Structural System in Architecture
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Lecture – 15
Theory of Column

Welcome to the NPTEL online certification course on Structural Systems in Architecture. This is module 3, and this is on structural mechanics; and we are in the last lecture on this module, the lecture number 15 in serial, and the topic of this lecture is theory of column.

So, in this particular lecture I am going to cover the following concepts:

- Introduction
- Classification of Column
- Euler's Theory of Long Column
- Concept of Equivalent Length
- Concept of Slenderness Ratio

The tentative learning objective of this particular lecture will be:

- Discuss the types of column and its structural implications.
- Outline the stability of long column.
- Apply the concept of slenderness ratio in analysis of column.

So, first let us go to the initial discussion on column. Columns are the vertical members also called pillars, and everybody knows what is that. Today in our society, there are buildings coming up here and there. They may be RCC building, sometimes it may be of steel, or wood. You might have seen these particular buildings coming up from the foundation to the roof. So, when constructed, first they construct the foundation and the columns.

Columns are nothing but the vertical element of the any structural systems and this is actually going to support the beams and the slab; and the prime objective of the column is to transmit the super structural load or load from the beam finally to the ground. It has to be finally transmitted through the foundation. Primarily, a column is a compression member in its nature. For example, if you have some load on a top of a particular stick, the stick acts as a column and that is definitely being under a compression.

It is not a tension member; but yes, depending upon the location of the column and depending upon the associated load between the two sides or the adjacent side or maybe there is a third side, column also experiences some bending. So, in case of the monolithic construction, when there is a two-side beam and the cross beams are monolithically casted with the column, there might occur some rotation in the column head. Due to the rotation at the column head, bending might occur; and such bending is uniaxial bending. Sometimes there might be biaxial bending too; but that is not in our scope.

In architecture columns also contributes strongly to aesthetics. It strongly acts as a visual element in elevations. Even in early civilizations from the Egyptian to Roman to the Greek, if you see even in the modern architecture also, columns have been used as prominent architectural element. Sometimes they are very dominating. For example, in Parthenon in Athens; and another one is the British Museum in London.

Parthenon, Athens

The British Museum, London

Figure 1: Parthenon, Athens and British Museum, London

Source: *https://www.ancient.eu/Athens/*, *https://interestingengineering.com/*

So, in those buildings if you see; the columns are huge and prominent. The columns give you an impression of monumental scale in an iconic structure.

So, again if you look into the history of architecture, then you can see that, there are different types of columns, based on its design, its capital, the proportions of length and shafts. There are three orders in the Greek column. They are Doric, Ionic and Corinthian order. I am not going to elaborate on that, but it shows the growth of design in different period and adoption of details and designs to achieve aesthetic satisfactions.

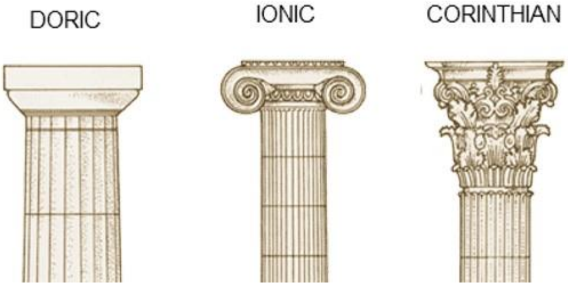


Figure 2: the three Greek orders of column

Source: https://imgur.com/

It also satisfies the situational or conditional use of different type and design of columns in respective buildings.

DORIC is the oldest column type. It is simplest and shortest; and looks heavy.

IONIC columns stand on a large base. The capital of the column is having simple form of decoration with two opposite ornamental spirals.

CORINTHIAN were the most having, most decorative and delicate capital. It is also extended to a larger portion below the capital.

In designs, there are always lot of experiments; sometimes the radius increases towards the feet, also different type of the capital came; capital of the column also changes because of the bearing length, because it also has to take care of the higher amount of load. As a result, different orders came.

So, from that point of view if we suddenly increase the capital, it will look very odd. So, you have to make some ornamentations to camouflage that in such a way that it creates a proportional beauty on the column.

Now, let us see some examples of modern columns. The photographs shown in Figure 3, are some examples of modern columns, which I have taken from online sources. They are completely different from what we have seen earlier, in case of early ages. Now, in the modern era, the columns are very slick, having different type of definitions, different type of dimensions and expressing a pleasing visual appearance. On the other hand, by virtue of these element, it will also create an amazing and wonderful interior experience with different expressions of the column capitals, like with different proportions, mushrooming or tree kind of designs.

