**Structural System in Architecture**
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**Lecture – 18**
**Indeterminate Beams and Frames**

Welcome to the NPTEL online certification course on Structural Systems in Architecture. So, today we are in the lecture number 18 of module 4, which is about Intermediate Beams and Frames. Earlier, we have already discussed about the intermediate beams and also the deflection of beams.

# Concepts Covered

- Analysis of Fixed Beam
- BMD and Deflected Shape
- Portal Frame

# Learning Objectives

- Analyzing of Fixed Beam.
- Evaluating the Bending Moment diagram of continuous beam.
- Assessing the deflected shape of continuous beam from BMD.
- Introducing the Portal Frame.

# Analysis of Fixed Beams

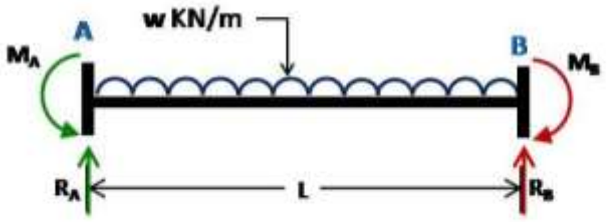



Figure 1 Fixed beam with UDL

We have,

Unknown Reaction = 2+2

Static Equilibrium equation available = 2

Degree of Indeterminacy of the beam = 2

Therefore, *two* Compatibility Equations are required to solve the third & fourth unknown reactions, and let it be Reaction at B ($R_B$ & $M_B$ )

To solve this, let us disintegrate the given beam in the following manner as shown in the Figure 2.

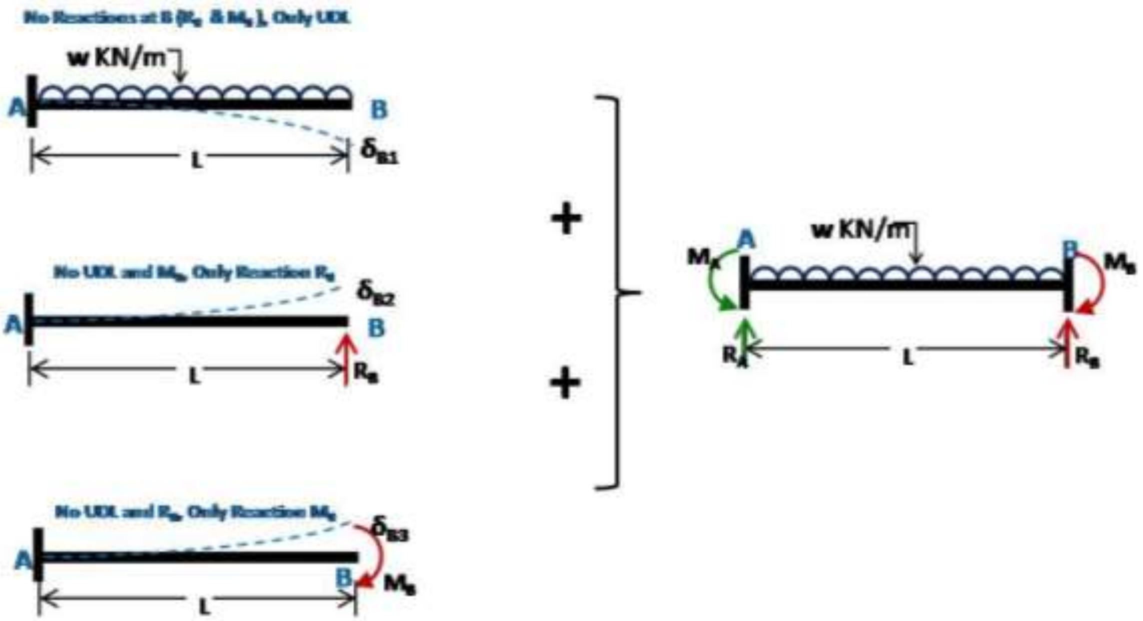



Figure 2 Disintegration of the given beam

Then we have,

$$\delta_{B1} = \frac{wL^4}{8EI} (\downarrow)$$

$$\delta_{B2} = \frac{R_B L^3}{3EI} (\uparrow)$$

And, $$\delta_{B3} = \frac{ML^2}{2EI} (\uparrow)$$

But we know that,

Deflection at B = 0

So, $\delta_{B1} = \delta_{B2} + \delta_{B3}$

i.e., $\frac{R_B L^3}{3EI} + \frac{M_B L^2}{2EI} = \frac{wL^4}{8EI}$

i.e., $\frac{R_B L}{3} + \frac{M_B}{2} = \frac{wL^2}{8}$

i.e., $8R_B L + 12M_B = 3wL^2$ [First compatibility equation]

