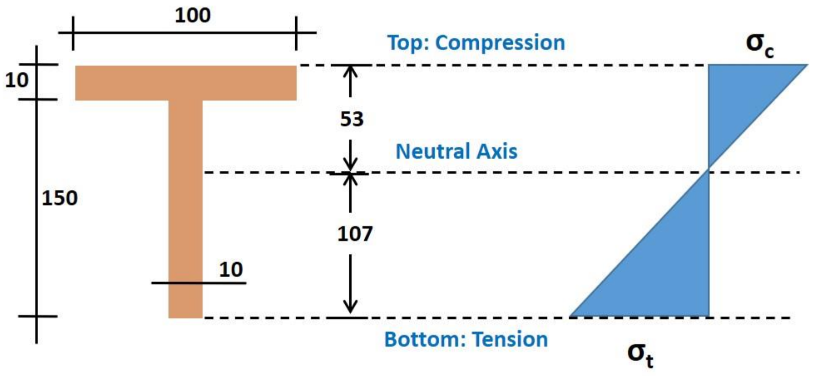


Figure 9: stresses in a T beam section

So, the stress will be smaller and the top and due to the funnel effect, it will be higher in the bottom.

$$\bar{y} = \frac{\{(150 \times 10 \times 75) + (100 \times 10 \times 155)\}}{\{(150 \times 10) + (100 \times 10)\}}$$

$$\bar{y} = \frac{267500}{2500} = 107$$

Now

$$\sigma_c = \frac{M}{I} \times y_{top} = \frac{M}{Z_{top}}$$

$$\sigma_t = \frac{M}{I} \times y_{bottom} = \frac{M}{Z_{bottom}}$$

Now,

$$Z_{bottom} = \frac{I_{XX}}{y_{bottom}}$$

$$Z_{top} = \frac{I_{XX}}{y_{top}}$$

Then,

$$I_{XX} = \frac{1}{12} \times 10 \times 150^3 + 1500 \times (107 - 75)^2 + \frac{1}{12} \times 100 \times 10^3 + 1000 \times (155 - 107)^2$$

$$I_{XX} = 6.66 \times 10^6$$

Assuming M=20 KNm

$$Z_{top} = \frac{I_{XX}}{y_{top}} = \frac{6.66 \times 10^6}{53} = 125.15 N/mm^2$$

$$Z_{bottom} = \frac{I_{XX}}{y_{bottom}} = \frac{6.66 \times 10^6}{107} = 62.24 \times 10^3 mm^3$$

Then σ_c and σ_t will be

$$\sigma_c = \frac{M}{I} \times y_{top} = \frac{M}{Z_{top}} = \frac{20 \times 10^6}{125.66 \times 10^3} = 159.15\ N/mm^2$$

$$\sigma_t = \frac{M}{I} \times y_{bottom} = \frac{M}{Z_{bottom}} = \frac{20 \times 10^6}{62.24 \times 10^3} = 321.33\ N/mm^2$$

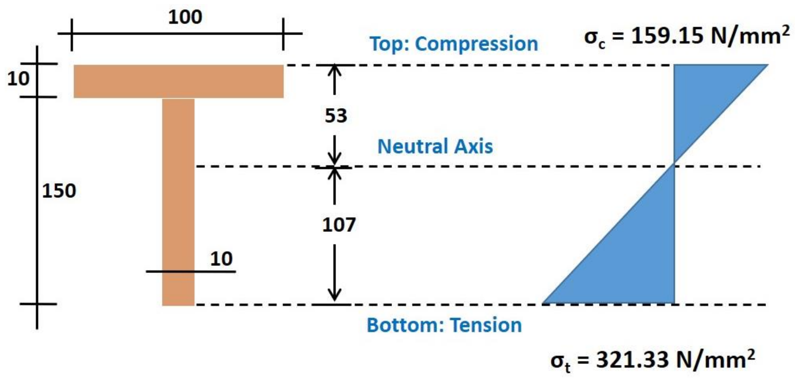


Figure 10: computation of stresses in a T beam section

So, next we will see another problem with an asymmetrical I section. This is to find the Moment of Resistance of the I- section. Given: Permissible Bending Stress in Compression and Tension is 250 N/mm^2 and 150 N/mm^2 respectively.

