Structural System in Architecture
Prof. Shankha Pratim Bhattacharya
Department of Architecture and Regional Planning
Indian Institute of Technology, Kharagpur

Module No # 02
Lecture No # 06
Theory of Elasticity-1

Welcome to the NPTEL online certification course on Structural System in Architecture. This is the second week of this particular course. And in the second week in the module number 2 we will discuss the strength of material. If you remember in the last week we had discussed about the different type of force and force system. So now we will go to the strength of material part of our syllabus of our course curriculum.

This is the lecture number 6 and we will discuss about the theory of elasticity that is part 1 in this lecture and in the next lecture 7 we will discuss the part 2.

In this particular lecture we are going to cover some concepts which include the properties of engineering materials initially. And we will understand or try to develop some of the concepts of elasticity and the theory of elasticity, Hooke's law and stress-strain diagram in general.

Learning Objectives:

The objective of this particular lecture will be:

- To outline the material properties.
- To discuss the concept of elasticity and it is application in the structures.

Introduction:

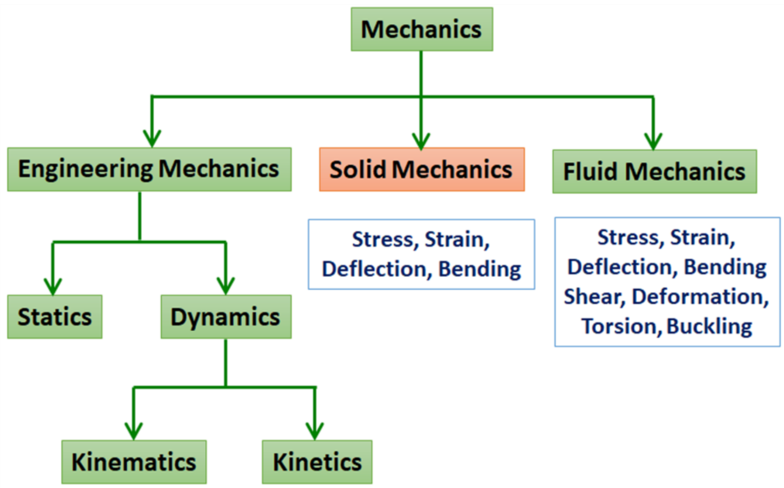


Figure 1 Classification of Mechanics

So if you remember this flow diagram we had introduced in the second lecture where we had mentioned that the solid mechanics is one of the integral part of the mechanics domain. And in the solid mechanics part we will deal with the stress-strain deflection, bending etc., which will come into play in any kind of structure when it is under some external loading. As a building is a composition of different structural elements, therefore we need to understand that due to various types of loading, the particular element or the group of elements will be under some kind of deformation. And as it is deformed there will be some kind of stress-strain and bending and so forth. From that point of view we discussed the static part initially in the first week so in the second and third week we will thoroughly deal with the solid mechanism part.

Besides, in this part we will have some mathematical formulae which are very simple, for small mathematical computations, just to know about the basic principles of the solid mechanics or strength of materials.

Properties of Engineering Materials:

So first of all, let us discuss about the engineering properties of the material. And those are elasticity, which is one of the most important properties, next, plasticity, ductility, brittleness, malleability, toughness and also the hardness. So each material has to actually undergo various types of test and properties had to be determined. And depending upon the quality or the character of these various parameters we can use the particular material in the different field or different types of applications.

Elasticity:

Now, what is the elasticity? The elasticity as you understand when a body is under some external loading, it is going to deform because its molecular structure will undergo some kind of transformation or in other words, the molecules readjust themselves and offer some kind of resistivity against the deformation or against that particular external loading. And by registering that particular external loading, if the body moves towards the molecular distribution, it redistributes itself in a deformed shape.

So all in all, a particular object or maybe a particular body will either shorten or elongate or it may get twisted angularly when it is subjected to external load. But, if you remove the load the deformed body will try to come to the original position. However, sometimes it regains its original shape fully and sometimes partially. This phenomena of regaining its original shape by the deformed body once the loading is withdrawn from the body, is called elasticity.

Next, we have to measure the elasticity in various ways to evaluate how much it has regained the original shape and how much is it going to deform under how much load so that we can quantify its elasticity.

