Structural System in Architecture
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Lecture No -33
Application of Tensile Structures in Architecture

Hello everyone, welcome to the NPTEL online certification course on Structural Systems in Architecture. Today we are in the module 7 and this is the seventh week and this is the third lecture on the seventh week. Today we will discuss about the applications of tensile structure in Architecture.

The following concepts to covered in today's lecture:

- Types of Cable Suspended System
- Numerical Problems on Suspension Cable
- Membrane and Cable Materials
- Case Studies of Tensile Structures
- Numerical Problems on Multi Cable Structures

The major learning objectives are:

- Outline the types of Suspended Cable Structure
- Apply the tensile structural concept in the Architectural projects
- Develop the numerical understanding of suspended and multi-cable structure

So, the types of cable suspended system if I see there are four typical types. The first one is called the Parallel Span System.

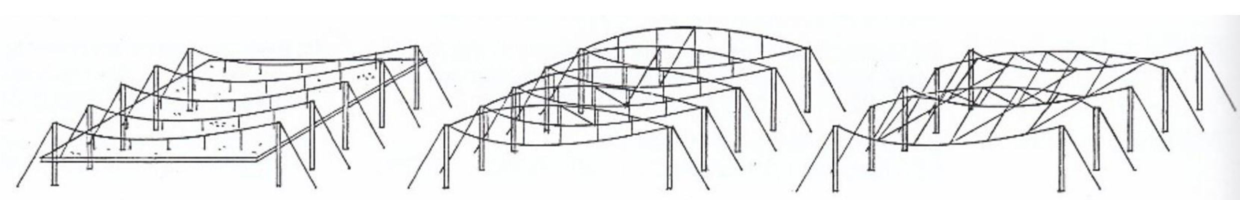


Figure 1 : Parallel Span System

In the parallel span system as you see in Figure-1, there are cables running parallel and those cables may be of sagging kind hogging or maybe the multi or the two cable systems. Those cables are simply suspended in form and they are pre-tensioned in either both the ways of maybe in the concave or convex way.

The second system is called the Biaxial Span System. In Figure-2 you can see that there are four images where through the cables the surface has been created. Towards the end or the edges of the surfaces there exists some rigid beam members. In between the beam members the cables are placed in different directions to make a surface or to result into a single surface and that surface as you know it can be synclastic or be anticlastic.

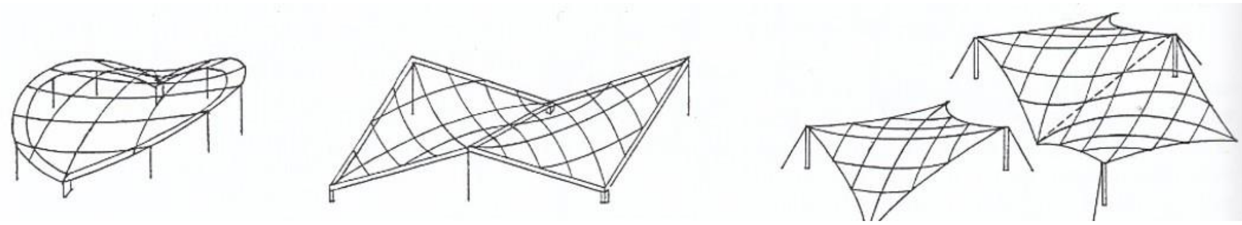


Figure 2 : Biaxial Span System

The third system is called the Radial System, where the surfaces are created by suspended cables; but here the suspended cables are placed in radial manner. There will be two compression rings, inner and the outer. In radial fashion when you place, it will be tied up in the compression rings. Sometimes there may be inner post and a compression ring outside. Those compression rings will take the final load and that that will be discharged to the various peripheral columns and the intermediate portion of the circular portion will be column free.

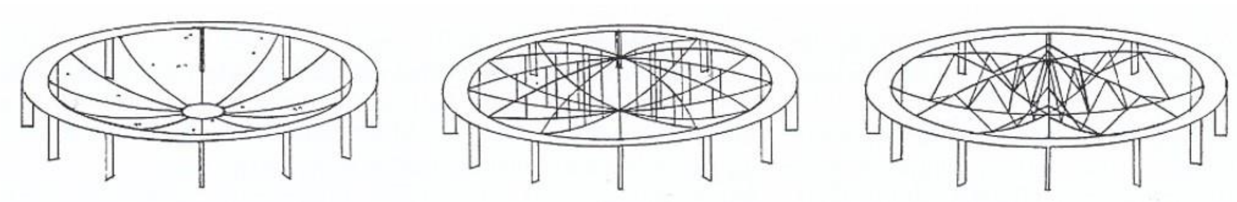


Figure 3 : Radial System

The final one is called the Cable Trusses or Cable Truss System.

