Structural System in Architecture
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Module No # 02
Lecture No # 09
Bending Moment Diagram

Welcome to the NPTEL online certification course on structural systems in architecture in the module number 2 i.e., the week 2. Strength of Materials is our chapter and today in the lecture no 9 we are going to discuss the bending moment diagram.

The concepts covered in this lecture are given below:

- Bending Moment
- Bending Moment Diagram
- BMD for Simply Supported Beams
- BMD for Cantilever Beams

Besides, the learning objectives for the lecture are as follows:

- Overviewing the theory of Bending Moment.
- Understanding the concept of Bending Moment Diagram.
- Illustrating the Bending Moment Diagram.

Bending Moment

If you remember in the last lecture we had briefly discussed about the bending moment in the introductory portion. There you had seen that because of external loading some changes occur in a particular body or a particular element of a structural component. This change is based on the orientation of the load with respect to the object and how is it applied on the concerned object. So out of the four kinds of forces, as discussed in the last lecture, one is compression, one is tensile force which results in elongation, the shear force which results in layer deformation and finally the bending moment which results in the bending of a particular component of a structure.

In this lecture we'll predominantly study the concepts related to the bending of a structure. As we know that external forces in any structural element produce support reactions. A stable structural member tends to bend under the external force and support reaction combination. This moment created by the external force causes bending of member, in general called the *Bending Moment.*

The combination of the external loads and the support systems together will give you the tendency of the bending and the magnitude of bending.

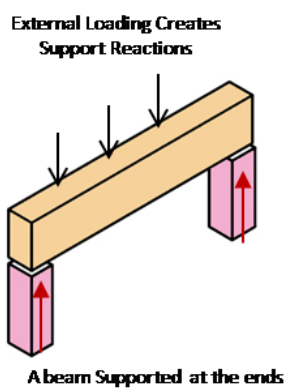


Figure 1 A simply supported beam subjected to external loading

Here in the Figure 1 we have a beam supported on pillars at both the ends. Also, the beam can be seen to be subjected by external loading from above (represented by black downward arrows). Additionally, we can see two red colored upward facing arrows at the supports. These are reaction forces offered by the supports in response to the external loading. As you must be aware of the third law of Sir Isaac Newton which states that "every action has an equal and opposite reaction".

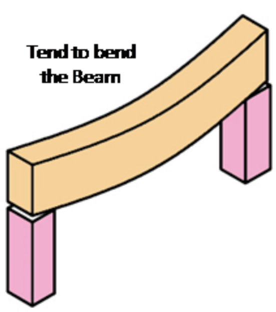


Figure 2 Bending in the beam

Consequently, we know that the structure is stable as the external loading is balanced by the reaction forces. However, though this stable structure will not collapse under the external loading, yet it will undergo some deformation in the form of bending in the beam.

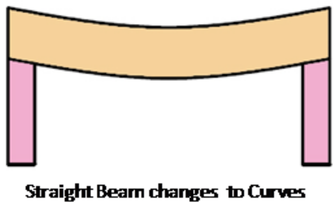


Figure 3 Bending in elevation

Due to the bending the straight beam will take a curvilinear shape as shown in the Figures 2 and 3. This bending creates a sagging moment in the beam. Though this moment will not result in any king of turning or rotation, but it will create a zone of compression at the top of the beam and tension at its bottom.

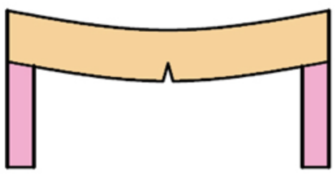


Figure 4 Crack in the beam

Due to the tensile force created at the bottom of the beam it tries to stretch this portion eventually creating a crack there.

Sagging Moment

As discussed earlier, the bending in a simply supported beam creates a sagging moment in it due to which there is a formation of compression zone at the top of the beam and tension at its bottom as shown in the Figure 5.

