Structural System in Architecture
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Lecture No -32
Membrane Structures

Hello everybody, welcome to the NPTEL online certification course on Structural Systems in Architecture. So, we are in the module 7 and the module 7 is all about the tensile and the plate structure; and this is the second lecture on the tensile structure. Today we will discuss the lecture number 32, and this is on membrane structure.

The basic concepts to covered here are:

- Introduction
- Numerical Problems on Catenary
- Basic Forms of Membrane Structures
- Support System of Membrane Structure

The major learning objectives of this lecture are:

- Outline the Structural concept of Membrane Structures
- Illustrate the types, parts and functions of Membrane Structures

Membrane structure are very common since history. The left-hand side image in Figure-1 is a membrane structure made of animal skin. It is called as Tipi, the traditional house we can find in North America. As you know the animal skins is nothing but a membrane; and that kind of a tent can be done through that.

Tipi: Traditionally Animal skin made tent, North America

Tourist Tent in Jaisalmer, Rajasthan

Figure 1 : the membrane structures

The next picture, the right-hand side image is a very recent picture. It is of Jaisalmer the desert of Rajasthan. The tents for the tourists are made by the membranes. It is a kind of canvas or cloth, basically a thick cloth. Hence, we can say that the membranes can be used well for structures which have been in use since early human civilizations.

So, our goal will be to understand how we will provide or use the cables, because the membrane alone cannot stay stable. Therefore, you need cable to make them support, keep it, straight and stretch it properly; and with that we can create the roof or maybe some form or structure out of membranes. Another fundamental goal is to minimize the number of internal columns or masts in a particular structure. We also have to see the profiles and strengths of both the membranes and the cables, so that they do not cross their permissible limits.

Now if you see in the nature, there are membrane structures do exist naturally. One such example is the net that is formed by the spiders or the cobweb. Refer Figure-2. In the Figure if you observe very minutely, then you can see that there are there of primary cables which are actually supporting it.

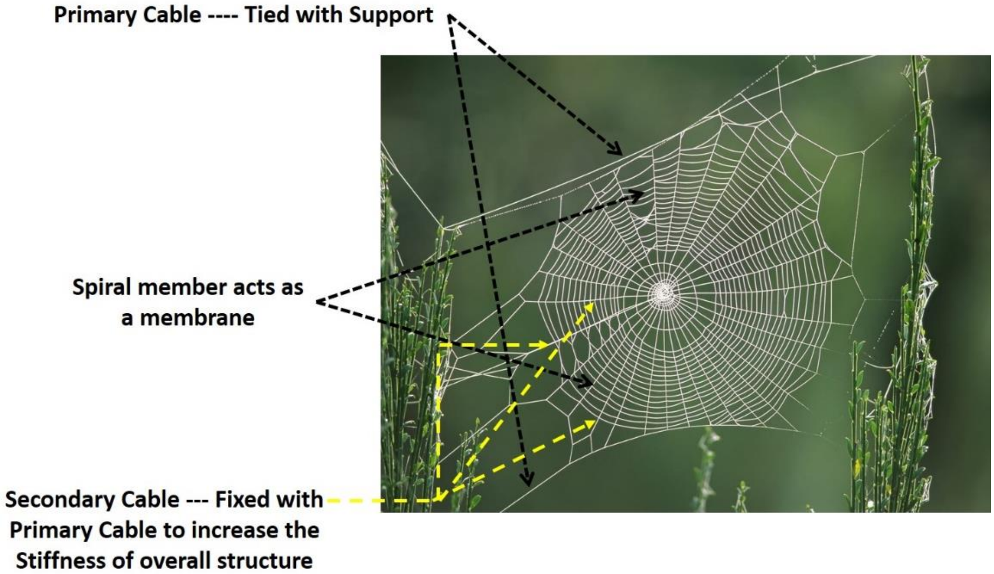


Figure 2 : the membrane structures in nature - the cobweb

It is also having the secondary cables, the radial one, which are connected to the primary one to increase the stiffness. Finally, there is a spiral member also in it, those are act as a membrane, so those cables are actually creating this kind of a membrane structures. See how beautifully they create this particular cable formations or the total structural formation based on the two set of cable and one set of the spiral membrane.

Next is catenary, we have already discussed that a catenary curve can be as presume or that can be assumed to be a curve which is created by its own weight. So, a membrane structure when it is a thin membrane, it is supported by the cable and it has to be very flexible and it will actually sag because of it is own weight.