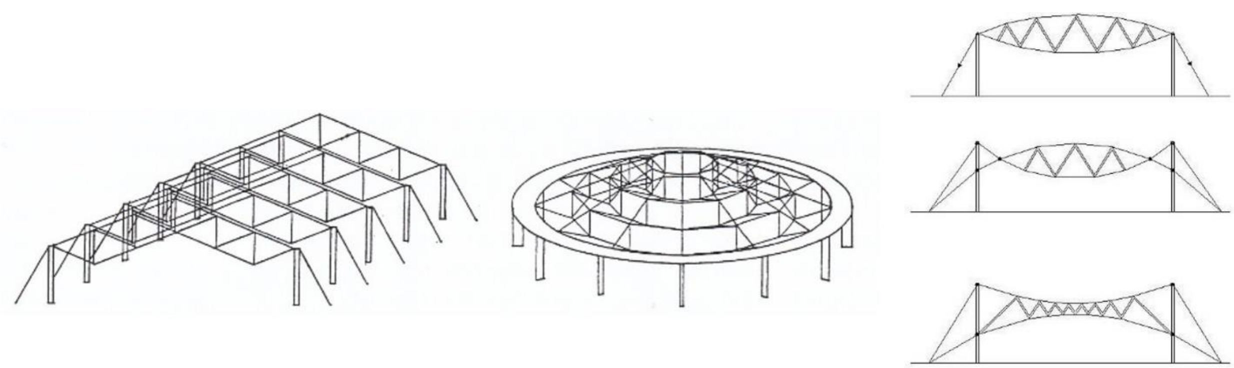


Figure 4 : Cable Truss System

This system is very similar to the first one where series of cables were placed; but instead of the single cable they can go for the multi cable system with some internal strut members which will act as compression member and overall, it will look like or it will behave like a truss. However, in this cable truss system primary and the secondary cables will be placed together and that will be separated by the internal trusses. This cable trusses system may be employed in the ring radial systems or may be in some other forms too.

Now, let us discuss some numerical problems on the suspension cables.

Now if you see, we have discussed a bit in the very first lecture on this particular week is that when you hang a cable if it is hanged by its own weight it is the catenary, it will give you a catenary shape. But in case of suspended cables, sometimes it may not be hanged by its own weight. Sometimes there may be some other link members, which are then connected to the primary cable. Therefore, the primary cable will have a series of pointed loads. In Figure-5, the green color downward arrows are the series of pointed load separated by some distance.

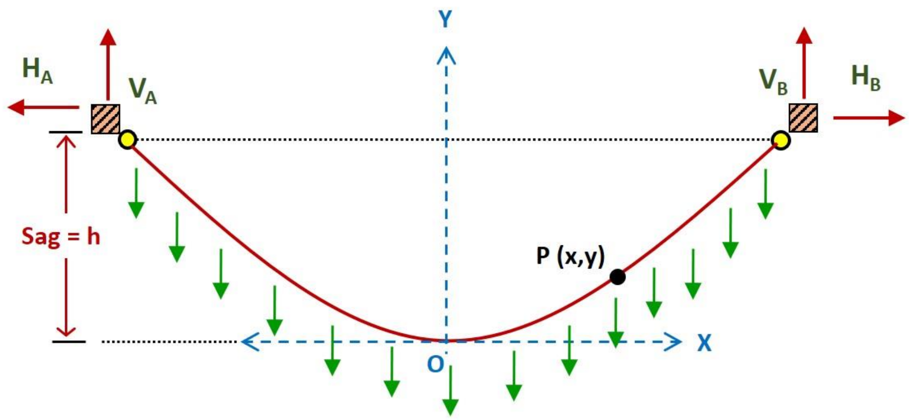


Figure 5 : suspended cable with catenary shape

Here if the total load is very much higher compared to the self-weight of the cable, then this cable will form a parabolic shape. The equation of this cable profile will be:

$$y = \frac{q_0 x^2}{2H}$$

q0 = Uniform distributed load KN/m

H = the horizontal thrust or reaction which is equal all over the cable

Tension in the cable at any point ‘x’ is given by:

$$T = \sqrt{q_0^2 x^2 + H^2}$$

Please always remember that in such formation we keep the origin in the most downward point or highest sag point. From the origin, right-hand side is the +ve X axis, left-hand side is the -ve X axis, upward is the +ve Y axis and downward from origin in -ve Y axis. In this case it is symmetrical, the amount of span in +ve X axis and -ve X axis is equal. But if it is not a symmetrical suspension cable like this, even than the segment's lowermost point will be the origin, and then the span on -ve X axis side and span on +ve X axis side will not be equal. However, our origin will be at that particular point itself. So, the value of x and y has to be given where we want to find out the tension or maybe any other parameters. Hence the tension will not be equal in this the whole cable from one support to the other support. The maximum tension will be where? The T_{max} It will be at the point where the value of 'x' is higher. In that case the highest tension will occur at the end A or end B where there are supports and we can easily replace the value of x = L/ 2.

