Structural System in Architecture

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Lecture No -27
Structural Principle and Application of Arch

Welcome to the NPTEL online course on *Structural System in Architecture*. We are in the module number 6, i.e. the 6th week and today we will be discussing about the structural principles and application of arch, which is the lecture number 27.

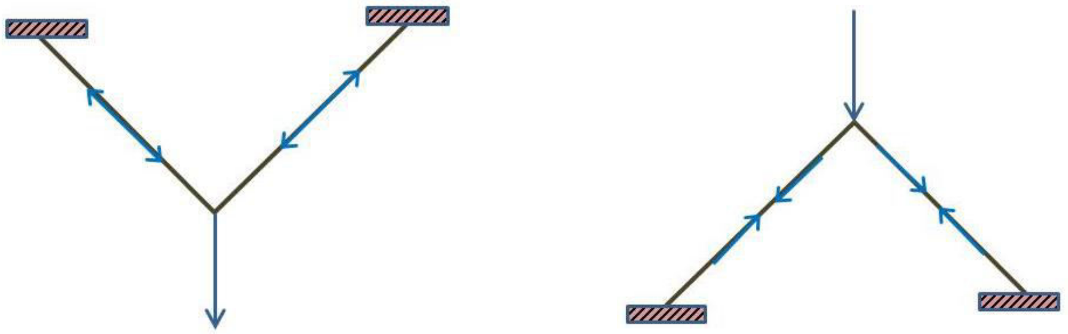
Concepts Covered

- ✓ Basic Structural Principles
- ✓ Arch Action
- ✓ Analysis of Arch
- ✓ Concept of Thrust Line

Learning Objective

- ✓ Illustrating the structural principle of arch.
- ✓ Analyzing the Arch circular and parabolic two hinged arch.
- ✓ To outline the arch stability through thrust line concept.

Basic Structural Principle



An external load creates tension in the sagging cable.

An external load creates compression in the hogging profile structure.

Figure 1 Basic structural principle in a cable (left) and an arch (right)

In the left side of the Figure 1 we have a cable suspended from two supports at the ends. Additionally, the cable is subjected to a load as shown in the figure. Clearly, the arrangement creates a tensile force in the sagging cable. Conversely, the arrangement shown on the right side is similar to the structural behaviour of an arch where the external load creates compression in the hogging profile structure.

