# Structural Systems in Architecture

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## Module 1
## Lecture – 3
## Moment, Couple and Static Equilibrium

Good morning students. Welcome to NPTEL online course on Structural Systems in Architecture. Today we will discuss the lecture 3 of module 1. This lecture will be on moment, couple, and static equilibrium.

The concepts to be covered here are:

- Moments and Couple
- Resultant of Force System
- Resolving of Force System
- Equilibrium of Force System

In the first lecture we have discussed about force system. Towards the end of last lecture, we have also discussed about the two important parameters in the structural loading; they are moment and the couple.

So today initially we will discuss about moment and couple a bit; and then we will again go to the force system and we will see how a resultant of the forces can be obtained. How in the force system, resolving of the force system can be done; and also, what are the equilibrium conditions for the force system. These three, the resultant, resolving and the equilibrium of the force system are very important for the statics point of view and most of the times in our building, in our architecture, in our structural engineering, we will go with this system of forces. So, based on requirements, sometimes we will find out the resultant, sometimes we will find out what is the equilibrium conditions etc.

The learning objectives of this lecture is:

- To conceptualization of Moment and Couple.
- To outline the Resultant and Resolving Force System.
- To deduce the Equilibrium Condition of Force System.

So, the learning objectives for this lecture will be; we will first try to conceptualize the moment and the couple; and then we will outline the resultant and the resolving of the force system. Here we will do some of the small computational or the mathematical problems and I hope that, you all are aware of them by virtue of your class 12th Physics. Finally, we will go to the equilibrium conditions of force system, which is actually applicable for the static equilibrium conditions.

Now, what is moment? Moment and couple are described as effect and the cause; the cause is the moment and the effect are couple of a force system of any kind of rigid body. So, it is a rigid body mechanics. When a force is applied to a particular point and then from another point when the other point is separated by a certain distance, we can find out how much is the turning effect of that particular force. How much is it going to turn? Turned means how much will be the rotational effect of the force because of that separating distance. So, the amount is directly proportional to the magnitude of the force and the perpendicular distance to the force from the point. The unit of the moment is Newton meter. Because Newton is the unit of force and the meter is the distance, so it is just multiplication of the Newton meter and both are directly proportional. In physical sense the moment is described as torque.

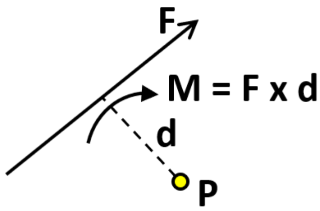



**Figure 1: force, distance, and moment**

Now in Figure 1, you can see that there is a point P and there is a force, line of action of the force is F, and the perpendicular distance is d, if this is the scenario, then I can say that moment is the force multiplied by the distance.

So, the force on the line of action of the force I have to identify and I have to see that what is the perpendicular distance. We always have to take only the perpendicular distance between the line of action force and the point of application, not any other possible arbitrary distances.

Next, we will talk about couple. Couple is a system of two forces, which are equal in magnitude but opposite in direction, and have parallel lines of action. In Figure 3, you can see that there are two equal forces F applied on a rigid body, where the left-hand side force direction is downward and right-hand side force is upward, both are moving in anti-clockwise direction, and they are having some separation distance $d_1 + d_2$ respectively from the point P. Now, I am going to find out what is the couple at point P.

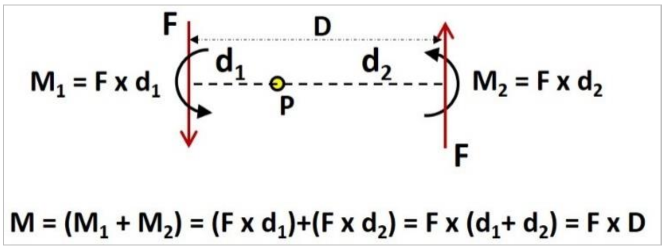



**Figure 2: force, moment and couple**

Here I have the point P and I have two perpendicular distances, so, I will get two moment generated, $M_1$ for the first or the left hand side force, it is F x $d_1$ and $M_2$ is the generated moment due to the second force the right hand side, and that is F x $d_2$. So now both the moments are in the same direction anti-clockwise directions. Therefore, we can add them arithmetically, and the total moment M will be $M_1 + M_2$.

Now if I finally do all the addition, that is sum of $F_1$ x $d_1$ plus $F_2$ x $d_2$, finally it gives me the F x D. So that gives me the formula to find out the couple.

Now we will see what are the difference between the couple, and moment. Moment is the force that measures the turning effect or the rotational effect by a force about a given point whereas couple are the two equal and opposite forces acting in a particular rigid body with parallel lines to each other. Nonparallel forces may not result into couple.

Now let us take an example. So, I have a bar and this bar is hinged or pinned at its bottom point, so that it can rotate, see in Figure 4. The bar can rotate under any kind of force. The length of the bar is 3 meters and it is having an inclination of $30^o$ with the horizontal. Now, I put a force, straight horizontal force F=20 Newton. After putting a force of 20 Newton, I have to find out the distance from the force to the point of hinge or pin; and this distance is 3 meters. But now I have to find out, what are the components of this particular force? So, if the angle of the bar is $30^o$ with the horizontal line, then the angle between the force and its cos component, which lies along the same line with central axis of the bar, will also be $30^o$ . The sin component of the force is perpendicular to the central axis of the bar, and downward. So, I have to find out what is the sin component and what is the cos component. So, this force, the cos component is 17.32N and the sin component is 10N.

Now if you see then you can see that the force 17.32 this is acting along with the axis, so this will not create any kind of moment because the perpendicular distance from this point to the force is 0. So, F x d that 17.32 x 0 = 0, so the moment is equal to 0; but the force 10N is having a distance of 3 meter. The final moment will be 10 x 3=30 Newton per meter.

