Structural System in Architecture
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Lecture No -31
Cable Supported Structures

Hello everyone, welcome to the NPTEL online certification course on Structural Systems in Architecture. Today we will start the module 7, that is the week 7 and here will discuss about tensile and the plate structures. The first three lectures will be on the tensile structures and the last two lectures will be on plate structures. Today we will start the first lecture of the week 7, that is the module 7 and the topic of this lecture 31 is Cable Supported Structure.

In this lecture four concepts will be covered. First, I will start with the tensile structure, the structural concept and the principles and then we will solve some numerical problems on the basis of the funicular polygon; and after that we will discuss some of the curvature of single and double curvature cable systems and their applications; and finally, we will end this particular lecture with the multiple cable supported system.

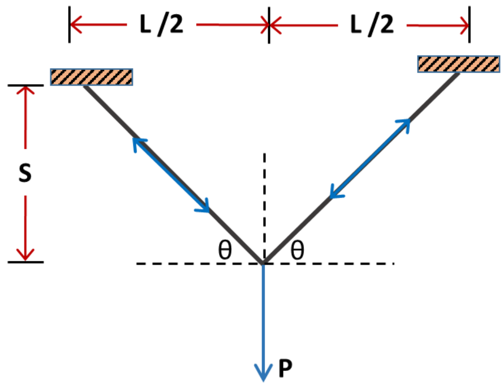
The two learning objectives of this particular lecture 31 will be:

- Outline the Structural concept of Cable Supported Structures.
- Illustrate the parts and functions of Cable Supported Structures.

When we think of any kind of tensile structure, which is a system made up of flexible and non-rigid matter, the tensile structure actually is going to form in such a way that it will try to stabilize the overall forces in that particular system. When the stabilization of the forces will occur, it will develop a particular subjective, a typical kind of a normal force or normal stress called tension.

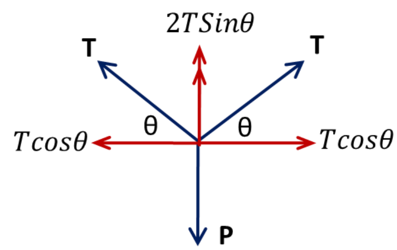
So, through the tension or the tensile stress, a total system of the structural system will stabilize the action; and this is one kind of the form active structural systems.

To begin with a small introduction of the cable structure, I have taken a very simple or the simplest type of cable structure where a cable is hanged from two supports and those cables are inclined in nature. As you can see in Figure-1, it is a symmetrical case under a load P and this inclination is angle theta to horizontal. So, by virtue of the suppose if I draw some freebody of this particular part, it has actually three forces applied on it, and they are the two tensions of equal in nature and opposite in direction inclined at theta degree with the horizontal and a downward force P.



$$2TSin\theta = P$$

$$T = \frac{P}{2.Sin\theta}$$



Very **Flat type of cable** arrangement (very less value of θ) provides very **high value of Cable Tension**

More the **sagging of cable** (θ close to 90°) provides very **low value of Cable Tension**

As the angle θ increases, Sinθ increases, Tension in the cable Reduces.
In an equal Span (L), more is the Sag(S), more is the angle θ, and less is the tension In the cable (T)

Figure 1 : the simplest cable structure

Under this particular three forces I can divide the result of force, this is the unknown force T and this T cos theta and this T cos theta of the two sides will cancel each other. So, I am left with the 2T sin theta which must be equal to this downward force P. So, I can write that-

$$T = \frac{P}{2Sin\theta}$$

From this equation there are some interesting conclusions that may be drawn. The first conclusion is that, if we can have the same span that is L is fixed and when the sag is higher, sag means the portion shown as S, is higher, the angle theta will be higher. Because the tan of theta will be higher, as the theta is higher the sin theta is also higher and the T will be lower. So, more the sagging of the cable, the lower value of the cable tension will appear. This is one of the most fundamental points that we should know before we start with.

With reference to the earlier one, the span and the sag was defined as the support-to-support distance, is the span and the sag is the lowermost tip of the cable to the particular line of the span, I mean line of the support that is the span. Generally, we keep this particular sag to span ratio is almost about 1 is to 8 to 1 is to 10, that means if the sag is 1, 8 to 10 times will be the span of that particular cable structure to make the tensile or the tensile force in a limiting order.

Now what we have discussed in the last example is one typical external force applied in a cable. If I increase the number of forces, suppose there are two such forces, sometimes there are three or maybe more. When we increase the number of forces, the triangle developed between forces will start moving and finally result in to a polygonal shape. There will be small breaks in the cable and this particular shape that exists because of the different loading in the different points of the cable and that particular shape is called the funicular shape. Refer Figure-2.