Triangular Arch

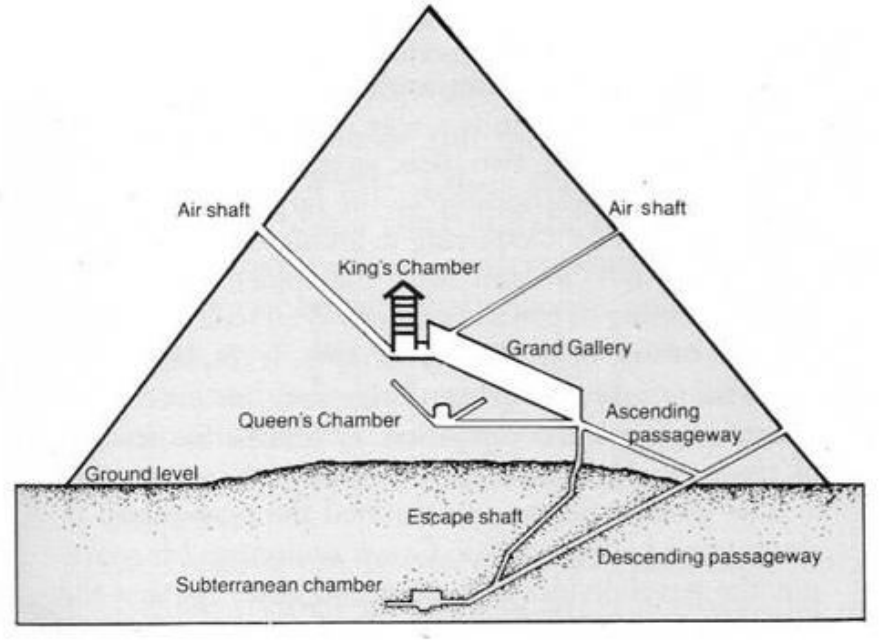


Figure 2 The Great Pyramid of Giza, 2560 BC

Figure 3 Queen's Chamber

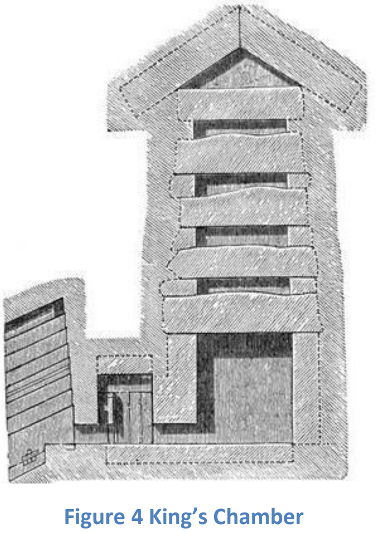


Figure 4 King's Chamber

The above figures show some of the examples of a triangular arch.

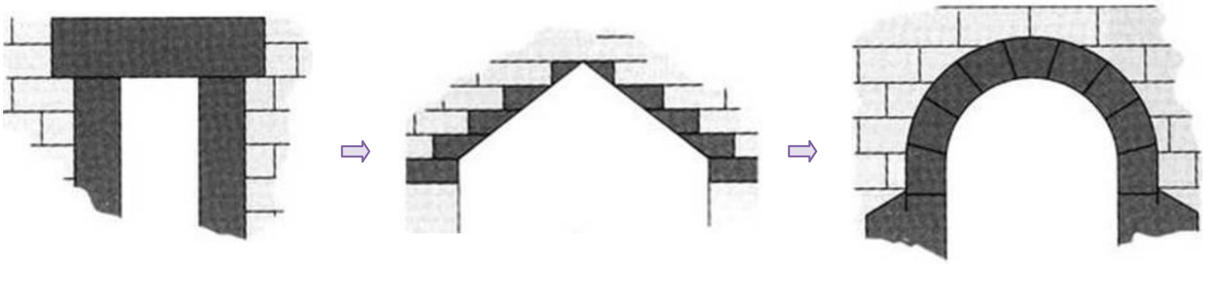
Corbelled Arch

A corbelled arch is developed by regular corbelling of the horizontal courses of the wall masonry. The horizontal corbel cantilever of 1/4th to 1/3rd of the block module is preferred. Corbelled arches can be built without support. For building such an arch, it is essential to pay attention to the balance of the masonry when courses rise. It is important to note that the center of gravity should not go beyond the limit of stability.

Stone corbelled arches at Borobudur, Java, Indonesia

Entrance of Sun Temple, Konark

Figure 5 Examples of a corbelled arch



Post and Horizontal Lintel   Corbelled Arch   Semi-circular Arch

Figure 6 Formation of the various arches

Arch Action

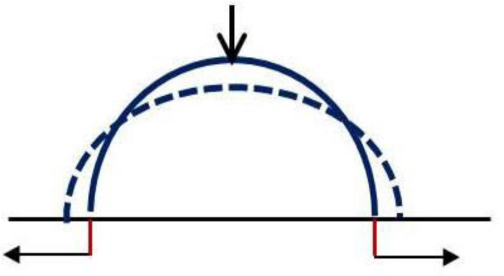


Figure 7 Arch action

So under the downward load the leg of the arch is going to spread outward. So to establish equilibrium along with the vertical reactions the horizontal reactions also exist. In other words, if there is external load in a particular arch it will create a horizontal reaction.

In the absence of the horizontal reaction the arch will become unstable because its legs will go apart. Besides, vertical reactions are also there because $V_A + V_B$ must be equal to the external loading P.

