Structural Systems in Architecture
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Module 1
Lecture – 4
Structural Supports and Reactions

Welcome to this NPTEL online certification course on Structural Systems in Architecture. So today we are in module 1; and this is the fourth lecture. The topic of today's lecture is Structural Supports and Reactions.

We will try to cover the following concepts:

- Types of Structural supports
- Type of Beam and Support Reactions

The intended learning objectives are:

- Understanding various types of structural support.
- To interrelate the types of support and its nature of reactions related to loading.
- To Establish the procedure to calculate the support reaction.

So, we will and try to understand the various structural supports, how it behaves and what is the mechanism? Then we will try to interrelate the types of support and the nature of the reaction, which is related to the loading, different types of loading; and the finally we will try to establish the procedure to calculate the support reaction; and how to find out the support reactions.

Supports are the external systems. They are associated with a structural member to ground the loading, which is actually applied as external loading. Suppose, I am now sitting on a chair, so the external load on the chair is my load, my weight, 50 kg or 60 kg, and finally the total load is getting distributed to the ground by 4 legs of the chair; and at the vertical legs are supported by the ground. The junction between the legs and the ground, the system is called the support system. Now, there are typically many types of support system; but out of that the following four types are very common in our buildings or may be in various structures.

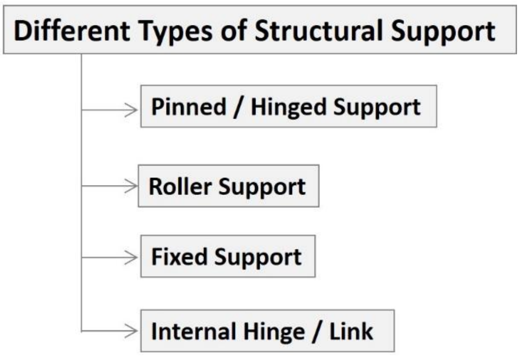


Figure 1: types of structural support

The first one is called pinned or hinged support. The second one is the roller support. Of course, the roller support may be the most general one then the pinned or the hinged support. Then there is a support called fixed support and then internal hinge or link support.

The roller supports are like a support, or may be the member is on a roller or a wheel. So, it is analogous to something like somebody is on a skating ring. Therefore, it can move. The support will allow the member to move in particular direction. So, in Figure 2, you can see that in left-hand side image is the skating and the right-hand side image is a roller support which is taken from a bridge.

Figure 2: the roller supports

The roller supports will allow some kind of thermal expansion and dynamic loading and because of that it may allow the movement.

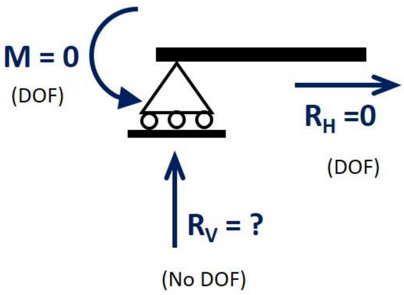


Figure 3: typical roller support

If I see a roller support, it will have 3 reactions. Out of the 3 reactions, it cannot move in the vertical direction because there is no possibility. So, as it cannot move in the vertical direction, it gives me a kind of a reaction in vertical direction, R_V is the reaction which unknown to me, but this particular support it can move in horizontal direction, both left and right. The reaction at horizontal direction will be 0. It cannot take any kind of horizontal moment, so moment in horizontal direction is always going to be 0. Again, the roller is is a hinged or a pinned, so it can actually move in horizontal way, so it cannot take any kind of a moment. Even if there is a certain amount or smallest amount of moment, it will move. No question of any horizontal rection can observe, only it can observe or give some kind of the vertical rection. So, out of 3; it has two degrees of freedom (DOF), this is the degree of freedom and one unknown reaction, this is the unknown reaction. So, the roller support enjoys two degree of freedom and it has one unknown direction.

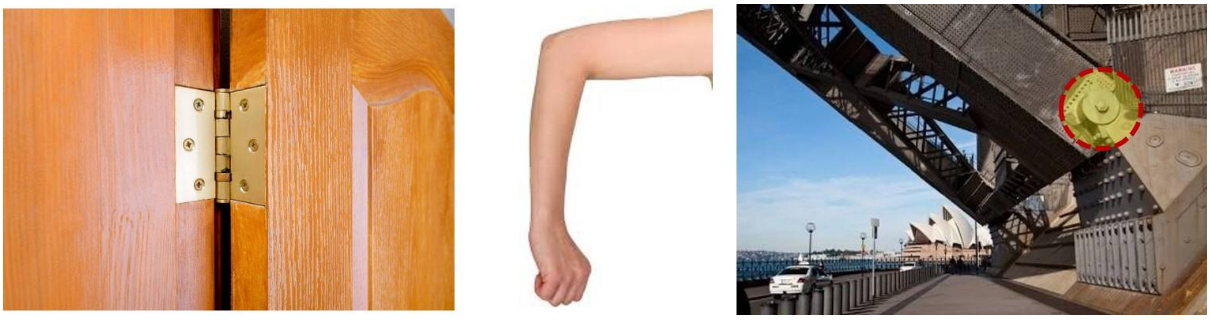


Figure 4:pinned or hinged support

Now next is the pinned or the hinged support. Here it is very similar but it does not have any kind of roller, it is fixed on the base. So, if I say a door hinge is a very common example, it has a hinge only, it does not have any kind of a roller. It is quite similar to our elbow joint. The right-hand side image of Figure 3, is the So this particular hinge support is very common in trusses and bridges and also sometimes in buildings.

Now, in the hinged support, let us see what is the degrees of freedom.