Elasticity of a material is one of its most important properties. Every object/ material in the world has some elasticity. For example, if you consider the human bone, it has some elasticity. The steel, copper, aluminum or other metals also definitely possess a fair amount of elastic property. The skin and the human hair, everything has some elasticity. Also, if you take some stone, it also has some amount of elasticity in it.

Nevertheless, different materials maybe either classified as high elastic material or low elastic material depending on the magnitude of elasticity exhibited by them under various loading conditions.

Plasticity:

Now the next property is plasticity which is just the reverse concept of elasticity. Hence plasticity is the property of material by which when it is subjected to external load, the body will go for a permanent deformation naturally which is a non-reversible kind of change. Most of the materials become plastic under the action of a large force and behave in a manner similar to a viscous fluid. Even if a load of very small magnitude is exerted over a plastic substance, it can cause non-reversible changes and that is called plasticity.

Then the next property is ductility which is another property of material and is very much in tune with the property of elasticity. It states that under external load (tensile) a material can be drawn out in small wires or small sections.

For example, a chewing gum or a rubber kind of material, if subjected under tension such as stretching, will become thinner. This property is called ductility. Metals like steel, aluminum etc. are very highly ductile.

Brittleness:

The next property is brittleness which is the lack of ductility. So when a particular material cannot be drawn into a smaller section by subjecting it to a tensile load then it is called brittleness. For instance if you take a chalk piece and subject it to a tensile load it will not cause any kind of elongation with a thin sectional increment and it is eventually going to break. This happens due to the property of brittleness.

Malleability:

Then the next property is malleability which is quite similar to the property of ductility. However, unlike the latter where a material is drawn into smaller sections by virtue of tensile force, in case of malleability it is defined as the property by which the body is uniformly extended under the compressive loading.

So suppose you take a small piece of a malleable material and you hammer it and subject it to some compressive load or maybe you roll it with another hard material then that material will become a very thin flat plate. This phenomenon is called the malleability of the material. Metals like gold, iron, copper and silver have wonderful malleable property, and that is why from gold and silver you can create lots of wonderful ornaments with various embossing and engravings etc.

Toughness:

The next property is toughness. It is the property of a material which enables it to absorb energy without fracture. So even if you hammer a tough material it will just absorb the load and won't break or deform. Hence this is another very important property of a material. It is especially beneficial while designing bridge like structures where there is fatigue loading acting upon it.

Finally, the last property is hardness which is the ability of a material to resist indentation or the deep recess. Suppose if you exert pressure on a hard material then after withdrawing the force the material would not elongate or form any kind of deep recess or notches as in indentation on its surface. For example, diamond is one of the hardest materials found on earth.

Now additionally, we have some other 2 properties along with the above mentioned ones. Those are homogenous materials and isotropic materials.

Homogenous:

The material can be said homogenous when it has a homogenous composition throughout. So if you take a particular material which is having 3 different dimensions of X, Y and Z. But if you check the composition from each corner of each of the dimensions and find out that it is similar then you can say that the material is homogenous.

Isotropic:

Lastly, the isotropic materials are the materials that have uniform elasticity in all the 3 directions.

Sometime a material may have different magnitude of elasticity in different directions. For example, let us consider a bamboo which is actually made of some kind of grains. So in the direction along that of the grains we will encounter some elasticity value due to which either it would compress, or it would shrink and or maybe elongate and come back to its original position in a different way. And if you just put the load in the other direction, i.e. across that of the grains, it may show some different phenomena. This shows that the elasticity is not equal in the mutually perpendicular direction of fibers or grains. So then we cannot say that the bamboo is isotropic. In order to be isotropic it has to have uniform elasticity in all the directions.