This is asymmetrical because, bottom flange is 120 by 20 and the top one is 60 by 10, as shown in Figure 11; also, the permissible bending stress in the compression and tension is not equal, compression it is 250 N/mm^2 and the tension it is 150 N/mm^2, where the tension is little less.

Now, calculating the CG of the section, we found that to be at 42.25 above bottom.
Moment of Inertia $I_{XX} = 8.29 \times 10^6 mm^4$

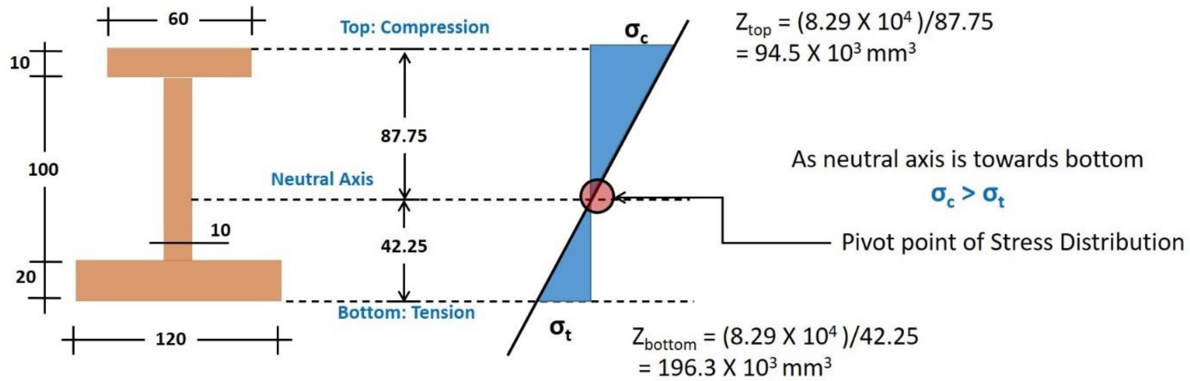


Figure 11: stresses in an asymmetrical I section

So, it will be pivoted at red dot point shown in Figure 11, and the neutral axis will lie towards the bottom or near to bottom. Therefore, the σ_c that is the stress in the compression zone will be higher with compared to the stress in the tensile zone. If remember in the last example we have seen that the neutral axis was towards the top that is why the top fiber stresses are less compared to the bottom. However, in this example the scenario is reversed.

Here it is given that the Permissible Bending Stress in Compression and Tension is 250 N/mm^2 and 150 N/mm^2 respectively.

Now, there is a relation between σ_c and σ_t which is governed by the two triangles above and below the pivoting points; and we can find the relation with respect to y_{top} and y_{bottom}. We can compute the relation as follows:

$$\frac{\sigma_c}{\sigma_t} = \frac{y_{top}}{y_{bottom}} = \frac{87.75}{42.25} = 2.08$$

$$\sigma_c = 2.08\,\sigma_t$$

So, from here we can see that σ_c is almost 2 times of σ_t; that means whatever may be the value of σ_c, it will be almost twice of that σ_t. It is higher because the neutral axis is towards the end or the bottom.

Now, assuming $\sigma_t = 150\ N/mm^2$ because this is the maximum permissible limit, I cannot put it this value as 151, because I know that beyond maximum permissible limit, it will fail. So, computing this, we can find

$$\sigma_t = 150 N/mm^2$$

$$\sigma_c = 2.08 \times 150 = 312\ N/mm^2$$

So, if we take the bottom tension limiting case as 150, then the top compression goes much higher than its limiting case of 250. Therefore, we cannot go with this, because with $\sigma_t = 150 N/mm^2$ the beam section will fail.