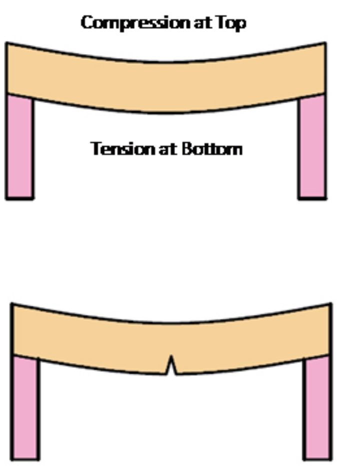


Figure 5 Sagging moment created due to bending in a simply supported beam

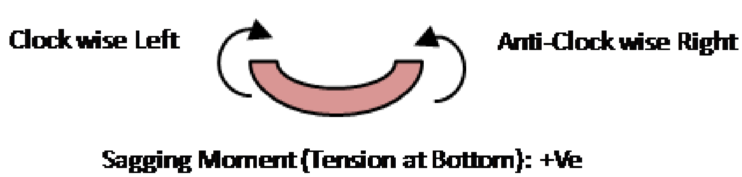


Figure 6 Sign convention of sagging moment

It is very important to understand the sign convention of the bending moment.

A downward bending is known as the sagging moment which is always positive.

Nevertheless, it is also important to recognize the sagging moment because often times the bending is so minute that it is seldom visible through the naked eye. In such cases consider a section of the member which goes from one end to the other end as shown in the Figure 6. Then if there's a clockwise moment on the left hand side and an anti-clockwise moment on the right hand side, the overall moment in the beam is sagging.

Here it is important to note that if one side of the beam has clockwise moment then the other side will have an anti-clockwise moment so as to balance the previous moment and make the structure stable.

Hogging Moment

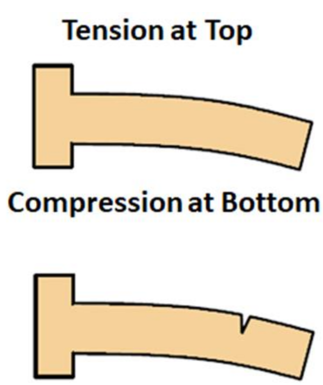


Figure 7 Hogging Moment

Hogging moment is the opposite of sagging moment where the bending occurs towards the upward direction. In case of cantilever beams where one end is fixed, if you place a load above it then the type of moment you see is a hogging moment.

Contrary to the sagging moment, here the tension zone is created at the top of the beam whereas the compression zone is created at the bottom as shown in the Figure 7. Hence the crack is formed at the top.

Figure 8 Sign convention of hogging moment

Here also it is very important to note the sign convention which is always negative for a hogging moment.

A hogging moment can be recognized if a structural member faces clockwise moment in the right side and anti-clockwise moment in the left side.

Bending Moment Diagram

Now let us understand the bending moment diagram for which you need to first understand how the bending takes place. For this let us do a small experiment (shown in the video).

Let us take a 12 inch scale and place it over two supports, one in the position of 1 inch and the other at the position of 11 inch. So, we have a span of 10 inch between the supports. Then you exert some pressure on the scale. You'll be able to see that the scale is bending downwards.

Now, move the supports closer, i.e., at the positions of 4 inch and 9 inch respectively. Now if you exert the same amount of pressure over the scale you won't see any bending. However, if you increase the pressure you'll now be able to see the scale bending. Hence it is important to note here that earlier when the span was bigger a small amount of load could cause the bending in the scale whereas, when the span was reduced considerably, a load almost double the previous one was required to create the bending of similar magnitude.

Therefore, it is rightly said that bending in a particular structural member is dependent on the geometry of the loading, magnitude of the load, position of supports etc. Furthermore, let us see how to draw the bending moment diagram for a simply supported beam subjected to point loads.

The Figure 9 shows a simply supported beam of span 5m supported at the ends A and B. the beam is subjected to two point loads of 15 units and 5 units at a distance of 1m and 4 m from the support A respectively.

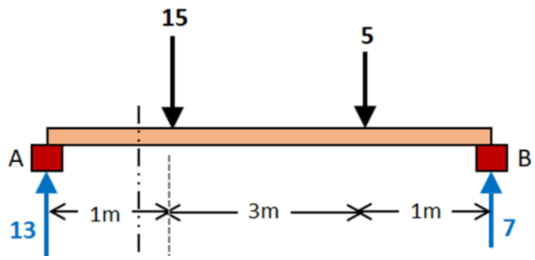


Figure 9 Simply supported beam subjected to two point loads

So, given

$P_1 = 15$ units

$P_2 = 5$ units

L = 1m + 3m + 1m = 5 m

Then,

Step-1: Find the support reactions

We know that,

$R_A + R_B$ = total downward load = 15 + 5 = 20 units

So taking moment about A,

$5R_B = 5X4 + 15X1 = 35$

i.e., R_B = 7KN

i.e., R_A = (20-7) = 13KN

Step-2: BM at X

Let X is any point on the beam at x units from A.