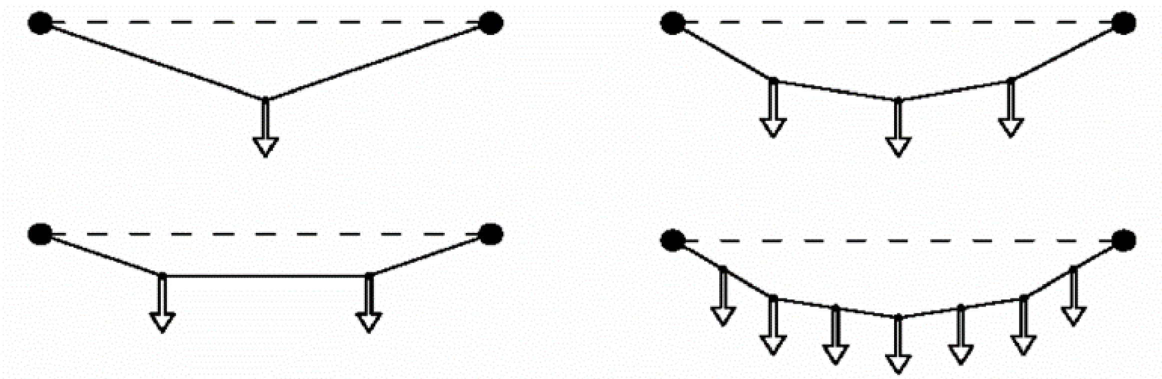


Figure 2 : the funicular shape of a cable

And this is one of the extremely important shapes in case of a cable supported structure and again each and every part of this cable, suppose if you take maybe any kind of cable, I mean any cable in any position of that funicular polygon will be under pure tension.

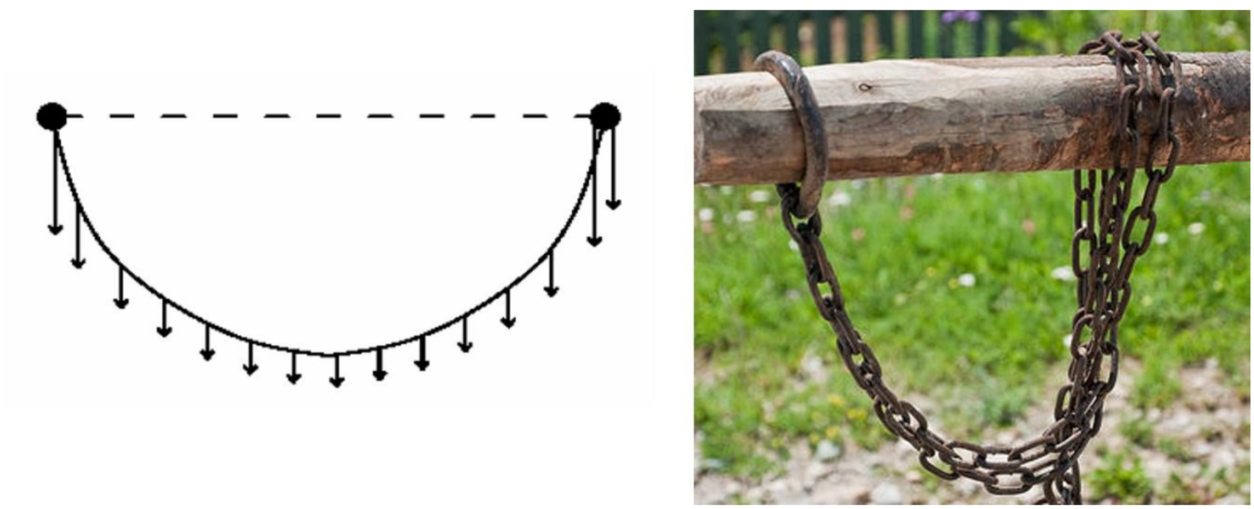
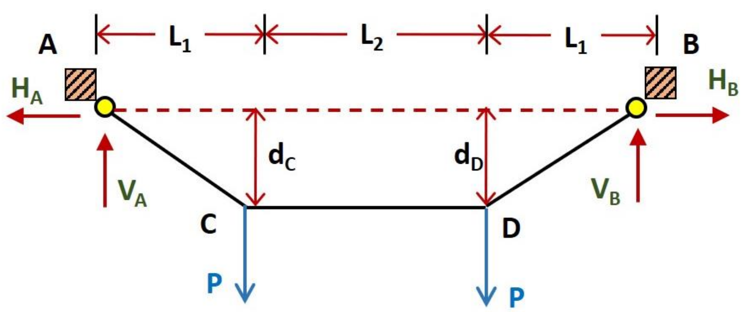


Figure 3 : Catenary shape

And now, if instead of that particular external force if I now put that particular cable with its own weight, that means it is not under any kind of external force, but its own weight, then this particular shape will take a mathematical kind of a shape or that shape can be represent by mathematically with a high parabolic function. We will discuss in the next class in that or next lecture; and that shape is called catenary. Suppose you just hang a metallic chain which is shown in the right-side image of Figure-3, the shape it will be created for its own weight. See, no any kind of load is given over there, by its own weight and under gravity it will give a very smooth kind of a shape which is a catenary. But please remember that this catenary is not parabola. Parabola can be drawn in this funicular polygon, if you increase the number of loading and each load is equal to each other and very congested one to next, and that external load is too high compared to the self-weight of the cable, then only it will lead to a parabola.