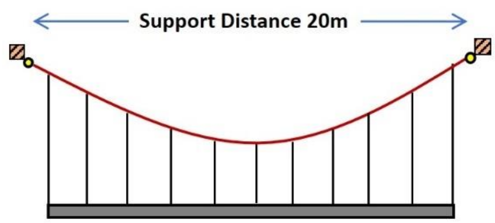
T_{max} will be given as:

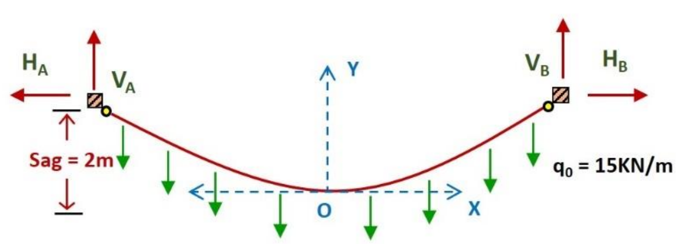
$$T_{max} = \sqrt{\left(\frac{qL}{2}\right)^2 + H^2} = \sqrt{V_A^2 + H^2} = \sqrt{V_B^2 + H^2}$$

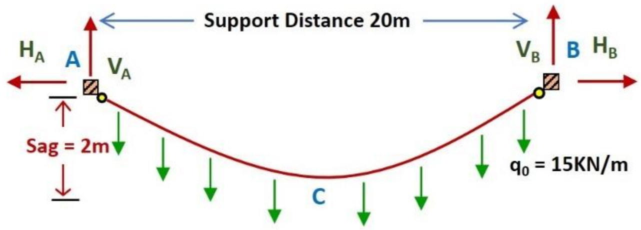
Whereas the T_{min} will occur for the minimum value of x, that is at x =0; and if i put x =0 , then

$$Tmin = H$$

So, now let us solve a problem. Suppose there is a suspended cable with supports at distance of 20 meter apart and the sag is 2 meters. Now I have to find out the support reaction and this particular suspension cable is actually taking care of a roof whose weight is given as 300 kilo Newton and then we have to find out the support reaction. So, from here I have to actually find out what is the uniform distributed load.







So, I know that this is the total 20 meter of the span and then the load is 30, so then I can assume that a 15 kilo Newton per meter uniform distributed load is acting on the primary cable. So, the q0 is 15 kilo Newton per meter and I know the length of this is 10 meter and the sag is 2 meters.

So, I can now easily compute the reactions and the tension in the cable. This is a symmetrical case, so left and right span will be equal.

$$V_A = V_B = 0.5\,(q_0\,L) = 0.5(15X20) = \mathbf{150KN}$$

Then taking the FBD

Taking FBD of left side of C &

Equation moment about C = 0

V_A X 10 = H X 2 + 15 X 10 X 5

H = (1500-750)/2

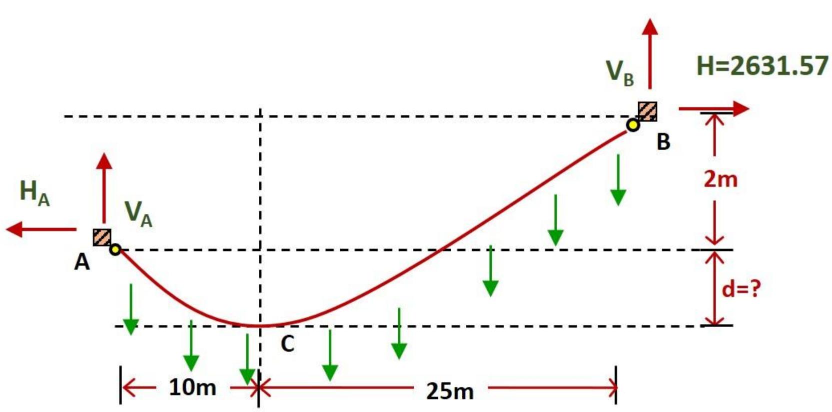
H = 375 KN

Then the tensions will be:

$$T_{max} = TA = TB = \sqrt{V_A^2 + H^2} = \sqrt{150^2 + 375^2} = 403.8KN$$

$$T_{min} = H = 375KN$$

Next I have given another problem. Here the situation of cable is asymmetric. Where I have kept the distance of the span as 35 meter and out of 35 meters 10 meters left hand side and the 25 meters towards right hand side from the origin point.