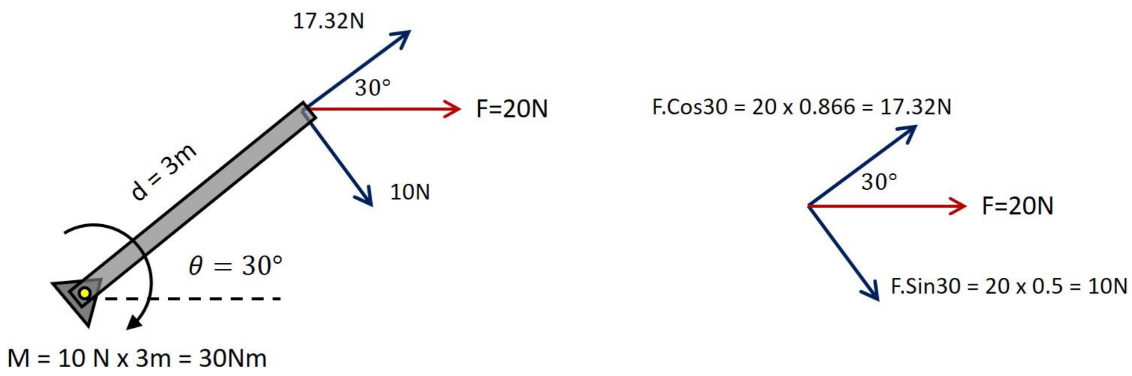



**Figure 3: force couple and moment**

Even though the applied force is 20N and it is acting on it, it is not going to be 20 x 3, it will be 10 x 3 because the perpendicular component of the force is 10; and the other component is going to be through the axis, so it will not be considered.

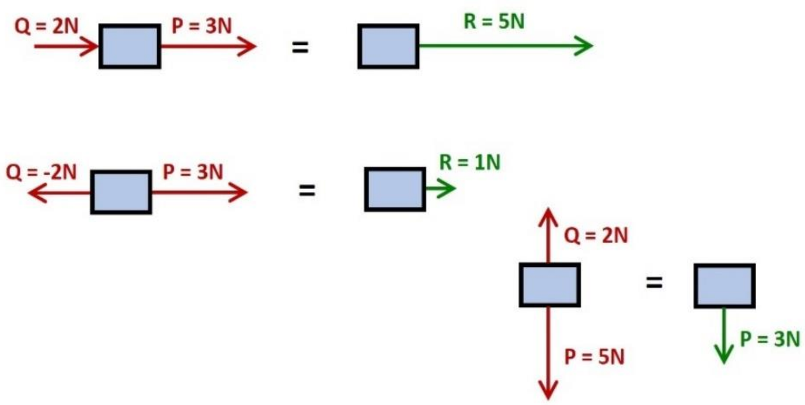



**Figure 4: resultant of force system in coplanar force**

Now let us go to the next agenda of today's lecture, that is the resultant of the force. Now let us see how the resultant of the force can be computed? At first, we will take the coplanar forces. So, suppose a body is under a force of P and Q which is 3 and 2 Newton respectively and both are acting in the same direction (see Figure 4). As it is acting in the same direction, this will just be added arithmetically, so the resultant will be 5. On the other hand, in second condition, the force P and Q are opposite in direction. On right-hand side it is 3 Newton and in the left-hand side it is 2N, this 2 will be now negative 2 and the resultant force will be in the right-hand side direction, 3-2=1 Newton. Similarly, if I have the forces in the vertical directions, as shown in 3$^{rd}$ case, I have downward resultant force as 3 because 5-2=3. So, these are the resultant force computation for the coplanar forces.

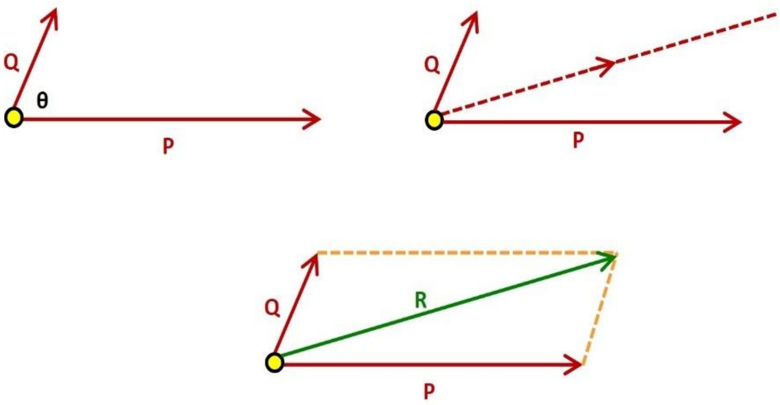
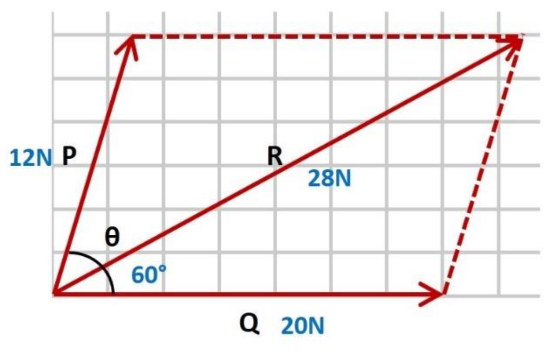



**Figure 5: coplanar non-concurrent**

Now the coplanar forces when it is concurrent. The concurrent forces that means that when the forces meet at a certain point and they may be of different directions. Then how to calculate the forces or the resultant forces of that? You can see in the Figure 5, that there is a common point, lets ay it is a yellow color ball; and at this point force P and force Q are now suppose inclined at θº. Now I want to find out what is the resultant forces?