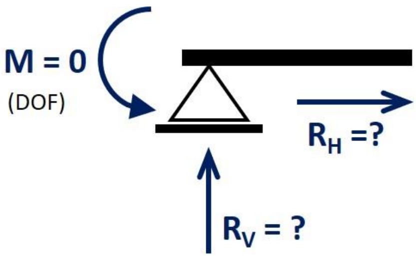


Figure 5: typical hinged support

So here you see it cannot take any moment because it is a hinge. So, it will again in a with certain amount of load it will move, or we can say that it will rotate, but it will not translate. So, this is a degree of freedom; but it cannot move like a roller or it cannot move in the upward or downward directions, so it has 2 unknown reactions. So, we will see that the degree of freedom is 1 and the number of unknown reactions are 2 in this case.

Figure 6: fixed supports

In case of a fixed support, it is like somebody is holding some stick in his/her hand. It is like something is very much grouted in a vertical or horizontal plane, see Figure 6. So, those are the fixed supports.

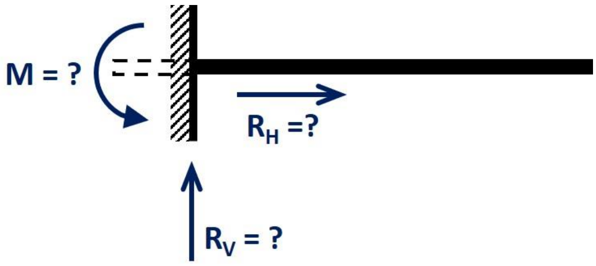


Figure 7: typical fixed support

The fixed supports are not under hinged or pinned condition; it cannot move. So, it will have some kind of a moment which I do not know, it cannot allow any vertical or horizontal movements as well. It will impart some kind of vertical and horizontal reaction. So, based on that, it has no degree of freedom and number of unknowns are 3. So, the degree of freedom plus unknown are almost about 3; if you add these two.

Figure 8: internal hinges

Now the internal hinge; they are sometimes also used for some parts of the machines as well as it is used in the trusses. It prevents relative displacement of the member and allows rotation. So, if you see the Figure 8; it is a truss, with internal hinge members, so they are widely used in the trusses too.

Figure 9: links

A connecting structural member is introduced as Link to connect two different members. It prevents the displacement in the direction of link. It allows rotation, displacement in specified direction. It is normally used in machines. Link are also something like, as shown in Figure 9. In buildings it does not have much use.

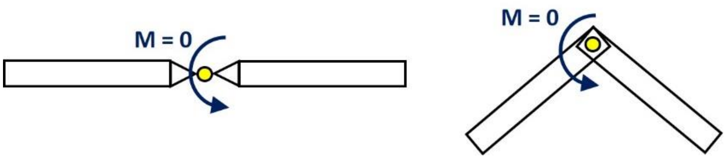


Figure 10: internal hinge

Sometimes a pin or hinge is introduced in the member to connect the two parts of it, called Internal Hinge. It prevents relative displacement of the member. It allows rotation, horizontal and vertical displacement. But in internal hinge or in this link, the moment is equal to 0, degree of freedom is equal to 1 over here. In this particular link on any hinge, it cannot take any kind of moment. So, degree of freedom equal to 0 and number of unknown equal to here 2, because of both the horizontal and vertical directions it has to take care of.

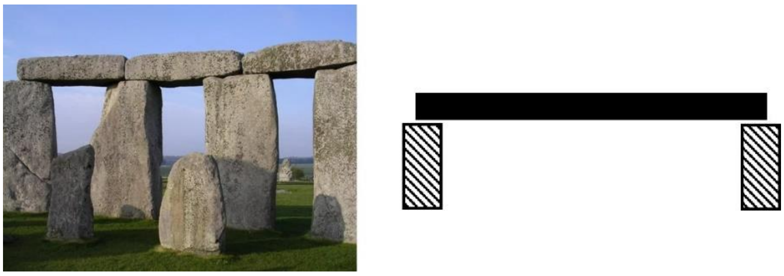


Figure 11: simple support

Now there is a support system called simple support. In this simple support, it will not allow any kind of a vertical movement and also sometimes the horizontal movement. Sometimes, may be in one direction but not in the other direction, so that the particular plank or particular beam is does not move from one point to the other. A plank of wood resting on two concrete blocks is the perfect example of the simple support.

In Figure 11, you can see the image of Stone Henge; where you can see the two-capital or pillar kind of fitting which is supporting a stone on top of it. It is a very nice example of simply support systems.

Table 1: types of structural supports and their degrees of freedom and unknown reactions

Name	Vertical Reaction	Horizontal Reaction	Moments	Degree of Freedom	Unknown reaction
Roller	YES	NO	NO	2	1
Hinged	YES	YES	NO	1	2
Fixed	YES	YES	YES	0	3
Internal Hinge	YES	YES	NO	1	2

Now if I come back to this 4 type of supports, so what I have is that, in the roller I have vertical reaction yes, I have to find out that, but there is no horizontal reaction or moment, degree of freedom is equal to 2 and the unknown reaction is 1. Similarly, for other support systems please refer Table 1. So, you can see that, only fixed support has vertical reaction, horizontal reaction, and moment; that means 3 unknown reactions.

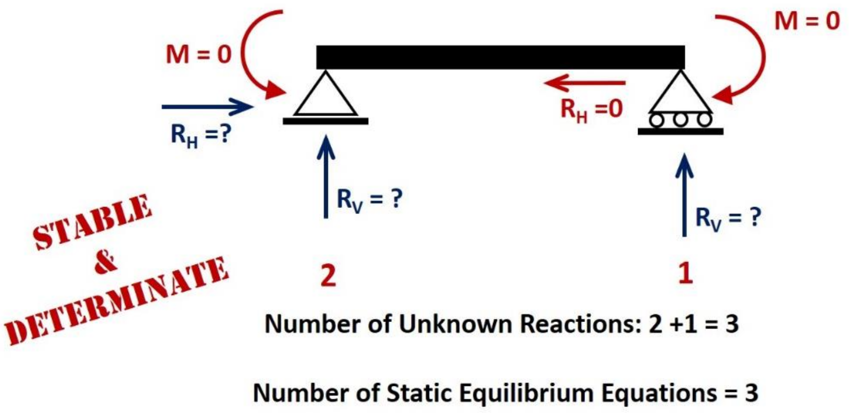


Figure 12: type of beam and support reactions

Now we will go to types of beam and their support reactions. Now this is a simply supported beam. In Figure 12, conceptually we can see that, a plank is hinged at one end and there is a roller support at the other end. Here, for the hinge it has 2 unknowns and 1 degree of freedom but for the roller it has 1 unknown and 2 degrees of freedom. So, what I see is that it has total 3 unknowns and we have 3 equation of statics, what we have discussed in the last week. They are: $\sum F_X = 0$, $\sum F_Y = 0$; and $\sum M = 0$.