I have also mentioned that the intensity of load is here 20 and the vertical separation of the two support is 2 meters. Now I have to find out what is the sag? The I have to find out the tension. So, to do that, I know that this is my origin, as I told before that this the lower most point will be my origin. Now solving this by using the respective formulas. The equation of the cable profile is

$$y = \frac{q_0 x^2}{2H}$$

Now for point A

For Point-A, y = d, x = -10 gives

d = (20 x 100) /2H

then for point B

For Point-B, y = 2+d, x = 25 gives

2+d = (20 x 625) /2H

So I have two unknowns, the 'd' and 'H'

Then solving the above two equations we get:

d = 0.38m

and hence

H = 2631.57KN

So, as I know sag and horizontal thrust, I can easily come back and then find out what is my VA and VB. Solving this:

Taking moment about A and equation to Zero:

$35V_B$ = 20X35X17.5 + 2X2631.57

V_B = 500.4KN

Taking moment about A and equation to Zero:

$35V_A$ = 20X35X17.5 - 2X2631.57

V_B = 199.6KN

Then, computing the Tension at A and tension at B

$T_A = \sqrt{V_A^2 + H^2} = \sqrt{199.6^2 + 2631.57^2}$ = 2639.12KN

$T_B = \sqrt{V_B^2 + H^2} = \sqrt{500.4^2 + 2631.57^2}$ = 2678.72KN

So next let us go to the cable and membrane material. As you all know, the primary material of the tensile structure is either a cable which is going to take care of the internal load from one compression ring to the other mast or maybe from one mast to the other mast, and then it is the membrane material that is the tensile fabric material. So, there are some property I have listed down on the requirements, being an enclosure element, it needs to be airtight, waterproof, fire resistant and durable. As a daily-use element, it requires to transmit daylight, reflect heat, control sound, and be easy to keep clean.

So, in that particular perspective, broadly we have three type of material.
The first is **PVC coated polystyrene fabrics**. They are very old type of material and this material is in use since last 20 years. Some key features of this material are:

- This is made with woven high tenacity polyester base fabric and a flexible plasticised PVC coating.
- These fabrics are durable, translucent and waterproof.
- It can withstand extreme environmental conditions, humidity and UV-radiation.
- Wide range of colours, most economic, useful life phase in excess of 20 years can be expected.
- These fabrics are suitable for retractable roofs, membrane façades and tensile fabric structures.

So, the next material is a bit new in the sense, that it has developed a lot. I mean initially it has started in 1950's probably for some other purpose but it has developed a lot and this is called PTFE. **Polytetrafluoroethylene** it is full name, but it is popularly known as the PTFE. This is a fiber glass coated material and it is straight name is Teflon which is very commonly used. The teflon sheet is nothing but this polytetrafluoroethylene. It is extremely durable and weather resistance indeed. It has a translucency of the 7% to a 15% and reflects almost about the 68 to 75% of the incident rays. So that means almost about 7 to 15% heat due to the radiation direct solar radiation can penetrate through this material and definitely whenever that particular percentage is penetrating the daylight is diffused and glare free.
The other material is called **ETFE**, it is **ETFE it is ethylene tetrafluoroethylene**. Some of the properties are:

- ETFE foil systems - a mechanically pre-tensioned single layer system or a multi-layered "air-pressured" ETFE foil cushion system.
- It does not degrade, discolour or turn yellowish under extreme environmental conditions or UV radiation.
- It has high degree of transparency or translucency.
- The rapid advancement of these fluoropolymer foils enables architects to design ground-breaking structures that are worlds apart from conventional façade and roof construction.

These three materials are quite common in the field of our membrane material application in the architectural structures.

Now we will see some other materials which are used for cables. The cables mostly are of high strength steel cable that will strain lot of stress, may be more than 500 N/mm^2. So those cables are actually put in bundles and in a spiral form. Sometimes there are cables available which are in a form. Refer Figure-6.

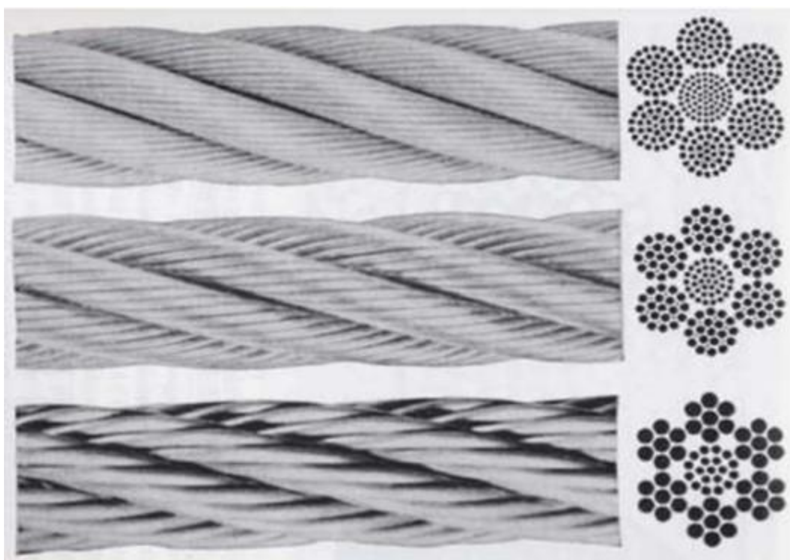


Figure 6 : the cable materials

But the **spiral cables** are definitely good and that will give you more durability and due to the friction, it can take care of some excess amount of load. Cables serves a number of functions in tensile structure applications:

- Reinforcement of the fabric where the spans and stresses get too large.
- Linear tension support elements along edges.
- Stays to stabilize rigid support elements.

If we see the types of cables, then a cable may be composed of one or more structural ropes, structural strands, locked coil strands or parallel wire strands. A strand is an assembly of wires formed helically around a central wire in one or more symmetrical layers.

Now let us see some case studies. At first, let us discuss about the National Gymnasium at Tokyo. The Architect is Kenzo Tange. The profile of the building is shown in Figure-7. If you see from Architectural perspective, the you can see that it is the fusion between Japanese Architecture and the modern structural expression that two has been fused together. It is an enormous, huge structure and that has been tackled very, wonderfully by the architect.

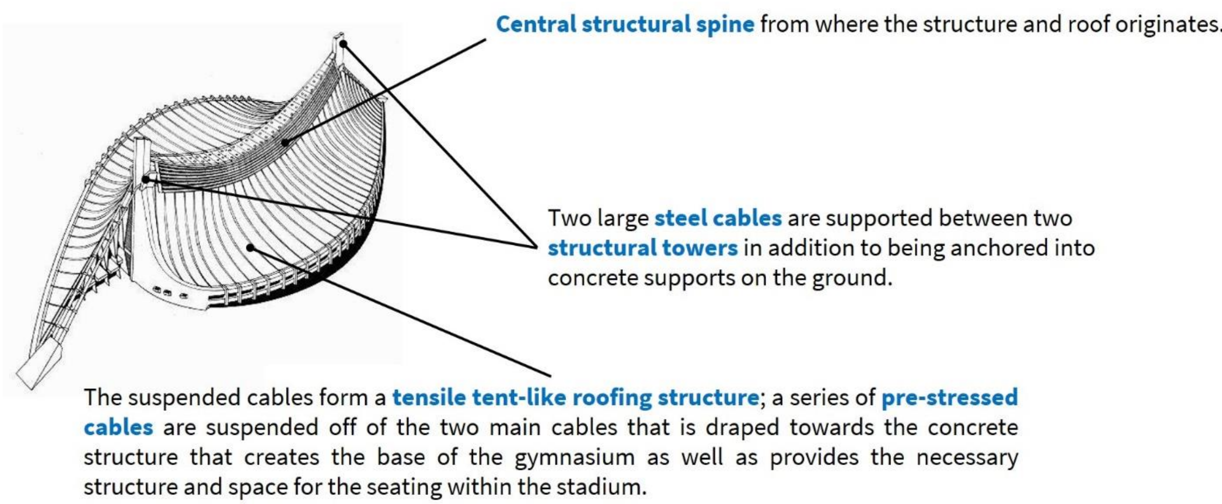


Figure 7 : National Gymnasium at Tokyo by Kenzo Tange

If you see in this particular figure, then it has a central the spine, from the central spine the roof is originated. But this central spine was actually nothing but the two very large steel cables which is actually going to support it at the end; which is can be treated as a pillar or maybe the mast. Then the tent like structure or tent like material or the flexible membrane material can now act as a roofing material. It is internally supported. The black lines are actually those internal supporting secondary membrane which is following the profile of catenary. Those are pre stressed cables which goes or merges with the profile of a hanging tensile membrane.

The second one, this is two bridges, both the bridges from our country. One is in Mumbai which is Bandra-Worli Sea Link. This is a cable supported bridge. there are lot of such examples of the cable supported bridges. Another one from Calcutta; it is the Vidyasagar Setu or we also called is as a second Hooghly bridge.

Figure 8 : cable supported bridges

In industrial architecture you will see lot of structures which are cable hanging or the cable supported. This is a cable hanging kind of a structure you may say, you see this particular the industrial building which is a warehouse in UK. Very wonderfully they have used the mast and they are placed at regular intervals and the roof is supported by hanging cables.