Thus,

BM at X = load on the left side of X × distance = $R_A \times x = 13x$

Step-3: BM at X when x = 0

BM at X = $R_A \times x = 13 \times 0 = 0$ = BM at A

Step-4: BM at X when x = 1m

BM at X = $(R_A \times x) - (15 \times 0) = (13 \times 1) - 0 = 13$ units

Step-5: BM at X when x = 4m

BM at X = $(R_A x) - (15 \times 3) - (5 \times 0) = (13 \times 4) - 45 - 0 = 7$ units

Step-6: BM at X when x = 5m

BM at X = $(13 \times 5) - (15 \times 4) - (5 \times 1) = 0$

Now, all the above values of bending moment can be plotted in a graph to obtain the BMD as shown in the Figure 10.

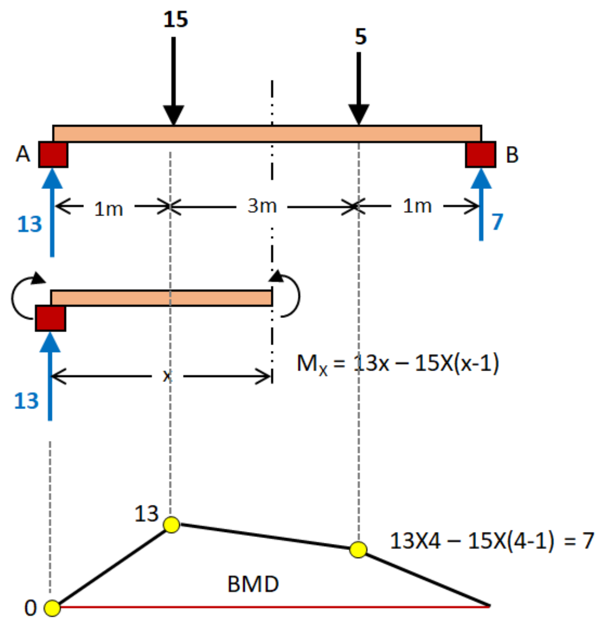


Figure 10 BMD of a simply supported beam subjected to point loads

BMD of Beams Under the Following Cases

Case-1: Simply Supported Beam Subjected to Point Load

Here in the Figure 11 we have a simply supported beam of span L, supported at its ends A and B. Further, this beam is subjected to a point load P concentrated at its geometric center C.

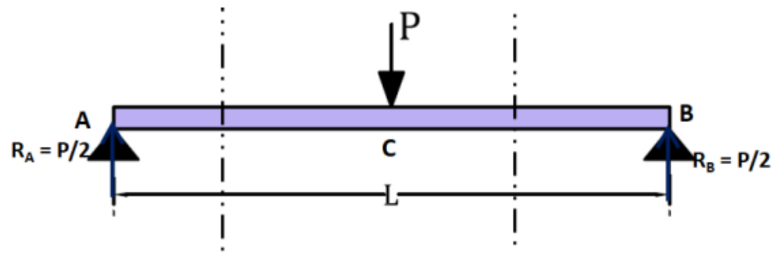


Figure 11 Simply supported beam subjected to a point load at the center

We have,

Load = P

Span = L

<u>Step-1: Find the reactions</u>

As we know that,

Sum of positive forces = sum of negative forces

i.e., sum of the reactions at both ends = P

i.e., $\frac{P}{2} + \frac{P}{2}$ = P [Since, the load P divides the beam symmetrically]

Hence,

Reaction at each end of the given simply supported beam subjected to a point load at its center is P/2.

<u>Step-2: Compute the BM</u>

Here,

Let X be a point anywhere on the beam at a distance x from A.

Then,

BM at X (when x = 0) = Load on the left of X × distance = $R_A \times 0 = 0$

BM at X (when x = L/2) = $R_A \times \frac{L}{2} = \frac{P}{2} \times \frac{L}{2} = \frac{PL}{4}$

BM at X (when x = L) = $(R_A \times L) - (P \times \frac{L}{2}) = 0$

<u>Step-3: Draw the BMD</u>

Now all the above values of bending moment can be plotted in a graph to get the BMD.