Let us solve some numerical problems. Here I have taken a funicular polygon with two such forces P and the supports are at A and B. There are supports and as you know these supports will have reactions, vertical reactions V A and V B, and the horizontal directions H A and H B. This H A and H B will be equal because otherwise there will be imbalance in the X-axis direction. Refer Figure-4. So here we can have some unknowns or the support on reactions may be unknown or the tensile forces are unknown.



From Symmetry : $V_A = V_B = P$

FBD at left part of C,
Equating moment about C is Zero:
$Hd_c = L_1 V_A$
$H = (V_A L_1)/d_c$

Figure 4 : Funicular Polygon; problem-1

From the symmetry we can say the VA and VB is equal to P because the forces are twice P downward and it is symmetrical; and if I take the free body diagram at that particular point C, this is the free body diagram and if I take the left part then I can equate and at point C, the moment equal to 0 and I equate it and can find out how much is the H. Because if you see the Hdc is the moment created by the H because this is separated by a distance dc from C.

And it must be equal to the moment generated by the force VA, this A and that is at a distance L1. From that I can find out the value of H. So, VA, VB and H can be computed and similarly from the free body at C. Refer Figure-5, and we can easily find out the tensile forces in the cable, that is written over as TCA that is the cable CA, the tensile force in the TCD the cable CD we can find out.

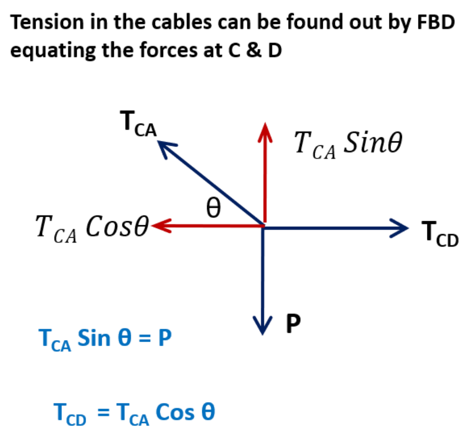


Figure 5 : free body diagram

If I now take the same problem with the values that P equal to 15 kilo Newton, L1 is 3 meters, L2 is 4 meters and then we can easily solve the equations. From the symmetry I can understand that these reactions will be the 15 kilo Newton and suppose if I take the moment about the HA into 1 meter, this dc is equal to 1 meter, L1 is equal to 3 and VA is equal to already I noticed 15.

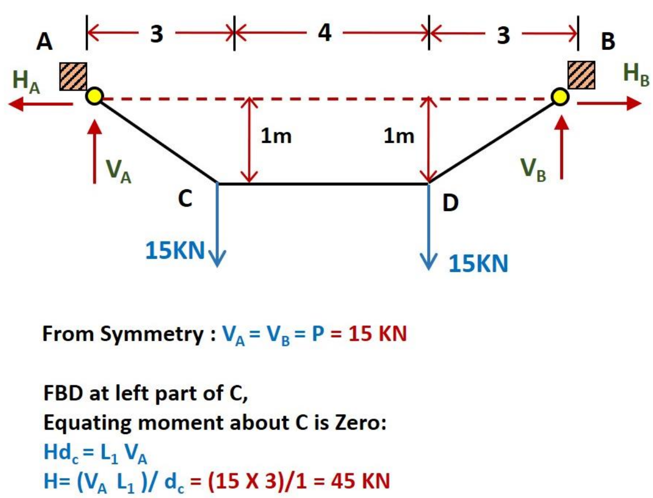


Figure 6 : numerical problem

So, the HA can be computed as 45 kilo Newton. Similarly, I can find out all the forces by the free body at particular point C. I have computed all the tan theta, tan theta is 18.4 degree. So, this is the tan of theta will be 1 by 3. So, I also find out the corresponding value of the sine and cos theta and with this equation if I try to find out the values, TCA is 46 and TCD is 44.53. Of course, one thing is very important that the tension at AC must be equal to the tension at BD; because of the symmetry.