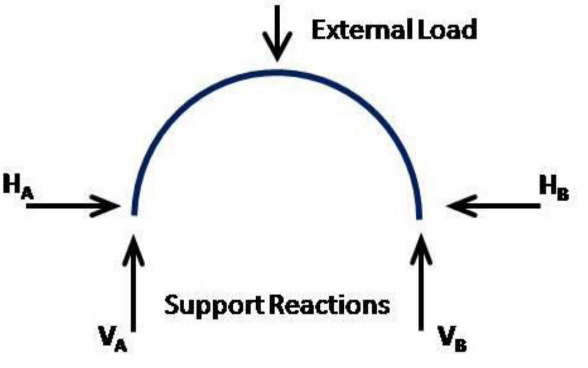


Figure 8 Reaction forces in an arch

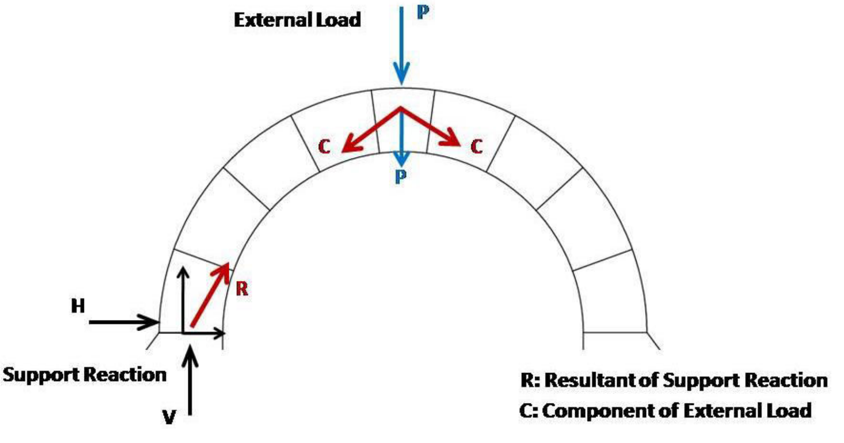


Figure 9 Arch actions

Each component of an arch is in compression under the action of gravity load.

So, brittle materials can be used in arch construction.

Here the load P will break into two components C and C as shown in the Figure 9. Additionally, the horizontal reaction H and the vertical reaction V will give rise to a resulting force R. The forces C and R will be spread across the whole of the arch and due to their directions being opposite, they will tend create compression zones throughout the arch. Therefore, it is said that each component of an arch is in compression under the action of gravity load.

Example of Arch Actions

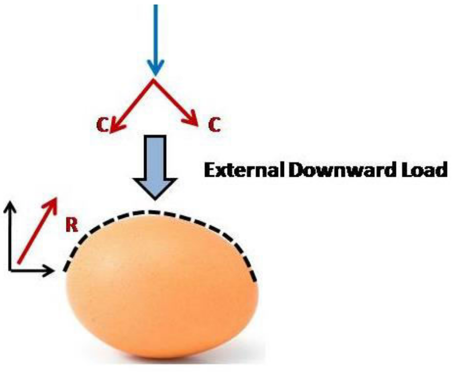


Figure 10 Arch actions in an egg

Arch action on the thin shell of an egg creates compression. As egg shell is a brittle material it will resist the compressive stress.

Hence, it will not break easily and it's tough to break the egg by external loading.

However, in case of an internal upward load arch action on the thin shell creates tension. As egg shell is a brittle material it cannot resist the tensile stress. It will break very easily. Therefore, it's easy to break the egg by internal upward loading.