Figure 12 shows the BMD of the given beam.

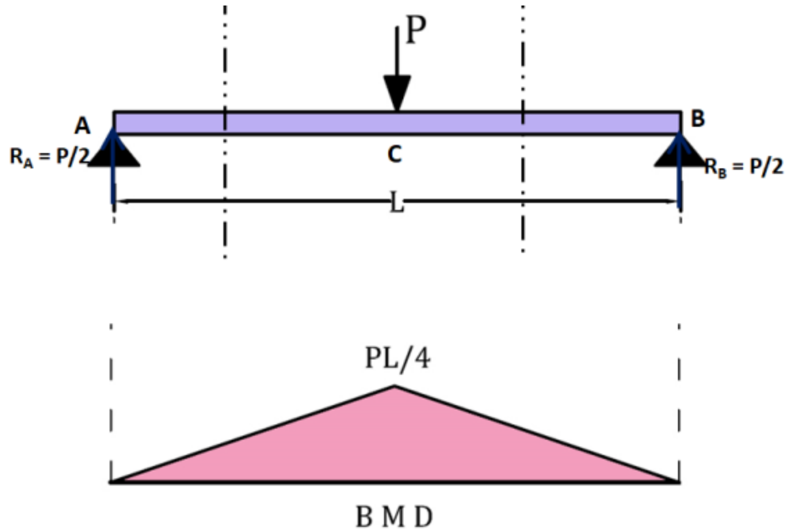


Figure 12 BMD of a simply supported beam subjected to a point load at the center.

Case-2: Simply Supported Beam Subjected to Two Point Loads Placed at Equal Intervals

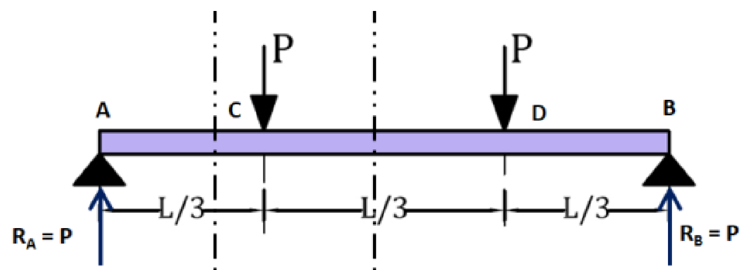


Figure 13 Simply supported beam subjected to two point loads placed at equal intervals

Here we have another simply supported beam of length L supported at the ends A and B. Two point loads of P units are placed at C and D dividing the beam equally by length L/3.

So we have,

$P_1 = P_2 = P$

Span = L

Step-1: Find the reactions

Clearly,

$$R_A = R_B = P$$

Step-2: Calculate the BM

BM at A = $R_A \times 0 = 0$

BM at C = $(R_A \times \frac{L}{3}) - (P \times 0) = \frac{PL}{3}$

BM at D = $(R_A \times \frac{2L}{3}) - (P \times \frac{L}{3}) - (P \times 0) = \frac{PL}{3}$

BM at B = $(R_A \times L) - (P \times \frac{2L}{3}) - (P \times \frac{L}{3}) = 0$

Step-3: Draw the BMD

Finally, all the above values can be plotted in a graph to achieve the resultant BMD as shown in the Figure 14.

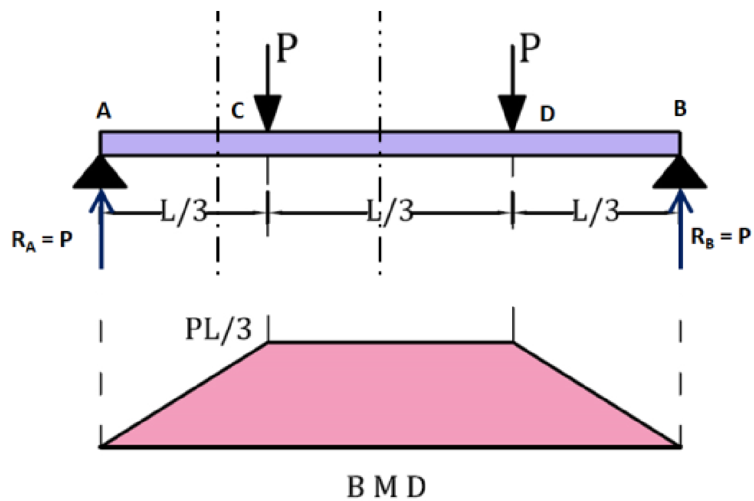


Figure 14 BMD of a simply supported beam subjected to two point loads placed evenly

