Structural System in Architecture
Prof. Shankha Pratim Bhattacharya
Department of Architecture and Regional Planning
Indian Institute of Technology, Kharagpur

Lecture - 22
Analysis of Truss-1

Welcome to the NPTEL online certification course on Structural Systems in Architecture. The week 5 is on Truss and Space Frames; and we are in the second lecture on this module which will be on the Analysis of Truss Part-1. This is lecture number 23 in sequence. Here we will discuss about two types of analysis.

The basic concepts to be covered are:

- Indeterminacy of Truss
- Methods of Truss Analysis
- Joint Method of Truss Analysis
- Section Method of Truss Analysis
- Examples

The intended learning Objectives are:

- To Outline the Various Methods of Truss Analysis.
- To Analyze Truss by Joint Method with examples.
- To Analyze Truss by Section Method with examples.

First of all, let us see what is the indeterminacy? Indeterminacy we have already discuss in the very first week of lecture, and if you have some doubts then you can go back to the first week lectures and get a revision.

Indeterminacy is the redundancy; that is how many unknown forces are there in a particular system that we have to first judge and then we have to actually find out whether it is possible to tackle the condition or to find out unknown forces by virtue of the 3 statically equilibrium equations like $\sum F_X = 0$, $\sum F_Y = 0$ and $\sum M = 0$ or not. If it is; then it is indeterminate; and if it is not it is indeterminate structure.

In case of Truss, it will have two type of indeterminacy. The first one is called external Indeterminacy and the second one is Internal Indeterminacy.

So, what is external indeterminacy (or the determinacy whatever you may call it)? In case of a Truss it is about the supports and support conditions of the Truss. We can also say that external indeterminacy depends upon the support conditions; like how many supports and what type of supports are there in a particular Truss; and it follows the rule adopted for beams. Because supports are the external conditions which is plugged into a particular system of Truss. Let us say that, there is a beam and beam is supported with 2, 3 or may be 1 support; and those are plugged-in systems. So, here we have to see how those support systems exists in a given Truss system. The applicability of the indeterminacy will be same as it is for beams.

Now, let us see some examples for external indeterminacy. Let us consider a Truss as shown in Figure-1. The black lines are internal members and yellow dots represents the joints or nodes of the Truss; and it is rested on a hinge and a roller support. The supports are placed on same base with same directions.

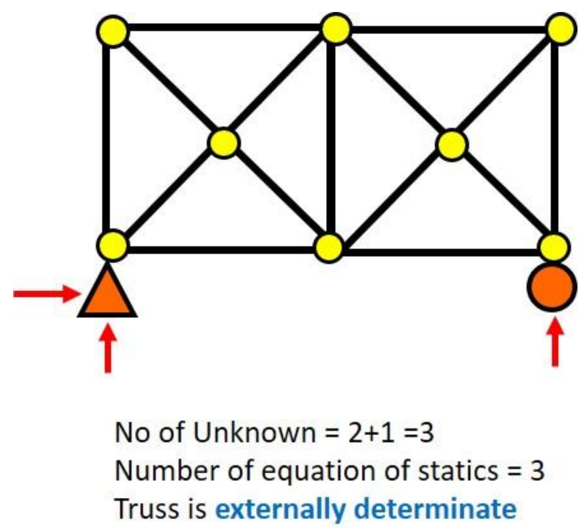


Figure 1 : external determinacy of a Truss system

Then, as you already know, the hinge support will have two unknown reactions, vertical and horizontal as given in Figure-1; and roller support will have only one unknown reaction in vertical direction. So, there will be 3 (2+1) unknown support reactions, and as we can see they are external to the Truss. Now, we have to see whether it will be stable or determinate or not. As there are 3 unknown reactions and we have 3 equations of static equilibrium with us, we can solve the unknowns; and therefore, the Truss is externally determinate.

On the other hand, what will be situation if we change the support type, support location and support orientation? Let us assume that we change the roller support to a hinge support and it is relocated and re oriented as shown in Figure-2. So, both are now hinge supports. Therefore, the unknown reactions at each support will be 2, resulting into a total of 4 (2+2) unknowns. But we have only 3 equations available to resolve this, which is not possible. So, compatibility equations are required to solve this. Therefore, it is not a determinate Truss system; it is an externally indeterminate Truss system; and the degree of indeterminacy will be 4-3=1 (no. of unknowns-equations of static equilibrium).

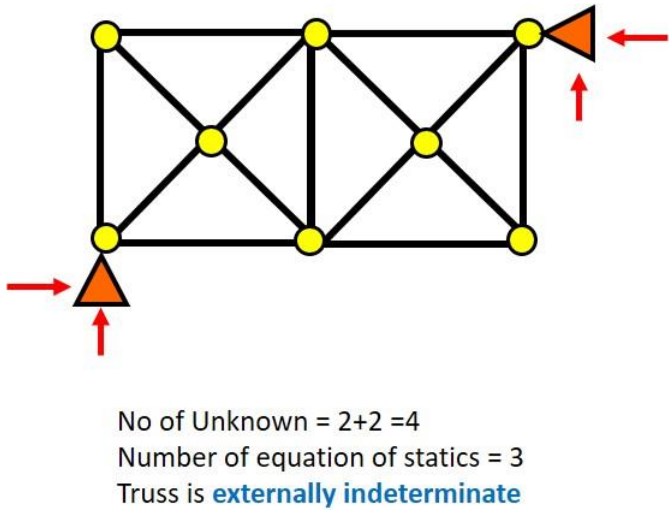


Figure 2 : external indeterminacy of a Truss system

So now let us discuss the what is Internal Indeterminacy? A structure or a Truss can be externally determinate or indeterminate, we can find it out by solving the reaction forces. If the reactions are more, then by virtue of some other methods like compatibility equations, we can find out 4 or 5 unknown reaction forces. If it is statically determinate type of structure, then we can find the reactions forces very easily with the help of 3 equations of static equilibrium.

But after that we have to see the internal members. How the internal members are geometrically linked with each and other so that we can easily compute those internal forces by virtue of those 2 or the other 3 equations of the static equilibrium.