Effect of Theory of Elasticity on the following cases:

CASE-1: When a spring and a metal bar are suspended

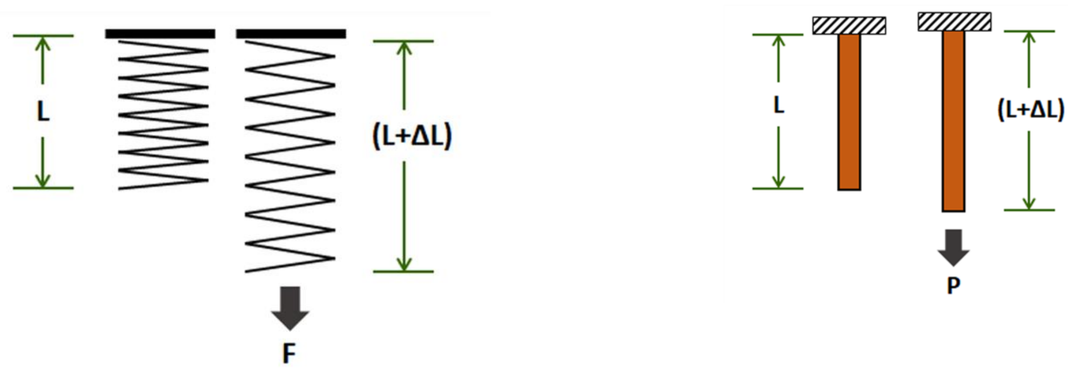


Figure 2 Theory of elasticity when a spring and a metal bar are suspended

Now let us go to the next concept, i.e. the theory of elasticity. Here, this first figure is very common to us; when a spring of a length L is hanged and subjected to a force P or F downward, then this spring will elongate. So here I have shown that the elongation is ΔL because the original length was L and Δ L was the elongation. So the final length of the spring is $L + \Delta L$.

Similar phenomena will happen if there is a bar. If there is a solid bar which is subjected to a particular force then even though it is made up of steel or any kind of material it is going to elongate because of the property of elasticity. As I have told you earlier, that when an elastic material is subjected to the external force there are some molecular distribution changes which takes place within the material in an attempt to resist the force and eventually elongates.

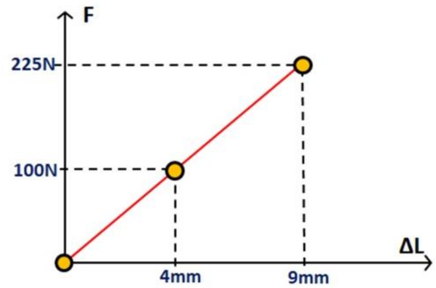


Figure 3 Graph showing theory of elasticity when a spring and a metal bar are suspended

Nevertheless, if you remove the force the bar will come back to its original position L and that is why the material is called as an elastic material. Now as I understand from the very common behavior of any particular spring or a particular body, if you increase the force it will increase the elongation. Hence, based on this observation, I have drawn a graph where let us suppose that given a 100 N load the elongation is 4 mm and if I exert 225 N of load I might get a 9 mm elongation. So more the tensile forces more will be the elongation.

CASE-2: When 2 bars of same length but different cross-sections are suspended

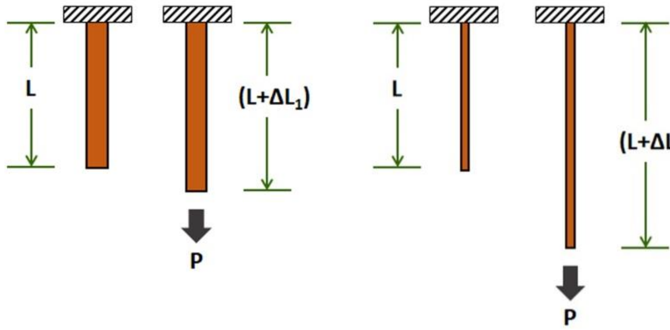


Figure 4 Theory of elasticity when 2 bars of same length but different cross-sections are suspended

Similarly, let us take 2 bars having same length L but different cross-sectional areas 'a' and 'A' respectively. Now if I put the load on the fat bar where the cross-sectional area is higher, it elongates by a length, say ΔL_1 whereas the thin one which has 'a' as the cross-sectional area elongates by ΔL_2. Here, the thin bar will exhibit lesser resistivity as compared to that of the thick bar. So the ΔL_2 will be higher than the ΔL_1. Hence, a fat or a thick bar will elongate less whereas the thin bar will elongate more if both the original length is similar.