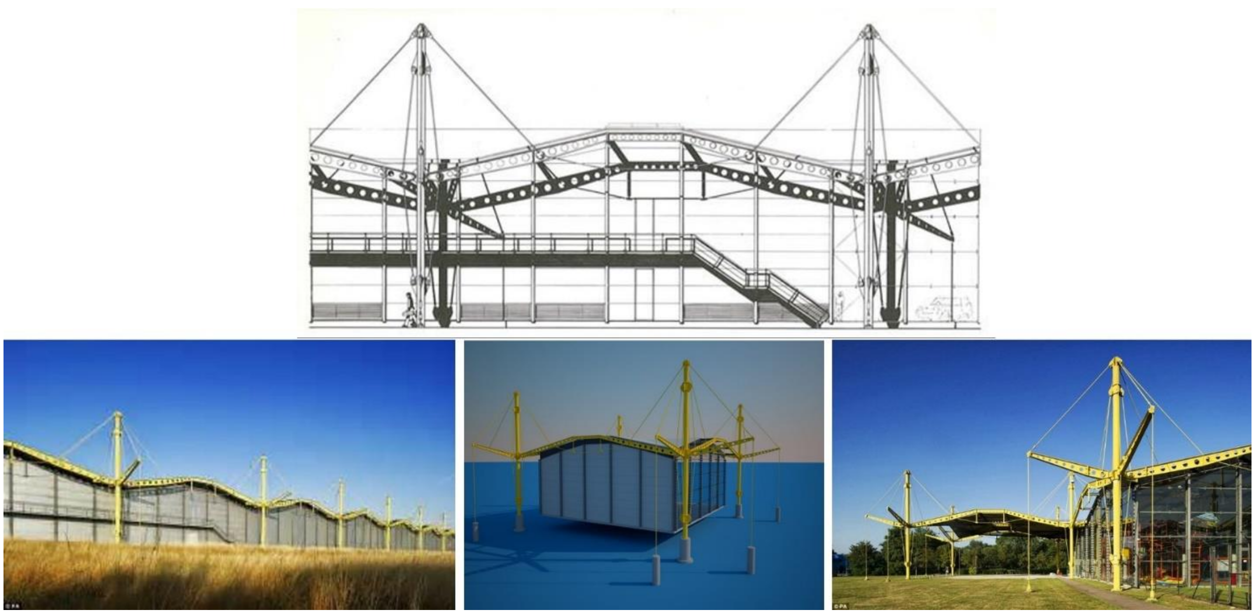


Figure 9 : warehouse in UK

The next example is Jawaharlal Nehru stadium, New Delhi. This was renovated during the time of the last commonwealth games, and now featured as Asia's largest tensile group structure. It has the total coverage of 54,000 square meter of the area, and that is treated with 88 segmental membranes. It spans over the 240 meters. The cable net system features 443 tons of pre-stretched cables with a length of 2,220 km in varying diameters from 40 mm to 95 mm

Figure 10 : Jawaharlal Nehru stadium, New Delhi

Now if you see the anatomy of this particular stadium, it has a stadium stands where the people sits. Then there is a steel this is the second one this is a steel compression ring, which just now we have discussed about this is 8500 tons of steel has been used and this red color is the ramp. This ramp is required for the vertical circulations or the different layer circulations from one layer to another layer of the spectators sitting area.

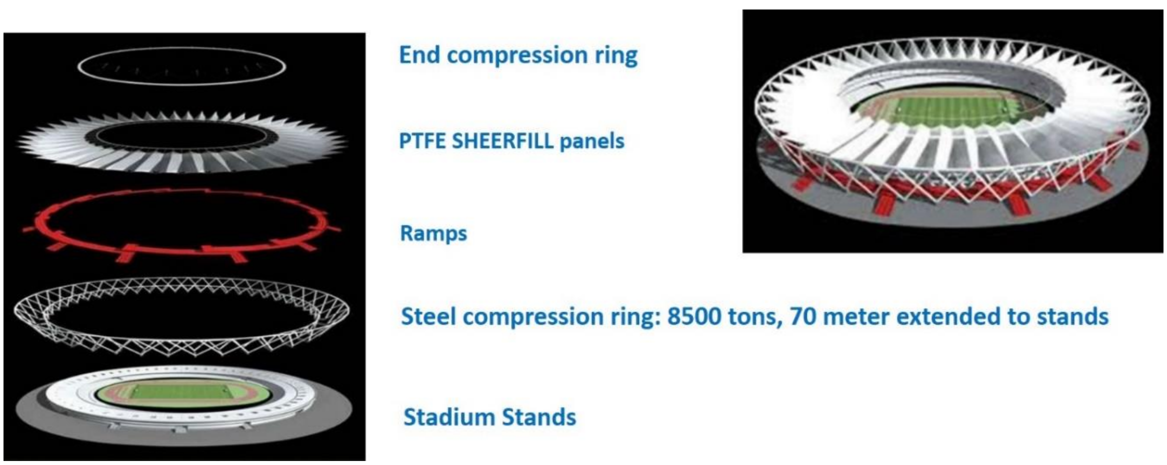


Figure 11 : anatomy of Jawaharlal Nehru stadium, New Delhi

And over this particular steel compression ring this PTFE sheer fill panel, which is also very energy efficient panel has been placed. Those are the panels of 88 segments, supported by the cables. Finally, at the end there is at the internal position. And at the end edge the compression rings have been placed., and that ring has thoroughly taken account of all different parts of the structural forms. It is a tensile structure, it has a membrane, it has three-dimensional truss that is the space frame systems.