Also we have,

$$\theta_{B1} = \frac{wL^3}{6EI}$$

$$\theta_{B2} = \frac{R_B L^2}{2EI}$$

And, $\theta_{B3} = \frac{ML}{EI}$

Again we know that,

Slope at B = 0

So, $\theta_{B1} = \theta_{B2} + \theta_{B3}$

i.e., $\frac{R_B L^2}{2EI} + \frac{M_B L}{EI} = \frac{wL^3}{6EI}$

i.e., $\frac{R_B L}{2} + M_B = \frac{wL^2}{6}$

i.e., $3R_B L + 6M_B = wL^2$ [Second compatibility equation]

Next,

Solving both the compatibility equations (multiplying the latter with 2 and the subtracting the resulting equation from the former) we get,

$$2R_B L = wL^2$$

i.e., $R_B = R_A = \frac{wL}{2}$ (↑) [Due to symmetry]

Now, substituting the value of $R_B$ in the first compatibility equation we get,

$$8R_B L + 12M_B = 3wL^2$$

i.e., $\left(8 \times \frac{wL}{2} \times L\right) + 12M_B = 3wL^2$

i.e., $M_B = M_A = -\frac{wL^2}{12}$ [hogging] [Due to symmetry]

Then,

Bending Moment at any section ‘x’ from end-A, $M_x = \frac{wL}{2}x - \frac{wL^2}{12} - \frac{w}{2}x^2$

At mid-span, x = L/2

$$M_{mid} = \frac{wL}{2}\left(\frac{L}{2}\right) - \frac{wL^2}{12} - \frac{w}{2}\left(\frac{L}{2}\right)^2 = +\frac{wL^2}{24} \text{ (Sagging)}$$

Finding the location of zero Bending Moment:

$$M_x = \frac{wL}{2}x - \frac{wL^2}{12} - \frac{w}{2}x^2 = 0$$

i.e., $6x^2 - 6Lx + L^2 = 0$

i.e., $x = \frac{6L \pm \sqrt{36L^2 - 24L^2}}{12} = 0.211L$

Therefore, the resulting BMD is given in the Figure

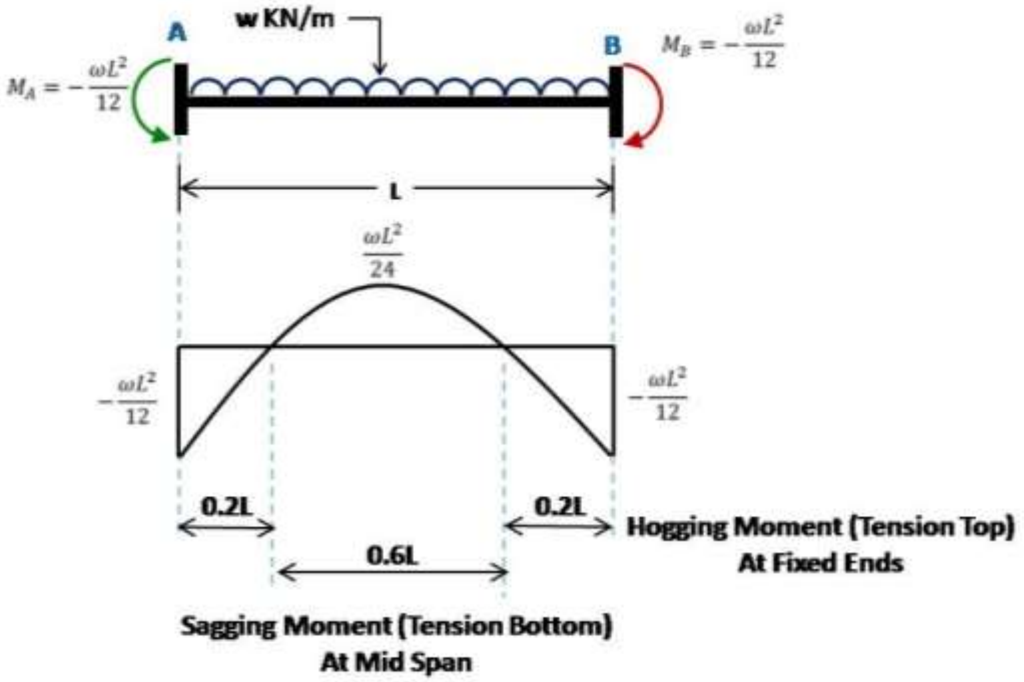



Figure 3 BMD of the given beam

Hence, the deformed beam would look as it is shown in the Figure 4.