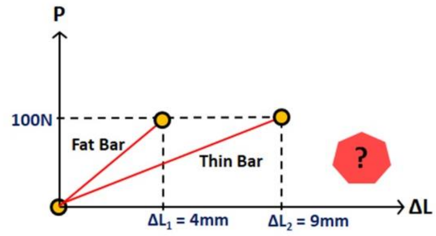


Figure 5 Graph showing theory of elasticity when 2 bars of same length but different cross-sections are suspended

Now let us assume that the load P is same and is 100 KN or 100 N. Then the thicker bar will give me an elongation of suppose, 4mm and the thinner one, say 9 mm respectively. So now we can say that not the load alone but the cross-sections also play a role vital role in determining how much it will elongate.

CASE-3: When 2 bars of same length but different cross-sections are suspended (computation with load intensity)

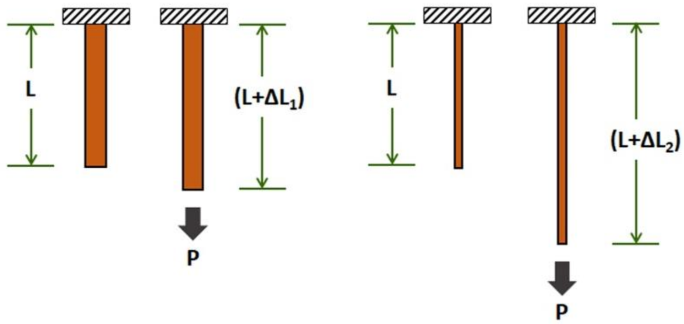


Figure 6 Theory of elasticity when 2 bars of same length but different cross-sections are suspended (computation with load intensity)

So next what I have decided that I will take a ratio of the load and cross-section. So the intensity of the load of the thicker bar is P/A whereas, that of the thinner bar is P/a. Now, since P/a is higher than P/A, intensity of load is higher here in the thinner bar and hence elongation is also high.

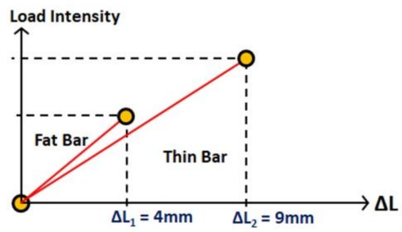


Figure 7 Graph showing theory of elasticity when 2 bars of same length but different cross-sections are suspended (computation with load intensity)

So we can conclude that load intensity is also equally important in determining the elongation of an elastic substance. Hence, not just the load or length but the cross-section of the material on which the load is acting upon is also very essential in the computation of the elongation.

CASE-4: When 2 bars of different lengths but equal cross-section are suspended

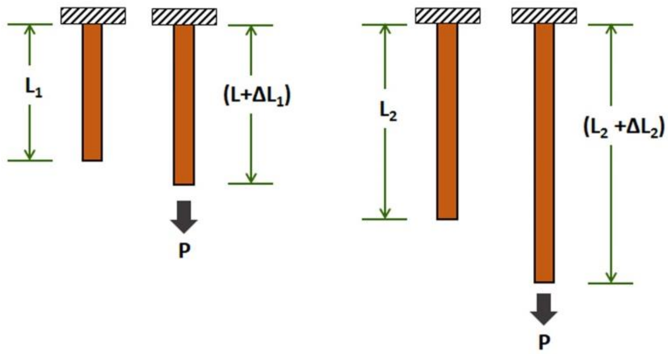


Figure 8 Theory of elasticity when 2 bars of different lengths but equal cross-section are suspended

Now let us assume another criterion where I have 2 bars of same cross-section but different lengths L_1 and L_2 respectively. L_1 is smaller than L_2. Here, also see that when the length is higher the elongation is also higher. In other words, ΔL_2 is greater than ΔL_1 as L_2 is greater than L_1.

Hence we may conclude that the absolute value of the elongation is not a prime factor rather, the ratio of proportionate of elongation and the original length is a more dependable factor in determining the elasticity of a material.