Here, suppose there are two pulling forces which are trying to pull the same yellow ball, where force P is higher than the force Q. The force Q is in θ direction with respect to P. So, we can imagine that the influence of P will be much higher as compared to influence of Q, because the amount or the magnitude of P is much higher.

So, the resultant will actually go like the red dotted line, as shown in the top right image of Figure 5. It will remain near to P because the influence of P will be higher than Q; and finally, we can have a parallelogram created by the 2 parallel lines of P and Q. Now we can say that the resultant forces will be along the green line R, which is the cord or the diagonal of that particular parallelogram.




**P = 12N, Q = 20N, θ = 60°**

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$R = \sqrt{12^2 + 20^2 + 2X12X20\cos 60}$$

$$R = \sqrt{12^2 + 20^2 + 2X12X20X0.5}$$

$$R = \sqrt{144 + 400 + 240}$$

$$R = \sqrt{784}$$

$R$ =28N

**Figure 6:the forces and the resultant**

So now to calculate this R we have a very general formula:

$R = \sqrt{P^2 + Q^2 + 2\,PQ\cos\theta}$

Now, if I put some values like P=12, Q=20 and θ is 60º, as shown in Figure 6, then using this formula I can find out the resultant value of R.

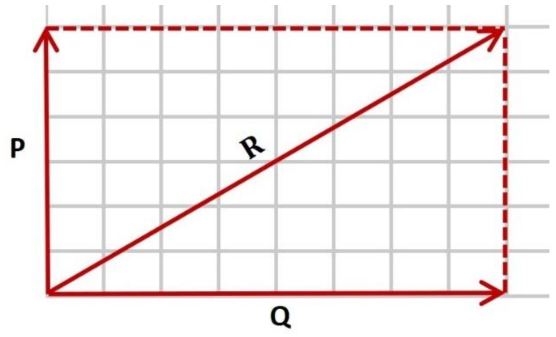



**Figure 7: forces perpendicular to each other and its resultant**

Now see the Figure 7. Now in this case, if the force P and Q are perpendicular to each other, that means the θ, the angle between P and Q will be 90º. So here I will put the cos θ to be 90º, and Cos 90 = 0. So, it will be:

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos 90}$$

$$R = \sqrt{P^2 + Q^2}$$

So, from the geometry you know that, it is the hypotenuse of a triangle, whose base and the heights are P and Q.

Similarly, we can compute for forces like 8 and 6 Newton which are in the perpendicular direction, the resultant will be 10 Newton, see Figure 8. So, this is for the coplanar concurrent forces.

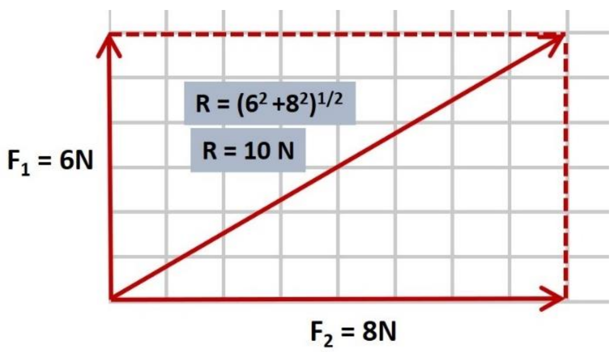



**Figure 8: resultant of two perpendicular forces**

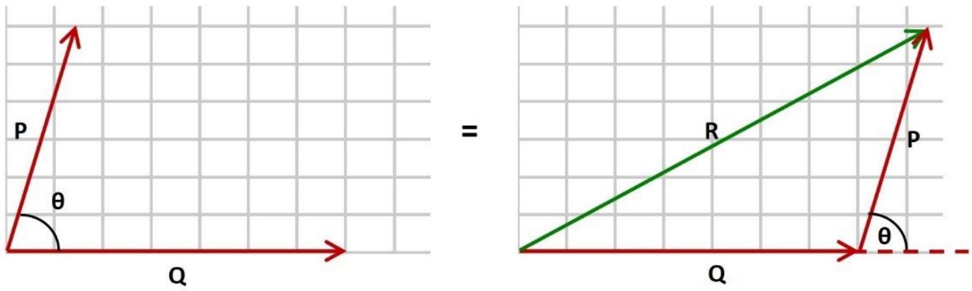



**Figure 9: resultant force: law of triangle**

These coplanar concurrent forces can also be cutoff of through the law of triangle. Suppose like the earlier example, if I take P and Q with an angle of θ, and Q is along horizontal direction and P is upward direction, making angle of θ with Q. Now, the P can be shifted to another point as shown in Figure 9, maintaining same inclination and angle of θ. Then, we can close this triangle, (the green line, as shown in Figure 9) and we can find out the resultant. When we close the triangle, we must remember that the direction of closing line that is the resultant should be in opposite direction.

Similarly, for the perpendicular forces also, we can find out the resultant. But we should always keep it in mind that, the resultant force is the reverse direction of closing triangle side.