Let us do some numerical problems on multi cable structure. We can use multi cable structure in stadiums, or bridges etc. If you remember, in our very first lecture we have discussed about four types of multi cable structures. Here in the first example, we have taken a radial shape multi-cable structure. It is actually supported at the top, in a single point and there are various equidistance points on the deck where it is connected. On the deck we are considering a load of 90 KN and assuming that the cables are placed equidistantly. So, there will be 30 KN load on each part, see Figure-12. But the thing is that, the A, B and C the three cables is going to provide TA, TB and TC those three tensile force. But those forces are not going to be equal because the angle of inclination with the vertical post or mast is changing.

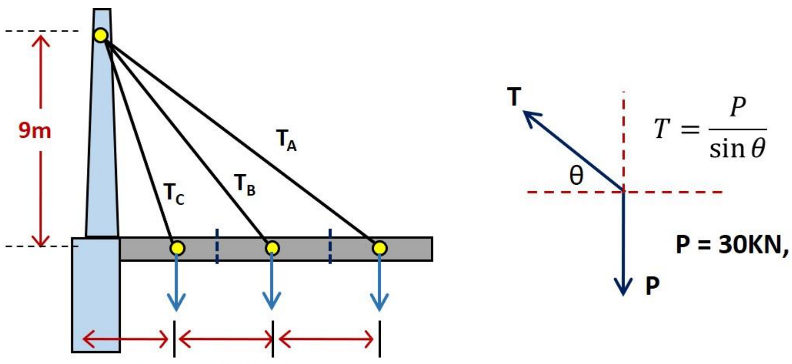


Figure 12 : the radial multi cable system

In the right-hand side image, you can see that how will it happen. So, if P is the downward force then due to that the tensile force T has to be there. Hence, I can find out what is the relation between T and P. That T and P is having this kind of relation:

$$T = \frac{P}{\sin\theta}$$

If that is so and I can now find out the $\tan\theta$ that is the $\tan^{-1}\theta$; from there I can find out the 45 degree. See the table below.

Table 1: the three points of cables

Cable	Tan^{-1}	θ	Sinθ
A	(9/9) =1	45°	0.707
B	(9/6) = 1.5	56.3°	0.83
C	(9/3) = 3	71.6°	0.95

If I know the angle theta, then we can use the T and P relation and find out the value of load. It is given in Table-2.

Table 2 : the loads at TA, TB and TC

Tensile force in each Cable		
T_A	30/0.707	42.4 KN
T_B	30/0.83	36 KN
T_C	30/0.95	31.6 KN

From here we see that the intensity of load is decreasing from point TA to TC

So, we see that if the angle is higher, suppose the angle is 45º for A and the tension is 42.4 KN, whereas for C the angle is higher 71.6º, the load is lower, it is 31.6 KN. Hence, we can say that, in case of the radial system as the sin increases the load decreases.

The next example we see with the harp shape. In harp shape instead of one point I put the cables in the three points in the vertical support; and now the inclination for each is same; that means all are at 45º, so the T A, T B and T C will be the same and that is 42.4. So, each cable will have to have taken care of same quantum of loads; but definitely the length of the cables is different.