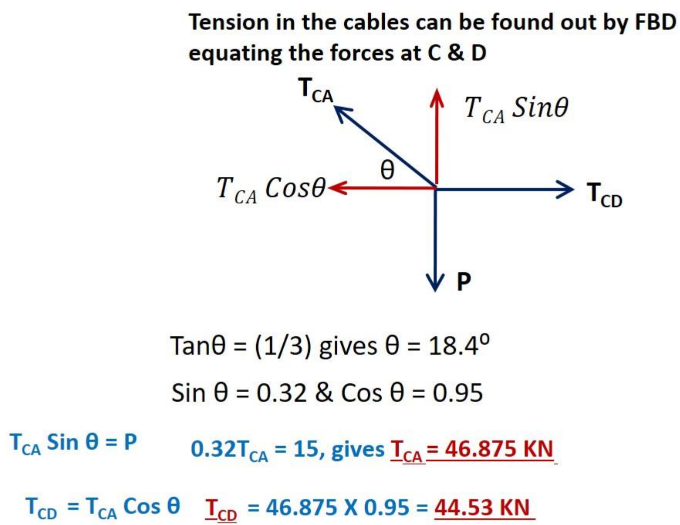


Figure 7 : The Free body diagram

Let us take another problem where this polygon is in different order, different order in the sense the two forces are now different, 20 and 10 kilonewton. There are equal distances apart; 3 meter each but the supports are not in the same level, one support B is almost 1.5 meter above the support A. This sag length is 1 meter at that particular point it is given. Refer Figure – 8.

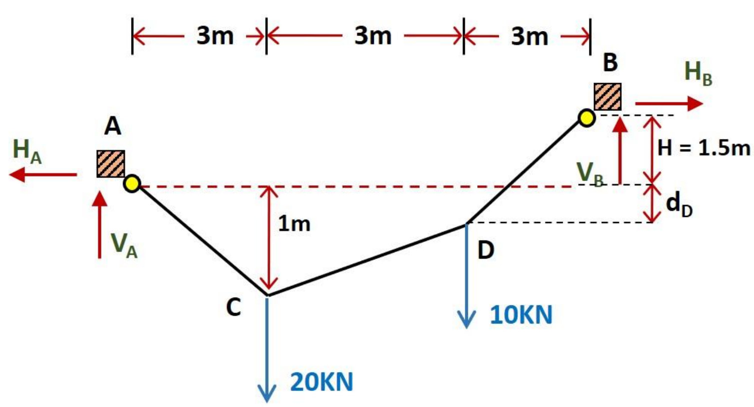


Figure 8 : with asymmetric loads

So now I need to find out what are the tensile forces in the cable and also first I have to find out what is the sag at D, that is also unknown.

If I take moment at B and put it equal to 0, from that point of view all the forces are coming to my picture in this moment equation, this VA is at a distance of 9 meter. So, VA into 9 that gives the moment in clock-wise direction; and this 1.5 into H at this point, the HA will again create a moment in the same direction (clock-wise) which is HA and distance between the point B to this line is 1.5 meter and must be equal to the moment generated by this points, load at C and load at D, that is 20 into 6 plus 10 into 3, so this is the first equation.

Moment at B = 0

$9V_A + 1.5H = 6X20 + 3X10$

$9V_A + 1.5H = 150$ ------------- Eq.1

Taking FBD at left side of C & put Moment at C =0

$3V_A = 1XH$

$H = 3V_A$ --------------- Eq.2

Solving Eq.1 & Eq.2

V_A = 11.11 KN, H = 33.33 KN

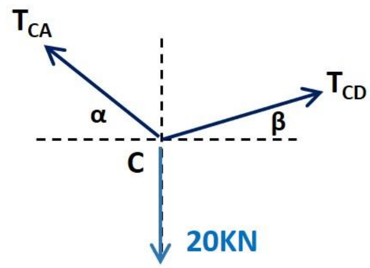
Hence, **V_B = (30-11.11) = 18.89 KN**

And if I take the free body at C and I can find out what is the relation between the VA and H, then 1 into H because 1 is distance into H, so that this moment and VA into 3 which will give me the equation 2, and this equation 2 is now used to find out the values, will help the equation 1 and then I can substitute and find out those two values of the VA and VB and gradually I can find out how much is the value of H and finally I can find all the value of dC by taking the free body at D. So the dC you see is three times into VB, so this particular moment must be equal to moment generated by H into 1.5 plus dC. Why 1.5 dC? Because from point D this H is separated at a distance dD plus 1.5, out of that 1.5 is known, so I can easily find out the dD at 0.2 meter and we can now go to each joint, join C and join D.