So, let us first see a triangle. The number of members (m) is 3, the black lines, and number of joints (j) is also 3, the yellow dots, in Figure-3. Next, we will see it with respect to the relation

$$m = 2j - 3$$

A Truss is internally statically determinate, if it satisfies the above equation.

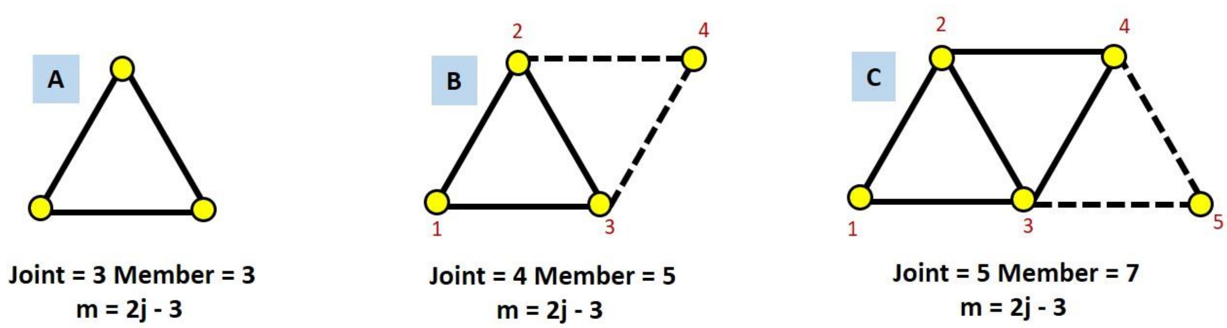


Figure 3 : internal determinacy of a Truss system

In case A of Figure-3, we have 3 members and 3 joints, then if we see it with respect to the relation of ‘m’ and ‘j’ then

$$m = 2j - 3$$

$$m = 2 \times 3 - 3 = 6 - 3 = 3$$

In this the relation of ‘m’ and ‘j’ holds good.

In case B, we have added two new members with one more joint. Then, number of members are now 5, and number of joints are 4; then

$$m = 2j - 3$$

$$m = 2 \times 4 - 3 = 8 - 3 = 5$$

In this the relation of ‘m’ and ‘j’ holds good.

In case C, let us go one step further; and we have added another joint with two new members. So now, number of joints are 4 + 1=5 and number of members are 5 + 2 = 7. Then;

$$m = 2j - 3$$

$$m = 2 \times 5 - 3 = 10 - 3 = 7$$

In this case also the relation of ‘m’ and ‘j’ holds good.

So, what we have seen here is that, at least two members are connected to a joint or node. It may be more than two, but it should contain at least two. Then, we can easily find out the forces in the members by using the two equations of static equilibrium, that is $\sum F_X = 0$ and $\sum F_Y = 0$. So, if we proceed like this, first by solving the nodes with two members, say joint-1 (as shown in Figure-1, central and right-hand side image) then next we go the joint-2, with three members. In joint-2, even if we have 3 members, force in one member linking joint 1 and 2 is already know by now; so, we have to find force in rest of the two members connecting 2 to 3; and 2 to 4; which can be done easily. Similarly, next we can go the joint-4; and so on.

So, gradually you will see that, if you start from a joint having 2 members, then you can easily go to the next joint and next joint like that and finally find out all the forces by virtue of the $\sum F_X = 0$ and $\sum F_Y = 0$ equations.

So, that means if the Truss holds the relation $m = 2j - 3;$ between the members and the joints, then the Truss is internally statically determinate. If the Truss is having more members than $2j - 3;$ then remember, you have a problem and internally somewhere you have to stop; you cannot actually find out those unknown forces by virtue of those equations of statics. Now, you have to think of some compatibility equations.

Now, let us see another condition.

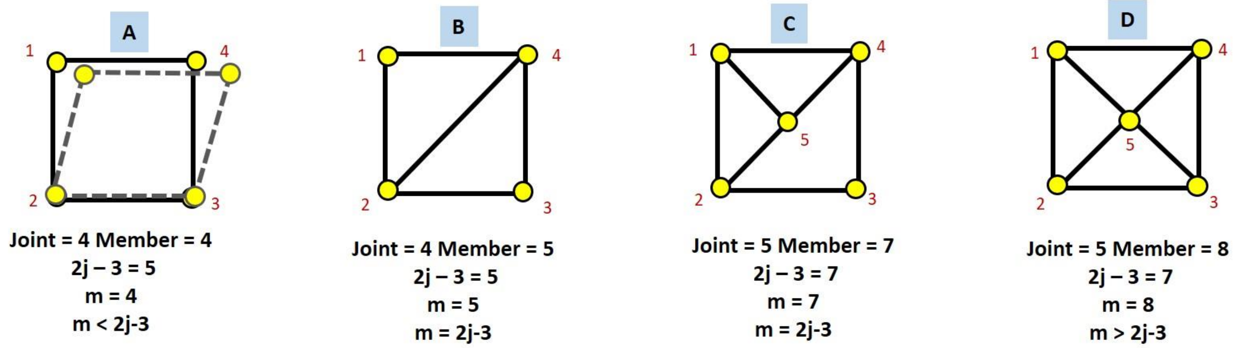


Figure 4 : indeterminacy in a square to triangular units

So, what is shown in Figure-4 is a square unit, and gradually we are adding members and joints developing it into triangular components. In case A, there are 4 members and 4 joints. Now, if we see the 'm' and 'j' relation, then

$$m = 2j - 3$$

$$m = 2 \times 4 - 3$$

$$m = 8 - 3 = 5$$

Here, $m < 2j-3$

In last lecture we have discussed about it with the example of ice cream sticks joined by pins; and we know that it is very unstable; and similar case is this. Case A, is statically indeterminate and unstable

Then in case B, we have introduced a new diagonal member, the fifth member connecting joint 2 to 4 as shown in Figure-4. Here the joints are 4 but members are 5; then the relation is:

$$m = 2j - 3$$

$$m = 2 \times 4 - 3$$

$$m = 8 - 3 = 5$$

It satisfies the relation and it holds good. So, I can say that case B is a statically determinate and stable condition, also internally determinate structure. Here, I can easily start from this joint-1, and I can find out the forces in the two links or members, and gradually I can find all unknown forces in all the members of the given structure.