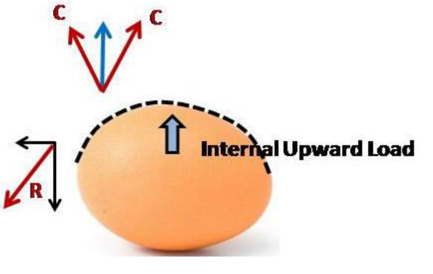


Figure 11 Arch actions in an egg

Revision: Beam Subjected to a Point Load

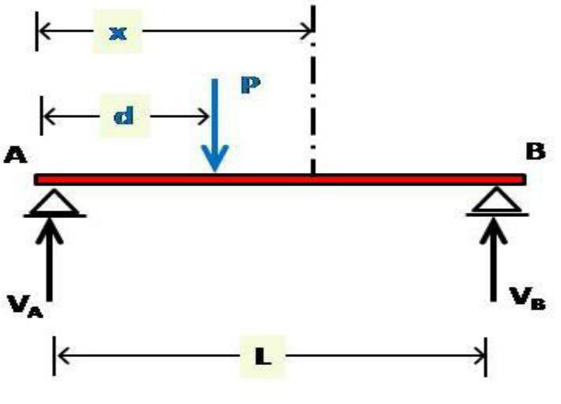


Figure 12 Flat arch subjected to a point load

In case of a beam there are no horizontal reactions. So,

$M_{BX} = (V_A \times x) - P(x - d)$

Arch Subjected to a Point Load

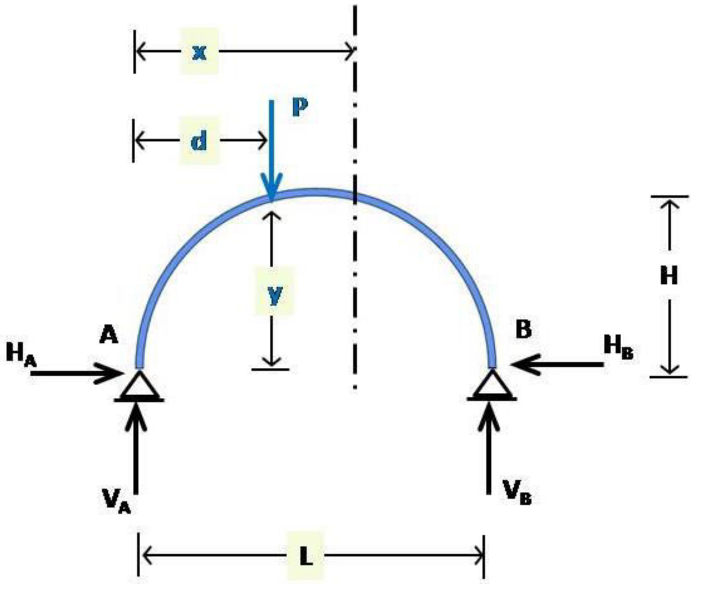


Figure 13 Arch subjected to a point load

Horizontal reactions exist in case of an arch.

So,

$M_{AX} = (V_A \times x) - P(x - d) - H_A \times y$

i.e., M_{AX} =[Beam Moment] - $H_A \times y$

Therefore,

Arch Moment < Beam Moment

Since an arch creates lesser moment when compared to a beam, it requires lesser material as well. Hence it is better to go for an arch as compared to a beam for a given span.

Analysis of Arch

Two-hinged Semi-circular Arch with Concentrated Load

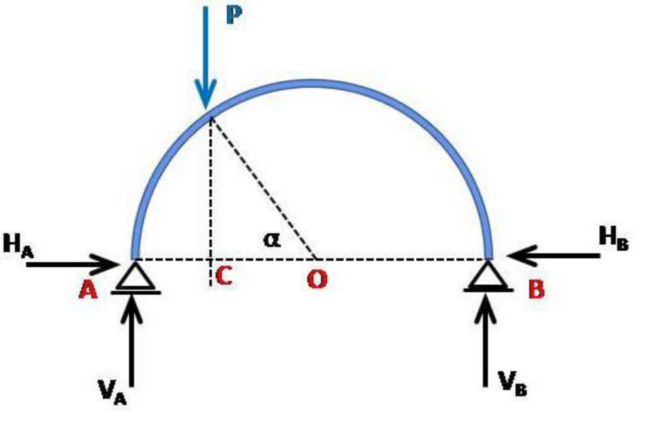


Figure 14 Analysis of two-hinged semi-circular arch with concentrated load

We have, $H = \frac{P}{\pi} sin^2\alpha$

Taking Moment about A

$$2RV_B = P \times AC = P \times R(1 - \cos\alpha) \qquad [AC = R\ (1\text{-}\cos\alpha)]$$