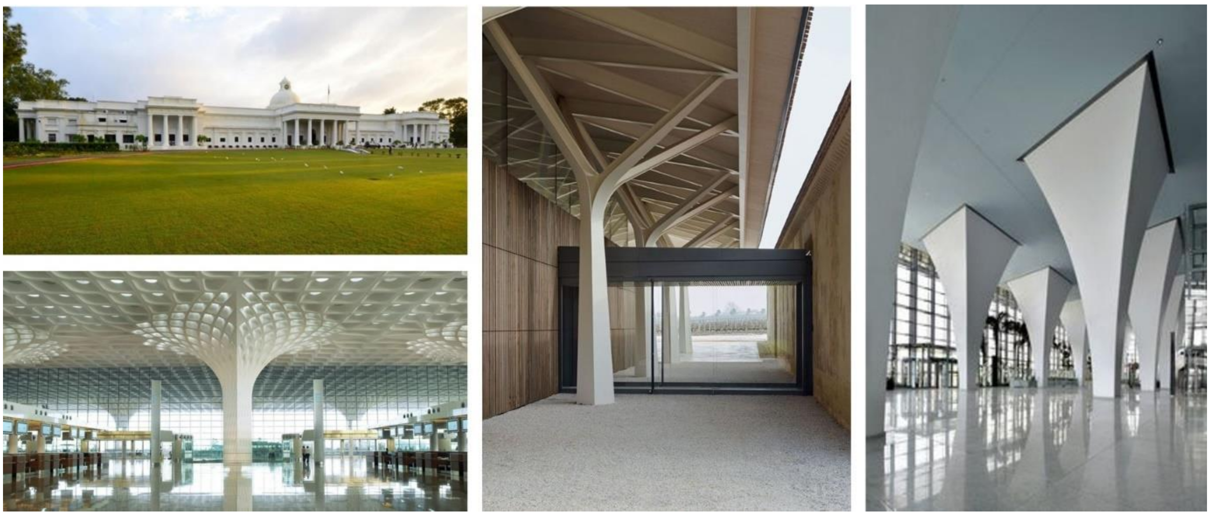


Figure 3: different modern columns

Source: https://in.pinterest.com/, https://www.dreamstime.com/, https://www.architectmagazine.com/, https://www.thoughtco.com/, https://www.thoughtco

Now, the classification of the column. Columns can be classified in many ways, and structurally there are two ways to classify the columns. The first is the short column and the long column. Short and long column are defined with the two dimensions of a column; one is the lateral dimension that is the thickness or maybe the diameter or maybe the cross-sections; these are lateral dimensions; and another dimension is the length. So, these two dimensions determine that whether the column is a long or short. If the ratio between the length and the lateral dimension, let's say 'L' is the length and 'D' is the diameter, and ratio $\frac{L}{D}$ is less than 12, then it is short column, and it will look very bulky; on the other hand, if the ratio is more than 12, then it is long column, and long column will appear slender. The short column is subjected to the compressive stress and it fails due to crashing. On the other hand, the long columns are subjected to critical stress and it fails due to buckling.

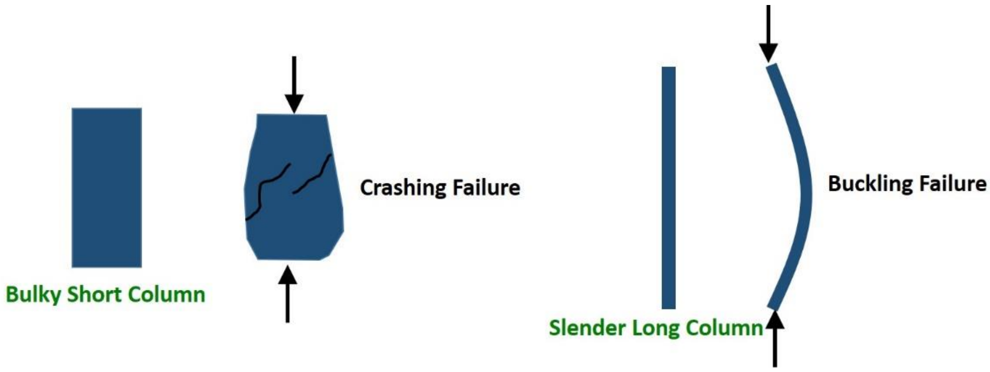


Figure 4: short and long columns and its failure

There is another way to classify the columns, that is braced column and the unbraced column. The braced columns have an additional member which will take care of the stability against lateral loads. The additional member is either a single diagonal member or maybe there is a brick wall. If it is an infill brick wall, it will act as a bracing or can take any kind of a lateral load, like wind or maybe some earthquake load.

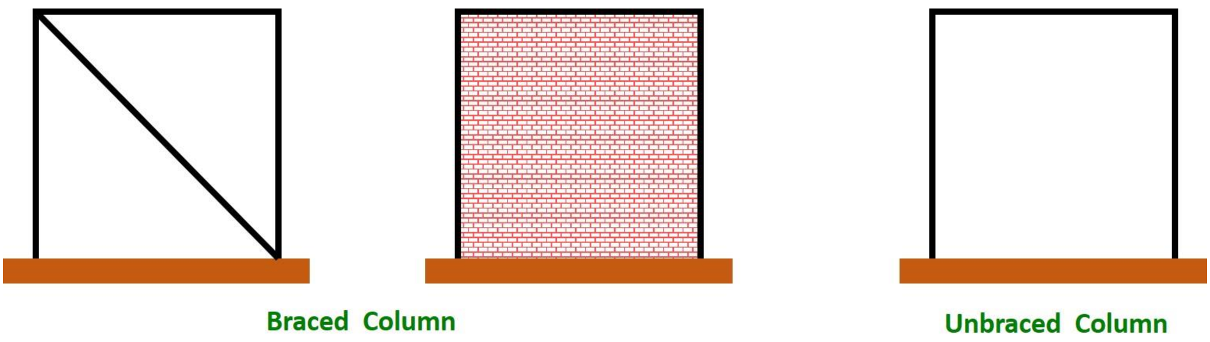


Figure 5: braced and unbraced column

On the other hand, sometimes columns are unbraced, there is nothing, it is empty. There is only the beams and the columns, no additional members exist, no filling or brick wall or neither there is diagonal member. They are the unbraced column. So, we have to make sure that the column itself is so strong enough to tackle the lateral load.