Next, in case C, if I join another member with one more joint, then m=7, and j=5; then

$$m = 2j - 3$$

$$m = 2 \times 5 - 3$$

$$m = 10 - 3 = 7$$

It also satisfies the 'm' 'j' relation; and with the help of static equilibrium equations we can find out the unknown forces. Here also, we will start with a joint with two members or links and gradually proceed to three member joints; and finally, we can find out the unknown forces in all the all the members.

But if introduce one more member, keeping number of joints as 5, as shown in case D of Figure-4, then it becomes m=8, and j=5. Then looking into 'm' 'j' relation:

$$m = 2j - 3$$

$$m = 2 \times 5 - 3$$

$$m = 10 - 3 = 7$$

Here, m > 2j-3.

So, the structure is statically stable but internally indeterminate. So, I cannot actually find out the forces in all the 8 members by virtue of the equations of static equilibrium, $\sum F_X = 0$ and $\sum F_Y = 0$.

In order to find all the 8 forces, I have to search for one extra equation; because the degree of indeterminacy (DOI) is 1 here.

$$DOI = M - (2j - 3)$$

$$DOI = 8 - (2 \times 5 - 3)$$

$$DOI = 8 - 7$$

$$DOI = 1$$

Therefore, I'll be in need of one more equation to solve this.

Now, if we see to conclude these conditions then we can say that;

If, m < 2j-3 .... The Truss is structurally unstable.

If, m = 2j-3 .... The Truss is structurally stable & internally determinate.

If, m > 2j-3 .... The Truss is structurally stable & internally indeterminate.

Now, let us see the "Joint Method of Truss Analysis". In this method what we will do is that, we will assume each and every joint is in equilibrium we have to go one by one. We have to draw the free body diagram of each joint or each node and we will apply the simple equations $\sum F_X = 0$ and

$\sum F_Y = 0$. In this analysis approach is applicable to the joints having maximum two unknown force members. So, we have to go like that.

In case of section method typically we will break the Truss into two typical parts and we will draw the free body diagram of the left or right side of the Truss. But what will happen here is that, we only either left side or the right side of the Truss; and then we will apply the equilibrium equation for moments in each part of the free body, $\sum M = 0$. and we will try to find out the unknown forces in it. The analysis is applicable to the free body portion having maximum three unknown force members; and this method is applicable only to internally statically determinate trusses.

Now, we will discuss some examples. First, we will discuss through Joint Method of Truss Analysis. Let us consider a parallel chord Truss as shown in Figure-5. The bottom chord nodes (or joints) are marked as L1, L2, L3, L4 and L5 and top chord nodes are marked as U1, U2, U3, U4 and U5. The top nodes are given with pointed loads of 10KN each, resulting total load to be 50 KN. The span of the Truss is ranging from L1 to L5. The intermediate distance between bottom nodes (L1 to L2, L2 to L3 etc.) and top nodes (U1 to U2, U2 to U3 etc.) is 2 meters, having total span of 8 meters; and the depth of the Truss is 1.5 meters.

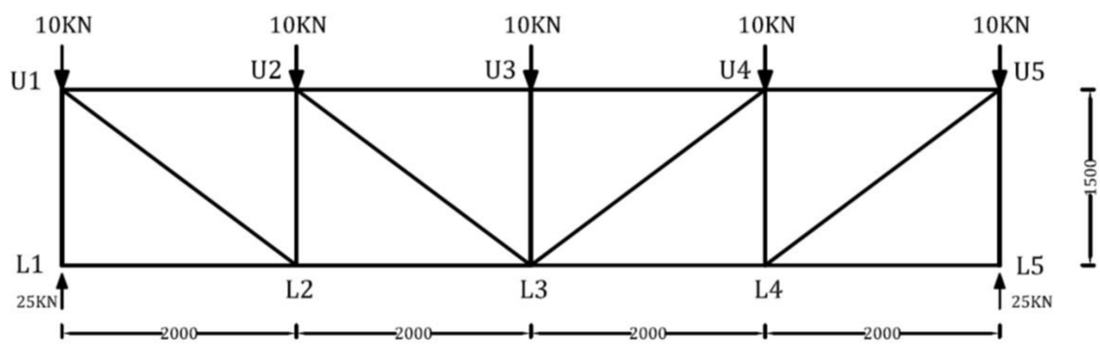


Figure 5 : a symmetrical parallel chord Truss

Now, I can easily find out the support reaction as 25 each, because it is symmetrical and there are total 50 KN loads. Now dividing the Truss in to two equal parts, from L1 to L3 one part; and L3 to L5 the second part; and then we can solve the unknown forces.

Then, first we will touch the node L1; because this is having only two members associated with it; and we can easily find out these two forces. When we can find these two forces then, we can jump into the next node U1; because at U1 the force in the member connecting L1 to U1 is already known to us and we can find out rest of the two forces associated with U1 (connecting U1 to U2 and U1 to L2).

Then we will go to L2, because here also one force is known, that is the force in the member connecting U1 to L2; and then we can find out the two unknown forces associated with node L2 that is force on L2 to U2 and L2 to L3.

Then we can go to U2; again, here also two forces are known to us, they are the forces in the member connecting U2 to U1 and U2 to L2. Then we are left with two unknown forces, and we can resolve those two; that is force on U2 to L3 and U2 to U3.

Finally, we can come back to the U3 and we can find out the force in the member connecting U3 to L3, because force in member connecting U3 to U2 is already known to us, and force in member connecting U3 to U4 will be same as force on member U3 to U2, because it is a symmetrical Truss. The navigation to proceed from one node to another is given in Figure-6.