Now, I have 3 unknowns and I have 3 equations available. So, I can solve these 3 unknowns by virtue of the 3 equations. Therefore, this beam is a very stable beam and it is determinate beam. Determinate beam means with these 3 equations, I can find out the unknowns.

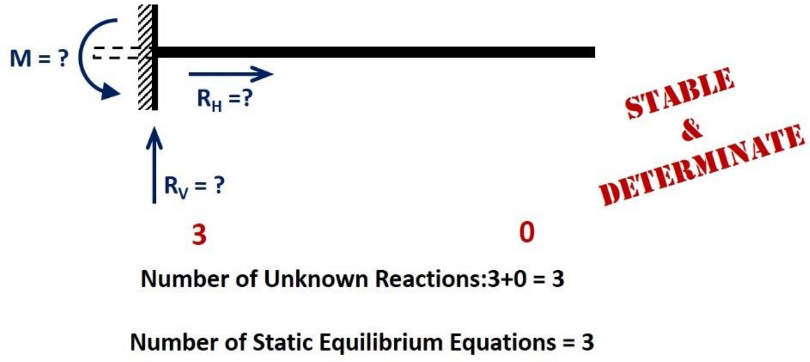


Figure 13: cantilever beam

Next let us consider a cantilever beam. Cantilever beams are those which is having one end fixed and the other end is free. So, it has M, R_H, and R_V are 3 unknowns, and the other end is free, so there is no question of any unknowns. Therefore, 3+0 = 3 unknowns are there; and you have same 3 equations, so it is also stable and determinate beam.

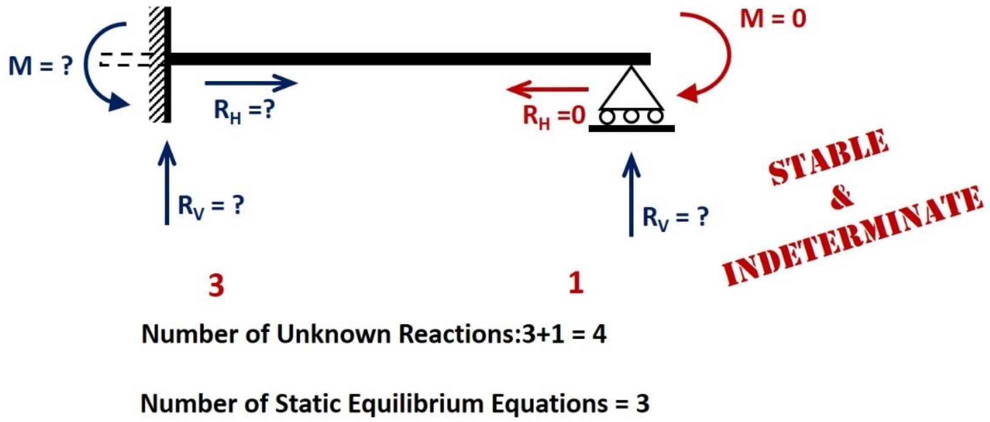


Figure 14: propped cantilever

Propped cantilever beam. Now, what I did is that in the cantilever one end is free and the other end I make a kind of a roller. Suppose it is going to bend, so I put some kind of a prop. So, 3 unknowns over here, but here due to the prop this end is not free now, this is having one unknown. So now the total number of unknowns are 3+1 = 4, but I only have 3 equations, so how can I find out the 4^{th} one? It is not possible. Therefore, it is stable but indeterminate beam. So, I cannot solve this beam very easily, I have to look for some alternatives; and we will discuss that afterwards.

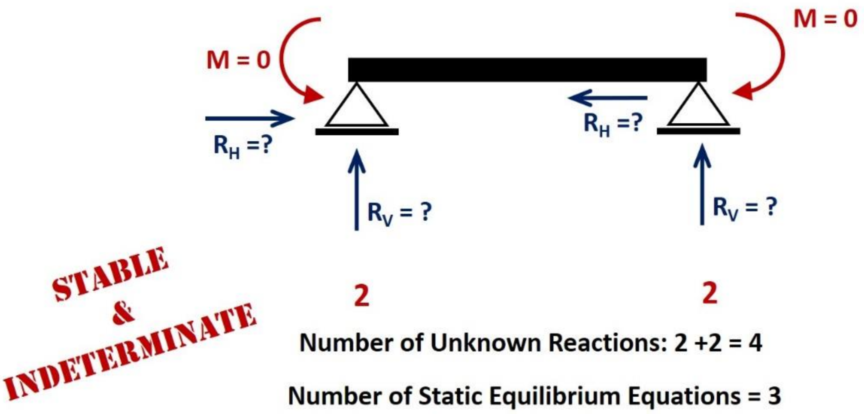


Figure 15: hinged support at both the ends

The Figure 15, is a beam with hinged support at both the ends. In this case there are 2+2=4 unknowns and we again have only 3 equations. Therefore, it will also be a stable but indeterminate kind of a structure.