Now, let us go to the Euler's theory. Euler's was a mathematician and he gave a theory on long column and he developed one mathematical equation for the long column. Till today we use that equation to design and analyse the columns.

The long column fails due to buckling and eventually, we have to find out the critical load. The critical load if we see conceptually, then if there is a very long and straight column, and if you just gradually increase the load on it, then at a particular point of time the column will buckle. The buckling will not be gradual, it will be sudden, within a fraction of second it will fail. If you just decrease the load, with slight decrease will be stable, it will not buckle. So, this particular point, with that amount of load which will result into buckling of the column is the critical load; and with that load the column becomes very, very unstable. So, to gain the stability, you have to lower the critical load. Therefore, we have to understand and we find out how much is the critical load of the column.

There are four major parameters which decides the critical load of a column, and they are:

1. Geometric property of the column cross section.
2. Property of column material.
3. Length of the column.
4. Support condition at the ends of the column.

There are different end conditions for columns. The columns can be with:

1. hinge and hinge at both the ends, it can be definitely constructed in steel but not in RCC;
2. fixed and the free
3. fixed and fixed; and
4. fixed and hinged

In the first case, at the hinge point there will be no moment, moment 0, and a slope will be generated from both ends as shown in Figure 6; and the equation of deflection is also given. In case of a fixed and free, there will be 0 slope and it will bend like as in the figure. The equation of deflection is also given. Similarly, for fixed and fixed ends and for fixed and hinged it is also given in Figure 6.

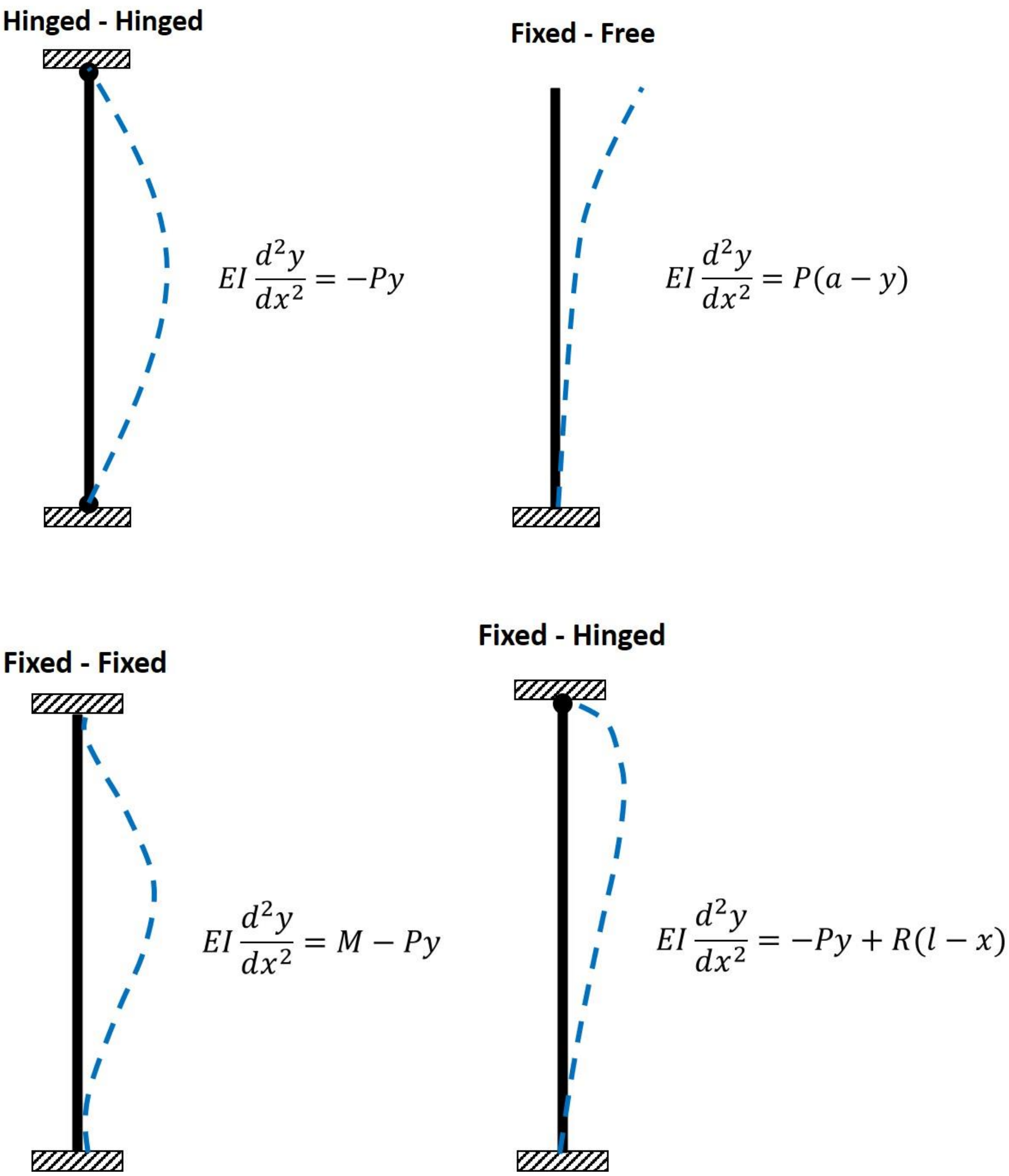


Figure 6: deflection equations of column

The differential equations for deflection are really tough. So, I am not going to solve those differential equation. There are books that you can go through and solve the same.