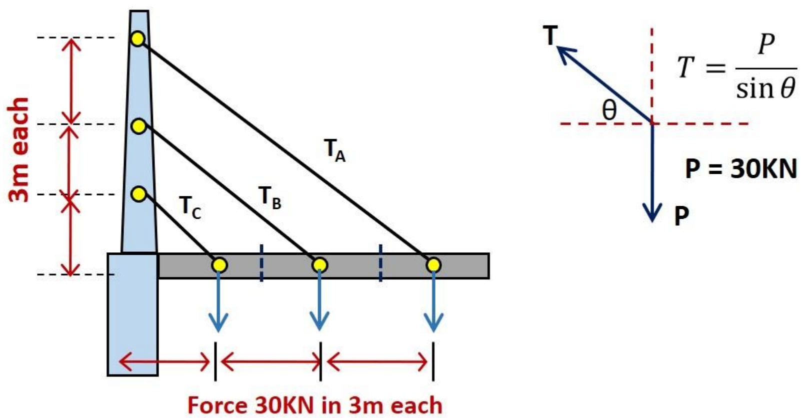


Figure 13 : the harp multi cable system

Here, $T_A = T_B = T_C = 30/\text{Sin } 45° = 30/0.707 = 42.4\text{KN}$

So, from this point of view of both are stable. Either radial or may be the harp we can go for. But I can have a kind of comparative analysis chart on both the system. Where I found out all the forces as I have explained earlier. Then I based on Pythagoras Theorem can find out the lengths of the cable, because I know the base and height. When the permissible tensile stress of the cable is given, let's say here we are taking it as 500 Mpa and the density of the cable is assumed as 7800 Kg/m3, then we can find out the cable c/s area and then multiplying it with the density I can find out the total weight of the cables. Hence I can analyze which is more economical.

Here, if I compare radial and harp system, then I found that radial is more economical than the harp. For the above-mentioned examples, the cable length required for radial is 33.03 meters, and total weight is 191.70 tons. Whereas, in case of harp system, the total cable length is 25.46 meters, and total weight is 168.38 tons. Hence, we can conclude that among the two examples considered above, the harp system is more economical compared to radial system. Refer Table-3.

Table 3: Comparative Analysis of Radial & Harp system (based on previous two examples)

RADIAL						
Cable	**Tensile Force (KN)**	**Base**	**Height**	**Cable Length (m)**	**Cable C/s Area (Sq mm)**	**Cable Weight (Ton)**
A	42.4	9	9	12.73	84.80	84.19
B	36	6	9	10.82	72.00	60.75
C	31.6	3	9	9.49	63.20	46.77
			Total Length of Cables:	33.03	Total Weight of Cables:	191.70
HARP						
Cable	**Tensile Force (KN)**	**Base**	**Height**	**Cable Length (m)**	**Cable C/s Area (Sq mm)**	**Cable Weight (Ton)**
A	42.4	9	9	12.73	84.80	84.19
B	42.4	6	6	8.49	84.80	56.13
C	42.4	3	3	4.24	84.80	28.06
			Total Length of Cables:	25.46	Total Weight of Cables:	168.38

Here, for this lecture I have taken the reference of these books.

- **Engineering Mechanics** by Timishenko and Young McGraw-Hill Publication
- **Structure Systems** by Heino Enge, Hatje Cantz Publisher
- **Structure and Architecture** by Meta Angus J. Macdonald, Elsevier Publication
- **The Structural Basis of Architecture by** Bjørn N. Sandaker, Arne P. Eggen, Mark R. Cruvellier, Routledge
- **Building Structure Illustrated** by Francis D.K. Ching, Willy
- ***Fabric Architecture:*** Issue: January/February 2008 by IFAI Publication

In conclusion we can say that:

Tension fabric is still a relatively new building material, despite its more extensive use over the past decade or so, in architectural applications. Tensile structures are the excellent option for architects and engineers for designing sustainable structures. Especially over large areas, it is an incredibly lightweight, material saving, and energy conserving solution for roofing systems.

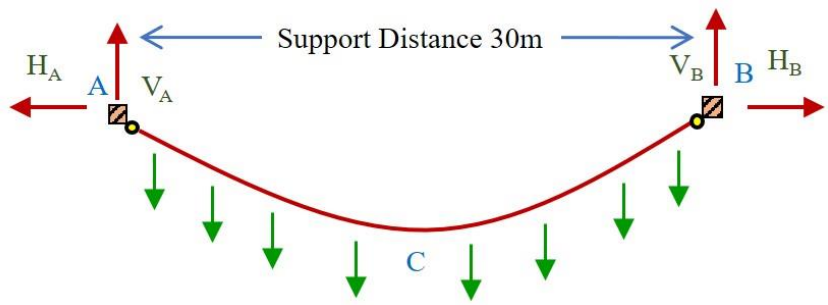
There are two homework for you,

Problem:1

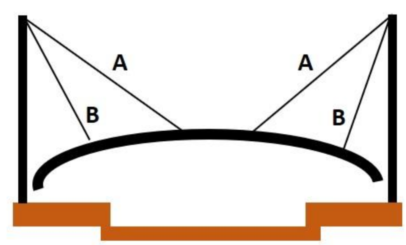
The Span of the cable is given as 30m

Find the Sag, if the maximum tensile force in the cable is 500KN

q_0 = 12KN/m



Problem:2



The roof of a stadium is supported symmetrically by two sets of cable (A & B) as shown in the figure. The total load of the roof is 240 Ton. Assume cables are supporting equal load. Find the tensile force in the cable A & B. Assume the following:

The inclination of cable A and B with vertical is 45^0 and 30^0 respectively.

Take g= $10 m/s^2$

That's all for this lecture.

Thank you very much.