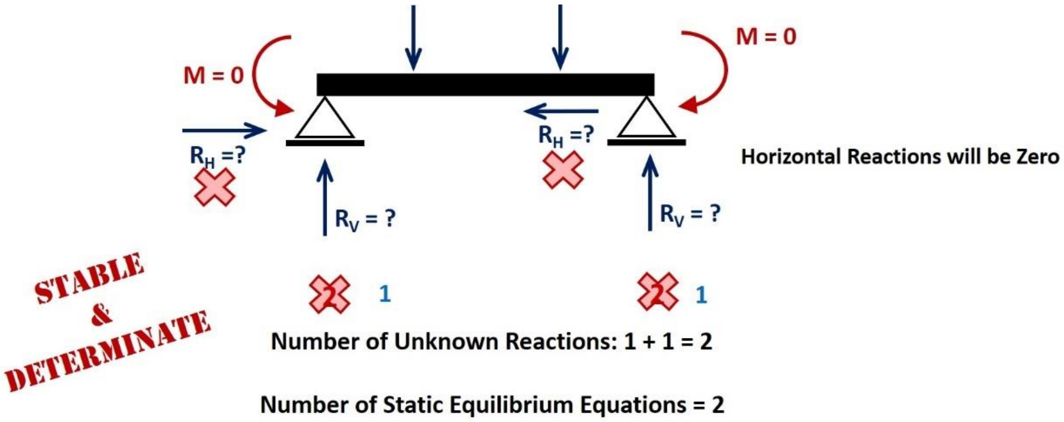


Figure 16: hinged support at both the ends with gravitational loads

But if that same beam as shown in Figure 15, is now under only two downward load that is only gravity load, then we can say that both the reactions $R_V = 0$; because if there is a downward load, there is no question of reaction from the other sides. So, in that case, this 2+2 will be now 1+1=2, because both the unknown R_H will be omitted by the downward gravidity forces. Now, the number of static equations is 3-1=2. Why did we subtract 1, because we cannot take $\sum F_X=0$, because there is no question of the $\sum F_X = 0$. So, the number of unknowns is equal to number of equations. Therefore, this will be a stable and the determinate beam.

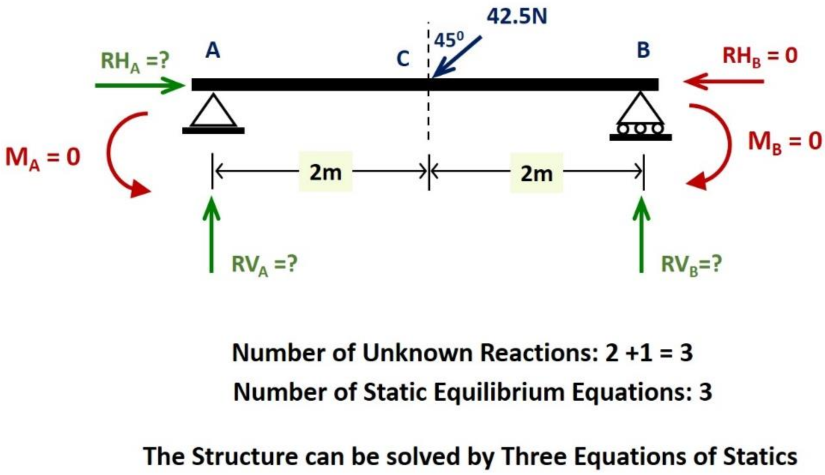
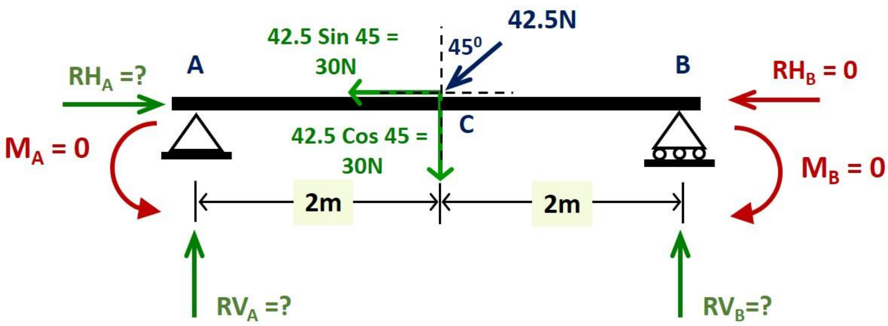


Figure 17: beam with roller and hinged support

So, suppose this is a beam with a hinged support at one end and a roller support on the other end, as shown in Figure 17. I have a force of 42.5 N at point C with an inclination angle of 45º. We have three unknowns, RV_A, RV_B, and RH_A. So, as you know you can resolve this force. I have 3 equations, unknowns are 3 and the equations are also 3, so this structure can be easily solved by 3 equations of the static. Now let us solve it.