i.e., $V_B = \frac{P}{2}(1 - \cos\alpha)$

Taking Moment about B

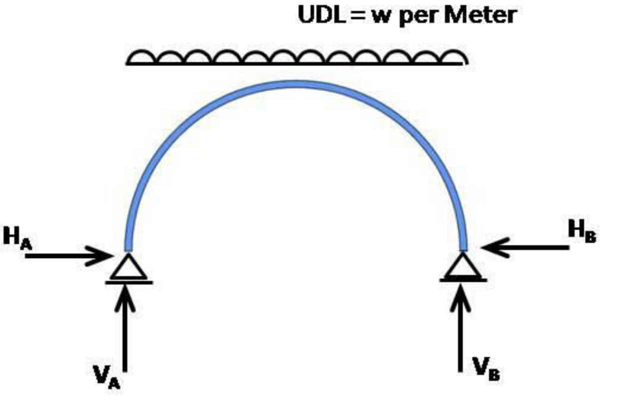
$2RV_A = P \times BC = P \times R(1 + \cos\alpha)$ [BC = R (1 + cos α)]

i.e., $V_A = \frac{P}{2}(1 + \cos\alpha)$

Therefore in case of central concentrated load (P) [α = 90°]

H = $\frac{P}{\pi}$ and $V_A = V_B = \frac{P}{2}$

Case-2: Two-hinged Semi-circular Arch with Uniformly Distributed Load



$$H = \frac{4wR}{3\pi} \qquad V_A = V_B = wR$$

Figure 15 Two-hinged semi-circular arch with uniformly distributed load

Profile Equation of a Parabolic Arch

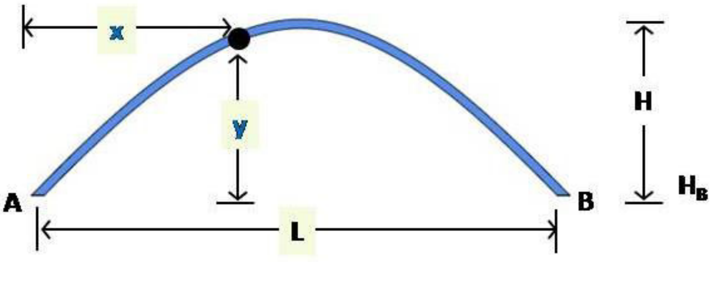


Figure 16 Parabolic arch

Equation of the parabolic arch:

$$y = \frac{4H}{L^2}(L - x)x$$

At x = 0,L; y = 0

At $x = \frac{L}{2}$; $y = \frac{4H}{L^2}\left(L - \frac{L}{2}\right)\frac{L}{2} = H$

Case-3: Fixed Parabolic Arch with Concentrated Load

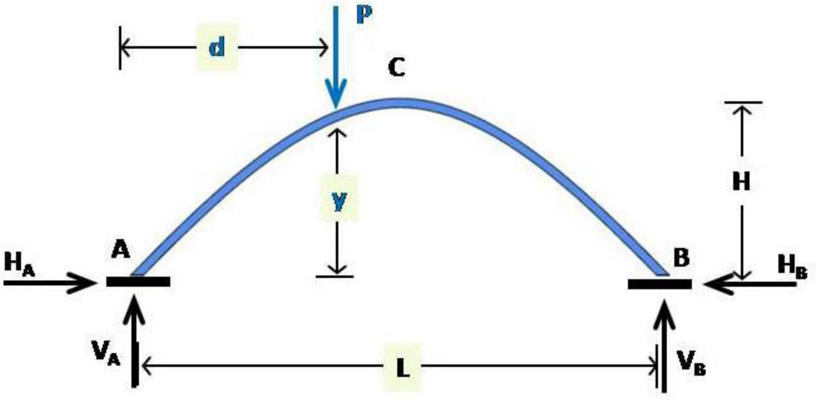


Figure 17 Analysis of fixed parabolic arch with concentrated load

$$M_A = PL\left(\frac{d}{L}\right) \times (\frac{L-d}{L})^2 \times (\frac{5d}{2L} - 1)$$