Case-3(a): Simply Supported Beam Subjected to UDL

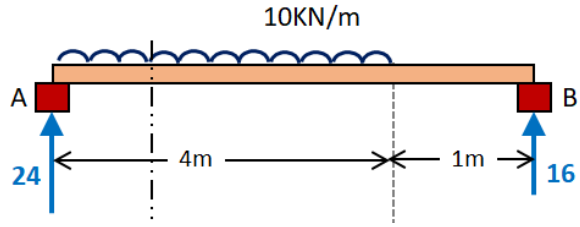


Figure 15 Simply supported beam subjected to UDL

The Figure 15 here shows a simply supported beam of span 5m supported at the ends A and B. the beam is subjected to a UDL of 10 KN/m up to 4m from A.

Step-1: Compute the reactions

Taking moment about A,

$5R_B = (10 \times 4) \times (2) = 80$

i.e., $R_B = 16KN$

Then, $R_A = (40\text{-}16) = 24KN$

Step-2: Find the BM

Let X is a point anywhere on the beam at a distance of x units from A.

Then,

$M_x = (R_A \times x) - (10 \times x) \times \frac{x}{2} = 24x - 5x^2$

Thus,

BM at X (when x = 0) = $(24 \text{ x } 0) - (5 \text{ x } 0^2) = 0$

BM at X (when x = 1m) = $(24 \text{ x } 1) - (5 \text{ x } 1^2)$ = 19 KN-m

BM at X (when x = 2m) = $(24 \text{ x } 2) - (5 \text{ x } 2^2)$ = 28 KN-m

BM at X (when x = 3m) = $(24 \text{ x } 3) - (5 \text{ x } 3^2)$ = 27 KN-m

BM at X (when x = 4m) = $(24 \text{ x } 4) - (5 \text{ x } 4^2)$ = 16 KN-m

BM at X (when x = 5m) = (24 x 5) – (10 x 4 x 3) = 0 KN-m

Step-3: Draw the BMD

Now, all the above values of bending moment can be plotted in a graph to obtain the requisite BMD. Figure 16 shows the BMD of the given beam.