$$\sum F_x = 0 \quad RH_A - 30 = 0 \qquad RH_A = 30N$$

$$\sum F_y = 0 \quad RV_A + RV_B = 30$$

$$\text{At } A, \sum M = 0 \quad RV_B \times 4 = 30 \times 2 \quad RV_B = 15$$

$$RV_A = 15$$

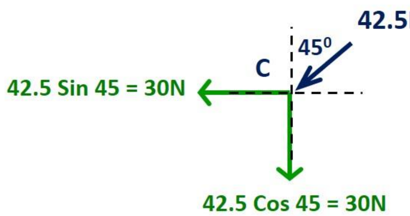


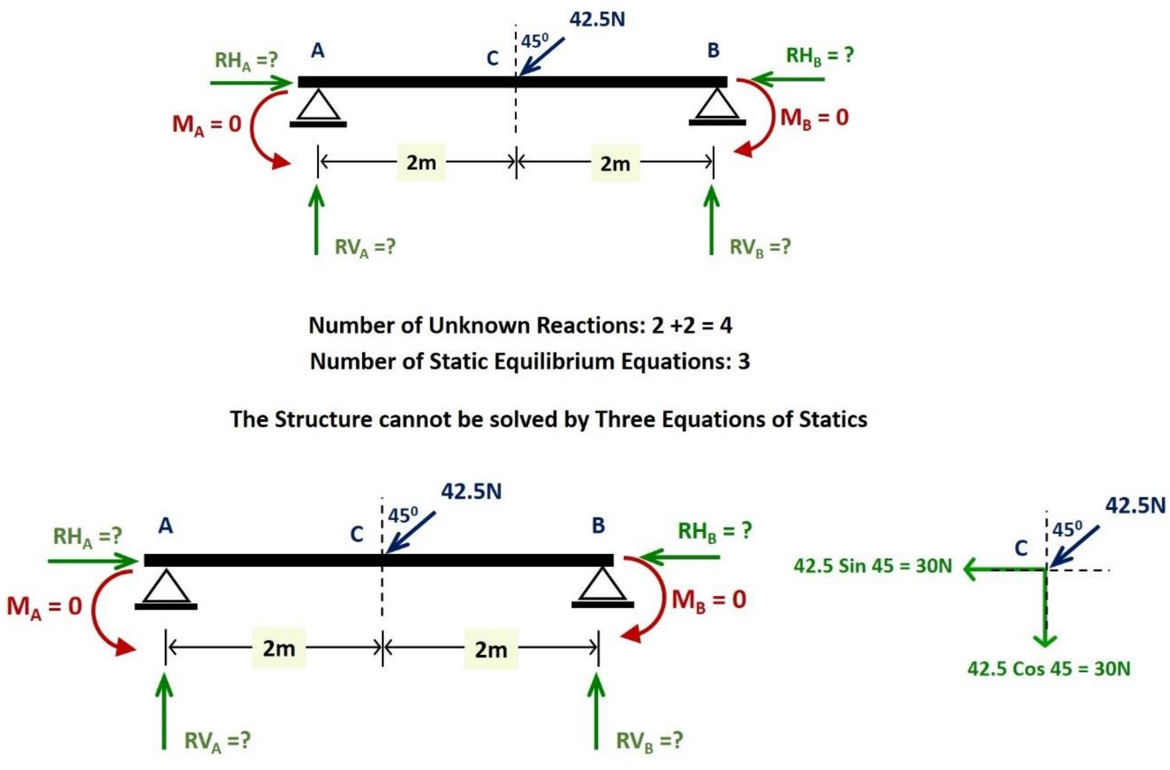
Figure 18: beam with a hinged and a roller support

Here the force 42.2 is now resolved into 45 degree, and I will get the sin component and the cos component. I have got 30N for both the components. Now, I may say that in F_X Direction, the $\sum RH_A$ + 30N in horizontal direction at C, must be equal to 0. So that gives me $\sum RH_A$ = 30 Newton.

Now, in vertical direction, I have and I have ∑ RH_A, ∑ RH_B, and 30 N downward at C; and these 3 must be equal to 30, this is second equation. I need third equation because I have 3 unknowns, one unknown I have already got. The third equation is the ∑ M, moment = 0 at point A, you can also take B but let us take at A.

The reaction RV_B this having a perpendicular distance of 4 meter, so this gives you an anti-clock wise moment at B, and downward force at C must be equal to 30 x 2 meter, which is in clock-wise diction. So, the sum of these reactions should be equal, to make the beam balance. So, I made it equal and I found out what is the RV_B = 15 and the RVA is also 15. So, from that I can find out that these two reactions.

In this case the beam or the structure is supported with hinged supports at both the ends, see top image of Figure 19. Here also, the unknowns are 2+2=4; but the number of equations remains same to be 3. So; this structure cannot be solved by the 3 equations of the static.



$$\sum F_x = 0 \qquad RH_A - 30 - RH_B = 0$$

$$\sum F_y = 0 \qquad RV_A + RV_B = 30$$

How to find RH_A & RH_B =?

$$\text{At } A, \sum M = 0 \qquad RV_B \times 4 = 30 \times 2 \qquad RV_B = 15$$

$$RV_A = 15$$

Figure 19: structure with hinged support at both the ends

Because, in the same way if you see that I can find out this RV_B and RV_A equal to 15, 15 just like other way around, as shown in lower image of Figure 19, but I cannot find out RH_A, RH_B because this is not 1 now, they are now 2 unknowns, and I have only one equations left with me, so how to find this RH_A, RH_B? So, this is a statically indeterminate kind of a case.

Now if we see a cantilever beam, here also I have the same representation, see Figure 20. Here also I have 3 unknowns, RH_A, RV_A, M_A; and number of equations are also 3. So, it can be solved by the equation of the static. Again, the RH_A, must be equal to the sin component of force 42.5, acting at C with an angle of 45^o, and similarly the RV_A also must be equal to the cos component of the same. The moment is also fine, because the force downward is 30 and the distance between them is equal to 2, so 30 x 2=60.

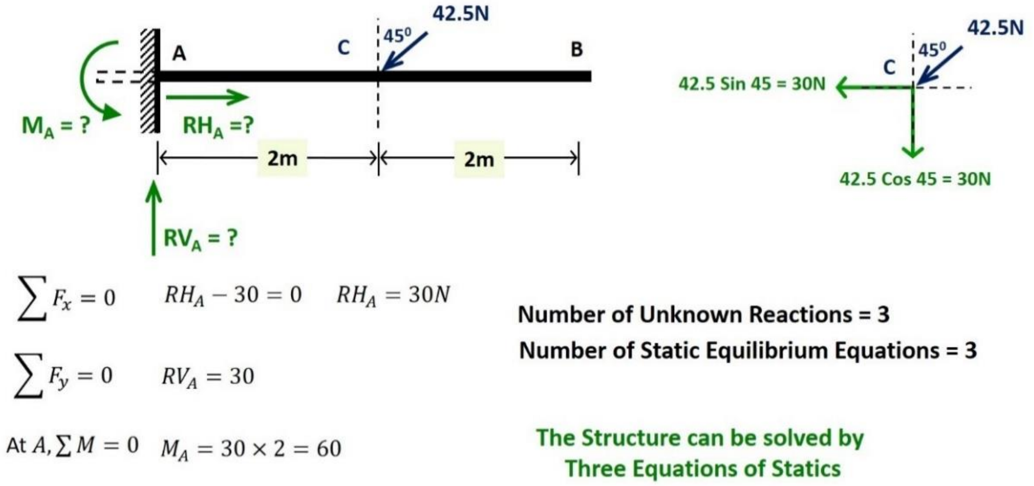


Figure 20: cantilever beam with one end fixed and the other end free

Therefore, these 3 equations can used to find out the 3 unknown values.