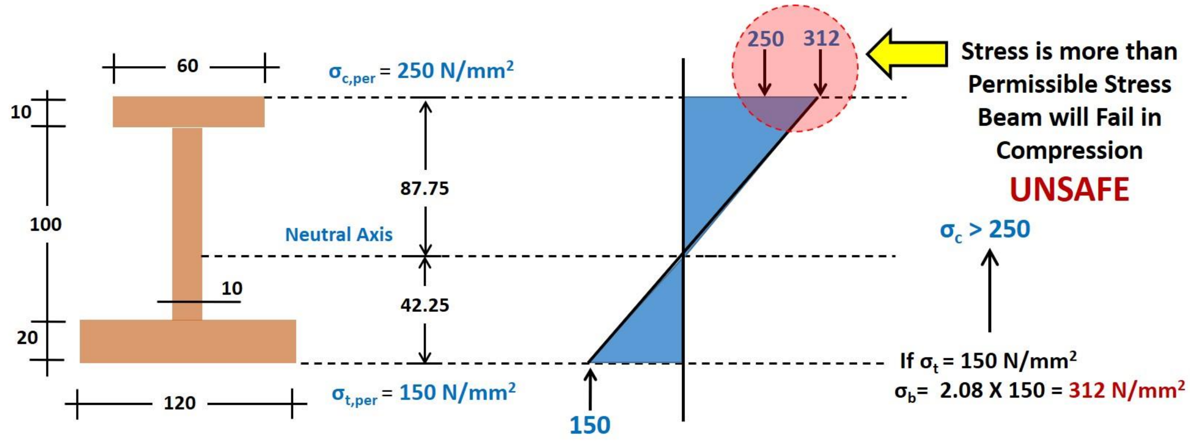


Figure 12: with limiting case of σ_t finding σ_c in an asymmetrical I section

As shown in the red highlighted circle in Figure 12, the permissible compression in the material is only 250, so if I put 150 as σ_t, then σ_c becomes 312, which is beyond the permissible limit of the compression; and the beam will fail.

So, let us go to the other case around. What is the other case? That is checking with the limiting case of compression. We know the relation that $\sigma_c = 2.08\,\sigma_t$; then computing this:

If

$$\sigma_c = 250N/mm^2$$

$$\sigma_t = 250/2.08 = 120\ N/mm^2$$

$$\sigma_t < 150$$

As 150 is a limiting permissible case for the tensile stress, and it is 120, then this is okay. Therefore, it is mark it in green colour in Figure 13; and it is safe.

In the earlier case it was not safe. So, in the safe case the topmost portion should catch the compression which is highest compression or the permissible compression that is 250 N/mm^2 and corresponding to that the bottom most portion will have 120 and that is safe. Based on that MR will be:

$$MR = Z \times \sigma = 94.5 \times 10^3 \times 2550 = 23.625 \times 10^6\ Nmm$$

$$MR = 23.625\ \ KNm$$

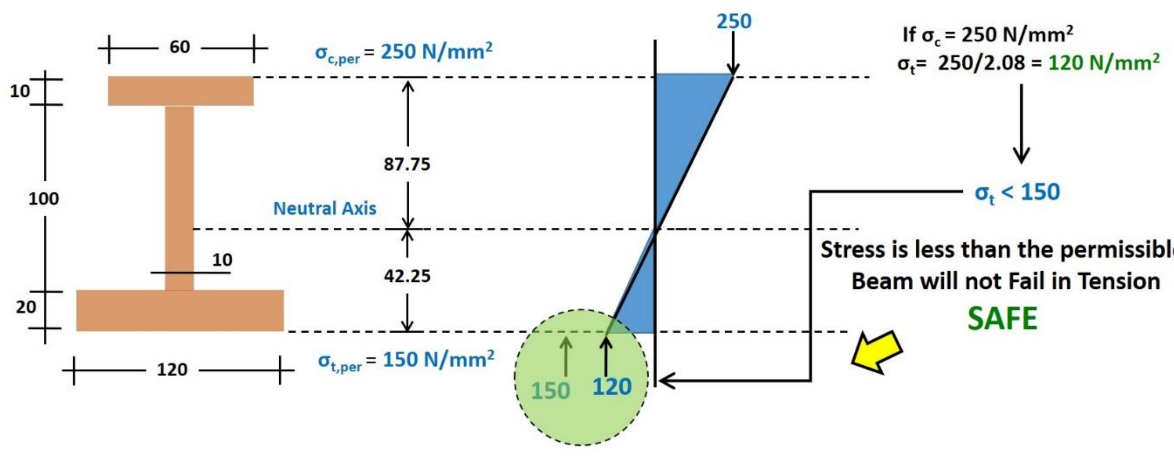


Figure 13: with limiting case of σ_c finding σ_t in an asymmetrical I section

So, now going to another problem. Here I have kept a particular channel sections as beam and the I have to find out the values of thickness ‘t’ such a way that these both compression and the tension achieve simultaneously. So, for that I have to achieve $\sigma_{c\ permissible}$ as 200 and achieved $\sigma_{t\ permissible}$ as 100, so what should be the value of x such a way that I can achieve, refer Figure 14.