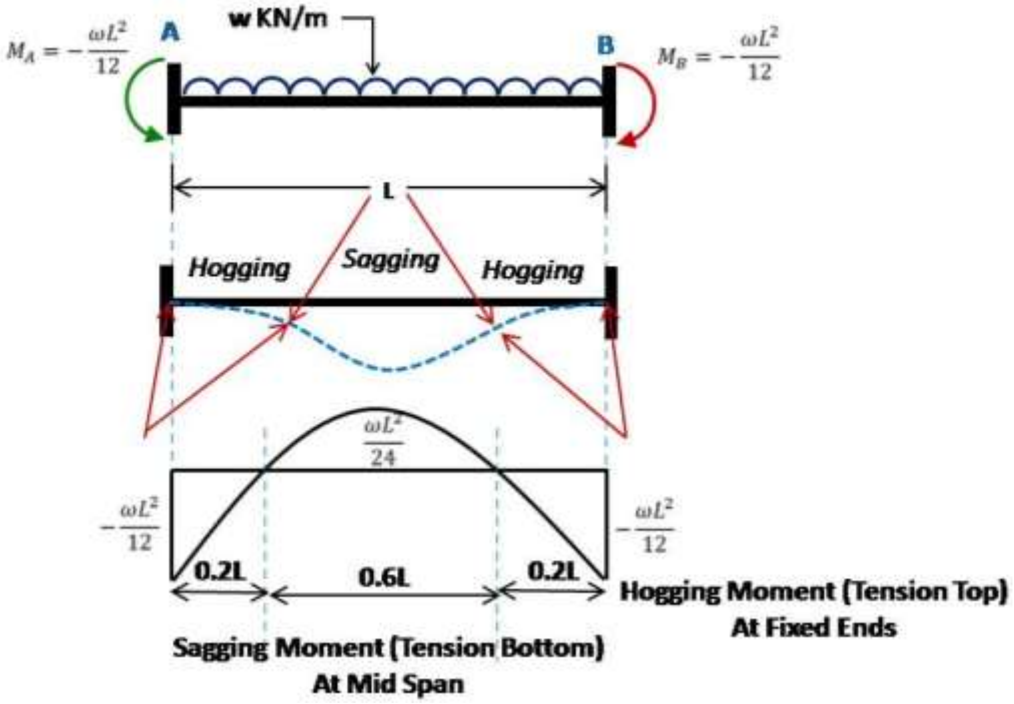



Figure 4 Shape of beam deflection



Then,

As a beam is likely to develop cracks at the tension zones, the position of the cracks for the given beam is shown in the Figure 5. Thus the reinforcement has to be provided in these areas perpendicular to the direction of cracks. As a designer you need to calculate how much amount of reinforcement is required in these areas so that the steel can take up the excess tensile stress over here. So that is why it is important to find the magnitude of bending moment and along with that you also need to find where it is hogging and where it is sagging kind of moment.