The general form of the standard differential equation is given by:

$$EI\frac{d^2y}{dx^2} = Ay + B$$

The solution of the standard differential equation for different conditions is given by:

Table 1: the Euler's Critical Load and Equivalent Length of column with different support conditions

Support Condition	**Euler's Critical Load**	**Equivalent Length**
Hinge - Hinge	$P_E = \frac{\pi^2 EI}{l^2}$	$L_E = l$

Fixed – Free	$P_E = \frac{\pi^2 EI}{4l^2}$	$L_E = 2l$
Fixed – Fixed	$P_E = \frac{4\pi^2 EI}{l^2}$	$L_E = l/2$
Fixed - Hinged	$P_E = \frac{2\pi^2 EI}{l^2}$	$L_E = l/\sqrt{2}$

When there are various end conditions or boundary conditions, we can find out the Euler's critical load, which is our intention to find. Here if we see:

PE = critical load of the column

E = Young modulus

I = moment of inertia

l = actual length of the column

L_E = equivalent length of the column

The PE and L_E varies with change in end conditions. The E is different for different materials, for example, for RCC it is 1×10^4, in case of steel 2×10^5

If you observe it, then you can see that there are always some numbers in each formula, like ¼, 4,2 etc. So, we can actually rewrite this all the Euler's critical equation or critical node equation as:

$$P_E = \frac{\pi^2 EI}{L_E^2}$$

So, you have to remember this formula and you have to remember these four such equivalent length and the length comparison for the 4 different type of support system, then we can solve the problem or we can actually go ahead.

Now, we have to understand what is equivalent length?

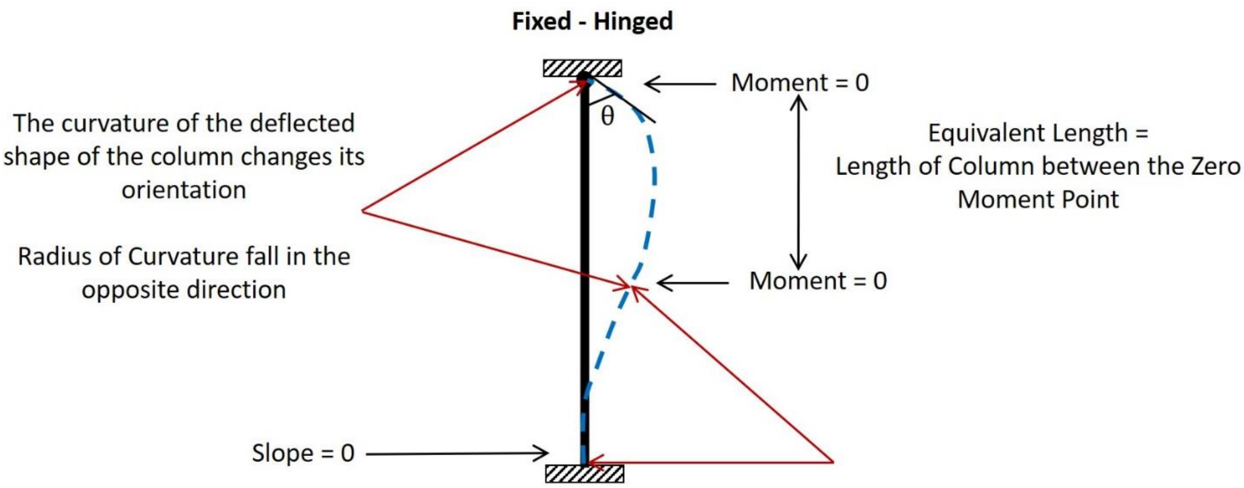


Figure 7: Equivalent length of a column with fixed and hinged support

So, I have taken a condition of fixed and hinge, as it is a fixed and hinge after deformation the slope will have some value, say θ, and θ tis not equal to 0; but here in the fixed end θ is going to be 0, so the deflected curvature will be as shown in Figure 7, and the slope will be 0 at fixed end.

When I see the deflection curvature, then I can see that the curvature of the deflected shape of the column changes its orientation. The direction of radius of curvature at fixed end and at the free end is different. That means at a particular point the curvature changes, as shown in Figure 7. Curvature changes means, if you remember second week lecture then, that is the point of contraflexure and the moment will also change. So, if you just put it in the horizontal way, it is from the hogging to sagging something like that. So, we may say that at point where the curvature changes, the moment will be 0. Why will this moment be 0? Because this is the point where the moment changes, as curvature changes this point changes from positive to negative, so definitely there is a sign change in the moment. Where there is this change of curvature, that length; the distance between this moment 0 points is nothing but the equivalent length of the column, refer Figure 7. So, first we have to find out where is the moment 0 point and we have to find out that the length between them.

So, if it is a hinge and hinge, we know the moment 0 will occur at the hinge itself, so the equivalent length will be same as actual length. The moment 0 points are shown in red dots in the Figure 7. In fixed and free case, the moment 0 point will be mirror image to each other, and the fictitious length is twice of ‘l’’. In the third case; this is fixed and fixed, the equivalent length will be 0.5 l. In case of fixed and hinged it is 0.7l.