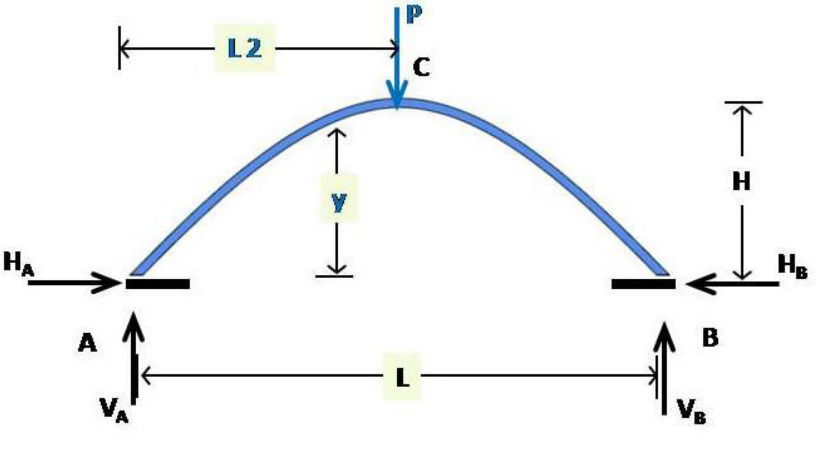
$$M_B = PL(\frac{d}{L})^2 \times \left(\frac{L-d}{L}\right) \times (\frac{5(L-d)}{2L} - 1)$$

$$M_C = \frac{PL}{2}(\frac{d}{L})^2 \times [1 - \left\{\frac{5(L-d)}{2L}\right\}^2]$$

$$V_A = (\frac{L-d}{L})^2 \times \left(1 + 2\frac{d}{L}\right)P$$

$$V_B = (\frac{d}{L})^2 \times \left(1 + 2\left(\frac{L-d}{L}\right)\right)P$$

$$H_A = H_B = \frac{15PL}{4H}(\frac{d}{L})^2 \times (\frac{L-d}{L})^2$$



$$M_A = M_B = \frac{PL}{32}$$

$$M_C = \frac{3PL}{64}$$

$$V_A = V_B = \frac{P}{2}$$

$$H_A = H_B = \frac{15PL}{16H}$$

Case-4: Fixed Parabolic Arch with Uniformly Distributed Load

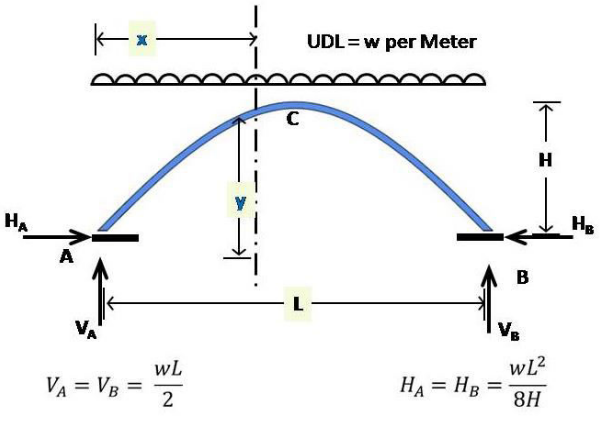


Figure 18 Fixed parabolic arch with UDL

$$M_x = \frac{wL}{2}x - \frac{wx^2}{2} - \frac{wL^2}{8H}y$$

$$\text{i.e., } M_x = \frac{wL}{2}x - \frac{wx^2}{2} - \frac{wL^2}{8H} \times \frac{4H}{L^2}(L-x)x$$

$$\text{i.e., } M_x = \frac{wL}{2}x - \frac{wx^2}{2} - \frac{w}{2}(L-x)x$$

$$\text{i.e., } \boldsymbol{M_x = \frac{wL}{2}x - \frac{wx^2}{2} - \frac{wL}{2}x + \frac{wx^2}{2} = 0}$$

Concept of Thrust Line

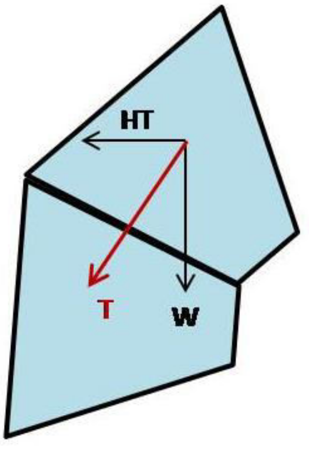


Figure 19 Concept of thrust line

A Thrust is generated in the masonry Arches due to its profile. The intensity and angle of the thrust may disturb the stability of the arch as a whole.