Now we will discuss about the resultant force for coplanar-parallel forces. The parallel forces are those two P and Q, as shown in Figure 10. They are the parallel force on a rigid body. When there are forces, there must be the resultant too. The resultant will be at opposite direction, here I am showing this as opposite force showing an equilibrium kind of condition. So, the R will be located in such a position that it will satisfy valid the equation below:

$$P \times D1 = Q \times D2$$

And, $R = P + Q$

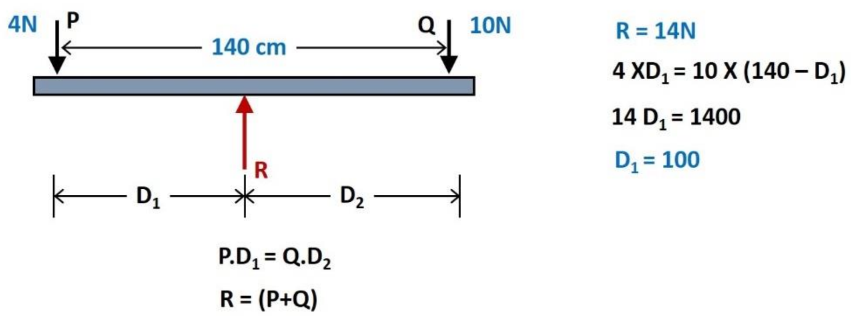



**Figure 10: resultant force: coplanar parallel forces**

So, if I know the distance between them and the values of P and Q; I can find out the position of the resultant force and what will be the magnitude. I can also find the magnitude of the resultant by simple arithmetic addition of P and Q, because they are in same direction.

Now, let us suppose P = 4N and Q = 10N, both are acting in downward direction; and distance between them is 140 cm, as shown in Figure 10. So, the resultant force can be simply P + Q =14 N. We can also calculate the position of the resultant force by applying the formula P x D1 = Q x D2. This will be:

$$4 \times D1 = 10 \times (140 - D1)$$

$$14\ D1 = 1400$$

$$D1 = 100, \text{ So, } D2 = 140 - 100 = 40$$

Now, next let us go to the resolving of the force system. Suppose there is a body, let's say a yellow color ball, and force P is acting on it, at an angle of θ, as shown in Figure 11. Now, I have to resolve this force in the X and Y direction. It is very easy, we just have to drop the perpendiculars from the end point, in both X and Y direction; and I can find out the cos component and the sin component of the P. Your cos component will be towards the θ, and the sin component will be on other side.

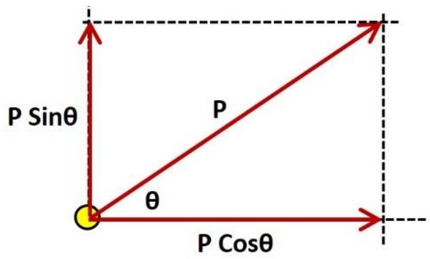



**Figure 11: resultant of force: cos and sin component**

Now, consider force P = 20N and the angle θ = 60°, see Figure 12.

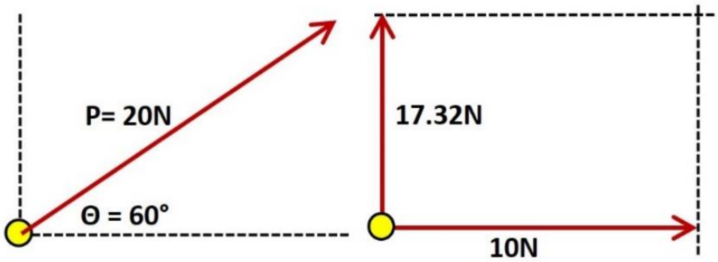



**Figure 12: resolving cos and sin components of force**

Resolving this, we get:

$P\ Cos\theta = 20 \times Cos\ 60° = 20\ X\ 0.5 = 10N$

P Sinθ = 20 x Sin 60° = 20 X 0.866 = 17.32N

So again, if I have these 2 forces, then I can cross check the resultant force, using the formula:

$$=\sqrt{P^2 + Q^2}$$

$$= \sqrt{10^2 + 17.32^2}$$

$$= \sqrt{100 + 300}$$

$$= \sqrt{400}$$

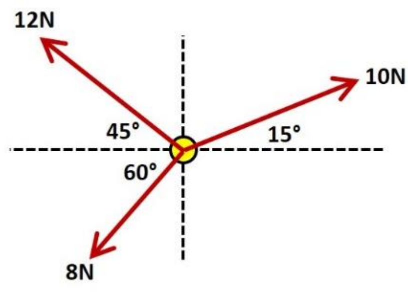
$$= 20$$

So, it proves that the if we have the resultant force, we can find the resolved forces and from resolved forces we can find the resultant force.

So now, let us suppose I have three forces having 15°, 45° and 60° inclination with the horizontal axis and 10, 12 and 8 kilo Newton magnitude respectively. The forces and the angles with the X axis, P cos θ, and the P sin θ are shown in top image of Figure 13.

Now if I consider the force 10 Newton and angle 15º alone, then the cos component is 9.7 and the sin component is 2.6 which is close to 2.7, actually consider it is 2.7. Secondly, for the force 12 Newton and angle 45º, the cos component is 8.5 and sin component is also 8.5. Finally, for the force 8 Newton which is the downward, and angle 60º with the X axis gives 4 as cos component and 6.9 is sin component.

Now the three forces are resolved into six; as shown in Figure 14; as red and green forces respectively, in directions of, X and Y. Now I can say that three red color forces are equal to six green color forces.




| Force | Angle | Horizontal | Vertical |
|---|---|---|---|
| P | θ | P.Cosθ | P.Sinθ |
| 10 | 15 | 9.7 | 2.6 |
| 12 | 45 | 8.5 | 8.5 |
| 8 | 60 | 4.0 | 6.9 |