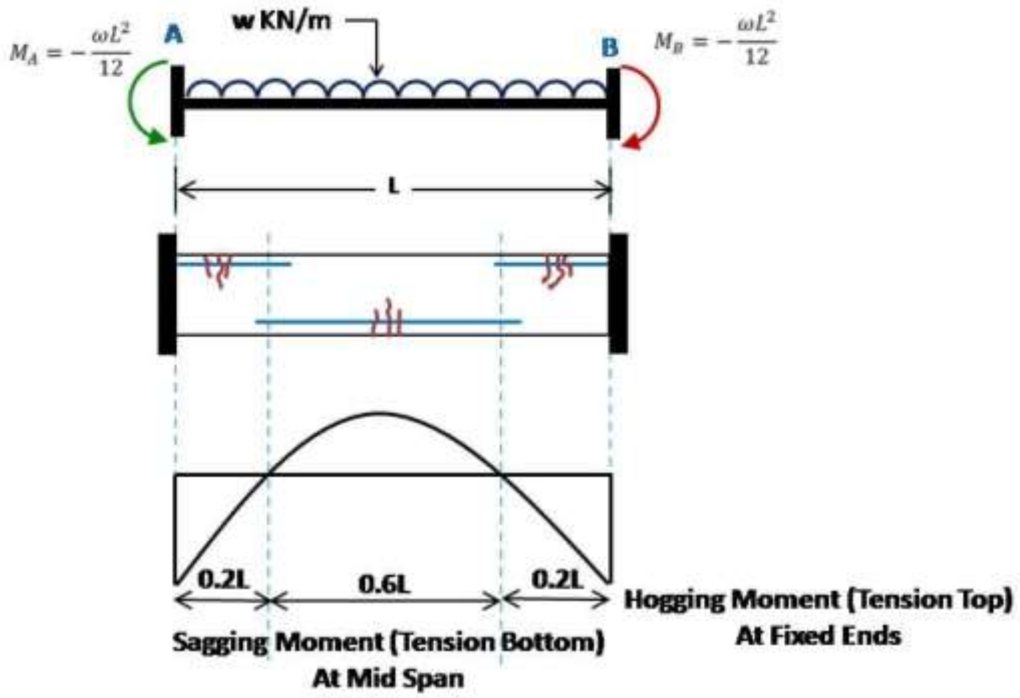



Figure 5 Position of cracks in the given beam

## Important Formulae

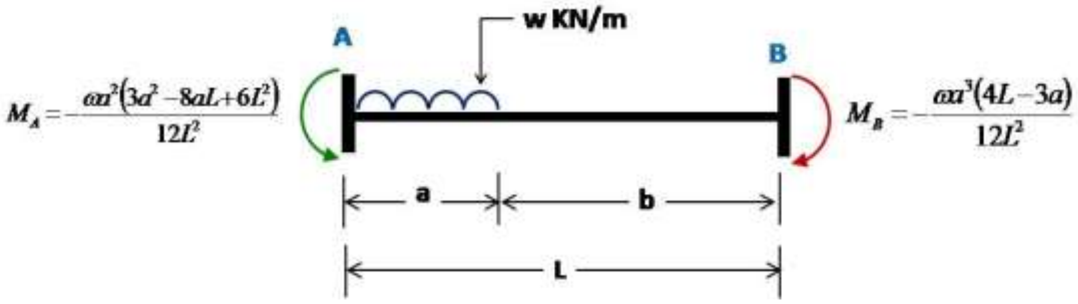



Figure 6 Case-1: Fixed beam partly loaded with UDL

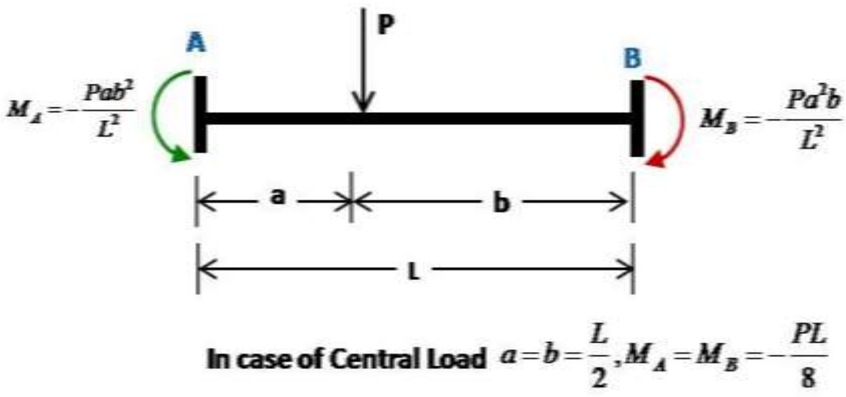



Figure 7 Case-2: Fixed beam subjected to point load

## BMD and Deflected Shape

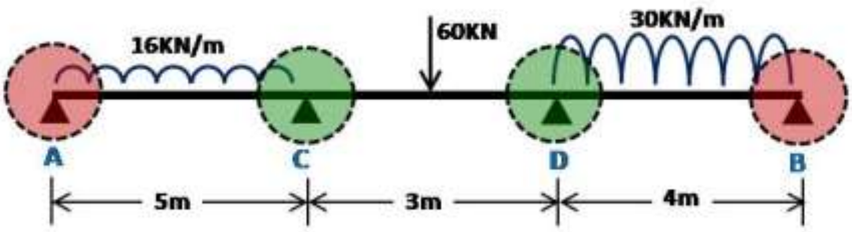



Figure 8 Continuous beam loaded with UDL and Point Load

To explain this concept we have a continuous beam here in the Figure 8, which is loaded with UDLs of different intensities at both the ends and a point load at the mid-span. Also, as evident from the figure, the spans of each of the sections are different.

Now,

As the ends A and B of the beam consist of hinged support, there'll be a sagging kind of a moment here as shown below in the Figure 9. And as the beam is continuous, there'd be a hogging kind of a moment at the supports C and D as shown below in the Figure 10.

Figure 9 Sagging(+ve) moment at the ends A and B

Figure 10 Hogging(-ve) moment at the mid-supports C and D

Next,

You'll encounter two kinds of moments in the given beam, viz. the support moments and the span moments. Considering the support moments, we know that the moments at the ends A and B will be 0. And the moments at C and D will be hogging moments for which you need to find the values by solving the unknowns. Here, we're taking the assumed values just to explain the concept to you. So suppose the moments at C and D are 20 and 40 respectively. Then the resulting BMD may be drawn as given in the Figure 11.