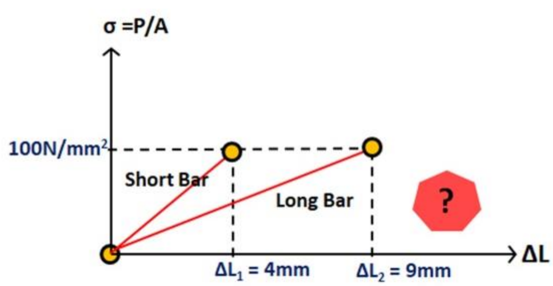


Figure 9 Graph showing theory of elasticity when 2 bars of different lengths but equal cross-section are suspended

Stress:

Instead of the absolute force if we take the intensity of the force i.e., the ratio of force and area, we get the stress. Hence, stress is nothing but the force by the perpendicular area.

Mathematically,

$$\text{Stress } (\sigma) = \frac{force}{area} = \frac{P}{A}$$

Its unit is N/m^2 or kN/m^2.

Strain:

The ratio of the change in length and the original length is known as strain.

Mathematically,

$$\text{Strain } (\varepsilon) = \frac{change\ in\ length}{original\ length} = \frac{\Delta L}{L}$$

Therefore, both stress and strain are very important in determining the deformation of the elastic body.

Hooke's Law:

The scientist Hooke had given a law some 250 years back but is very much applicable till date. The law states that within the elastic limit, the stress is proportional to strain.

Hence, if you increase the amount of stress proportionally the strain is also going to increase to some extent of some limit. This limit is called the proportionated limit or maybe the elastic limit.

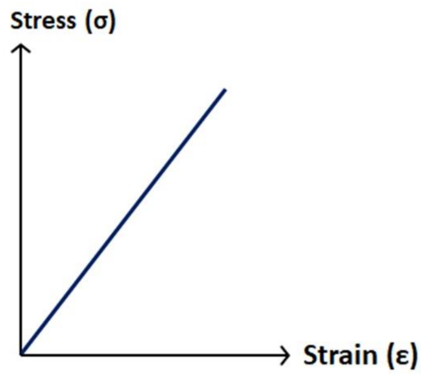


Figure 10 Hooke's Law

Mathematically,

Stress α Strain

i.e., $\sigma \propto \varepsilon$

i.e., $\sigma = E\varepsilon$

where, E is a constant called Modulus of Elasticity or the Young's Modulus of Elasticity. This is a prime property of the material which signifies its elastic property.

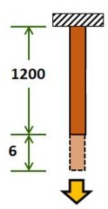
Therefore,

$$\frac{Stress}{Strain} = \frac{\sigma}{\varepsilon} = E = \text{Modulus of Elasticity}$$

So now, as you know from the earlier slide that stress is P/A (force by area) and the strain is ΔL/L, we can rewrite the above equation of elasticity as given below:

$$\frac{P}{A} = \frac{\Delta L}{L}$$

Numerical-1:



Let us solve a small problem. Suppose I have given a bar of original length, L = 1.2 m. The area of the cross-section of the bar is also given as 100 mm^2. The elongation is 6 mm, and external force, P is 75 KN. We have to find out the magnitude of elasticity of the material of the bar.

Solution: So, we have

Original length, L = 1.2 m = 1200 mm

Area of cross-section, A = 100 mm^2

Elongation, ΔL = 6 mm

Force, P = 75 KN

Then,

Step-1: Find the Stress.

$$\text{Stress } (\sigma) = \frac{P}{A} = \frac{75 \times 10^3}{100} = 750 \text{ N/mm}^2$$

Step-2: Find Strain.

$$\text{Strain } (\varepsilon) = \frac{\Delta L}{L} = \frac{6}{1200} = 0.005$$

Step-3: Calculate Young's Modulus of Elasticity.

$$E = \frac{\sigma}{\varepsilon} = \frac{750}{0.005} = 1.5 \times 10^5 \text{ N/mm}^2$$

Therefore, the value of the Young's Modulus of Elasticity of the material of the bar is 1.5 x 10^5 N/mm^2.