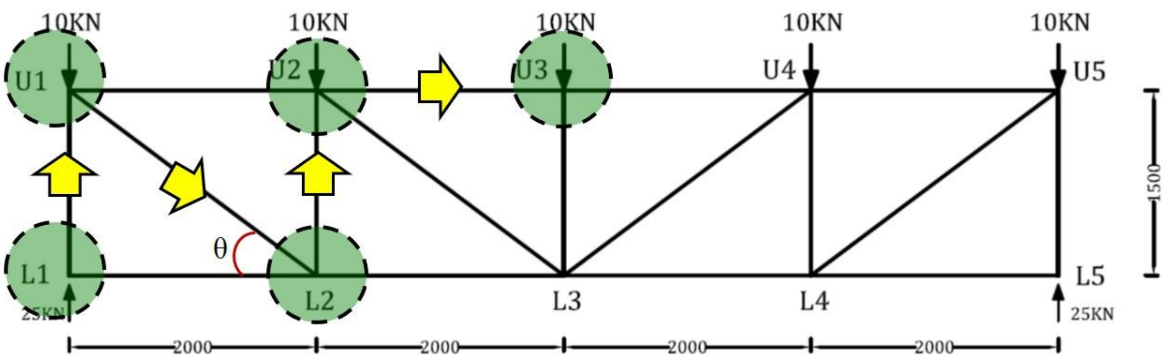


Figure 6 : navigation to calculate unknown forces in different nodes of a Truss

Now, if we find the value of θ, then from trigonometry and from the values of rise and span we can get it as

$$\sin\theta = \frac{1.5}{2.5} = 0.6$$

$$\cos\theta = \frac{2}{2.5} = 0.8$$

Then, if we take the joint L1, putting $\sum F_X = 0$ and $\sum F_Y = 0$ we will get the values F_{L1U1}=25 (C) and F_{L1L2}=0. F_{L1U1} is under compression because in the free body this particular joint is under compression.

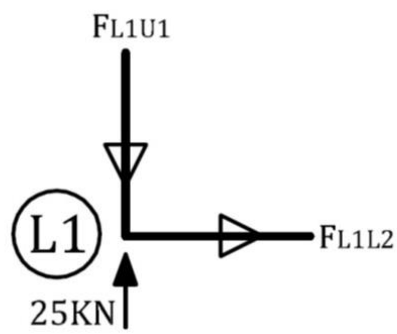


Figure 7 : joint L1

So, if the force is towards the joint that means joint will be under compression. Please don't be confused with force diagram, because in force diagram it will look like tension.

Next is joint U1.

Then if we see the figure,

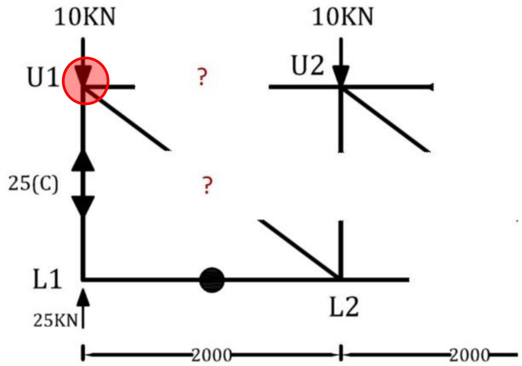


Figure 8: the forces

Now, we already have L1U1 is 25 and L1L2 is 0 which is shown in Figure8, the direction of the force will be equal and opposite. Next, in joint U1 and we need to find force in U1, U2 and U1, L2.

Then solving the unknown forces in U1, with respect to $\sum F_X = 0$ and $\sum F_Y = 0$

$$F_{U1L2} = \sin\theta = (25 - 10) = 15$$

$$F_{U1L2} \times 0.6 = 15$$

$$F_{U1L2} = 25\ (T)$$

$$F_{U1U2} = 25\ Cos\theta = 20\ (C)$$

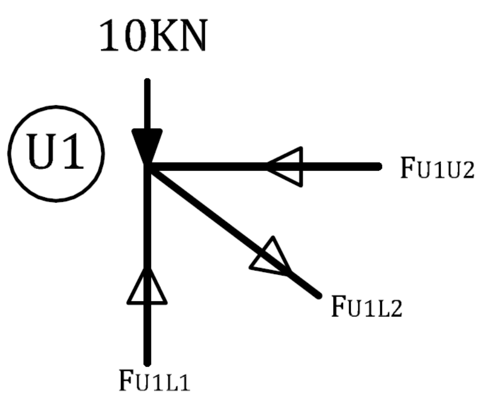


Figure 9 : joint U1

Finally, I can find F_{U1L2} is 25 (T) and F_{U1U2} is 20 (C). F_{U1L2} is under tension because at this joint it is force is actually trying to take out the joint; it is under tension and other F_{U1U2} is towards the joint, the notation considered is also correct, so it is compression. So, as F_{U1L2} is under tension, the notation of force will merge and in compression the notation will diverse.

Now we will go to joint L2. Similarly, solving the unknown forces in U1, with respect to $\sum F_X = 0$ and $\sum F_Y = 0$

$$F_{L2U2} = 25\sin\theta = 15\ (C)$$

$$F_{L2L3} = 25\cos\theta = 20\ (T)$$

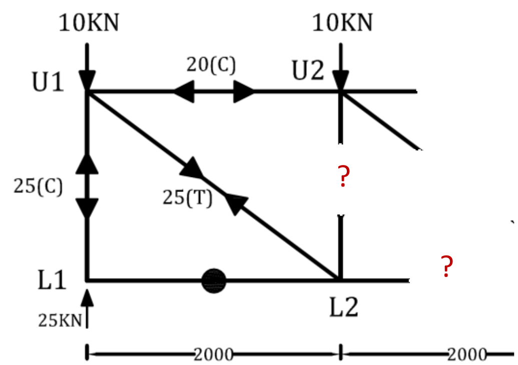


Figure 10 : forces in the Truss

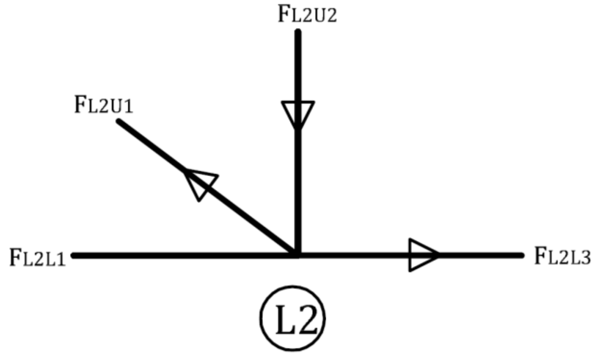


Figure 11 : the joint L2

Then, similarly at U2.