The Thrust (T) is the resultant of two forces:
(i) the Weight of the Arch (W) and
(ii) the Horizontal Thrust (HT).

Intensity and Line of Action of Thrust depends on the profile of arch and the weight of voussoirs. Arches with greater depth generate more horizontal thrust.

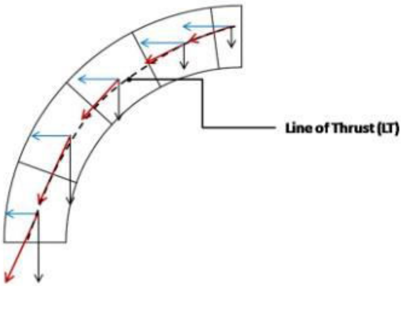
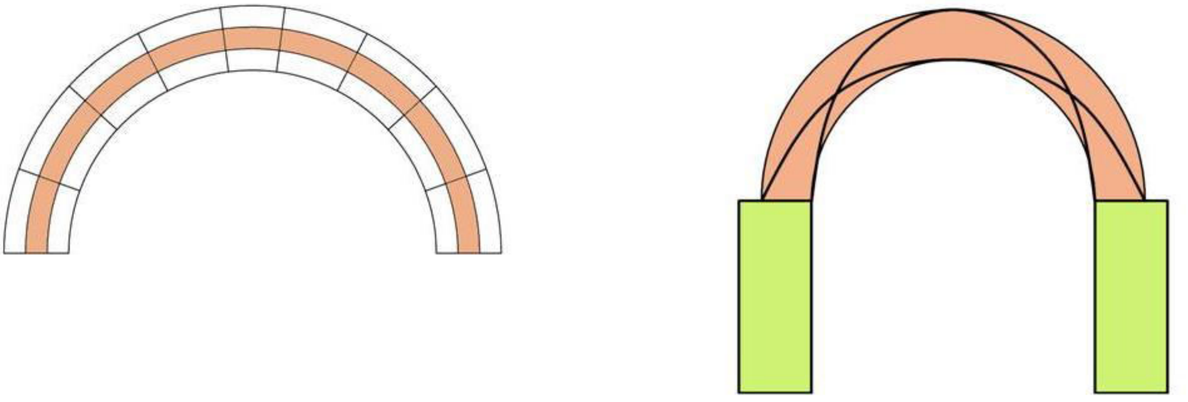
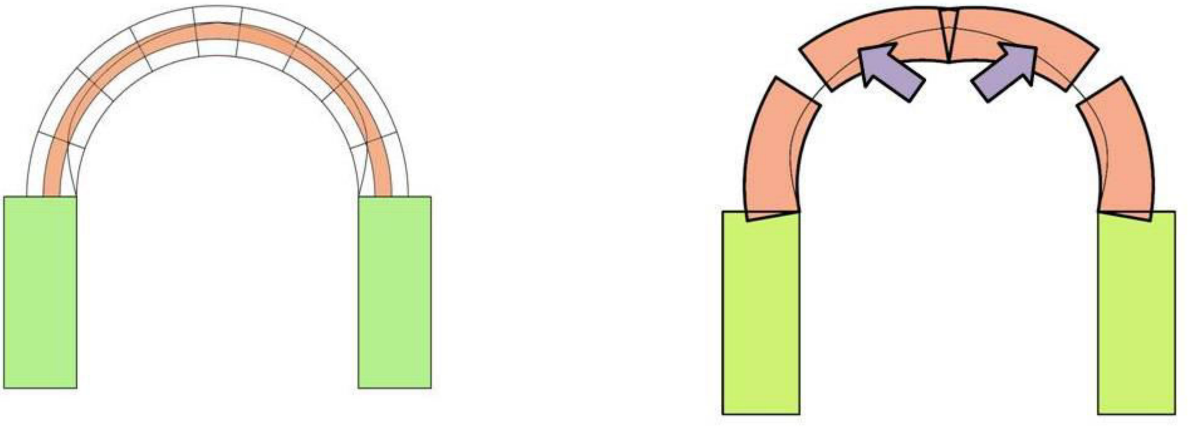