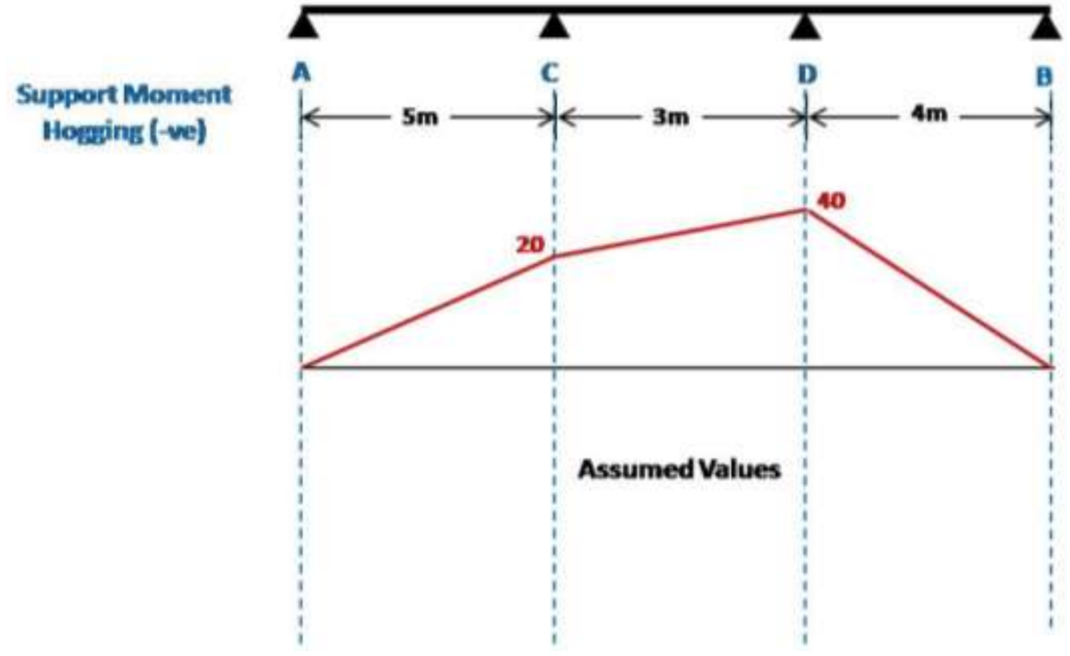



Figure 11 Support moments

In addition, for finding the span moments, each of the sections of the beam viz., AC, CD and DB maybe considered as simply supported. Nonetheless, the ends A and B in the given beam are already hinged, but had they been fixed, even then you may consider them as simply supported for the ease of computation. However, the mid-supports C and D could never have been fixed as those are parts of a continuous beam.

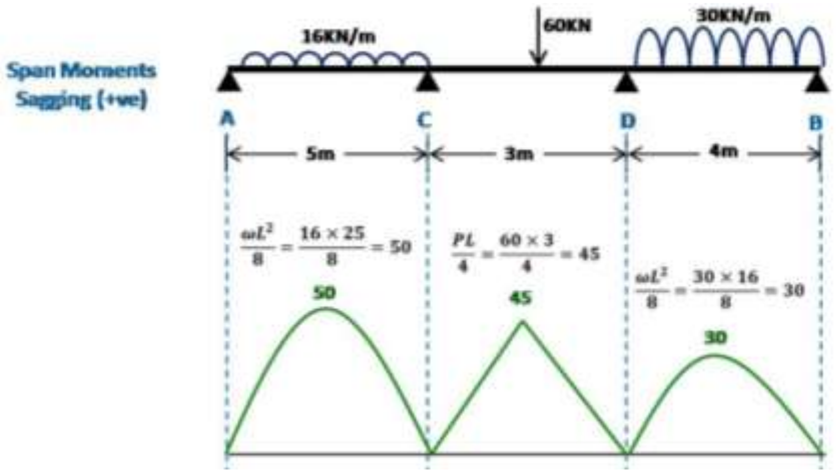



Figure 12 Span moments

Then, after considering each of the sections AC, CD, and DB as simply supported, you've to find the simply supported bending moment for each of them which is also known as the span moments which is a sagging kind of a moment. The span moments of the corresponding sections and their BMDs have been shown in the Figure 12.

Next, you've to superimpose both the BMDs shown in the Figures 11 and 12. Here, please note that the former has been shown in red colour as it is a hogging moment and hence negative. Whereas the latter has been shown in green colour as it is sagging moment; hence it is positive. These colour differentiation has been done solely for the ease of representation. The superimposed BMDs have been shown in the Figure 13.