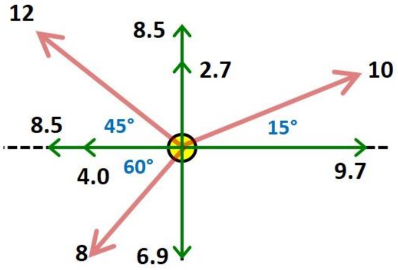



**Figure 13: resultant of more than two forces with different angles**

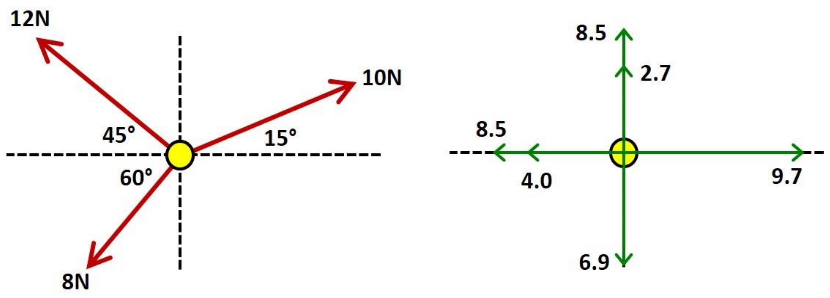



**Figure 14: the three forces resolved into six forces**

After this I can calculate forces along X and Y direction. Now, for forces in X or horizontal direction, it is:

9.7 - (8.5+4) = - 2.8

And for Y or vertical direction; it is:

8.5 + 2.7 – 6.9 = + 4.3

From this, I can find out the resultant force virtue of the R = $\sqrt{}P^2 + Q^2$.

$$R = \sqrt{(-2.8)^2 + (4.3)^2} = 5.13$$

Finally, I can also find out the angle θ; by virtue of this trigonometry and it will be:

Tan α = 4.3/2.8 = 1.53

$\alpha = \tan^{-1} 1.53 = 57^o$.

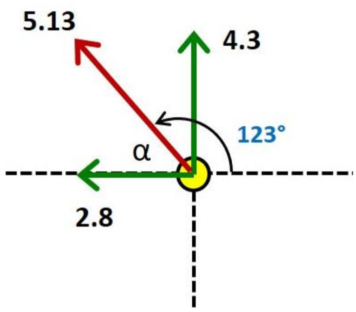



**Figure 15: angle of inclination**

So, if α is $57^o$, the other angle is $180^o$-$57^o$ = $123^o$. Therefore, I can say that this force system is nothing but a resultant force as shown in Figure 15, which your 5.13 and angle of inclination is $123^o$. Hence, through the resultant, again I can again come back to the final resolving and vice versa.

Now, let us discuss about the equilibrium of forces. Suppose there is a green ball, and this ball is at equilibrium condition under the five forces; as shown in Figure 16, left-hand side image. So, what I do? Can I break all the forces in to X and Y direction? Yes; I can break those five forces in X and Y direction, similarly as we did in the previous example, with the help of angle of inclination and amount of force.

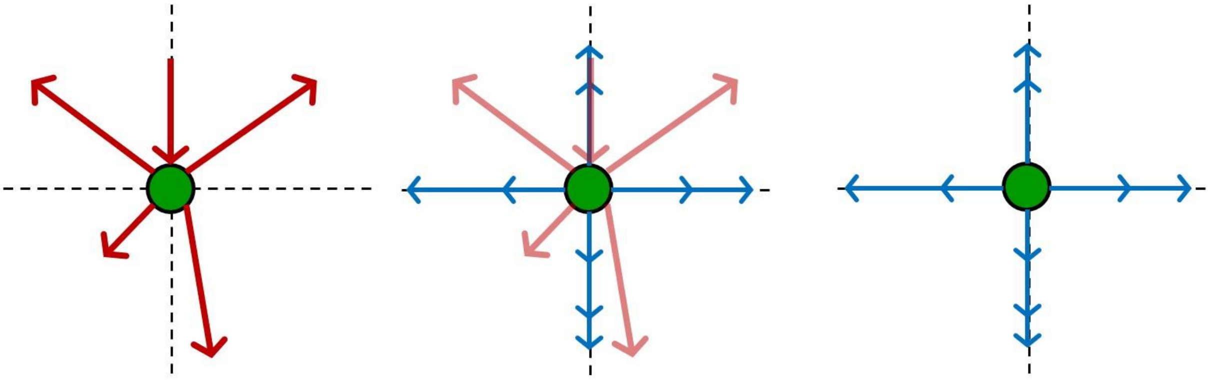


**Figure 16: equilibrium of an object under five forces**

Here, to maintain the equilibrium condition of the body, the sum of the forces in X direction must be equal to zero; as well as the sum of the forces in Y direction also must be equal to zero; as shown in Figure 17. Otherwise the body will move with respect to higher force or in any positive or negative axis. So, I can say that:

$\sum F_X = 0$; and

$\sum F_Y = 0$

So, these 2 are the very basic consideration for the equilibrium.

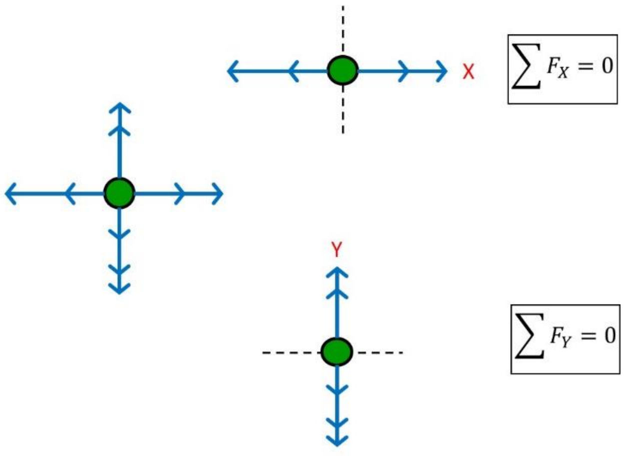



**Figure 17: equilibrium state of a body**

Now we will see the next example. Here a 200 Newton load is applied on a particular body through a rope, so I want to find out this tension in the wire or the rope $T_1$ and $T_2$ (see Figure 18).