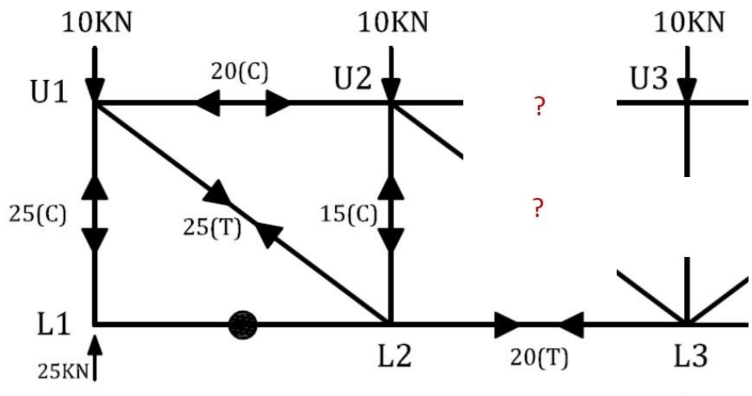


Figure 12 : the forces

Then solving the unknown forces in U1, with respect to $\sum F_X = 0$ and $\sum F_Y = 0$

$$F_{U2L3} Cos\ \theta = (15 - 10)$$

$$F_{U2L3} = 8.33\ (T)$$

$$F_{U2L3} = 20 + 8.33\ Cos\theta = 26.67\ (C)$$

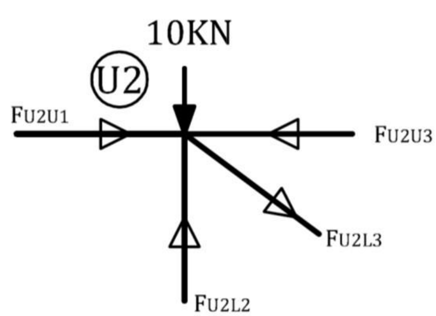


Figure 13 : joint U2

Then, similarly solving joint U4 we will get the final resultant as shown in Figure 14.

Now, the whole result is with us and we can see that it is very symmetrical. So, if you say L1 is 25, compressions then L5 has to be 25 compressions too. Because it is a symmetrical Trusses. We can solve half the portion of the Truss and it can be mirrored to the other half portion.

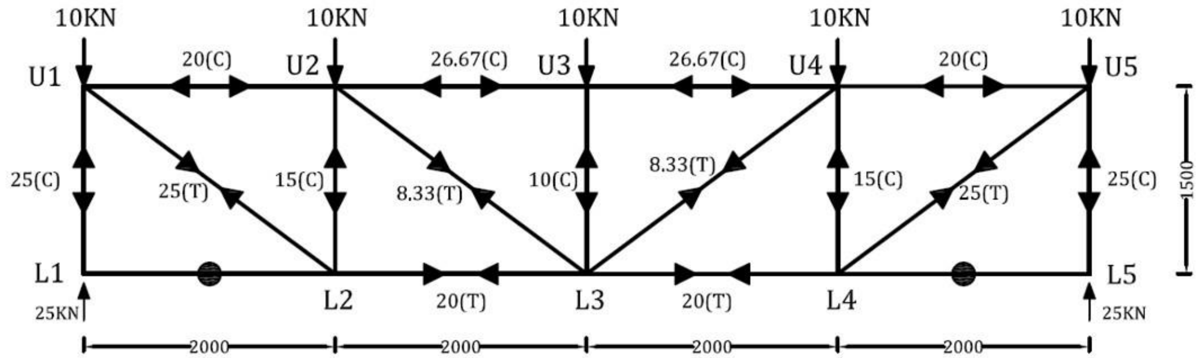


Figure 14 :all the resultant forces

If the force in L1, L2 is 0, then force in L4, L5 will also be 0. Similarly, if force in U2, L3 is 8.33, the force in L3, U4 will also be 8.33 as shown in Figure-14. So, in this way all the forces in the Truss can be solved by virtue of solving the half portion.

Now, we will see the section method to analyze a Truss. Here also we have considered the same Truss as in Figure-5. We already have solved that one, so we can actually crosscheck the answers by the section method. But here I have formulated the problem as: "Given a Lattice Truss of 8 meter of span, 1.5 meters depth. The truss is simply supported at the ends (L_1 & L_2). Find the forces in the members: U_2U_3, U_2L_3, & L_2L_3."

So, I have to find out only these three members. Then I will create a section over there, as shown in Figure-15; and I will bifurcate the Truss into two portions. I will either I consider the left or the right portion and try to solve the Truss by virtue of moment.

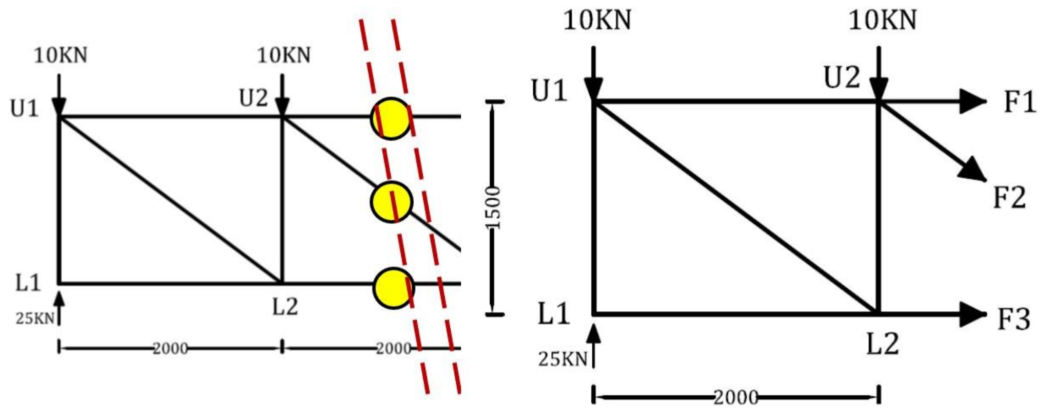


Figure 15 : considered portion of the truss with three members to compute

As you can see in the Figure-15, U1, U2 and L1, L2 is visible. There are three cutting forces or the cutting members which is written F1, F2 and F3.