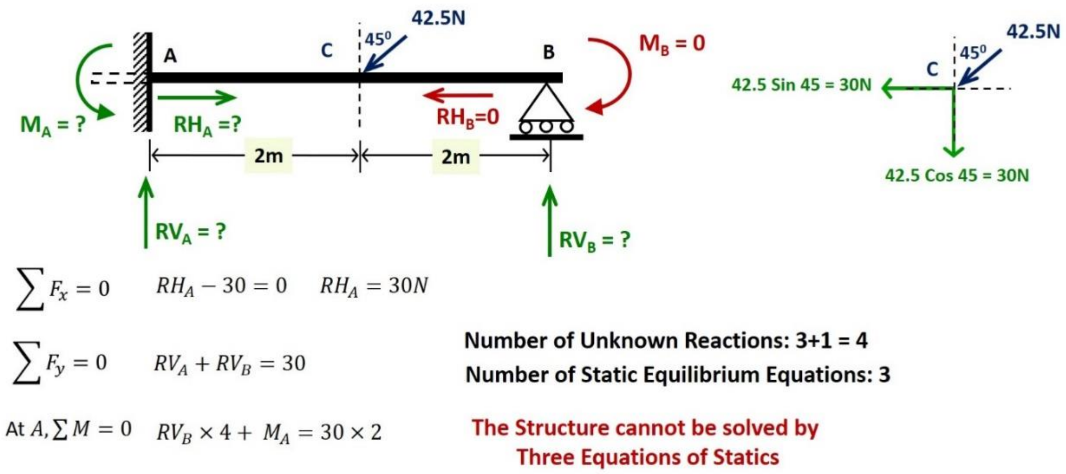


Figure 21: propped cantilever with one end fixed and other with roller support

Propped cantilever, again I have a problem because I have 3 unknowns at the fixed end, they are RH_A, RV_A, M_A; and the 1 unknown at roller support that is RV_A. I have 3 equations only, so it cannot be solved by the static equations. So here if you see these two, that is RV_A and RV_A is 30. Then the moment equation, by virtue of only one reaction in the horizontal direction, I can find out the RH_A that is horizontal at A = 30, but how to found the other 3, the RV_A, RV_B, and $M_{A;}$ I have no idea; because I have only 3 equations but I have 4 unknowns.

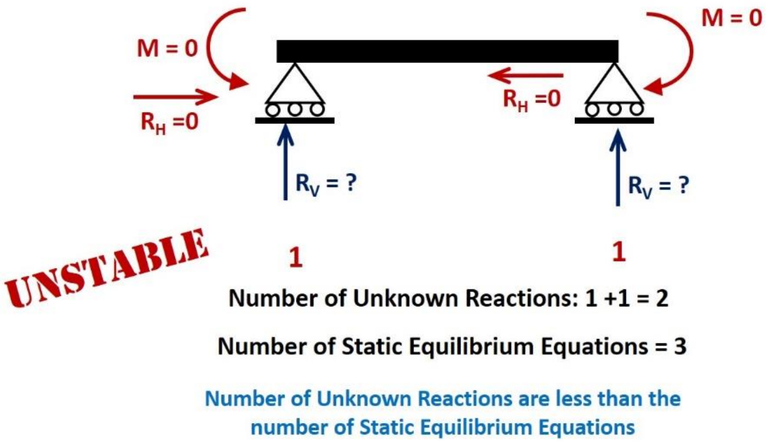


Figure 22: beam with two roller hinges

The simply supported beam with the roller hinges, see Figure 22. So, here it is 2 equations and 3 unknowns. So, if there are 2 roller hinges, then I have 2 unknowns; R_V and R_V; and total equation is 3. In this case I have 2 unknowns, but I have 3 equations available. The number of unknown reactions is less than the number of static equilibrium equations. Therefore, it will give me a very unstable kind of situation. As you all know, if I put a load at the centre, with some inclination, then what is going to happen? This roller will move, so that gives me the unstable condition.