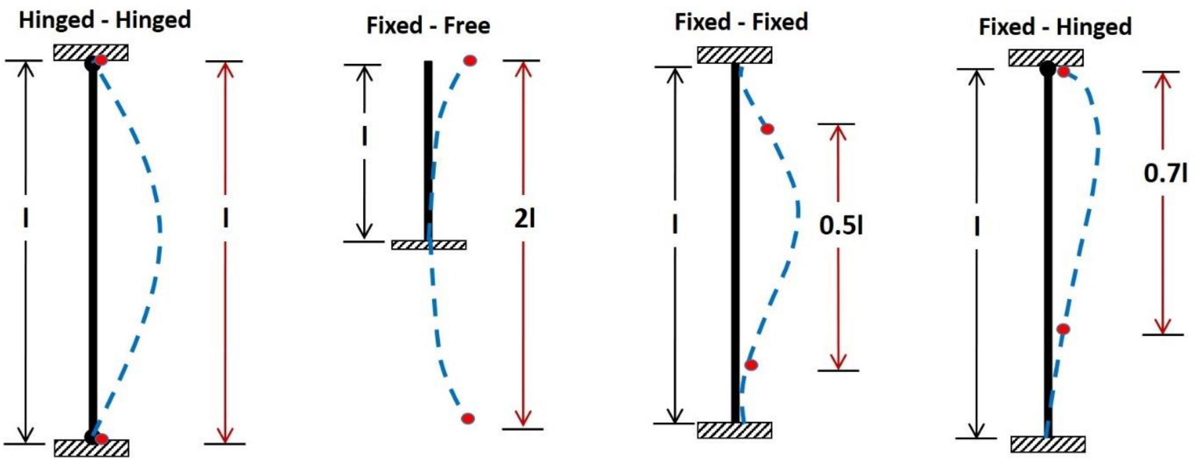


Figure 8: equivalent length of columns with different supports

So, we now have understood about the whole theory and philosophy associated with the equivalent length and the original length of a column.

Next, let us go to the small problem. I am considering a column of 100×300, as shown in Figure 9; and I have I_{XX} and I_{YY} values. I_{xx} and I_{yy} values. The length of the column is 3 meters; and value of E for concrete is $1 \times 10^4 N/mm^2$. Here out of these two, I have to consider the minimum one. Because the column will buckle in the weakest direction. So, computing I_{xx} and I_{yy}:

$$I_{XX} = \frac{1}{12} \times 100 \times 300^3 = 225 \times 10^6\ mm^4$$

$$I_{YY} = \frac{1}{12} \times 300 \times 100^3 = 25 \times 10^6\ mm^4$$

$$I_{min} = I_{YY} = 25 \times 10^6\ mm^4$$

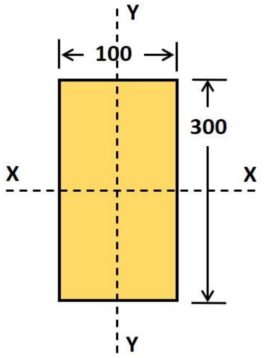


Figure 9: plan of the column

Now, calculating P_E:

$$P_E = \frac{\pi^2 EI}{L_E^2}$$

First, computing with consideration of both ends hinged:

Given that $l = 3000mm$ and in case of both ends hinged, $L_E = l$

So,

$$P_E = \frac{\pi^2 \times 1 \times 10^4 \times 25 \times 10^6}{3000^2}$$

$$P_E = 274156\ N = 274\ KN$$

Second, let us consider with both ends fixed:

Given that $l = 3000mm$ and in case of both ends fixed, $L_E = 0.5l = 1500\ mm$

So,

$$P_E = \frac{\pi^2 \times 1 \times 10^4 \times 25 \times 10^6}{1500^2} = 1097\ KN$$

Now, if we compare both the conditions then we can see that, when both the ends are fixed, the column do have more capacity of taking load; because l is decreasing, that is the equivalent length is decreasing.

When both ends are hinged, the column can tackle 274 KN, after that it will buckle. Whereas, when both ends are fixed, the column can tackle 1097 KN of load; and may be beyond that it will fail or buckle and it will become unstable.

Next, let us consider another problem with similar conditions. The problem states that:

Length of the Column is 3 meter and both the ends are hinged. The Column needs to carry 500 KN. Assuming E = 1 X 10^4 N/mm^2; Find the cross-section dimension of column with B:D =1:3

As both the ends are hinged, $L_E = l = 3000\ mm$.

Now, computing it with using the same formulas:

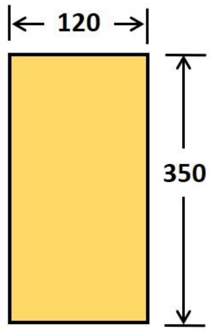


Figure 10: the new dimension of the column

$$P_E = \frac{\pi^2 EI}{L_E^2}$$

$$\frac{\pi^2 \times 1 \times 10^4 \times I}{3000^2} = 500 \times 10^3$$

$$I = 45.59 \times 10^6\ mm^4$$

Then, keeping $B = x\ and\ D = 3x$

$$\frac{1}{12} \times 3x^4 = 45.59 \times 10^6\ mm^4$$

Solving this, we get

$$x = B = 116\ mm, \qquad and \quad D = 348\ mm$$

So, approximating it, the design dimension of the column will be considered as: $120\ mm \times 350\ mm$.

Now, I can see that the if dimension of the column instead of 100 by 300, is 120 by 350, it can easily take the load of 500 KN.