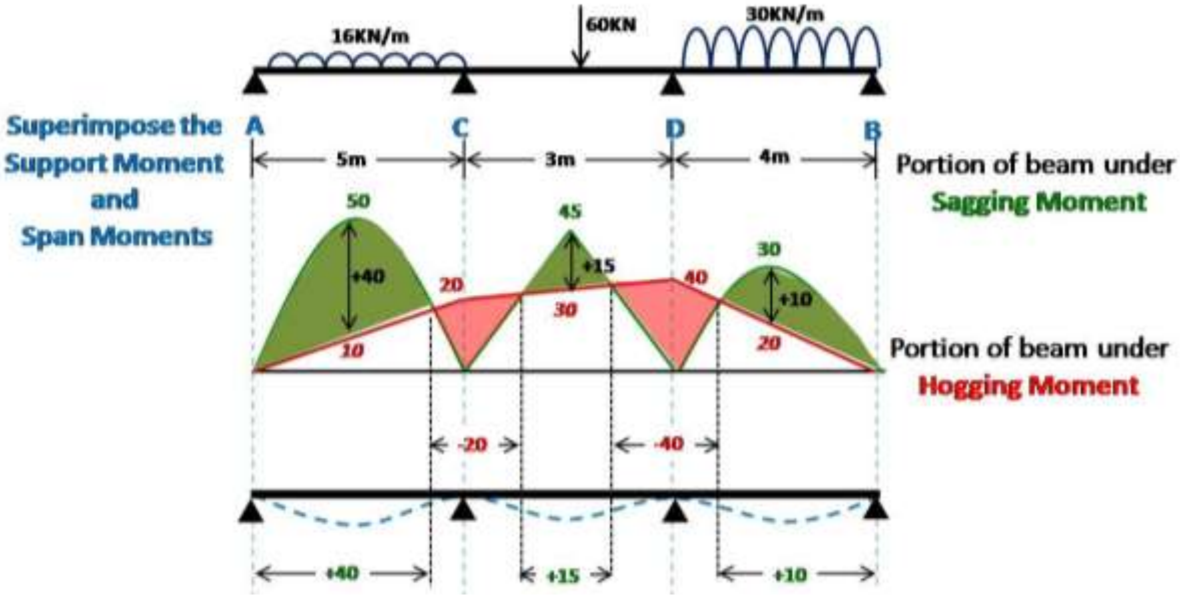



Figure 13 Superimposed BMDs

In the Figure 13 the areas coloured in green represent the zones of positive moment or sagging moment or the areas where the tension zones lie in the bottom whereas the areas coloured in light red represent the zones of negative moment or hogging moment areas or support moment areas or the areas where the tension zones lie at the top.

Consequently, you've to find the average of the adjacent support moments. Here it is 10, 30 and 20 (Figure 13). Now from these points you need to find the difference from the maximum span bending moments of the corresponding span. Here it is +40, +15 and +10 respectively. These are the central span moments with these you need to design the beam.



Now, the Figure 13 also shows the deflected shape of the moment. You can verify the tension-top and tension-bottom zones from here as well. The values of the corresponding sagging and hogging moments are again mentioned below and above the beam respectively. As you already know that the position of the tension zones will determine the position of the reinforcement, the reinforcement drawing for the given beam is shown in the Figure 14.

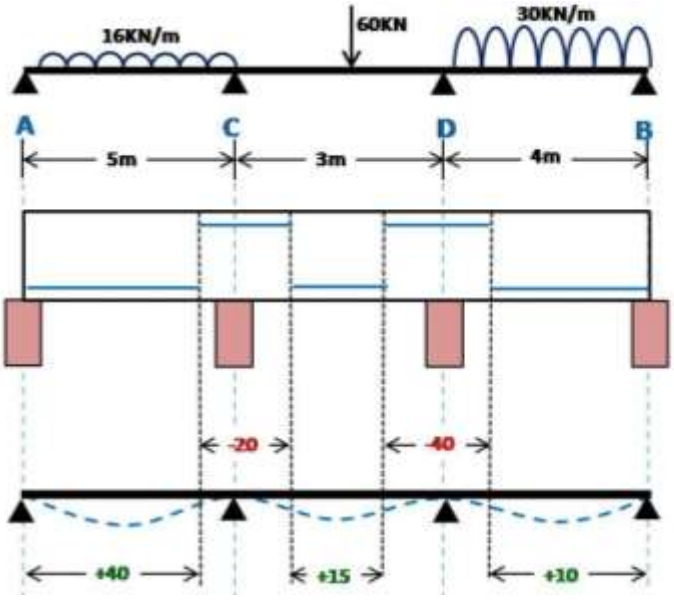


Figure 14 Position of reinforcement for the given beam

All things considered, the zones of sagging and hogging, the points of contra-flexure etc., all these together constitute the outcomes of the structural analysis and hence are carried forward to the design stage. For this reason, structural analysis is extremely important prior to the designing of the structural elements.

## Bending Moment Coefficients

In some of the handbooks you may find the bending moment coefficients mentioned for easier calculation in case of structures with equal spans such as the hotels, hostels etc., which you have to multiply with the total span load to get the corresponding moments or the reactions.

There'd be three types of coefficients mentioned viz., the support moment coefficients, span moment coefficients and the support reaction coefficients, which you have to distinguish according to the space where they're mentioned. The following figure shows such an example wherein each of these coefficients are mentioned with different colours for your better understanding.