First of all, assume that all are tensile forces and they are coming out from the joint. I also have to check the balance of the left portion and find out the force F1, F2, F3. What should be the correct values of F1, F2, F3 and what should be the correct direction of F1, F2, F3 so that this total half system on the left part of the system comes under equilibrium.

So, for that I have to use some moment equations in each joint. I have to take the moment either at L1 or L2 or at any point among the four points that is L1, L2, U1 and U2. Or it can be any node point, like meeting point of U2 and L2. But I cannot take the middle point of the members.

First, I will take the moment about U2. Why U2? Because automatically the moment created by F1 and F2 will be 0, because F1 and F2 passes through U2, as the distance is equal to 0. Therefore, the moment created by the force F1 and F2 at U2 = 0.

Then if see F3 at L2, then it is having a distance 1.5 meters from U2, so the F3 X 1.5 meter, the moment goes in clock-wise direction at L2; and this must be equal to 25 X 2 (here 2 is the distance between node L1 to L2 or U1 to U2), and its direction is anti-clock-wise at L1, and minus counter moment by load 10 and distance 2. So,

Moment about U2:

$$1.5 \times F_3 = 25 \times 2 - 10 \times 2$$

$$F_3 = 20(T)$$

So, F_3=20 (T), direction is right, so it is tensile direction.

Next, what I will have to do is that I have to find out the F1 and F2.

Then, I have taken the movement about U1. Why U1? Because force F1 passes U1 along the same line, so this will be automatically 0. So, I have to compute the perpendicular distance from U1 to F2, that we can find out by extending the lines as shown in Figure-16, and then compute it by trigonometry.

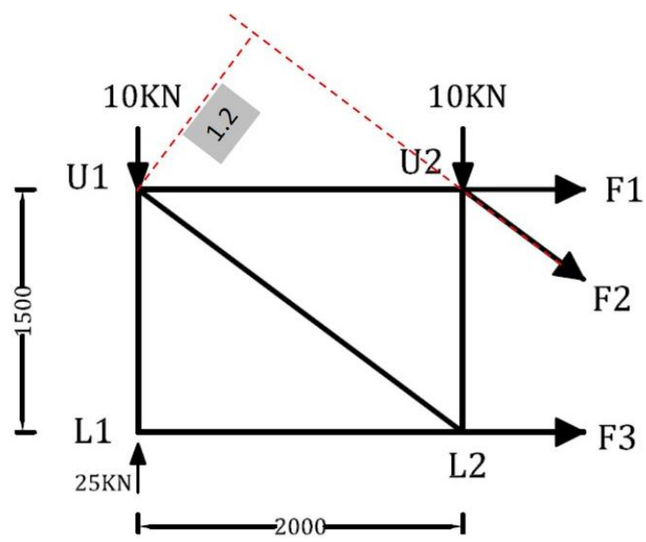


Figure 16 : the unknown forces

So, similar to F3, we can compute Moment at U1 as:

$$1.2 \times F_2 + 10 \times 2 = F_3 \times 1.5$$

$$= 20 \times 1.5$$

$$1.2 \times F_2 = 20 \times 1.5 - 10 \times 2$$

$$F_2 = 8.33\ (T)$$

The F_2 moment will be anti-clockwise. The force 25 KN and 10 KN will vanish because of collinearity. The F_3 X 1.5 (the rise), this moment will be in clock wise direction. So, we get the value of F_2 = 8.33, and it is tensile.

Finally, I have to take a moment from L2, and for this I have to find F1 and it will be very easy, because F_2 and F_3 is already computed. So, moment about L_2 is:

$$1.5 \times F_1 + 1.2 \times F_2 + 25 \times 2 = 10 \times 2$$

$$1.5 \times F_1 = 20 - 50 - 10 = -40$$

$$F_1 = 26.67\ (C)$$

Now, we can check with the values of the section method with the values of joint method and we see that these three vales valid.

In this way the section method has been taken into consideration. Here we have to take the each and every point and try to locate the point such a way that one of the forces can be computed by nullifying the other forces. Therefore, I have taken the U2 as they first joint, U1 as the second joint and the L2 in the third; and gradually I have computed the forces.

Next, we will see another example with joint method analysis of Truss. It is given with a Warren Truss of 8 meter of span, 1.5 meters depth. The truss is simply supported at the ends (L_1 & L_5). The intermediate distance between nodes is 2 meters, and at the bottom chord, the nodes are given with pointed load of 12.5 KN each.

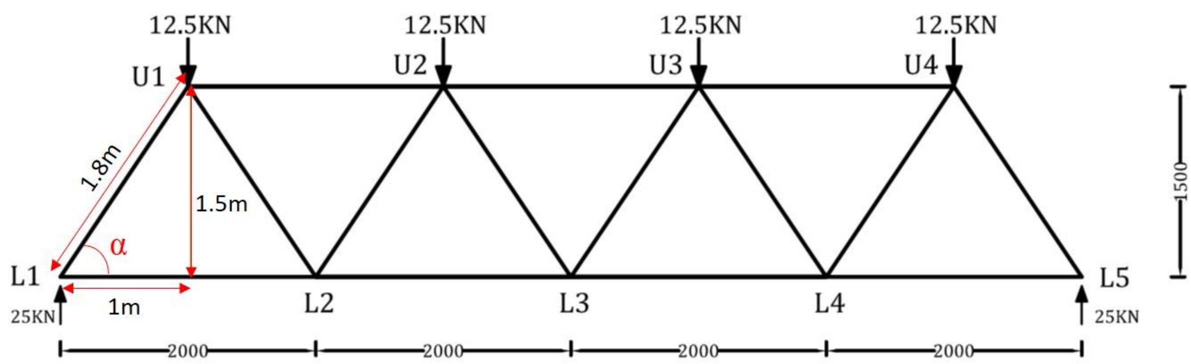


Figure 17 : Warren Truss

Here the reaction forces at each support will be half of total load that is 25 each. Again, I can find out the half portion of the Truss and I will start with a node having not more than two unknowns;

and here we will take L1. Then we will find out the values of Sin α and Cos α.

$$\text{Sin}\,\alpha = \frac{1.5}{1.8} = 0.83$$

$$\text{Cos}\,\alpha = \frac{1}{1.8} = 0.55$$

Then to find different unknown forces, we will proceed following the concept of not resolving the nodes with more than two unknowns. So, first we will start with L1, because it is having only two unknowns which we can resolve. Then following the same concept, we will proceed as the navigation shown in Figure-18.