Taking FBD at right side of D & put Moment at D =0

$3V_B = H\ (1.5 + d_D\ )(1.5 + d_D\ ) = (3\ X\ 18.89)/33.33 = 1.7$

$d_D\ = 0.2m$



$Tan\alpha = (1/3) = 18.4°$
$Cos\alpha = 0.95$
$Sin\alpha = 0.32$

$Tan\beta = (1\text{-}0.2)/3 = 14.9°$
$Cos\beta = 0.97$
$Sin\beta = 0.26$

$T_{CA}\ Cos\alpha = T_{CD}\ Cos\beta$

$T_{CA} = (0.97/0.95)\ T_{CD}\ = 1.02\ T_{CD}$

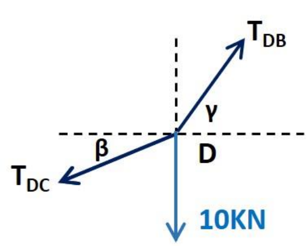
$T_{CA}\ Sin\alpha + T_{CD}\ Sin\beta = 20$

$1.02\ T_{CD}\ X\ 0.32 + 0.26\ T_{CD} = 20$

$T_{CD} = 34\ KN$

$T_{CA} = 34.8\ KN$

You can calculate the angles cos and sine component and find out the forces CD and CA in this 2 beam and also you can go to the joint D and to the free body diagram and find out the angles beta and gamma.



$Tan\beta = (1\text{-}0.2)/3 = 14.9°$
$Cos\beta = 0.97$
$Sin\beta = 0.26$

$Tan\gamma = (1.5+0.2)/3 = 29.5°$
$Cos\gamma = 0.87$
$Sin\gamma = 0.49$

$T_{DC}\ Cos\beta = T_{DB}\ Cos\gamma$

$34\ X\ 0.97 = T_{DB}\ X\ 0.87$

$\underline{T_{DB} = 37.9KN}$

So, those angles and from the geometry everything I can find out, including the tension at DB. The funicular polygon problems can be solved by virtue of the rest force balance and the moment which we have known from our beam calculations and the analysis of beams and it is very simple also.

Now let us discuss the single and the double cable curvature also. Refer Figure-9. In this particular cable if we see there are three such components, the first one is called the suspension cable which is the long cable, which is going to suspend and take the load by virtue of some other support reactions.

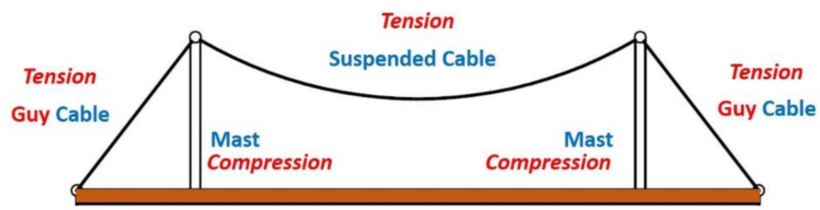


Figure 9 : single cable structure

There are guy cables which is also called stay cables which are actually connected with the suspension cable and a neutralizing kind of a force is acting towards the ground. The third component is the central column which is called mast and this mast are should be in compression, the cables will be in tension, both the stay or guy cable or maybe the suspension cable are in the tension and the mast will be definitely under compression.

But when you design the single cable, a particular roof can be over laid; or the bottom portion (the brown line) can be the roof; and we can take support from the suspension cable. If I take the support from the suspension cable, it will create a funicular polygon. That is the interesting thing, suspension cable sometimes can have a catenary because if you just leave it with its own weight, and put the roof over there or sometimes it may go with a funicular polygon, then we are taking the pointed load from beam. But the problem with this particular single cable is that it will be very prone to uplift when there is a wind generated and that will be a creating a serious kind of a concern if it is a very lightweight structure. This kind of tensile structure basically is a lightweight structure and the problem will occur in that cases. The guy cable will absorb the horizontal thrust, transfer the load to the ground and the mast is under the compressive force: designed for buckling and all those as you know it is a column remember.

Now, here you see the guy cable, as shown in Figure-10, if I put the mast perpendicular, you know the guy cable and the suspension cable, the forces will be in typically in the directions of two blue arrows, so the resultant will be like the red color arrow. So, what happened in this red color arrow with respect to the mast? It will invite a bending, because you can see that the red color arrow (the reaction) is not along with the axis of the mast. So, definitely this indicates that it will invite some bending.