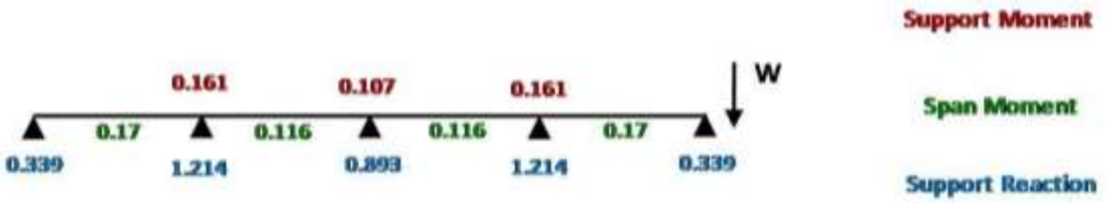



Figure 15 Bending Moment Coefficients

The beam shown in the above figure is a continuous beam with four equal spans of 4m and carrying a UDL of 12KN/m throughout.

Then,

Total Span Load = W = 12 X 4 = 48 KN

Therefore, now if you keep multiplying each of the support reaction coefficients with this total span load, you'll get the reactions at each of the supports. Similarly, you can also get the corresponding span moments and the support moments. The following figures show the support reactions, the span moments and the support moments respectively for the given beam.

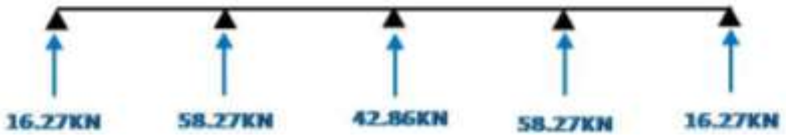



Figure 16 Support reactions

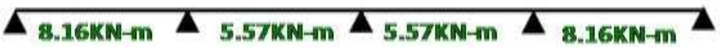



Figure 17 Span moments


Figure 18 Support moments

## Portal Frames

A simple Portal Frame consists of two columns and one connection beam, rigidly fixed at the junctions. They are also called "Moment Resisting Frame". Portal frames can be analyzed for gravity and lateral loads. Under external loading, portal frames produce Axial Force, Shear Force and Bending Moment unlike the beams and the trusses where the former carries shear forces and bending moments and the latter carries only axial forces.

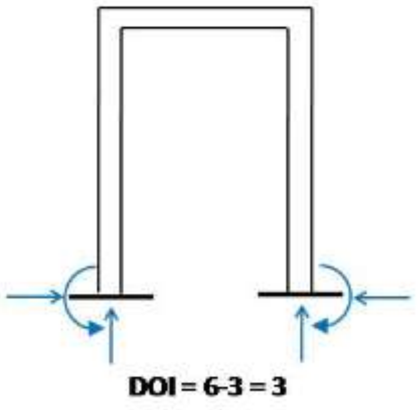



Figure 19 A simple portal frame

Given here is an example of a simple portal frame. As it is fixed, it will have three unknowns. Besides, there will also be 6 known reactions comprising of 2 upwards, 2 horizontal and two moments. Thus, the degree of indeterminacy here is 3.

A portal frame comprises of two components viz., bays and storeys. The former is the horizontal space between two columns whereas the latter is the vertical space created within. The following figure shows the bays and storeys in a typical portal frame.

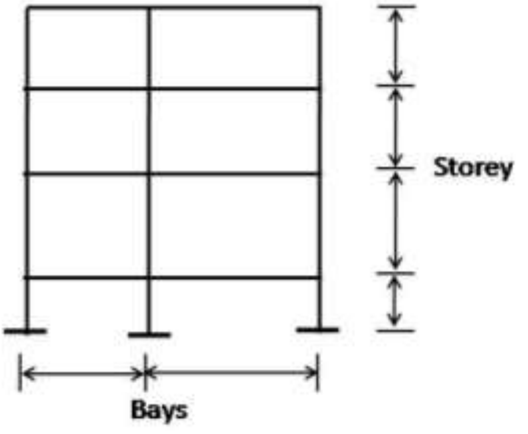
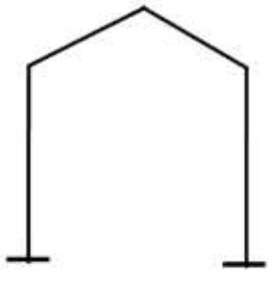
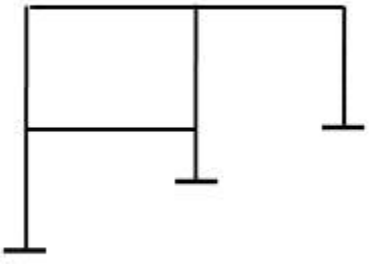



Figure 20 Bays and storeys in a typical portal frame

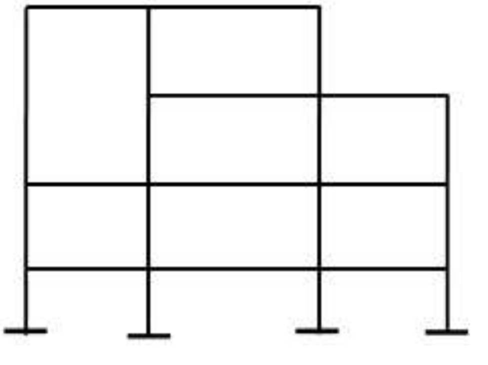
Following are some more examples of the portal frames and their uses.