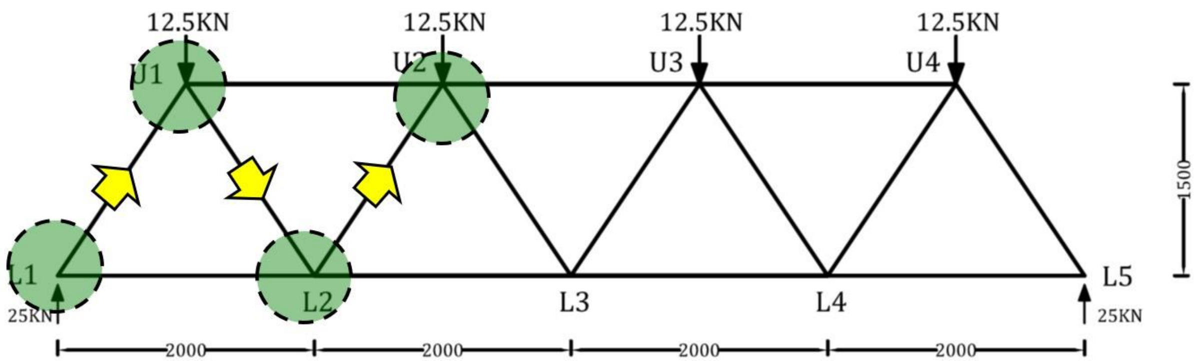


Figure 18 : sequence of resolving unknown forces in a Warren Truss

We will start with L1, and then we can go to the U1, the upper chord first point, where we know the force on U1, L1, and we will find out the two unknowns associated with it, (the U1, L2 and U1 U2). After U1 we will go to L2. Can we go to U2? We cannot go to the U2, because at that joint we only know the force on U1, U2, but we do not know rest of the three forces associated with this joint. There will be three more unknowns, which will be tough to resolve. At L2, we have two known forces, on L2, L1 and L2, U1; and two unknown forces, L2, U2 and L2 L3. Sotwo unknowns can be resolved. Then from L2 we will move to U2. At U2 we have two known forces, U2, U1 and U2, L2; and two unknown forces, U2, L3 and U2, U3. So, we can resolve the two unknowns. The symmetrical other part of the Truss will have mirrored values of the resolved part.

Now, at joint L1:

Let us assume the force directions are as shown in Figure-19. Then by resolving it, we can find,

$$F_{L1U1} Sin\ \alpha = 25$$

$$F_{L1U1} = 30(C)$$

$$F_{L1L2} = 30\ Cos\ \alpha = 16.5\ (T)$$

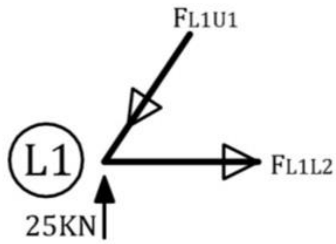


Figure 19 : joint L1

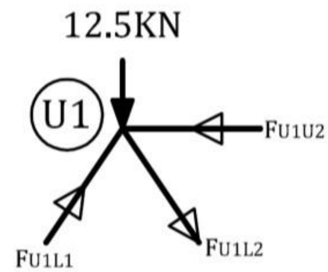


Figure 20 : joint U1

Then at joint U1

$$F_{U1L2} Sin\ \alpha = 30\,\mathrm{Sin}\,\alpha - 12.5$$

$$F_{U1L2} = 15\ (T)$$

$$F_{U1U2}\ 30\ Cos\ \alpha + 15\ Cos\ \alpha = 24.75\ (C)$$

Then the resolved forces till now will be as given in Figure-21.

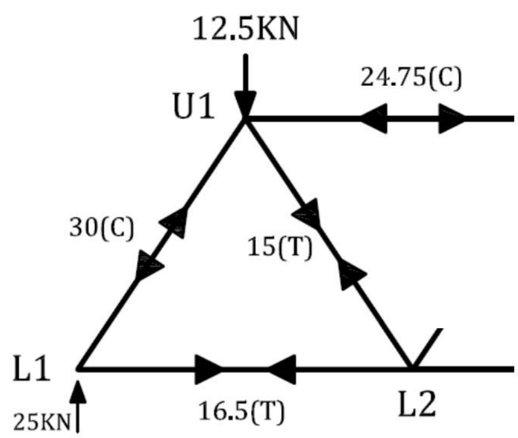


Figure 21 : resolved forces after considering L1 and U1

Then at joint L2, we will resolve it using $\sum F_X = 0$ and $\sum F_Y = 0$

$$F_{L2U2} Sin\ \alpha = 15\,\mathrm{Sin}\,\alpha$$

$$F_{L2U2} = 15(C)$$

$$F_{L2L3} = 15\ Cos\ \alpha + 15\ Cos\ \alpha + 16.5$$

$$F_{L2L3} = 33\ (T)$$

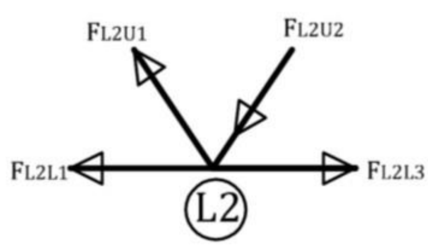


Figure 22 : joint L2

Then looking at joint U2, and solving it by applying $\sum F_X = 0$, and $\sum F_Y = 0$

$$F_{U2L3} Sin\,\alpha = 15\,\mathrm{Sin}\,\alpha - 12.5$$

$$F_{U2L3} = 0$$

$$F_{U2U3} = 24.75 + 15 Cos\,\alpha$$

$$F_{U2U3} = 33(C)$$

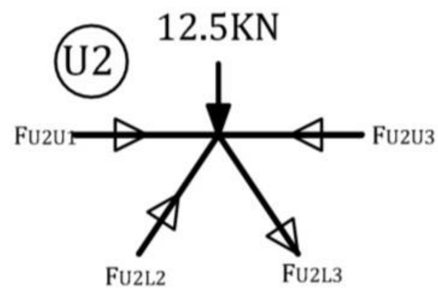


Figure 23 : joint U2

Then after solving these four joints, we get the resolved forces for half of the portion of given Truss; and values of forces of another half of the Truss will be just mirror image of it.