Graphical Representation Of Elasticity

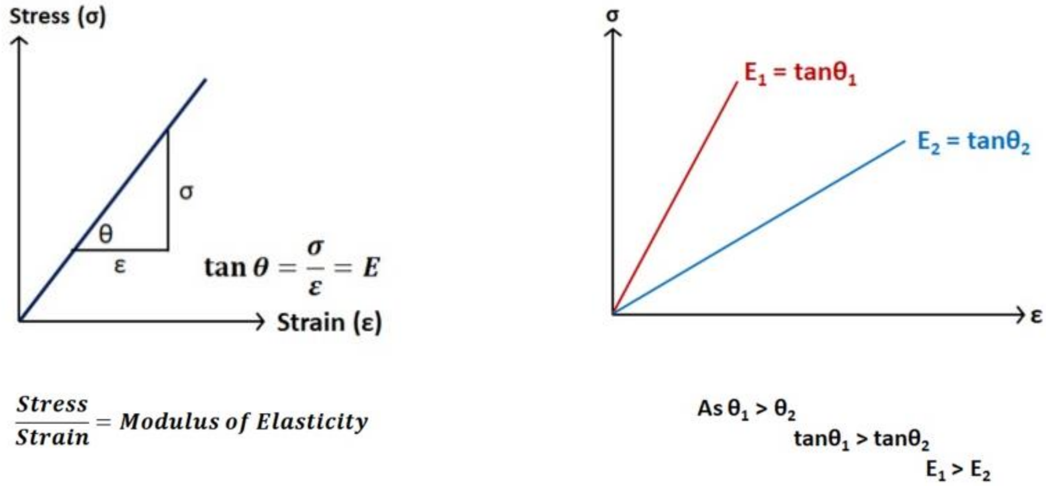


Figure 11 Graphical representation of Elasticity

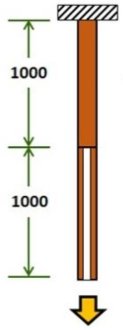
Since elasticity is computed as the stress by strain, so the latter is represented in the X-axis in the graph and the former is represented in the Y-axis. Now I can say that the slope of the graph is going to give you the elasticity.

Mathematically,

$$\text{Modulus of Elasticity (E)} = \frac{Stress}{Strain} = \frac{\sigma}{\varepsilon} = \tan\theta$$

Because the tan θ is nothing but the height divided by the base; so if you have the different material. So by either knowing the value of θ or that of both σ and ε of a particular material, you can find the Modulus of Elasticity of the corresponding material.

Numerical-2:



Now I have another problem for you. Here I have shown that this is a 2 m long bar in which the lower 1 m is hallow. In the hollow part, the outer diameter is 20 mm and inner diameter is 10 mm. In the upper solid portion the diameter is 20 mm. External force, P is 120 KN. Young's modulus is 2 x 10^5 N/mm^2 which is equal to the value of steel. So you have to find out the extension of the bar. [Hint: Find out the elongation separately for each portion as change in area results in change in stress.]

Solution: Given,

Total length of the bar, $L = L_s + L_h = 1\text{ m} + 1\text{ m} = 2\text{ m}$

In the solid part of the bar,

Diameter, $d_s = 20$ mm

In the hollow portion of the bar,

Outer diameter, $d_1 = 20$ mm

Inner diameter, $d_2 = 10$ mm

External force, P = 120 KN

Young's Modulus of Elasticity, E = 2 x 10^5 N/mm^2

Now,

Step-1: In the solid portion of the bar

Step-i: Calculate the cross-sectional area

Cross-sectional area of the solid portion, $A_s = \pi \times \frac{d_s^2}{4} = 314.28$ mm^2

Step-ii: Find the Stress

Stress in the solid portion, $\sigma_s = \frac{P}{A_s} = \frac{120 \times 10^3}{314.28} = 381.97$ N/mm^2

Step-iii: Compute the elongation

Let elongation in the solid portion = ΔL_s

We know that,

Modulus of Elasticity, $E = \frac{stress}{strain} = \frac{\sigma_s}{\varepsilon_s}$

i.e., $E = \frac{\sigma_s}{\frac{\Delta L_s}{L_s}}$

i.e., E = $\frac{\sigma_S \times L_S}{\Delta L_S}$

i.e., ΔL_s = $\frac{\sigma_S \times L_S}{E}$ = $\frac{381.97 \times 10^3}{2 \times 10^5}$ = 1.909 mm

Step-2: In the hollow portion of the bar

Step-a: Find the cross-sectional area

Cross-sectional area of the hollow portion, $A_h = a_1 - a_2 = \frac{\pi}{4}[d_1^2 - d_2^2]$ = 235.71 mm^2