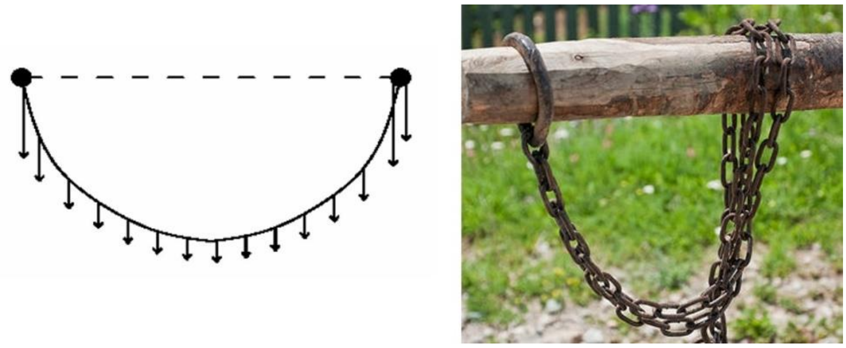


Figure 3 : Catenary

A chain can act as a catenary, because of its own weight it will sag, where no external weight actually has been introduced. Let's consider a gold chain which can form a catenary shape. Another very interesting thing is that in this catenary if you see, the load is going to densify as you go to the support. Assume that your one hand is one support and another hand is another support. So, if you go towards the support the loading intensity will be magnifying. Another very important thing is that, suppose, if I increase the span, then the sag is smaller and when the span is smaller the depth of sag will be very large.

Now let us go to the some of the numerical examples on catenary.

We know that the catenary shape can be mathematically established with some equation forms. In a catenary, suppose 'O' is the origin that is the zero-zero point, with x-axis and y-axis. There are two supports, A and B and both the supports are in the same level. Refer Figure-4. I have taken one point in the cable which is point x, y and which is at a curved distance 's'.

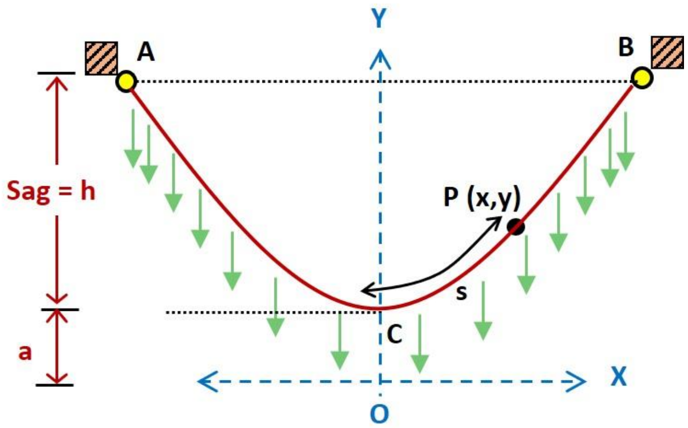


Figure 4 : example catenary

This is the distance of the particular arc or this particular point or the portion of the chain 's' that is the curve length. This length is under loading, if you take the uniform each weight of each millimetre or each centimetre this uniform distributed loaded are now gradually densify towards the end; and because of that the shape is going to form. The equation of the catenary it can be given like:

Equation of Catenary is given by: $y = a\cosh\left(\frac{x}{a}\right)$

Where, $a = \frac{T_0}{\rho A g}$

ρ = Density of Thread / Chain / Cable

A = Cross section area of Thread / Chain / Cable

g = Acceleration due to gravity

T_0 = Horizontal component of tensile force (Constant)

$$T_0 = \omega a$$

ω = loading intensity = ρAg, in N/m or KN/m

This is the equation where 'y' equal to a cosh means the hyperbolic cos x
The other relationship is:

$$s = a\sinh\left(\frac{x}{a}\right)$$

$$y^2 = a^2 + s^2$$

's' is the curved / actual length of cable from bottom most point (C) to the point of interest.
This a is a very tricky thing the 'a' is actually is the distance from the tip of this catenary to the origin and thus 'a' has to be defined or the measured by above mentioned equation.
Now next is, you can also find out the tension in the cable with the formula given below. If you take the free body of this cable, then you will see that there are three loads acting on it.

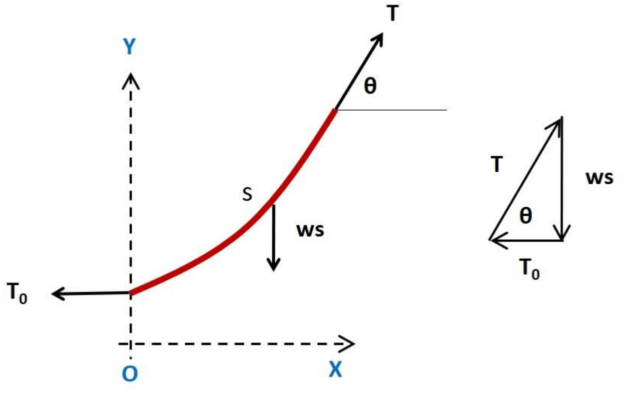


Figure 5 : the tension on a part of the cable

T = Tensile force at any point on cable (Variable)

θ = Slope of the cable where the tensile force is measured

$$T = \sqrt{T_0^2 + \omega^2 s^2}$$

$$T = \omega \sqrt{\left(\frac{T_0}{\omega}\right)^2 + s^2}$$

$$T = \omega\sqrt{a^2 + s^2}$$

$$T = \omega y$$

One is the horizontal tension; another is the tension in the inclined portion of the cable and the third one is at the end portion of the cable. Refer Figure – 5. Here ‘w’ is the loading intensity and ‘s’ is length. So, it is the total weight of the cable of that particular part.