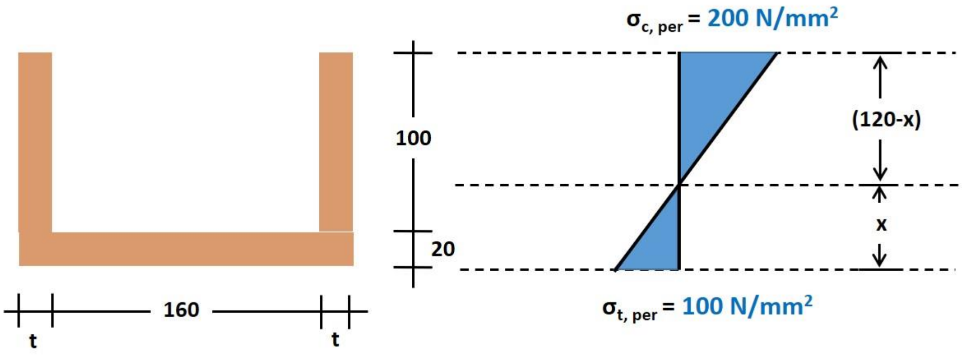


Figure 14: stresses in case of a channel section

So, for that I know the total depth of the section is 120, so if the depth below pivoting point is x, then above pivoting point will be 120 minus x; and computing this, we can find the value of x as:

$$\frac{120 - x}{x} = \frac{200}{100}$$

$$120 - x = 2x$$

$$x = 40mm$$

So, x has to be 40 and this the upper portion has to be 80. The hinge or pivoting point should be such a way that the sigma compression at the top will be 200; and at the same time, the sigma tension will be 100 in the bottom. After that I have then found out the CG of the section and the value of t.

$$CG = \frac{\{(160 \times 20 \times 10) + (2 \times 100 \times t \times 70)\}}{\{(160 \times 20) + (2 \times 100 \times t)\}} = 40$$

$$\frac{32000 + 14000t}{3200 + 200t} = 40$$

$$32000 + 14000t = 12800 + 8000t$$

$$6000t = 96000$$

$$t = 16mm$$

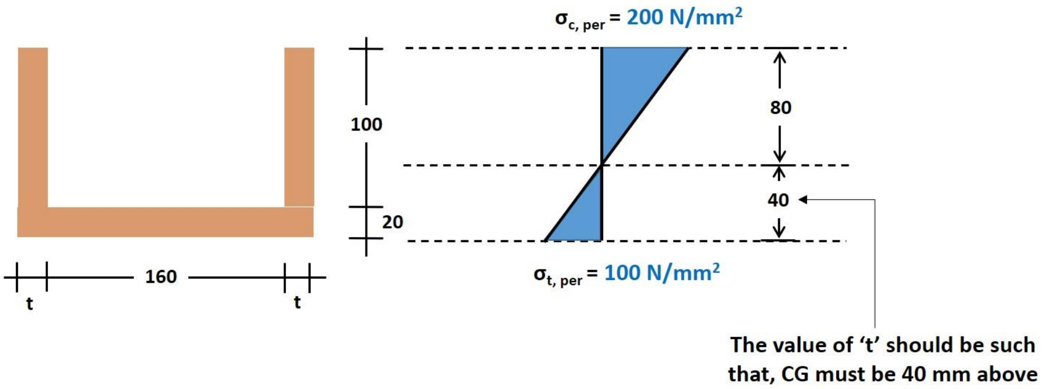


Figure 15: stresses in case of a channel section and its thickness

So, to achieve achieve $\sigma_{c\ permissible}$ as 200 and achieved $\sigma_{t\ permissible}$ as 100, the thickness 't' should be equal to 16mm

Now, let us see the last example with composite material. Let us suppose a wooden timber beam, and the timber beam of cross section 100 X 200 mm is subjected to a bending moment of 75 KN-m.