Step-b: Compute the Stress

Stress in the hollow portion, $\sigma_h = \frac{P}{A_h} = \frac{120 \times 10^3}{235.71}$ = 509.29 N/mm^2

Step-c: Calculate the elongation

Let elongation in the solid portion = ΔL_h

We know that,

Modulus of Elasticity, E = $\frac{stress}{strain} = \frac{\sigma_h}{\varepsilon_h}$

Therefore, $\Delta L_h = \frac{\sigma_h \times L_h}{E} = \frac{509.29 \times 10^3}{2 \times 10^5}$ = 2.546 mm

Step-3: Find the total elongation

Total elongation, $\Delta L = \Delta L_s + \Delta L_h$ = 1.909 mm + 2.546 mm = 4.455 mm

Therefore, the total elongation of the bar is 4.455 mm.

The Universal Testing Machine

The Universal Testing Machine is used to find the value of the Young's Modulus of Elasticity for various elastic materials.

Figure 12 shows a universal testing machine.

Figure 12 Universal Testing Machine

Stress-Strain Diagram

Following are some salient features of a typical stress-strain diagram, explained through that of steel.

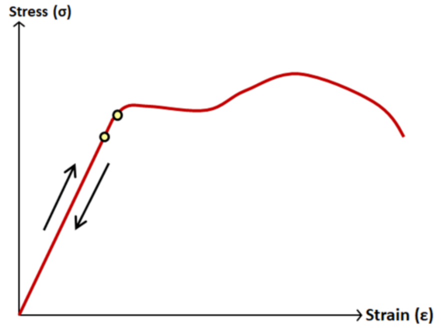


Figure 13 Stress-Strain Diagram showing Limit of Proportionality & Elastic Limit

Limit of Proportionality:

It is the stress at which a stress-strain diagram ceases to be a straight line. The increase in stress is directly proportional to strain. Hooke's law is valid within this zone of stress-strain combination.

Elastic Limit:

Elastic limit represents the maximum stress in a tensile test sample that causes no permanent or residual Deformation after the removal of the load.

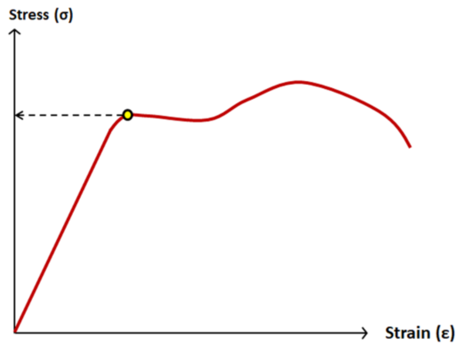


Figure 14 Stress-Strain Diagram showing Yield

Yield Point

Yield point is the point at which there is an appreciable elongation or yielding (flow) of material without any corresponding increase of stress (or loading). Ductile materials possess a definite yield point.

Yield Strength

It is the lowest stress at which the extension or yielding of the test specimen increases without increasing the load. It is also

Point & Yield Strength

called Yield Stress (σ_y). This yield stress is an important parameter to design any structure.

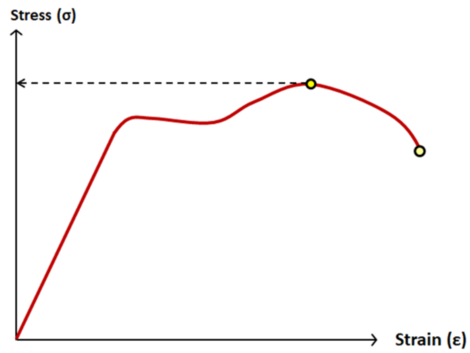


Figure 15 Stress-Strain Diagram showing Ultimate Strength & Rupture Strength

Ultimate Strength

Ultimate strength or ultimate stress corresponds to the highest point of the stress –strain curve.

Rupture Strength

Rupture strength is the stress corresponds to the failure of test sample under the tensile loading.