Figure 20 Thrust Line

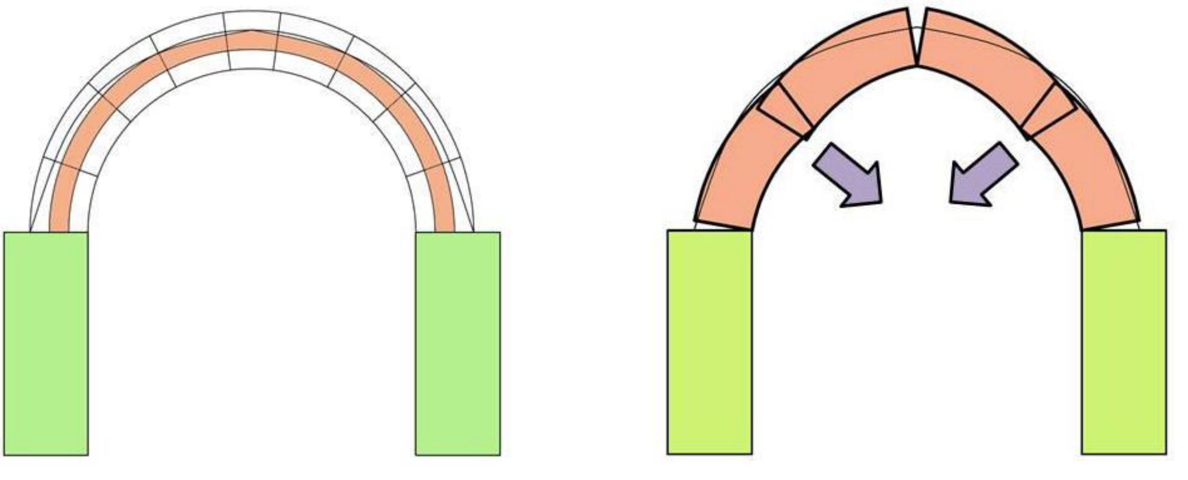
Each voussoir provides a Horizontal Thrust (HT) and Weight (W). Finally, the resultant of HT & W ends up in a Thrust (T). A line tangential to all the Thrusts provided by the voussoirs is called Line of Thrust (LT)



An arch is stable as long as Line of Thrust (LT) remains in the middle third of the arch section.



When LT goes in the inner third of the arch, the arch will tend to burst outwards.



When LT goes in the outer third of the arch, the arch will tend to collapse inwards.

References

- ✓ **Structure as Architecture** By Andrew W. Charleson, Elsevier Publication
- ✓ **Basic Structures for Engineers and Architects** By Philip Garrison, Blackwell Publisher
- ✓ **Structure and Architecture** By Meta Angus J. Macdonald, Elsevier Publication
- ✓ **AVD Construction** by Auroville Earth Institute

Conclusion

- ✓ Arch is a special type of structure. The curvilinear profile establishes a compression character in its parts.
- ✓ It shows less bending moment compared to traditional flat beam and hence makes it possible to be used as long span solutions.
- ✓ The stability of arch should be considered by the action of the line of thrust.

Homework

Q1. A fixed parabolic arch of span 12m and rise 3m is under a uniform distributed load of 10 KN/m. Find

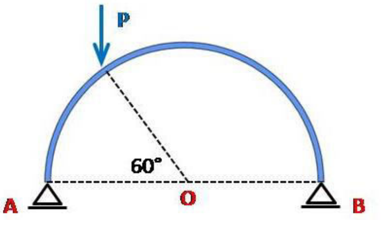
(i) The Support reactions and

(ii) The bending moment at one-third span (i.e. a section 4m from any support)

Q2. A circular two hinged arch of radius 'R' is under a single concentrated load 'P' as per the figure below. Find

(i) The Support reactions and

(ii) The bending moment at the crown of the arch, if P = 20KN & R = 5m



Q3. Two circular two hinged arches are placed 10m apart and support three straight beams as per the figure below. Each beam is carrying a UDL of 12 KN/m intensity. The three beams are symmetrically placed on the arch. The central beam is at the crown. The other two beams are connected to the arch radially at an angle 45°. Find

(i) The Support reactions and thrust at support

(ii) The bending moment at the crown and at the other two beam points.