Next, we will discuss the concept of the slenderness ratio. We have found out the Euler's critical load.

$$P_E = \frac{\pi^2 EI}{L_E^2}$$

So, now based on this particular critical load, I want to find out the critical stress, so stress is nothing but the load by area, so I divide the critical load by the cross-section area of the column that is A.

$$\sigma_{cr} = \frac{P_E}{A}$$

$$\sigma_{cr} = \frac{\pi^2 EI}{AL_E^2} = \frac{\frac{\pi^2 E}{L_E^2}}{\frac{I}{A}}$$

Then, let

$$\sqrt{\frac{I}{A}} = r$$

And this is called Radius of Gyration.

Then,

$$\frac{\frac{L^2{}_E}{I}}{A} = \frac{L_E^2}{r} = \lambda^2$$

Finally;

$$\sigma_{cr} = \frac{\pi^2 E}{\lambda^2}$$

Then

$$\sqrt{\frac{I}{A}} = r$$

This is called as Radius of gyration. Radius of gyration of a body about an axis of rotation is defined as the radial distance to a point which would have a moment of inertia the same as the body's actual distribution of mass, if the total mass of the body were concentrated.

Then, λ is the slenderness ratio.

$$\lambda = \frac{L_E}{r}$$

If λ is very high, the column is long column and λ is smaller that means, it is not slender it is bulky. Radius of gyration is smaller means it is very cosy, very small section, and if radius of gyration is very high then it’s a fat column.

Now, let us see how this lambda values will help us. So, I am comparing the strength of two sections; one is a solid circular section and another one is a hollow section, as shown in Figure 11.

So, the area I have computed almost similar area, I have computed the I, the moment of inertia of the hollow and the solid sections, I also computed the radius of gyration.

So, I can say that this particular 100 mm diameter solid circular section, the mass is concentrated almost about 25 mm around the center. On the other hand, in the case of hollow section, I may say that it has similar comparable area but the mass is concentrated almost about 53.29 millimetre away from the centre. So, it is much wider. This ring or hollow section may have almost same area with solid section, but here the mass is distributed towards outward. Whereas, in case of a solid section, it is very cosy. Then I have computed the r value. If I assume that here the column is 2 meter long, and the E value, the Young modulus is 2×10^5, then I can find out the λ value; which is 80 for solid section, and 37.5 for hollow section.

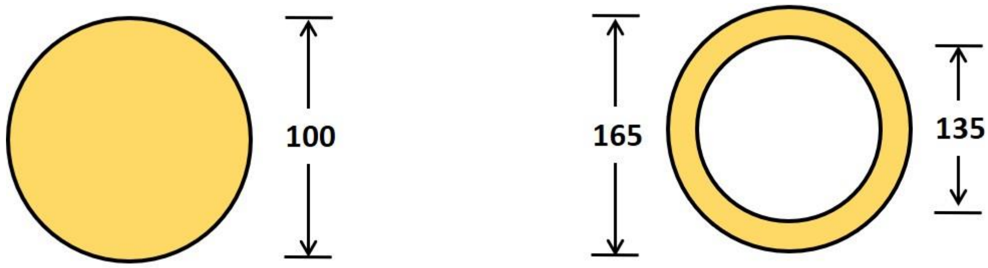


Figure 11: a solid and a hollow column

$$A_S = \frac{\pi}{4}(100)^2 = 7854mm^2 \qquad A_H = \frac{\pi}{4}(165^2 - 135^2) = 7069mm^2$$

$$I_S = \frac{\pi}{64}(100)^4 = 4.9 \times 10^6 mm^4 \qquad I_H = \frac{\pi}{64}(165^4 - 135^4) = 20 \times 10^6 mm^4$$

$$r_S = \sqrt{\frac{4.9 \times 10^6}{7854}} = 25mm \qquad r_H = \sqrt{\frac{20 \times 10^6}{7069}} = 53.29mm$$

Now, Assuming Effective Length of Column as 2-meter & E = 2X10^5 N/mm^2

$$\lambda_S = \frac{2000}{25} = 80 \qquad \lambda_S = \frac{2000}{53.29} = 37.5$$

$$\sigma_S = \frac{\pi^2 \times 2 \times 10^5}{80^2} = 380N/mm^2 \qquad \sigma_H = \frac{\pi^2 \times 2 \times 10^5}{37.5^2} = 1401N/mm^2$$

So, I may say that, in this particular column, the solid column is much longer as compared to the hollow column. Therefore, finally the critical stress also can be found out, so in case of the solid column I computed the critical stress is $380\ N/mm^2$, whereas in case of the hollow it is $1401\ N/mm^2$. If I now multiply this with the area, I can find out the critical load that PE.

So, we can say that, if we use almost same amount of material, it is always better to have a hollow column and distribute the mass outward such a way that the slenderness ratio is lower, why it is lower, because I have the higher amount of radius of gyration.

Next, I have computed the slenderness ratio for steel and aluminium, putting their values of E, 2×10^5 and 0.8×10^5 respectively. Then I have computed that how the slenderness ratio has drastically decreased the critical stress. Refer Figure 12. So, I may say that it depends upon the material, E values and also upon the lambda values.