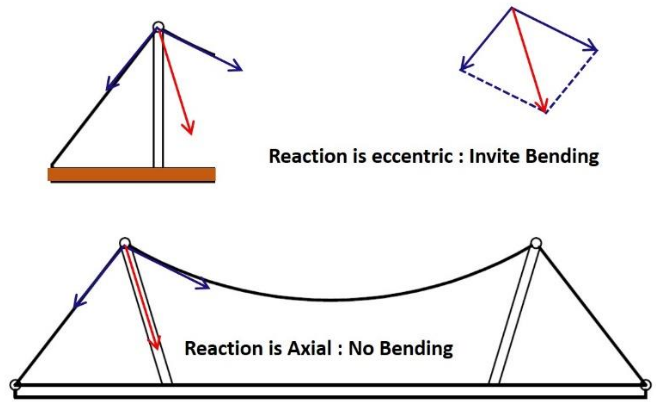


Figure 10 : reaction forces and position of the mast

But if I go with inclined mast, the inclination in such a way that, because of the tension in the cables that is a suspension cable and the guy cable, the reaction passes through the axis of the mast (refer bottom image of Figure-10), then this reaction will be axial. So, no bending will occur here and you can have a very economical or the slender design for the mast. It is one of the fundamentals that is why the mast sometimes is kept inclined, not perpendicular or perfectly vertical.

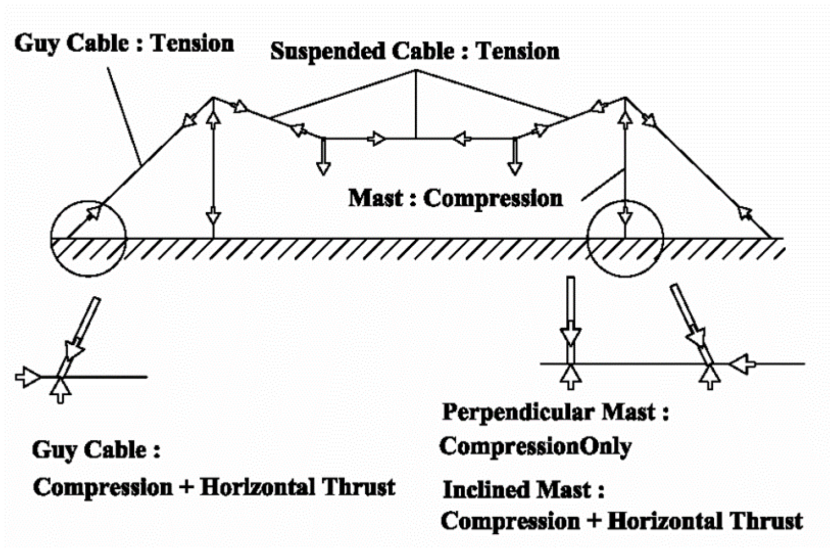


Figure 11 : the members and types of forces in a simple cable structure

I have already talked about the cable members, what is under compression and what is under tension. The Figure-11 will give you a broad illustrated idea and details about it.

In case of the double curvature the benefit is that the upper curvature will take care of the gravity and lower curvature will take care of your wind kind of scenario as shown in Figure 12. This structural system is more rigid to take care of the wind flutter.

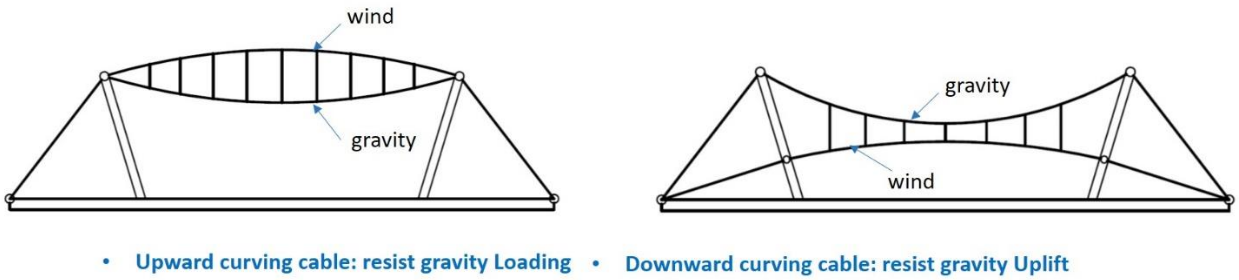


Figure 12 : double curvature cable structure

In this case sometimes or many times we have to link these two cables when there is a double curvature cable and those links are called separated or stirred because those also come under compression. Here we have two cables, we have two names also, the primary cables are cables which are taking care of your gravity so these two are the primary cables, this is secondary which is taken care of the wind.