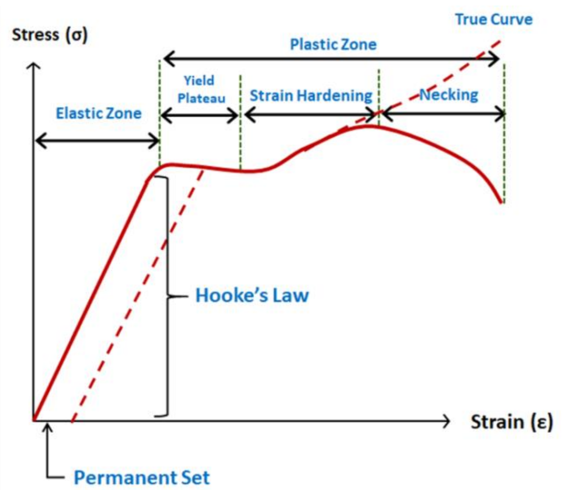


Figure 16 Stress-Strain Diagram showing different zones of material properties.

In the above figure you can clearly see that in the stress-strain diagram the portion till the Limit of Proportionality is the *Elastic Zone*. After the elastic zone we see the *Yield Plateau* because the sudden elongation or the strain takes place without increase of the load or the stress. And after that it again can take up some stress if you consider the redistribution of the molecular structure in the particular body or the particular object and this zone is called as a *Strain Hardening* portion going to harden again.

However, during this process if you stretch the material or if you increase the load a large amount of strain localize disproportionately over the particular part of the material. This results in a decrease in the cross-sectional area of the concerned portion. This eventually leads to the formation of a neck and the corresponding position in the curve is known as the zone of *Necking*. The stress strain curve above clearly shows that necking starts after the ultimate strength of the material is reached and sometimes after that point the material may rupture. This point is known as the *Ultimate Stress Point* or the *Ultimate Tensile Strength*. Therefore, the zones corresponding to the yield plateau, strain hardening and necking combined together, is called the *Plastic Zone*.

Furthermore, you might have noticed that the material ruptures at a point having lower stress rather than at the Ultimate Stress Point. Why does it happen so?

This happens because the cross-sectional area reduces locally due to necking. While the stress on the neck may continue to go up; the reduction in area meaning the load can decrease. If we keep track of the reduction in sample area, we can divide the force by the *actual* area (rather than the original area), and the displacement by the *actual* length. This leads to quantities called *True Stresses and Strains*. Indeed, the true stress-strain graph doesn't show any negative gradient.

So stress is increase enormously but what we can do is we can actually find out the area of cross section before the test. We do not have any kind of testing system that will every time we can find out the cross section in the duration of the particular this thing particular test. So if you can actually find out the neck areas so actual the true curve will be something like this and this will be the point of rapture and that will be the actual highest point of that.

But unfortunately, we cannot measure the necking and we cannot recalculate the stress as we do not possess any such instrument which would be able to track the change in the cross-section during the experiment.

Besides, Hooke's law is applicable in this phenomenon till the yield point as you know. And if you put the load till this point and then if you remove the load the curve will come back but it will not come back to the original strain or the original position. There will be some kind of permanent deformation in the material.

References:

- **Engineering Mechanics** by Timishenko and Young McGraw-Hill Publication
- **Strength of Materials** By B.C. Punmia, Ashok K.Jain & Arun K.Jain Laxmi Publication
- **Basic Structures for Engineers and Architects** By Philip Garrison, Blackwell Publisher
- **Understanding Structures:** ***An Introduction to Structural Analysis*** By Meta A. Sozen & T. Ichinose, CRC Press

Conclusion:

- Elasticity is a very important property of the material.
- During an application of external load a material initially undergoes the principle of elasticity.
- Further, it undergoes large or plastic deformation.

Home Work:

Q1. A solid bar (*diameter 'D'*) is replaced by a hollow bar (*outer diameter 'D' and inner diameter '0.5D'*). If all other geometric, material and loading parameters remain unchanged, then find the percentage increase in elongation in the hollow bar w.r.t. the solid bar.

Q2. A composite member is formed by connecting steel and aluminium bars with following specifications. Calculate the magnitude of the axial compressive load should be applied on steel bar so that total length of the composite member decreases by 0.3 mm. Given: $E_S = 2X10^5$ N/mm^2 and $E_A = 0.7X10^5$ N/mm^2.