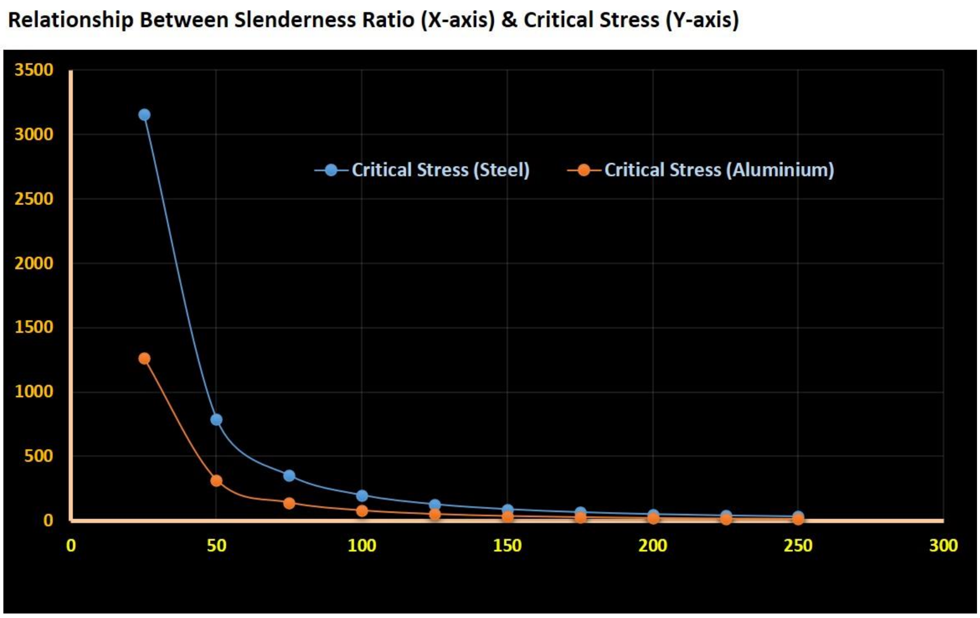


Figure 12: relation between slenderness ratio and critical stress

So, now if you see the Egyptian temple, for example the temple in Karnak or maybe the Luxor or maybe the Abu Simbel, anywhere, they always followed a particular pattern or particular formation. The central aisle will have a clerestory, the clerestory allows the daylight, so it has to be higher, refer Figure 13.

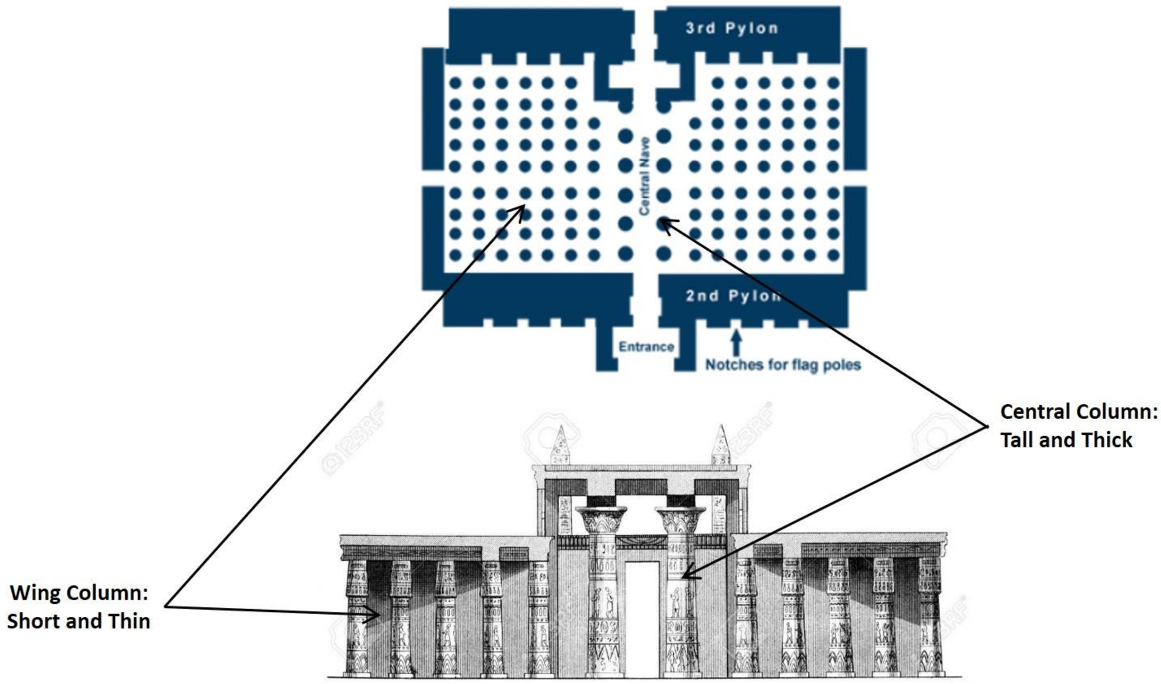


Figure 13: typical Egyptian temple

So, you need a higher column and these columns will have papyrus capital, and others will have the lotus capital, as you know from your history of architecture. So, you have two types of column; the central one is high and the wings or the flanks are the lower or smaller column.

In the plan if you see carefully then you will notice the variations in dimensions of the columns. The columns at central aisle is with higher diameter and in the flank the column dimensions are less. Why so? Because you have to understand that if you want to put the higher, the longer column you have to increase the diameter to take care of your critical stress, take care of the slenderness ratio and take care of the P_E that is the critical values of the forces.

So, we will see one small problem. The problem states that:

A 150 mm thick square slab of 5m X 5m dimension is supported by four columns of size 300mm X 300mm. The RCC slab supports a water tank of 15000 litre capacity. The length of the column is 3-meter. Assume both the ends of the column are fixed. The unit weight of RCC is 25KN/m^3. Estimate the Actual and Critical Stress of each column.