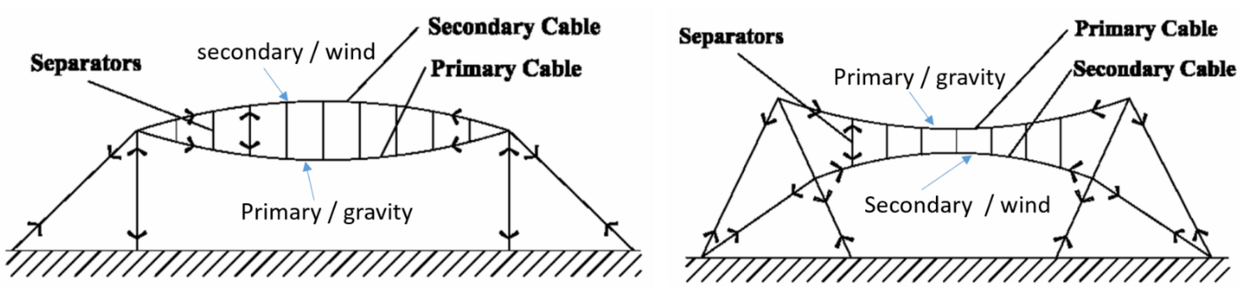


Figure 13 : types of cables in double curvature cable structure

In Figure-14, you can see how the particular roof have been supported by the supporting cables. The back-stay cables take care of the resolving or the neutralizing the forces and also this is having some of the supporting cables and finally there are mast, that is the central pillar which is actually taken care of the compression. In a particular system, other than the tensile material system will have some component which has to take the compression. So, mast actually takes care of the compression and there is Anchor also at the end of the stay cable.

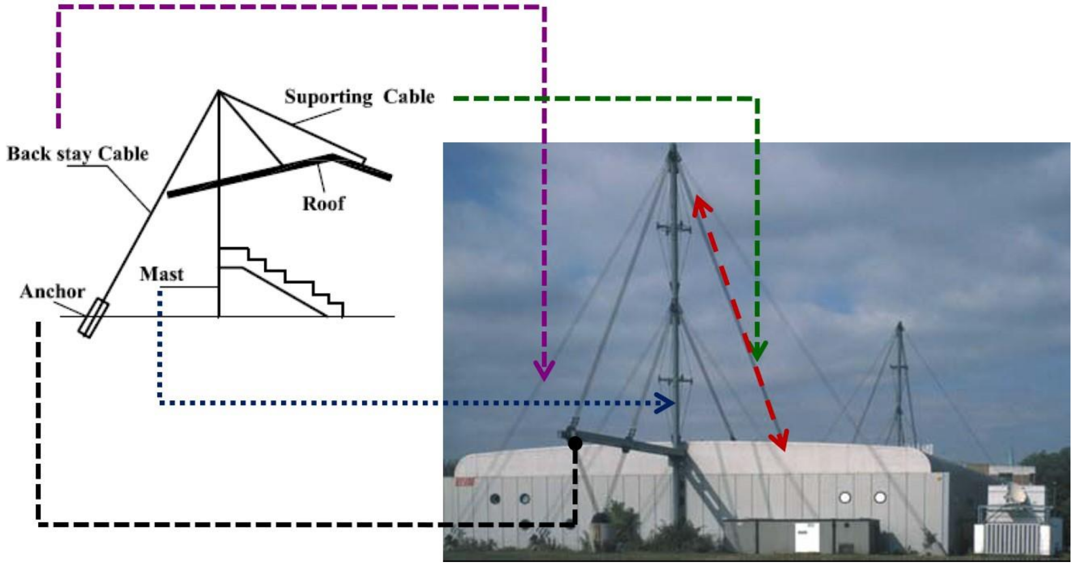


Figure 14 : types of cables in a structure

Then, there are multi-cable supported systems. Here, instead of one cable we have multiple cables. As reference you can see the Figure-14, where there are multi cable systems. But multi cable in the sense we will discuss now, there are types of multi cable which are there because of the different types of the deck and pylon that support both. It is applicable in the bridges where there is a cable stay bridge. But, instead of the bridges we can use it for stadiums, airports, arenas etc. where we need column free constructions. The multi cable system has four types:

i. Radial
ii. Harp
iii. Fan
iv. Star

The first one is radial. In radial system, there is a deck and there is a pylon. The cables connected evenly throughout the deck, but converges on the top of the pier, as shown in Figure below.

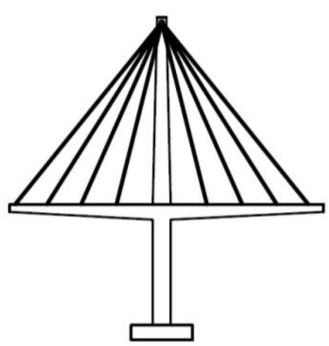


Figure 15 : multi-cable support system - Radial

The deck has to be supported and the deck is cantilevered out. It has to be supported by even cables which are actually connected in the deck at a uniform distribution, but they are actually pointed or connected at the top single point and converge from the deck to the pylon or the pier. This is called the radial system.

Another system is, the Harp system. The basic difference is that here the cables are parallel and evenly spaced along the deck and pier. They are again equally distributed and it is connected in different heights in the pier.