So that comes in a triangular the force geometry and from that force geometry I can find out what will be the tension of cable at any particular point. any particular point can be determined either by ‘s’ or by ‘y’ because ‘y’ changes as for the coordinate changes and ‘s’ changes also the distance changes so the length of s is also going to change.

Now the hyperbolic sine and cosine functions can be given with this particular two formula:

$$\sinh x = \frac{(e^x - e^{-x})}{2}$$

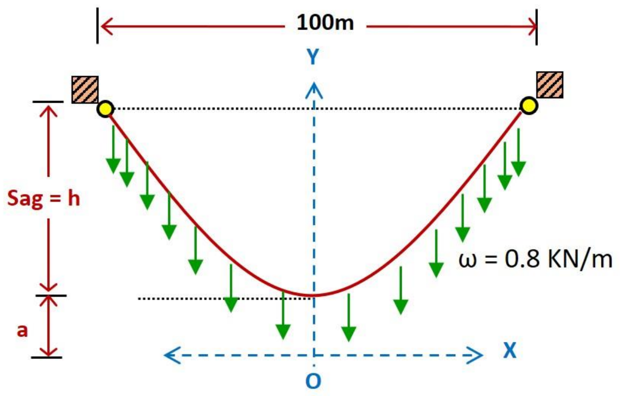
$$\cosh x = \frac{(e^x + e^{-x})}{2}$$

But in the calculator also we can find out. There is a hyperbolic button and from that if you start with that hyperbolic button first and then go to sign it goes in the hyperbolic sign and then put the actual whatever maybe the angle and then equal to gives you the value. Refer Figure-6.

Figure 6 : the hyperbolic function in calculator

So, you take the first on switch on and then you go to this hyperbolic and then go to the sine or cosine whatever and then put the values and then equal to, so here you get the sine hyperbolic. You clear it, and then you go to the hyperbolic suppose you want to do the cos, now give the values and then equal to then cos h. Then if you want to do inverse, then you go to hyperbolic, then to shift, then give the sin inverse then give the value (eg: 1.2) then equal to; this will give the value. Similarly, if you want to do the cos inverse, the go to hyperbolic then go to their cos inverse and then put the values and then put is equal to and then you get the values. So, like that you can use the calculator also in the hyperbolic function mode. For more details, please refer the video lecture, time 00:12:08 to 00:12:58

So, from here let us go to solve some problem.



Given:

Cable Length = 120m

Support distance / Span = 100m

Cable material density = 7800 Kg/m3

Cable Cross section area = 0.01m2

Find: Sag, T_{Max} and T_{Min}

So here I first find out how much is the Omega that is the intensity of the loading.

ρ = 7800 Kg/m^3 , A = 0.01 m^2 g = 9.81 m/s^2

$\omega = \rho A g$ = 7800 X 0.01 X 9.81 = 765.18 N/m

ω = 0.8 KN/m

What I do next is, I first use this first use this equation:

$$s = a \sinh\left(\frac{x}{a}\right)$$

Half Span (x= 50m) is equal to half curve length (s = 60m)

So put the s equal to 60 and x equal to 50. Now we have to solve this particular equation by hit and trial method, that means you put some values of a, and just try whether it is coming 60 or not.

$$60 = a \sinh\left(\frac{50}{a}\right)$$

For: a=55, s = 57.17

For: a=45, s = 60.9

For: a=47, s = 59.97…. Close to answer

By solving with hit and trial method, if I go for 47, 'a' = 47, I got 59.97 which is very close to 60 so I say that 'a' = 47.

Now as I got the a =47, now I use the second equation,

$$y^2 = a^2 + s^2$$

$$y^2 = 47^2 + 60^2$$

$$y = 76.216m$$

Sag (h) = (y-a) = 76.216 – 47 = 29.216m

So, the sag will be of 29.216 meter.

Similarly, I can find out the T_{Max} and T_{Min}:

$T_{Max} = \omega y_{Max} = 0.8 \text{ X } 76.126 = 60.973KN$

$T_{Min} = \omega a = 0.8 \text{ X } 47 = 37.6KN$

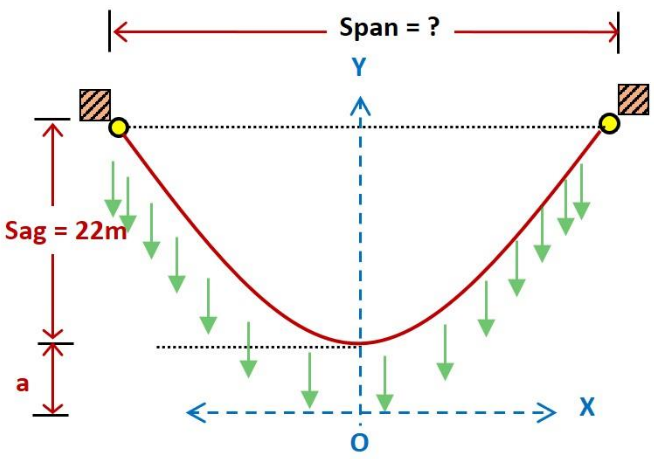
Now let us take another example. It is given that:

Cable Length = 150m

Loading Intensity = 1.2 KN/m

Sag = 22m

Find: Support distance / Span, T_{max} and T_{Min}



Now solving this with the equation,

$$y^2 = a^2 + s^2$$

$$(22 + a)^2 = a^2 + 75^2$$

$$484 + 44a + a^2 = a^2 + 5625$$

$$a = 116.84$$

Then,

$$s = a \sinh\left(\frac{x}{a}\right)$$

$$x = a\ \sinh^{-1}\left(\frac{s}{a}\right) = 116.84\ \sinh^{-1}\left(\frac{75}{116.84}\right)$$

$$x = 70.62$$

So, Span is

$$2 \text{ X } 70.62 = 141.24\text{m}$$

Then

$$T_{Max} = \omega y_{Max} = 1.2 \text{ X } (116.84+22) = 166.6\text{KN}$$

$$T_{Min} = \omega a = 1.2 \text{ X } 116.84 = 140.2\text{KN}$$

So, now let us go to the some of the basic forms of membrane structure. In the basic forms of membrane structure there are five such forms. There may be some the hybrid type of forms, which can be made. But in general, there are five forms:

i. Hyper
ii. Conical
iii. Wave
iv. Arch
v. Cushion

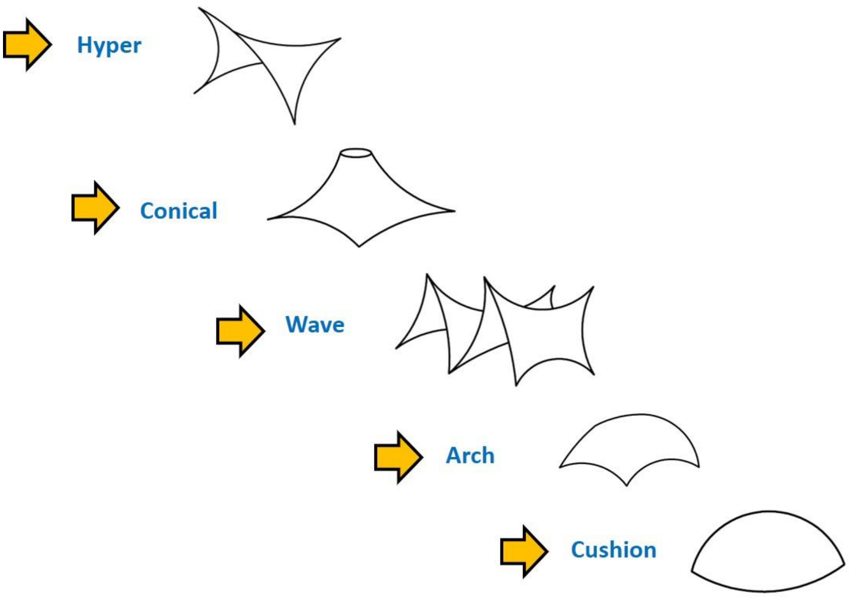


Figure 7 : basic forms of membrane structures

The Hyper form is anticlastic. There are two different curvatures and the terminating point should be placed in different levels.