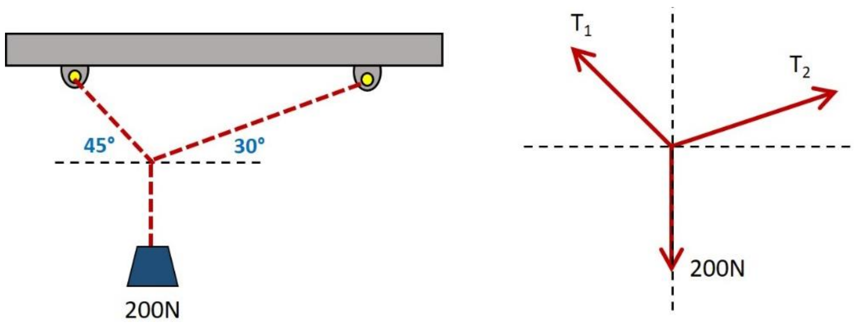



**Figure 18: equilibrium of a load through a rope**

Here, I know the inclination. So, I will resolve this $T_1$ sin θ and T1 cos θ, as well as for the other rope, that is the right-hand side rope $T_2$ as shown in Figure 19. Now, what I have is that, I have only the horizontal and the vertical component of the rope.

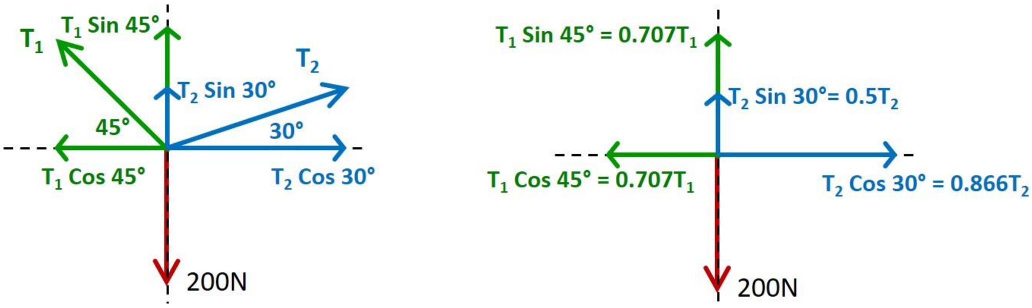



**Figure 19: resolving forces**

So, if I equate the X axis, then some of its forces must be equal, otherwise it will move in X direction. The 1st relation we get as:

$$0.707\ T_1 = 0.866 T_2$$

$$T_1 = 1.225\ T_2$$

Similarly equating equilibrium on Y axis:

$$0.707\ T_1 + 0.5\ T_2 = 200N$$

Solving them we get:

$$T_1 = 179.3N, \text{ and } T_2 = 146.4N$$

So, from those two equations I have found out the two unknowns $T_1$ and $T_2$, by virtue of the linear equation, I can find out what are the tension in the $T_1$ and $T_2$**.**

So, equilibrium of the forces also has a third component, that is the moment, which also should be in equilibrium. So, as shown in Figure 20, I have rigid body which is experiencing force P1 and P2 at a distance D1and D2 from a particular pin. Now the moment as this particular point of pin will be:

$M_L = P_1 \times D_1$ (Moments towards left)

$M_R = P_2 \times D_2$ (Moments towards left)

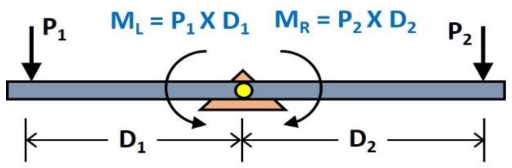



**Figure 20: equilibrium of moment**

Similarly, if I have two forces, 2 kg and 2 kg, with distance of 3 meter and 5meter from its hinge, as shown in Figure 21.

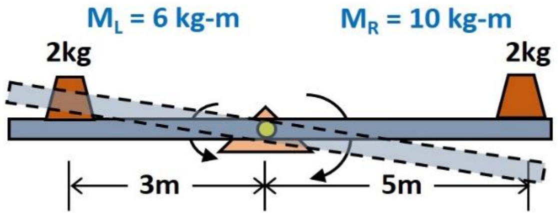



**Figure 21: equilibrium of moment due to equal load and un equal distance**

Computing this, we find

$M_L = 2 \times 3 = 6$ kg-m

$M_R = 2 \times 5 = 10$ kg-m

Here the $M_L \neq M_R$; that means it is not under equilibrium. Therefore, this bar will definitely rotate in clock-wise direction.

Similarly, in another case, if I have 2 kg and 1 kg load at distances 5 meter and 5 meter respectively as shown in Figure 22.

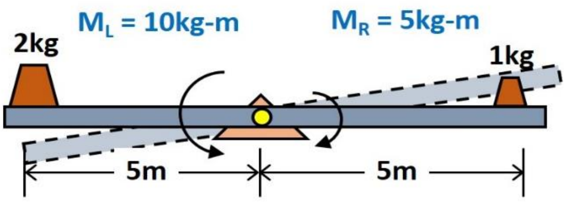



**Figure 22: equilibrium of moment due to unequal load and equal distance**

Computing this, we find:

$M_L = 2 \times 5 = 10$ kg-m

$M_R = 1 \times 5 = 5$ kg-m

Here also the $M_L \neq M_R$; that means it is not under equilibrium condition. So, the bar will rotate in anti-clock wise direction.

Now, let us see the sum of total moment generated by the applied forces.