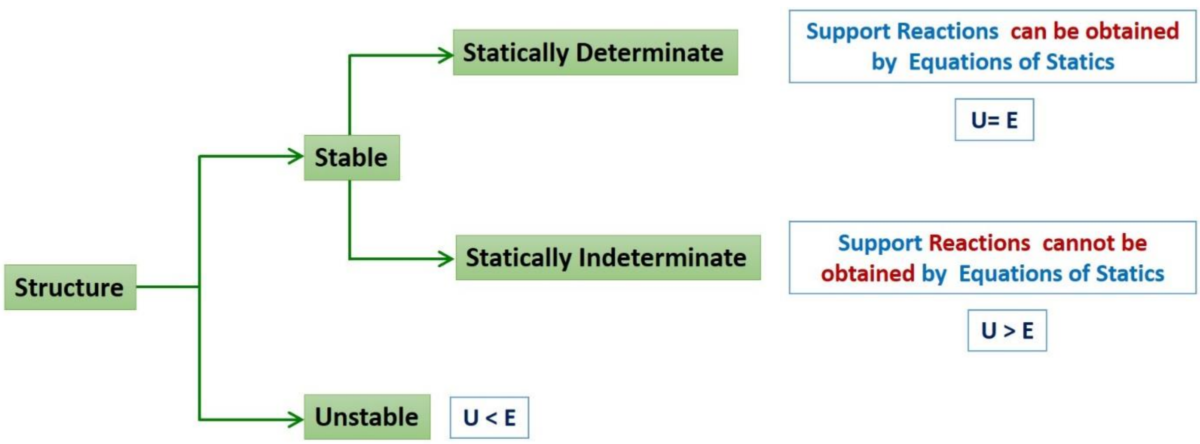


Figure 23: structures and its stability conditions

From the figure above, Figure 23; we can say that the structures can have stable or unstable kind of a nature. It can have statically determinate or statically indeterminate in nature. What is statically determinate or indeterminate, again let us discuss. Statically determinate structures are the one, where the support reactions can be obtained by the equation of statics, the 3 equations of statics. Statically indeterminate structures are those the support relation cannot be obtained by equation of statics. So, there we need some more equations because as you have seen that there are 4 unknowns and there are 3 equations, so you need one more equation. How to get that one more equation? For that you have to do lot of other things also, some of the compatibility conditions and etc.; that I will discuss afterwards, but those structures are statically indeterminate. So, from the unknown reactions and the number of equation point, if I see then, if the number of equations is equal to the number of unknowns, suppose 3 and 3, then it is statically determinate. Suppose the support reactions are 4 and number of equations are 3, so you cannot actually find out what are all the reactions, so it is statically indeterminate. On the other hand, when the number of unknowns are 2, and equations available is 3, then you have more equations you do not have that many of unknowns, so that will give you an unstable case.

Now, I have some figures for you. In Figure 24, at number 1, the structure has 3 hinges, one is roller other two are the hinges, and vertical loads as shown. So total number of unknowns are 3 why 3; because I have taken only the vertical forces, so no role of horizontal comes, so the equation is also 2, and therefore this is a statically indeterminate structure. So next is number 2, a cantilever beam with two loads and two props, one is roller and another is hinge. So, there

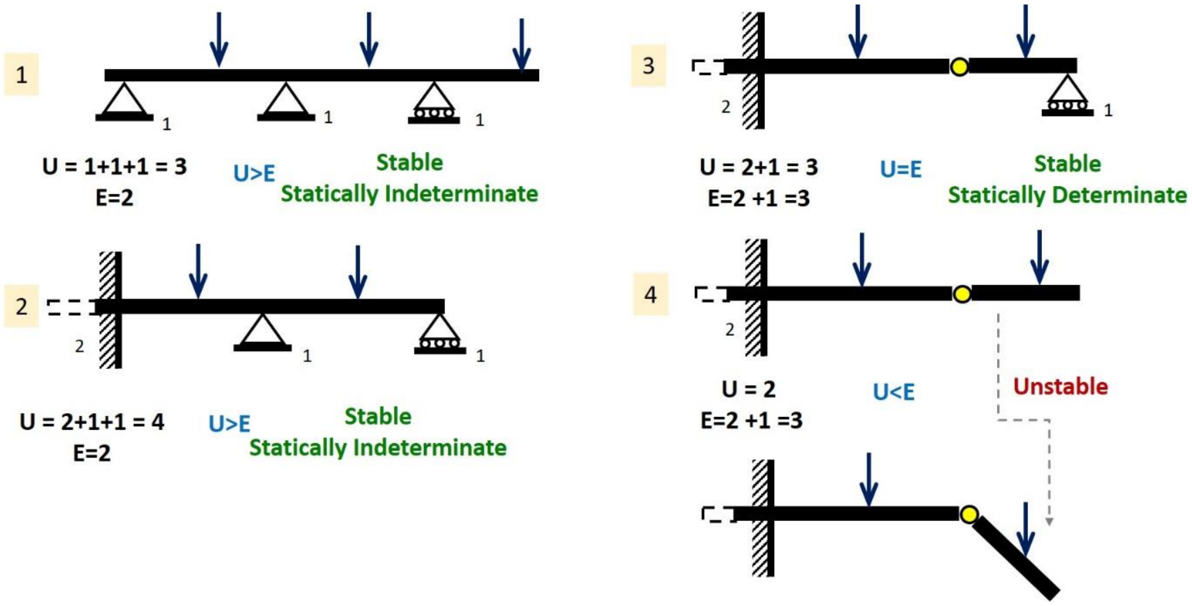
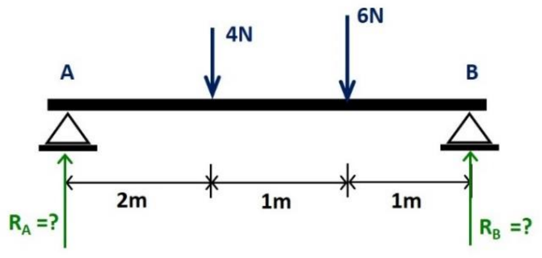


Figure 24: conditions of stability of a structure

are 4 unknowns. Again, the equations are 2 because $\sum F_X$, I am not taking, because I am not considering any inclined force or any force in horizontal direction. It is also an indeterminate beam. Now suppose in case 3, the beam has an internal hinge, shown as the yellow dot. The one end of the beam is fixed and another end has a roller, and beam has 2 loads as shown. Here, this is having 2 unknowns, 2 is at fixed end and 1 is at the roller, so total unknowns are 2+1=3. The equations are also 2+1, where 2 are general equations of static, $\sum F_X=0$; $\sum M=0$; and the other 1 is at hinge, it will be $\sum M$ at hinge=0. This is the additional equation I am getting here; and that is why this is statically determinate because 3 = 3. In the next one, that is 4 th, if it is only with the fixed end and no any other support, then at fixed end the unknown is 2, so the unknowns are 2 and equations are 2, and the equations are 2+1=3. Again, the number of unknowns are less compared to the number of equations, so it is unstable case. As you can see, if you put a kind of a load towards the end, it will become unstable.