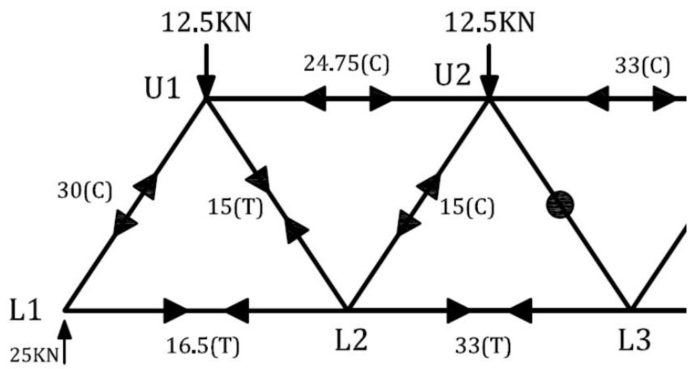


Figure 24 : the resolved forces for half portion of the given Truss

Then final resolved forces for the given Truss will be as shown in Figure-25.

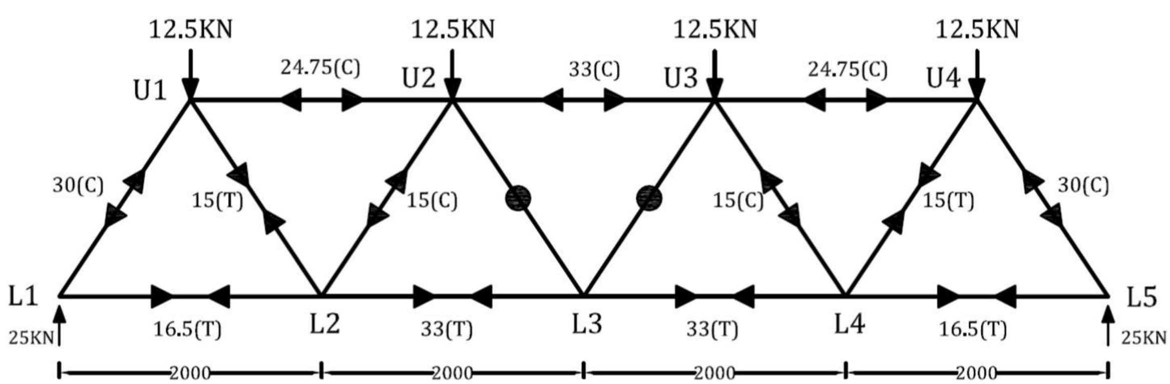


Figure 25 : the resolved forces for the whole Truss

So, for joint method of analysis of Truss, each and every joint has been taken into account as the $\sum F_X = 0$, and $\sum F_Y = 0$ and in that manner, we computed all the forces.

I have taken the reference of the following books for this lecture.

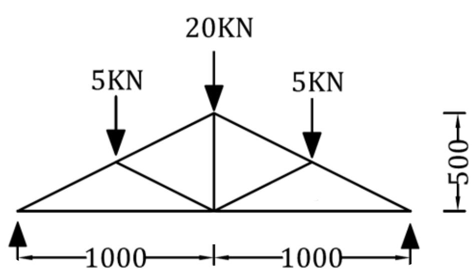
- **Structure as Architecture** by Andrew W. Charleson, Elsevier Publication
- **Basic Structures for Engineers and Architects** by Philip Garrison, Blackwell Publisher
- **Structure and Architecture** by Meta Angus J. Macdonald, Elsevier Publication
- **Examples of Structural Analysis** by William M.C. McKenzie
- **Engineering Mechanics** by Timishenko and Young McGraw-Hill Publication
- **Strength of Materials** By B.C. Punmia, Ashok K.Jain & Arun K.Jain Laxmi Publication
- **Understanding Structures: *An Introduction to Structural Analysis*** By Meta A. Sozen & T. Ichinose, CRC Press

Now, let us conclude this discussion. In conclusion we can say that structurally a truss consists of external and internal determinacy. So, the first thing we have to see is whether it can be solved by the equations of statics or not; and a determinate truss is analyzed by joint method or section method.
In the next lecture we will continue this analysis of Truss part-2 in lecture number 23.

Now, I have given you some homework.

Problem 1:

Find the forces in the truss shown below by joint method

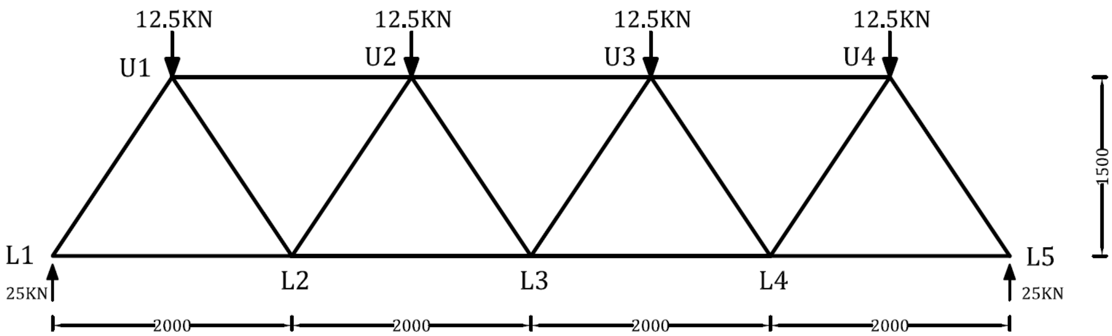


Problem 2:

Find the forces in the following members in the truss shown below using section method

(i) U1U2, U2L2 & L2L3

(ii) U2U3, U3L3 & L3L4



I think that is all for this lecture.

Thank you very much.