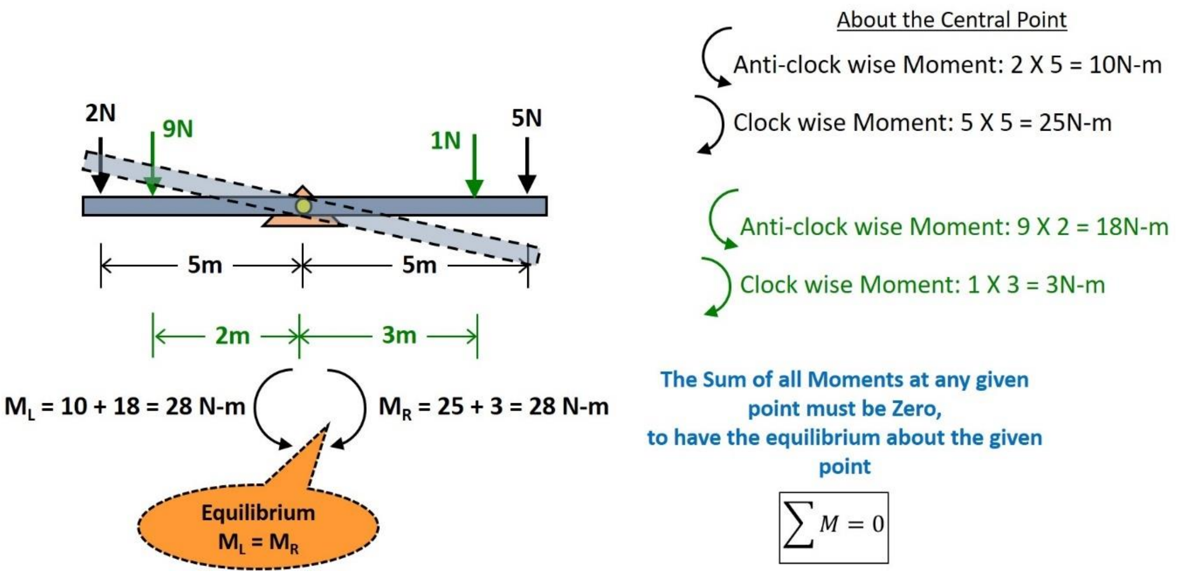



**Figure 23: clock wise and anti-clock wise moment**

So in case one, if I have 2 Newton and 5 Newton, and separating distance is 5 meter from the central pin, I may say now, $M_L$= 10 N-m and $M_R$ = 25 N-m. So, it will move towards heavier side that is 25 N-m, so, it will result to clock-wise moment. But, if I put 9 and 1 kilo Newton load at distance 2 meter and 3 meter respectively, then I will get the green color anti-clockwise moment and the clockwise moment as shown in Figure 23. Here, it will move in anti-clock wise direction.

Now if I add up all the anti-clockwise moment that is: 10 + 18 = 28 and the clockwise moment as the 25 + 3 = 28, both are equal, so it will be under equilibrium. So, in such cases the bar will not move; it will be in state of equilibrium. Hence, in the equilibrium conditions also, I may say that the sum of all the moments must be equal to 0, if so, then it will not rotate under any kind of motions.

Finally, what we found is that, there are 3 equations of the equilibrium.

$$\sum F_X = 0$$

$$\sum F_Y = 0$$

$$\sum M = 0$$

Now, let us see another example, as shown in Figure 24.

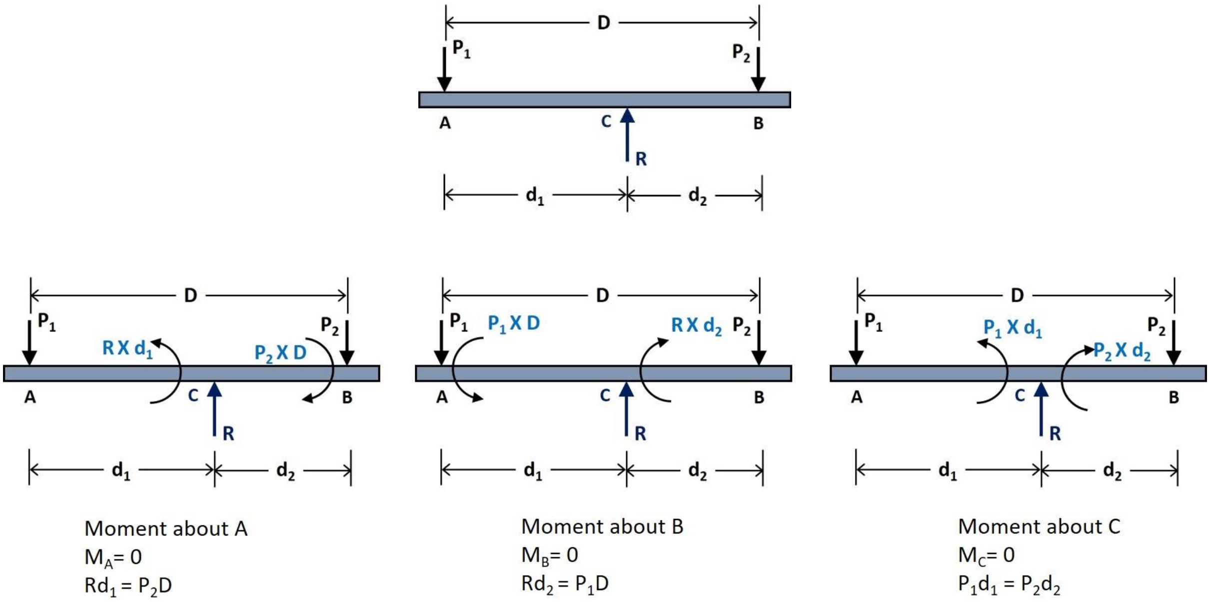



**Figure 24: equilibrium with different forces**

There are 2 such points and now I can find out what are the forces. Here, $F_Y = 0$, so the $R = P_1 + P_2$. If I take the moment about A, $M_A = 0$, I have this force R at a distance of $d_1$, so $Rd1=P_2 \times D$, so these two has to be equal to 0. If I take the moment about B, the equation changes like $R \times d_2 = P_1 \times D$.