So now if I have a beam as shown in Figure 25, and there are loads at different locations, I can find out the R_A and R_B, the reactions at A and B. So, I know the first equation is that the $\sum F_Y = 0$, so $R_A + R_B$ must be equal to 4+6 = 10, this is my equation number 1. I cannot take the equation number 2 as $\sum F_X = 0$, because there is no inclined load, so forget about that. Now I have to take the third equation as sigma moment equal to 0 at A. So, if I take the moment equal to 0 at A, this will not give any kind of a moment because it passes through that point. The distance between this force and the point is 0, but I have R_B, which is having a distance of 4 meters, so this gives me the moment at B, in anti-clockwise direction. It must be equal to sum of moment at A and moment at B. The load of 4 N is having a distance of 2 meter plus the 6 N is having a distance of 3 meter, so this is equation 2. Solving this I can directly find out the R_B = 6.5, and form first equation I can find out that R_A = 3.5.



$\sum F_y = 0$ $\quad R_A + R_B = 4+6 = 10$

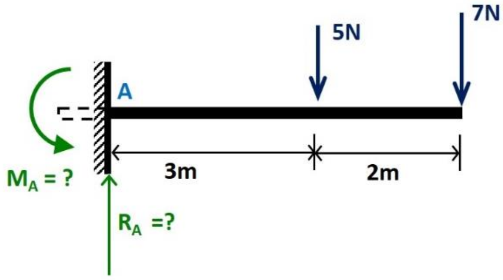
$\sum M_A = 0$ $\quad 4R_B = (4X2) + (6X3) = 26$

$R_B = 6.5N$

$R_A = 3.5N$

Figure 25: reactions of a hinged beam

Similarly, in Figure 26, if I say that this is how much will M_A and R_A, there is no question of horizontal direction. So, $\sum F_Y$ will give me the R_A, where R_A = 12 because R_A must be equal to 5+7.



$$\sum F_y = 0 \quad R_A = 5+7 = 12$$

$$M_A = (5X3)+(7X5) = 50N\text{-}m$$

Figure 26: reactions of a fixed beam

And this will be the upward 12 because otherwise this total beam will fall down. Now, how to find out the M_A? The R_A is passing through this point A, so, there is no question of the moment generated by the R_A. The first moment of M which is 5 x 3, and the second moment due to the 7 is 7 x 5. So those if you add it will be 50 Newton-meter.

The references for this lecture was taken from the folllowing books.

- **Reinforced Concrete Design** by Pillai & Menon, Tata McGraw Hill Publisher
- **Basic Structures for Engineers and Architects** by Philip Garrison, Blackwell Publisher
- **Understanding Structures: *An Introduction to Structural Analysis*** by Meta A. Sozen & T. Ichinose, CRC Press

So, based on these fundamentals I am concluding this particular lecture on the support and its reaction.

In conclusion, I may say that the different type of support provides different character of the reactions as I know. The hinge support gives different character, this fixed support gives different character of the reactions, but the reactions are always 3. It has one reaction upward in direction, one is horizontal direction and another is moment. Some of the support may have moment 0, some of the support may have the horizontal reaction 0 etc.

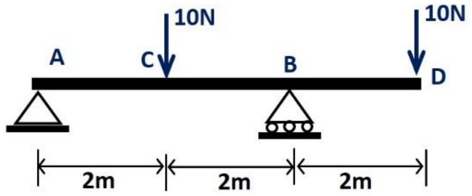
So, based on this the release of the supports, of the reaction of the supports, we can find out what is the degree of freedom and what are the number of unknowns for the particular support.

The structure may fall under statically determinate or statically indeterminate due to the nature of its support system, yes because if there are a lot of support system and there are lot of unknowns are there and you have only 3 equations of statics. So, based on that if the equations or number of unknowns are equal to the number of equations then it is determinate, if it is not if it is a greater number of unknowns are there, then it is indeterminate. If the number of equations is much higher with respect to the unknown, it is an unstable kind of a structure.

So, my next lecture or the last lecture on this particular module, we will discuss on structural loading and in that we will discuss some of the loading principles of the structure.

Now in this particular lecture also I have 2 home works for you. In 1st question, the beam is a statically determinate beam, you can easily find out the reactions at A and reactions and B, both are the upward or the vertical reactions. Here, there are 2 loads of 10 Newton each at point C and point D and what you have to do is, you have to find the reactions at A and B due to those load, if load is only at C and there is no load at D. In second case, if the both C and D is having load, and in third case if the D is only loaded with that.

1. Find the Reactions at A & B due to 10 N force at (i) C only, (ii) C & D both points (iii) D only



2. A beam AB is loaded as per the figure below. Find the Reactions at A & B

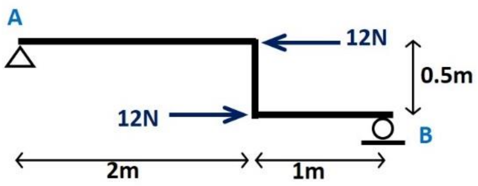


Figure 27: the homework

The next homework is that a beam is not a straight beam, it is a kind of a beam and there is a step like thing having 2 meter, 1 meter is two horizontal portion and 0.5 meter of the vertical portion, is under a two equal and opposite kind of load as 12 Newton, 12 Newton, then what should be the reaction at A and B? Now you see the A will have both reactions that mean it will have the reactions in the horizontal and the vertical direction, this is having both reactions and this is a roller, so it will not have any reaction in the horizontal direction, it will have the reaction in the vertical direction only. So, you have 3 unknowns are over here and you have 3 equations also, the $\sum F_X = 0$, $\sum F_Y = 0$ and $\sum M = 0$.

So, by virtue of the 3 equations, you have to find out the 3 reactions at A and B that is the second home work for you.

That is all for today.