Then

$$I = \frac{1}{12} \times 100 \times 300^3 = 225 \times 10^6$$

The stress can be given by:

$$\sigma = \frac{M}{I} \times y_{max} = \frac{75 \times 10^6}{225 \times 10^6} \times 150 = 50N/mm^2$$

Then the stress distribution diagram can be given as:

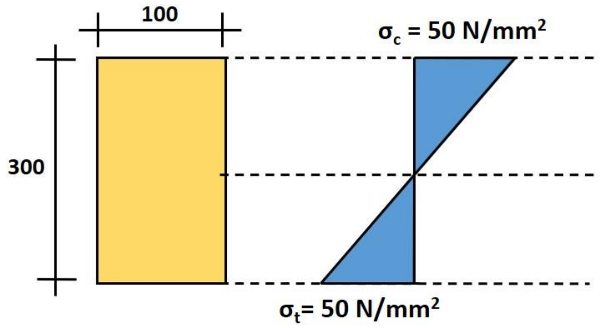


Figure 16: stress distribution diagram in a composite material beam section

On the other hand, if I put two steel reinforced plates with thickness 10mm each, on the top and bottom of this section, then, assuming Es = $2X10^5$ N/mm^2 and Et = $0.1X10^5$ N/mm^2, we will have two compatibility conditions. First is strain compatibility equation and second is radius of curvature compatibility equation, refer Figure 17.

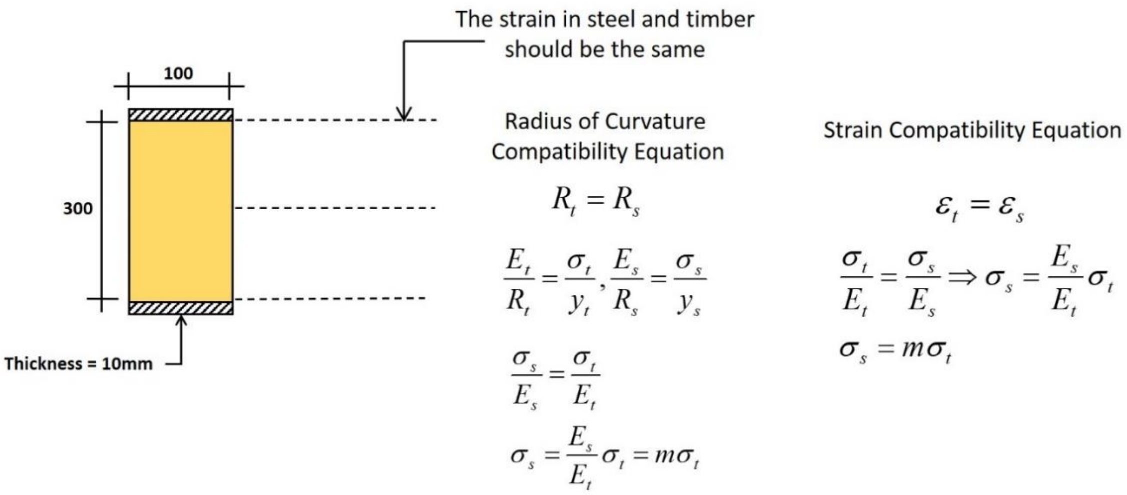


Figure 17: composite material beam section and compatibility equations

And from this strain compatibility, you may say that the stress in steel, σ_s must be equal to product of m and σ_t . What is m? If you remember m is E_s/E_t that is the modular ratio. If you go for the radius of curvature compatibility, because radius of curvature also has to be equal and these two interfaces of the timber and steel; otherwise one will slip with other. So, in that case also we can say that σ_s, the stress in steel is equal to modular ratio into the stress in timber. So, in that case what I have done is, our original section has to be enlarged to equivalent to timber sections where I increased the sections of the steel by multiplying the dimension of 100 by 20; and 20 is nothing but the modular ratio. So, now the equivalent timber section looks like as shown in Figure 18. The dimension 300 by 100 remains same, but at top and bottom there are two such enlarged portion of 20×100=2000.