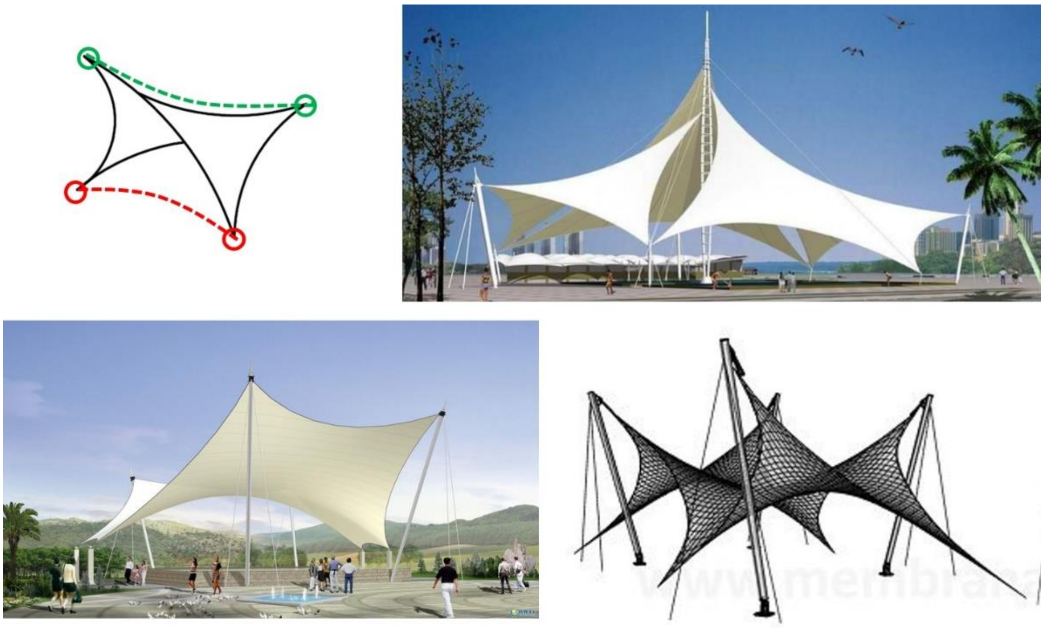


Figure 8 : The Hyper form

See in the Figure-8, there are two in down and these two just next are in the top so that kind of a formation can be made.

Then the conical form, it is Synclastic form where both the curvature in the same side and the Gaussian the curvature is positive and in this case you see what happened is the terminating points are placed at the bottom level and there is a ring and it will be connected to points.

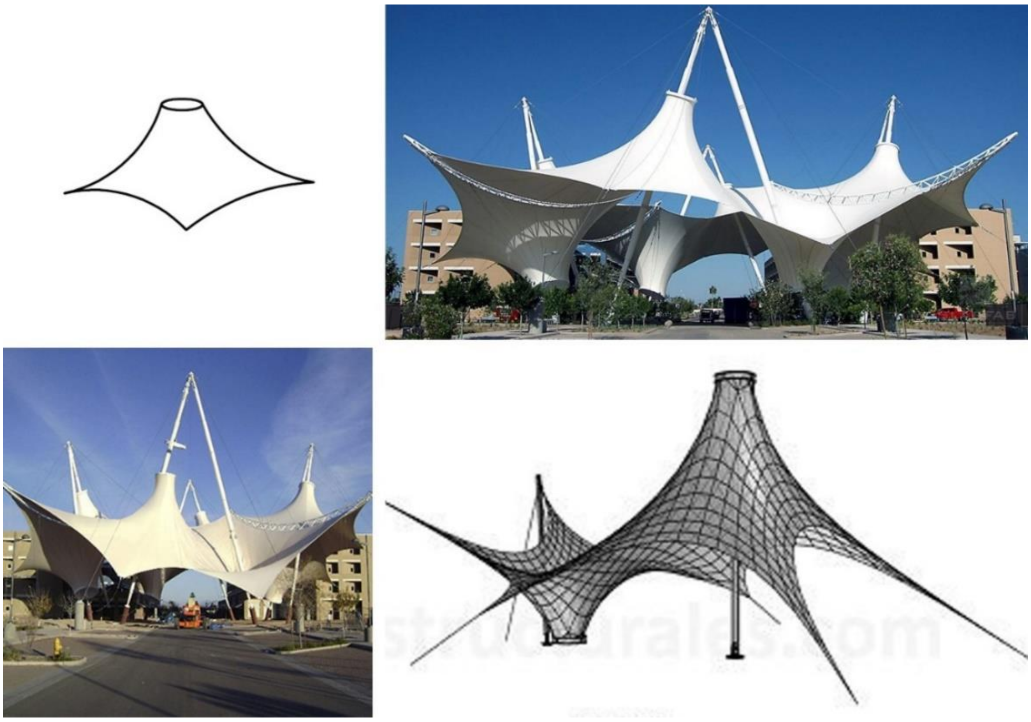


Figure 9 : The Conical form

The next one is the wave. Wave is basically an array of the some units or shell kind of a thing and it is not hyper. It is looks like a hyper because the levels are in different positions.