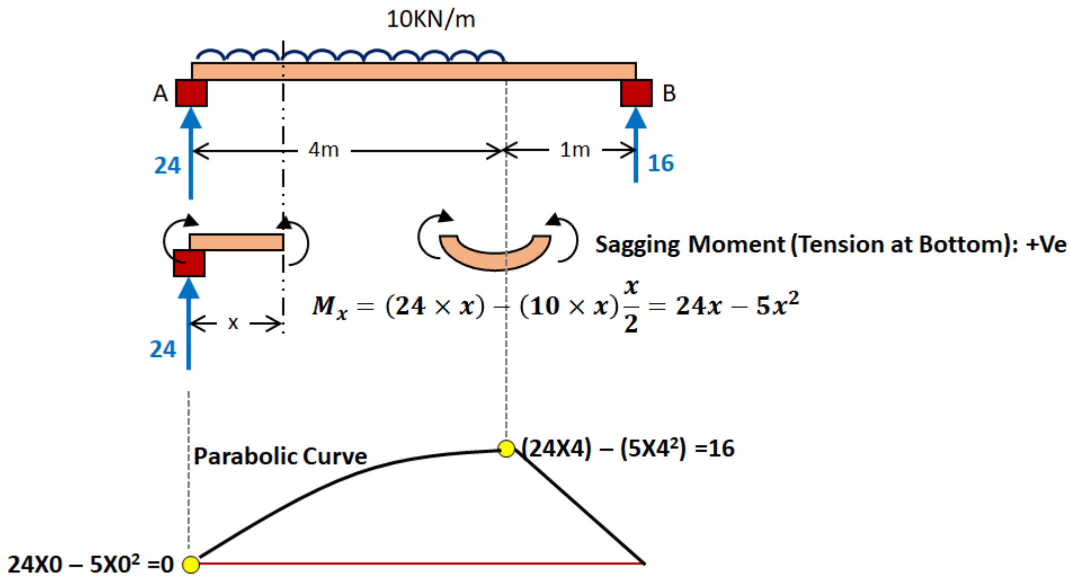


Figure 16 BMD of a simply supported beam subjected to UDL

Case-3(b): Simply Supported Beam Subjected to UDL

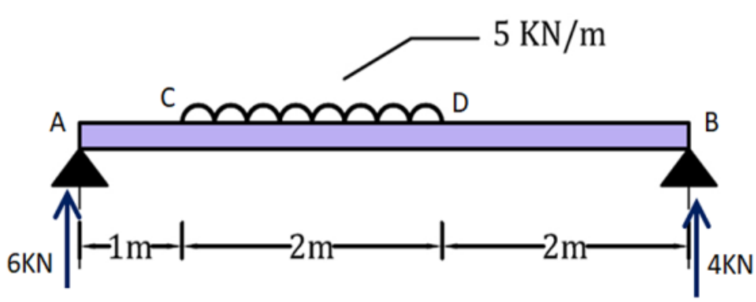


Figure 17 Simply supported beam partly subjected to UDL

Here in the Figure 17 we have a simply supported beam of span 5m supported at the ends A and B. A UDL of 5KN/m is spanning over 2m length at a distance of 1m from A.

Step-1: Calculate the reactions

We know that,

$R_A + R_B = 5 \text{ x } 2 = 10$ KN

But Taking moment about A,

$5R_B = (5 \times 2) \times (2) = 20$

i.e., $R_B = 4$KN

i.e., $R_A = (10 - 4) = 6$KN

Step-2: Compute the BM

BM at A = $R_A \times 0 = 0$

BM at C = $R_A \times 1 = 6$ KN-m

BM at X (Let X is a point at such that x = 2m from A) = $(R_A \times 2) – (5 \times 1) = 7$ KN-m

BM at D = $(R_A \times 3) – (5 \times 2) = 8$ KN-m

BM at X (Let X is a point at such that x = 4m from A) = $(R_A \text{ x } 4) – (5 \times 2 \times 2) = 4$ KN-m

BM at B = $(R_A \times 5) – (5 \times 2 \times 3) = 0$

Step-3: Draw the BMD

Lastly, all the above values of bending moment can be plotted in a graph so as to get the corresponding BMD as shown in the Figure 18.