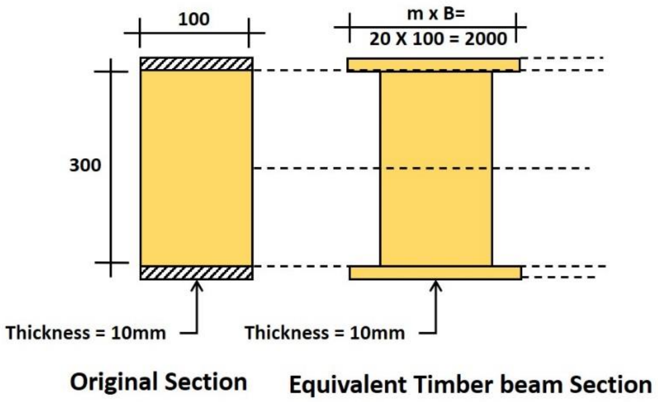


Figure 18: original and equivalent timber beam section

From that you can find out the moment of inertia, stress in steel at top, stress in timber at junction, and stress in steel at junction.

Moment of inertia of equivalent section:

$$I = \frac{1}{12} \times 100 \times 300^3 + \frac{1}{12} 2000 \times 10^3 + (2 \times 2000 \times 10) \times 155^2$$

$$I = 231.53 \times 10^6 mm^4$$

Stress in timber junction

$$\sigma_t = \frac{M}{I} \times y = \frac{75 \times 10^6}{231.53 \times 10^6} \times 150 = 48.6 N/mm^2$$

Stress in steel junction:

$$\sigma_s = m \times \sigma_t = 20 \times 48.6 = 972 N/mm^2$$

Stress in steel top:

$$\sigma_s = m \times \sigma_t = m \times \frac{M}{I} \times y = 20 \times \frac{75 \times 10^6}{231.53 \times 10^6} \times 160 = 1036.6 N/mm^2$$

These will be important for your RCC design of course, but RCC design is not included in this course.

The references taken for this lecture is:

- **Structure as Architecture** by Andrew W. Charleson, Elsevier Publication
- **Basic Structures for Engineers and Architects** by Philip Garrison, Blackwell Publisher
- **Structure and Architecture** by Meta Angus J. Macdonald, Elsevier Publication
- **Examples of Structural Analysis** by William M.C. McKenzie
- **Engineering Mechanics** by Timishenko and Young McGraw-Hill Publication
- **Strength of Materials** By B.C. Punmia, Ashok K.Jain & Arun K.Jain Laxmi Publication
- **Understanding Structures: *An Introduction to Structural Analysis*** By Meta A. Sozen & T. Ichinose, CRC Press

So, in conclusion we can say that the flange beam is effectively used as a main section which is very much important and very much effective; and the composite beam can be used to improve the moment of resistance or the capacity of the beam.

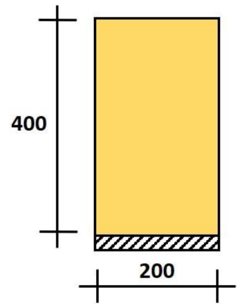
At the end of this lecture, I have given three questions as homework.

1. An I-Section is having following dimensions and act as a simply supported beam over a span of 12 meter. Estimate the maximum impose UDL it can carry. Assume the Self Weight of the beam as 1.2 KN/m. The permissible stress in compression and in tension is 250N/mm^2

   Section Dimension of I-Section: Top Flange: 150mm X 20mm
   Web: 600mm X 10mm
   Bottom Flange: 100mm X 10mm

2. Re-estimate the impose load (UDL) if two plates of 150mm X 20mm is fixed at top and bottom flanges. The self-weight and permissible stress of the beam remains unchanged.

3. A 200mm X 400mm wooden beam is further reinforced with a steel plate of 200mm X 15mm at bottom. Find the Moment of Resistance of the (i) Wooden beam only (ii) wooden beam with steel plate. The permissible bending stresses in steel and wood are 250 N/mm^2 and 150 N/mm^2 respectively. E_s = 2X10^5 N/mm^2 and E_t = 0.1X10^5 N/mm^2



So, thank you very much. This is the end of lecture 13.