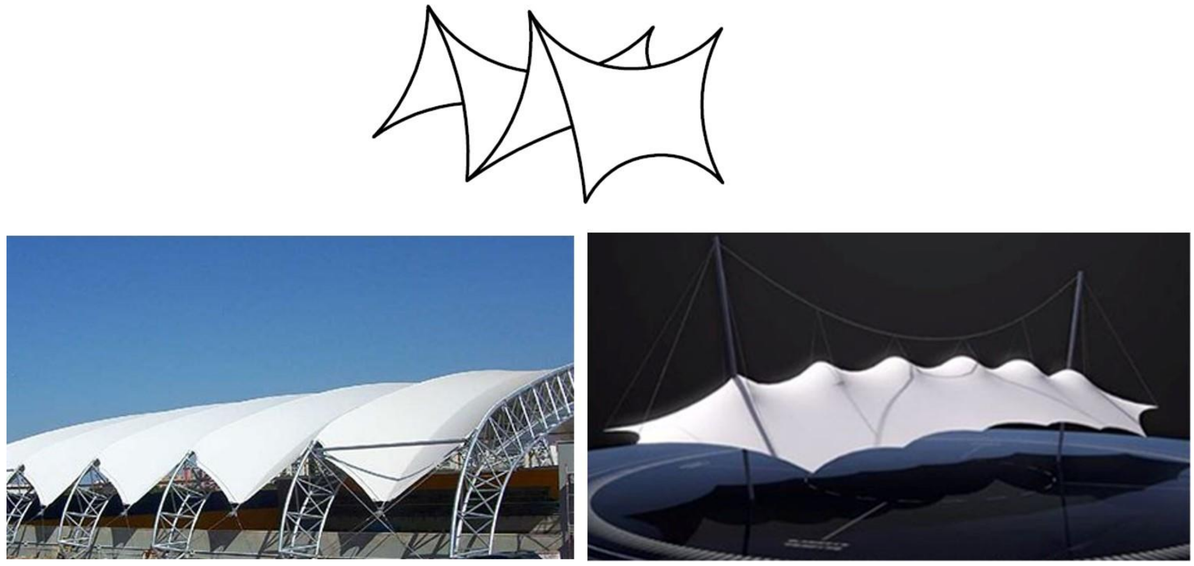


Figure 10 : The Wave form

The next one is the arch form where an arch has created by stiffened members. Sometimes the arch has been created one after another arch but those arches are truss or the steel truss. Those steel truss creates a stiffening attribute to the overall structural systems and then over that you create a membrane. So, it will be now considered to be anarch. Sometimes it can be composition of middle arches too.

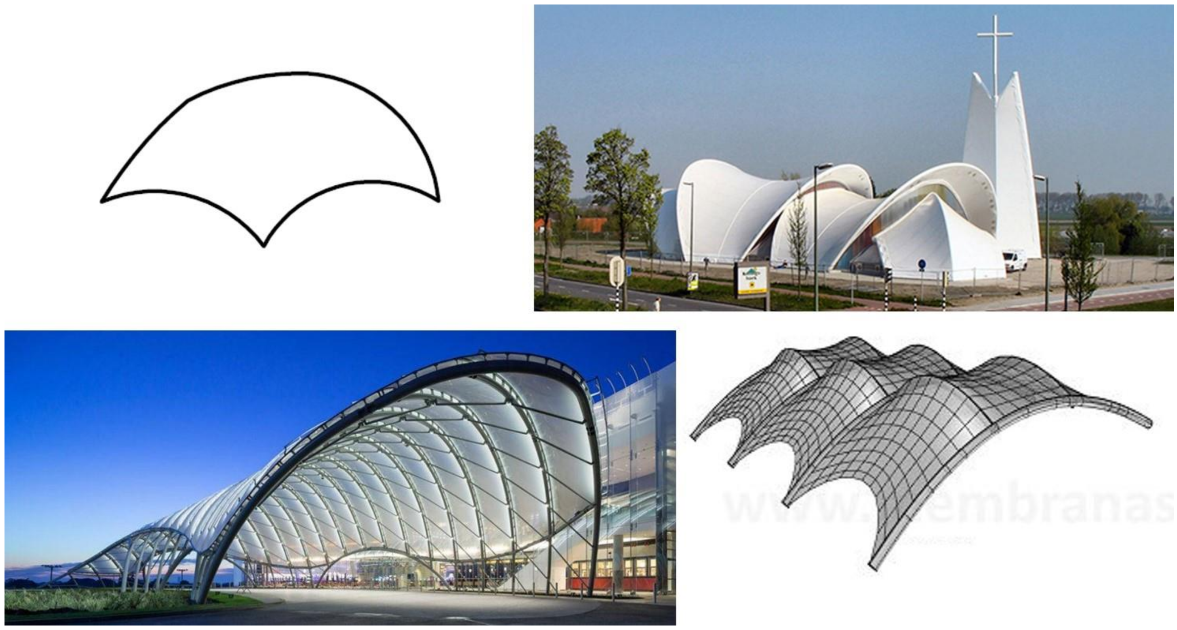


Figure 11 : The Arch form

It can be synclastic or sometimes it can also be anticlastic kind of a profile.

And the last one is called the cushion. The cushion is mostly used in the air inflated or pneumatic kind of a structure where the membrane is thick and there are less number of supports. As they are inflated, it is actually very light and it can retain that particular shape by the air pressure itself.