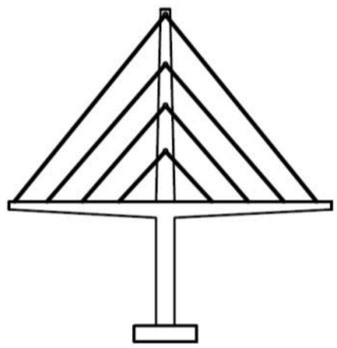


Figure 16 : multi-cable support system - Harp

In the lecture number 33 we will discuss about how the system generates forces and what will be the economic way to handle the forces for any kind of the architectural applications.

The next system is Fan system. It is the Combination of Radial and Harp, converges towards the top of the pier and evenly spaced at deck

Next one is a fan system, where you see it is something like a combination of the radial and harp where it has taken from the various points of the deck uniformly distributed.

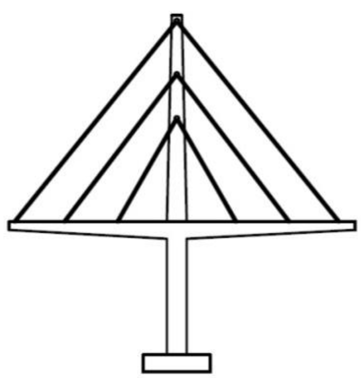


Figure 17 : multi-cable support system - Fan

But also, it is attached in the different point in the pylon in a vertical level but converged towards the top. It is exactly not going to converge in the same point but it is converged towards the top. See from one-point distance, the next distance is smaller or the nest distance is smaller.

The last one is the star. The star is just another reverse kind of a scenario, till now what we have seen is that we put the deck, there are a lot of points or the divisions in the deck. But here we take the load of the deck from a single point but we distribute the connectivity at the pier level in various or equal spacing different vertical levels and that creates another convergence not in the pylon but in the two extremities at the deck, as shown in Figure-18.

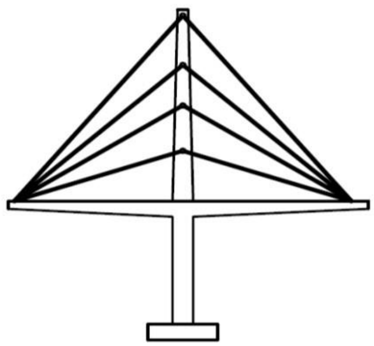


Figure 18 : multi-cable support system - Star

We can say that the star is almost the reverse of the radial kind of a thing.

So, these are the references I have took for this particular lecture.

- **Engineering Mechanics** by Timishenko and Young McGraw-Hill Publication
- **Structure Systems** by Heino Enge, Hatje Cantz Publisher
- **Structure and Architecture** by Meta Angus J. Macdonald, Elsevier Publication
- **The Structural Basis of Architecture by** Bjørn N. Sandaker, Arne P. Eggen, Mark R. Cruvellier, Routledge
- **Building Structure Illustrated** by Francis D.K. Ching, Willy

The conclusion of this today's lecture is that the cable supported structure is one of the structures that we can use for large arena. It is the extremely effective kind of a structural system and it is actually the way to support the building roof and this particular cable supported system can be analyzed by through this funicular polygon approach. The load path mechanism shows that the structure element of this cable supported system mostly tensile except for one or two.

Mast is always going to be in compression but other cables definitely will be under tension, so this tension has to be taken into account how much tension that particular cable can be according to its permissible stress also. We have some homework for you.

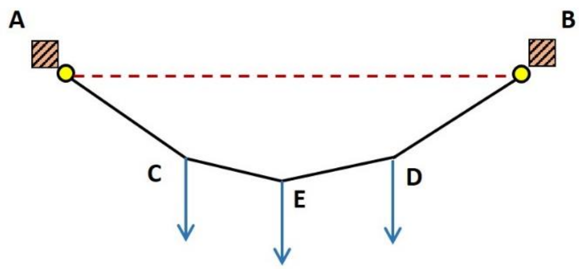
There are two home works for you.

Home Work

1

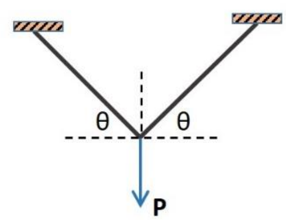
The figure shows, three equal load of 10KN each forms a funicular polygon. Each horizontal segments of the cable is 2m. The Sag at point C & D is 0.5m. Find:

i) Support Reaction
ii) Sag at central point E
iii) Tensile force at each cable



2

Two steel cables support a load 10KN as shown in the figure. The angle θ = 45°. Determine the diameter of the cable to be required to support the load, if the permissible tensile stress of cable is 500 MPa



That is all for this lecture number 31.

Thank you very much.