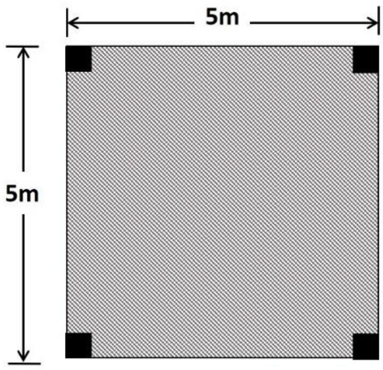


Figure 14: the plan of the concrete slab

So, I have found out how much is the total load on each column and I also find out the what is the compressive stress and then I again found out the how much is the critical stress, I found out the I vale, A value and all those, the dimensions of the columns are also given 300 by 300, I found out the area and the moment of inertia I by A under root.

Load of RCC Slab: $(5 \times 5 \times 0.15) \times 25 \ = \ 93.75KN$

Load of Water:

$$\frac{(15000 \times 9.81)}{1000} = \ 147.15\ KN$$

Total Load: $(93.75 + 147.15) \ = \ 240.9KN$

Load in each RCC Column: $0.25 \times \ 240.9 \ = \ 60.225KN$

Actual Compressive Stress:

$$\frac{60.225 \times 1000}{300 \times 300} = 0.67\ N/mm^2$$

Moment of Inertia of the Column:

$$I = \frac{1}{12} \times 300^4 = 675 \times 106\ mm^4$$

Area of Column: $300 \times 300 = 90000\ mm^2$

Effective Length of the Column $= 0.5 \times 3000 = 1500m\ (both\ end\ Fixed)$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{657 \times 10^6}{90000}} = 86.6mm$$

$$\lambda = \frac{L_E}{r} = \frac{1500}{86.6} = 17.32$$

$$\sigma_{cr} = \frac{\pi^2 E}{\lambda^2} = \frac{\pi^2 \times 1 \times 10^4}{(17.32)^2} = 329N/mm^2$$

Here the compressive stress for 3000mm tall column is $329N/mm^2$, but my actual compressive strength is very less, which is $0.67\ N/mm^2$. So, it is very much safe.

Now, the next condition we are assuming that the column length is 8 meters instead of 3 meters. So here length is changing and the λ will change.

Then computing this:

Moment of Inertia of the Column:

$$I = \frac{1}{12} \times 300^4 = 675 \times 106\ mm^4$$

Area of Column: $300 \times 300 = 90000\ mm^2$

Effective Length of the Column $= 0.5 \times 8000 = 4000m\ (both\ end\ Fixed)$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{657 \times 10^6}{90000}} = 86.6mm$$

$$\lambda = \frac{L_E}{r} = \frac{4000}{86.6} = 46.18$$

$$\sigma_{cr} = \frac{\pi^2 E}{\lambda^2} = \frac{\pi^2 \times 1 \times 10^4}{(46.18)^2} = 46.3\ N/mm^2$$

So, in earlier case the critical stress was $329N/mm^2$, but when height is changed to 8m, the critical stress is changed to $46.3\ N/mm^2$

In the Figure 16 if you see, the structure that is the image of a water tank, some intermediate beams are introduced. But there is no slab, it is not holding anything. So, do we need that? Are they unnecessary?

Yes! we need them and they are nit unnecessary. If we do not put them, then these long columns will buckle. These intermediate beams create the small diversion of the long columns and effective length of the column is reduced, one sense, it also acts as bracing.

Figure 15: references of water tank designs and its elements

. Now, you can calculate the effective length of the columns of these images, which is now maybe 2 meter or 3 meters. But here it is 12-meter, 15 meters? Definitely with the huge amount of load of water, it will buckle. So, for better stability reduce the slenderness ratio of the columns.

For this lecture, the references taken are:

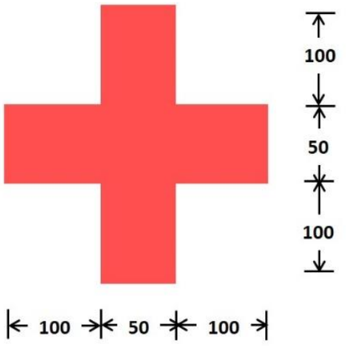
- **Structure as Architecture** By Andrew W. Charleson, Elsevier Publication
- **Basic Structures for Engineers and Architects** By Philip Garrison, Blackwell Publisher
- **Structure and Architecture** By Meta Angus J. Macdonald, Elsevier Publication
- **Examples of Structural Analysis** By William M.C. McKenzie
- **Engineering Mechanics** by Timishenko and Young McGraw-Hill Publication
- **Strength of Materials** By B.C. Punmia, Ashok K.Jain & Arun K.Jain Laxmi Publication
- **Understanding Structures:** ***An Introduction to Structural Analysis*** By Meta A. Sozen & T. Ichinose, CRC Press

In conclusion, I can say that the column can be classified into the short and long column. The shear is predominant for the short, buckling is in the long, slenderness ratio is a prime geometric factor for designing a column. So, in the next lecture we will go to the frame structure analysis and design.

Next, I have two homework for you.

1. Find the critical load and critical stresses for the column section given below for following four conditions of supports. The length of the column is 3-meter. E = $2X10^6$ N/mm^2.

    i. Both ends are Hinged
    ii. Both ends are Fixed
    iii. One end fixed and other is fixed
    iv. One end fixed and other is free



2. Compare the ratio of strength of a solid steel column to that of a hollow of the same cross section area. The internal diameter of the hollow column is 0.75 times the external diameter. Columns are of same length hinged at both the ends.

So, try out this two homework.

Thank you very much.