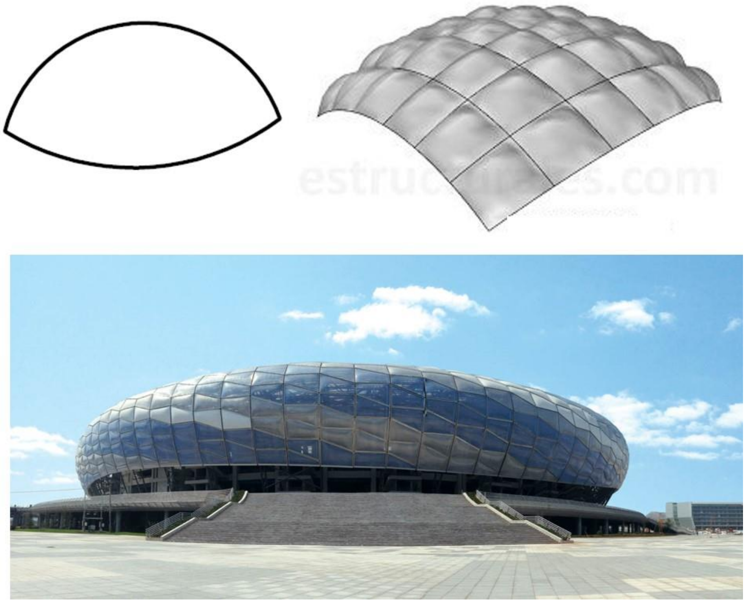


Figure 12 : The Cushion form

Now, if I take a flat panel membrane, suppose it is a handkerchief. Then raising the two-opposite corner of it can create a hyper and raising the middle part of that particular the handkerchief and placing the outer perimeter below a certain level with respect to that central level is going to give you a conical form. Then introducing a series of curvature and clamping edges or series of those in a symmetrical order can create the arch form or the wave form.

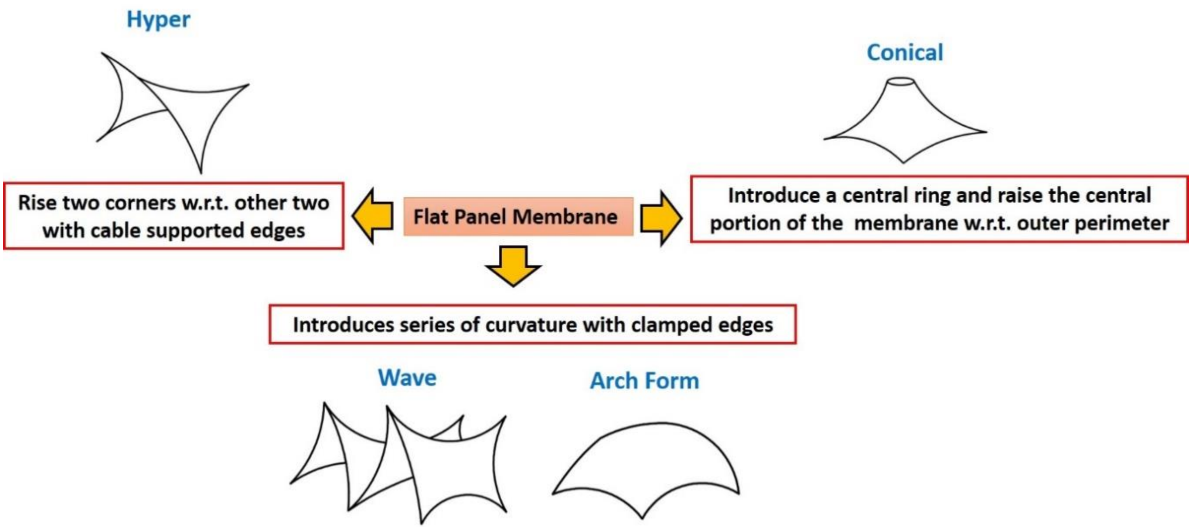


Figure 13 : The forms from a flat panel membrane

So, let us now discuss about some support systems for member structures. See in Figure-14, that is a typical sketch of a tensile structure. Membranes always need support; therefore, we need cables or some rigid truss members to support it. Within this supports also we have primary and secondary subgroups.

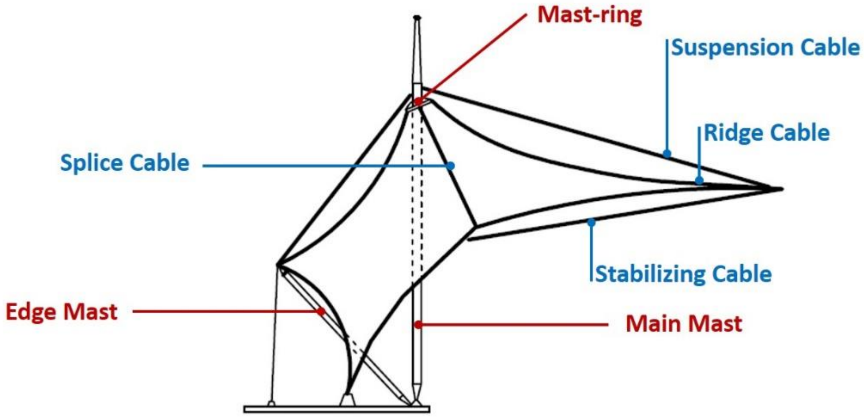


Figure 14 : the supporting members of a membrane structure

So, the mast -ring is the main member which is actually ground the force. There may be some more masts, which helps in anchoring the other sets of cables. As we have already discussed, the mast is always under compression.

The next set is the cables. Suspension cables and splice cables act as primary one and there may be other secondary cables like stabilizing cable and ridge cable which takes care of the formation. The ridge cables give the effect of a ridge and the splice cable actually changes the typical slope of one part of the membrane to another part by virtue of wide-angle change. We also need dome stabilizing cable because only the splice cable may not be sufficient to stabilize the structure. The cable must be connected to the main mast or to the edge mast to maintain the stability and the form. The stabilizing cable gives an additional support to the structure.