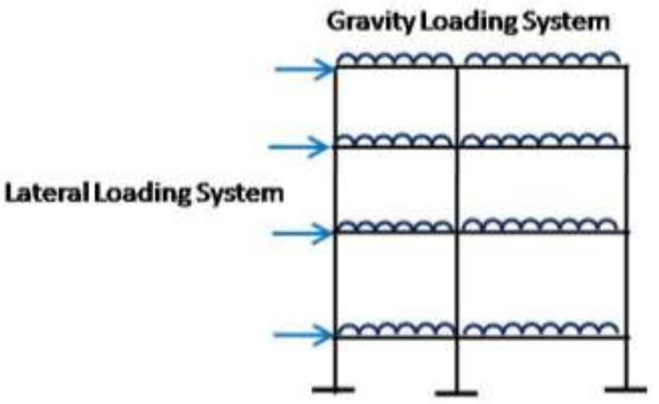
Suitable for factories



Suitable in contours



Discontinuity created for more headroom. Suitable for banquet halls etc.




The portal frames are created to take the gravity loads as well as the lateral loads such as the loads due to wind, earthquakes etc.

Figure 21 Various types of portal frames, their uses and the loading system

## Bending Moment Diagram in Portal Frame

We'll not deal with this topic in detail here but only give you a basic idea through the diagrams below.

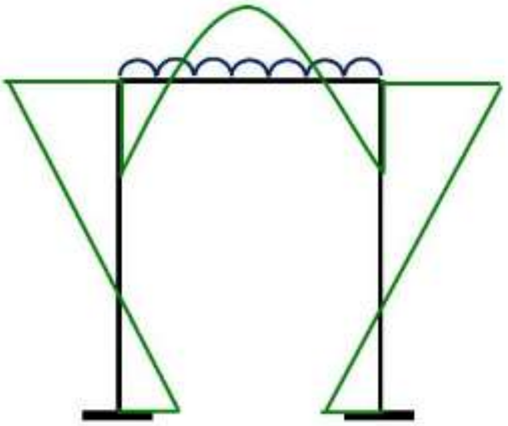


Figure 22 Bending due to gravity load

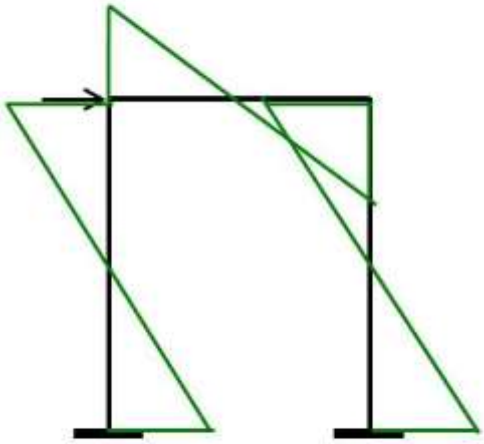


Figure 23 Bending due to lateral load

## References

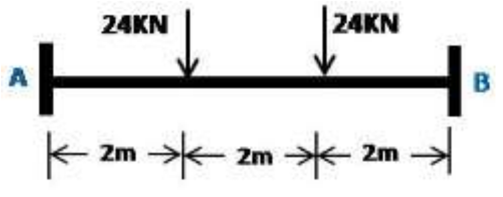

- **Engineering Mechanics** by Timishenko and Young McGraw-Hill Publication
- **Strength of Materials** By B.C. Punmia, Ashok K.Jain & Arun K.Jain Laxmi Publication
- **Basic Structures for Engineers and Architects** By Philip Garrison, Blackwell Publisher
- **Understanding Structures: *An Introduction to Structural Analysis*** By Meta A. Sozen & T. Ichinose, CRC Press

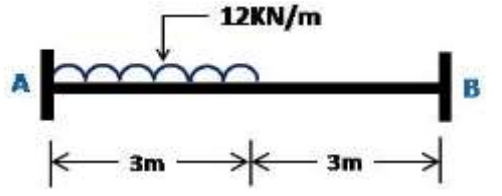
# Conclusion

To summarize, I'd like to state that the bending moment diagrams of fixed and continuous beams consist of both sagging and hogging. The nature of bending moment provides the deflected shape of beam and hence the reinforcement zones can be marked accordingly.

# Homework

Q1. Using formula (mentioned in the PPT) and superimposition method, find the 'Fixed End Moments' of the beam shown below. Also sketch the Bending Moment Diagram of the following beams.




Q2. Draw the BMD of the beam given below. Assume the support moments at A & C are 20KN-m & 30KN-m respectively. Also sketch the possible zone of beam reinforcement.