So, if I take a moment about C, also then, $P_1 \times d_1$ must be equal to $P_2 \times d_2$, so that way also I can find out the forces and all the moments.

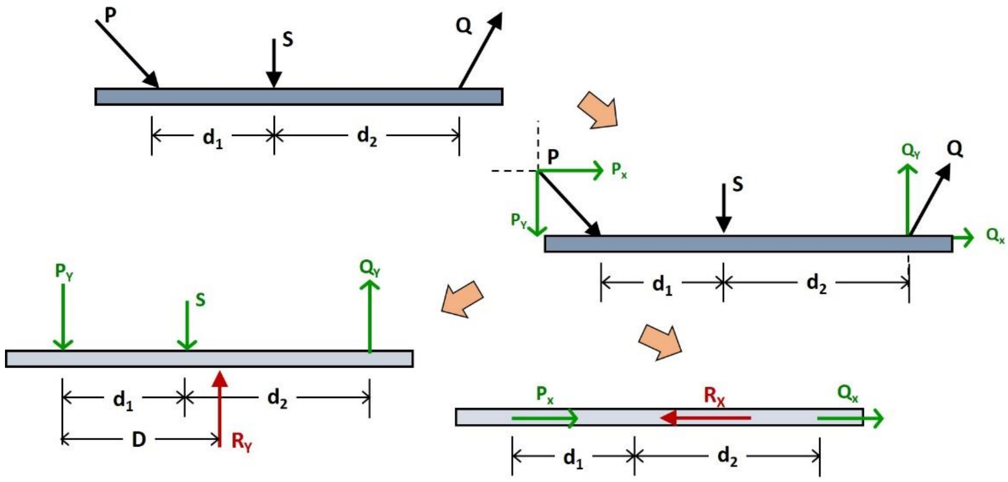



**Figure 25: resultant of forces in different orientations**

Suppose in a body the forces are acting different orientation or with different angles, as shown in Figure 25. Now, I can resolve the forces, I can resolve force P as $P_X$ and $P_Y$ and similarly I can resolve the force as $Q_X$ and $Q_Y$. Now, all the forces are either in X direction or in Y direction. Here, Y direction forces are $P_Y$, $Q_Y$ and S and that gives me some R value the resultant value in Ry direction. On the other hand, the X axis direction forces are Px, and $Q_X$ there is no S here. So, Rx will be calculated from here.

$$R=\sqrt{(R_X)^2+(R_Y)^2}$$

$$\theta=\tan^{-1}\left(\frac{R_X}{R_Y}\right)$$

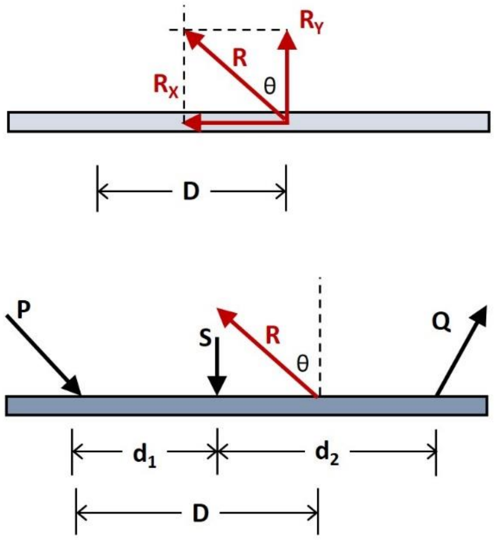



**Figure 26: resultant of forces in different orientations**

Then finally, I can find out what is the reaction or the resultant direction between them and with this formula I can also find out what will be the angle θ for that.

So, for this particular lecture, I have taken the difference as these 3 references like earlier one. They are:

- **Reinforced Concrete Design** by Pillai & Menon, Tata McGraw Hill Publisher
- **Basic Structures for Engineers and Architects** by Philip Garrison, Blackwell Publisher
- **Understanding Structures:** ***An Introduction to Structural Analysis*** by Meta A. Sozen & T. Ichinose, CRC Press

In conclusion we can say that, the moment and the couple are the two products of the force system, associated with the distance and condition of the support. Why condition of the supports? Because, if you want to figure out the moment, you need a pin support in between. The resultant of the force, and the resolving of the two forces into two specific orthogonal directions is essentially required, because lot of forces are in different angle or different way to act upon, so to resolve in two orthogonal direction.

The three equations of static equilibrium hold the key for the analysis of structure, because otherwise we cannot have any kind of equation what is required, boundary equation is required to find out what will be the total nature of the forces. So, based on that, my next lecture, lecture, the lecture number 4 will be structural supports and the reaction. Here, before we end today, I have some homework for you.

Today, I have three homework for you. All are very easy. In 1st problem you have to find out the resultant force for the force system. The second one is that, a particular inclined body is under a force 5 kilo Newton, red color force, and angle is 60º, so definite it will move; and if it is going to move then what should be the value of force Q at B which is 30 centimeter from O; which will give you equilibrium.

And the last one is that, a mass of 100 Newton is hanged by 2 ropes of AC and AB; and AB rope is 45º inclined but AC rope has these 5 inclinations, they are: 0º, 30º, 45º, 60º or 90º. So, what will be the change in tension in AB and AC, due to the change of orientation in AC only. Because AB is not going to change, AB will remain as 45º. So, what will be the nature of force? Also try to draw a graph for the changing force A to C by the virtue of the angles. Suppose you may take a graph and by virtue of various angles you can plot what is the changing pattern of the forces because of this changing angles. Thank you very much.