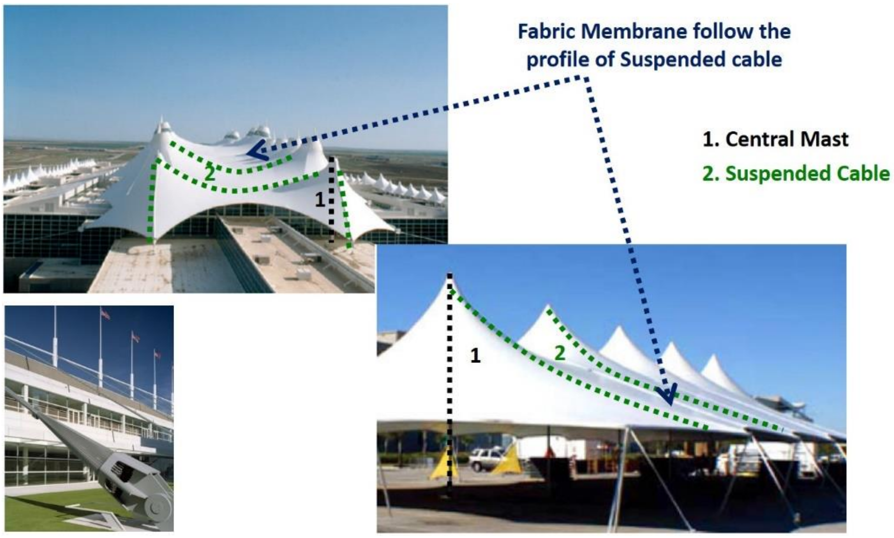


Figure 15 : examples of membrane structure

There are some of the photographs of different buildings that I have taken from internet, which are membrane structures. See Figure-15 carefully to know details of the members.

So, there are some more small examples. In the Figure-16, the left-hand side image is of a bus-stop which is composition of tubular truss and membrane, resulting into an interesting membrane structure. In between the tubular truss the membranes are stretched.

In between two steel tubular truss stretched membrane roofing

In between two curved cantilever column-beam stretched membrane roofing

Figure 16 : examples of membrane structure, a parking lot and a bus stop

The right-hand side image in Figure-16 is another very interesting structure. There is a column or you can say flying beam from the ground and then go with a parabolic way or non-linear way converting from column to a beam and they are placed with respect to some standard distance. Then, the membrane is stretched over it, covering them all. So, these are typical examples on use of membrane structures.

The next example we see is in Figure-17. The left-hand side image of Figure-17 is of reverse conical form or we can say it as inverted conical form where the ring is downward, supported by the suspension cables from the ground and flying upward supporting the membrane. The structure results in to a beautiful aesthetic of trees covering a huge area.

The right-hand side image in Figure-17 is another of membrane structure covering a large space. But here the membrane is supported by the edge support system, by a three-dimensional truss structure; where the mast is not touching the ground. The very interesting fact here is that, the mast is hanging and the hanging mast is then connected to the primary truss members through some supporting or connecting cables. As the mast is hanging, the total load of the structure is grounded by the inclined truss embers which are connected to the columns, and those columns which are touching the ground, finally transfers the load to the ground.

- Inverted Conical Membrane Roofing
- Central Mast & Mast-ring
- Radial Cable from Mast-ring

- Conical Membrane Roofing
- Suspended Central Mast supported by hanging cables
- Split roofing supported by edge space-truss

Figure 17 : examples of membrane structure exhibition grounds

So, I have taken the following reference for this lecture.

- **Engineering Mechanics** by Timishenko and Young McGraw-Hill Publication
- **Structure Systems** by Heino Enge, Hatje Cantz Publisher
- **Structure and Architecture** by Meta Angus J. Macdonald, Elsevier Publication
- **The Structural Basis of Architecture by** Bjørn N. Sandaker, Arne P. Eggen, Mark R. Cruvellier, Routledge
- **Building Structure Illustrated** by Francis D.K. Ching, Willy

In conclusion I must say that:

- Membrane structure acts as a skin to a building.
- The cables are hanged freely and rapped with the tensile fabrics.
- It provides strong built forms with excellent solutions for long span Architectural spaces.

It is definitely an excellent solution for the long span architecture. So, in the next lecture we will go to the last part of the tensile structure. That we will do some more numerical problems and also the application part form of numericals on tensile structure in architecture; and we will also discuss some of the real-life examples of the tensile structure by some famous architects.

So, now I have one homework for you:

Q. A cable of length 100m is fixed at two supports at same level. The load intensity of the cable is .5KN/m

Find the Span, T_{max} and T_{min} if the sag is

(i) 10m,

(ii) 20m,

(iii) 30m

Draw a graph showing the changes the respective values w.r.t. the sag and draw a conclusion.

Thank you very much that is the end of the lecture number 32.