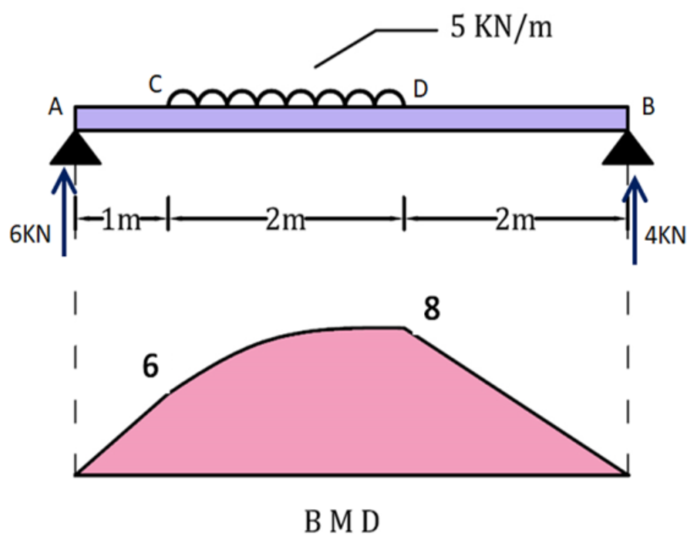


Figure 18 BMD of the simply supported beam partly loaded with UDL

Case-4(a): Cantilever Beam Subjected to UDL and Point Load

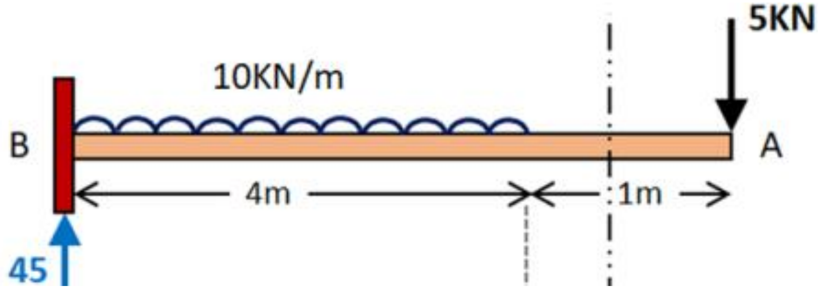


Figure 19 Cantilever beam subjected to UDL and point load

Figure 19 shows a cantilever beam AB of span 5m, fixed at B and A being the free end. A point load 5 KN is acting on the free end A of the beam. A UDL of 10 KN/m is acting on the beam from B and reaching up to 4m distance.

Step-1: Compute the reaction

$R_B = (10 \text{ x } 4) + 5 = 45$ KN

Step-2: Calculate the BM

Let X be a point on the beam at a distance x meters from A.

Then,

$$M_X = -(5 \times x) - [10 \times \frac{(x-1)^2}{2}] = -5x - 5(x-1)^2$$

Now,

$M_A = -(5 \text{ x } 0) = 0$

M_X at $(x = 1) = -(5 \text{ x } 1) = -5$ KN-m

M_X at $(x = 2) = -(5 \text{ x } 2) - (10 \text{ x } 0.5) = -15$ KN-m

M_X at $(x = 3) = -(5 \text{ x } 3) - (5 \text{ x } 2^2) = -35$ KN-m

M_X at $(x = 4) = -(5 \text{ x } 4) - (5 \text{ x } 3^2) = -65$ KN-m

M_X at $(x = 5) = -(5 \text{ x } 5) - (5 \text{ x } 4^2) = -105$ KN-m

Step-3: Draw the BMD

Plot the above values of bending moments in a graph so as to obtain the BMD such as shown in

Figure 20.

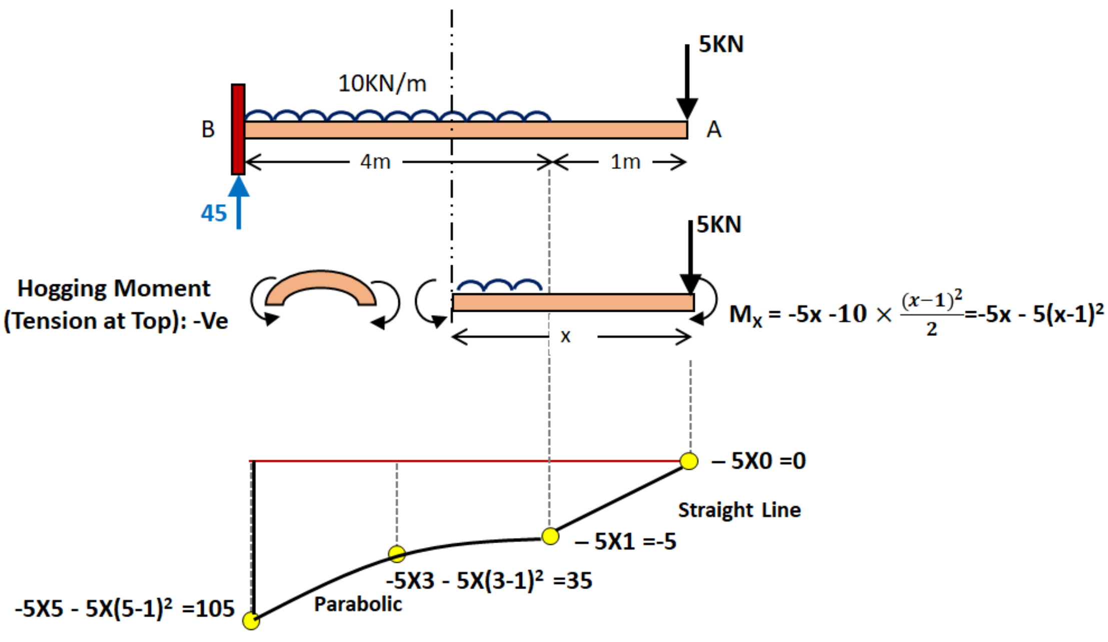


Figure 20 BMD of cantilever beam subjected to UDL and point load

Case-4(b): Cantilever Beam Partly Loaded with UDL

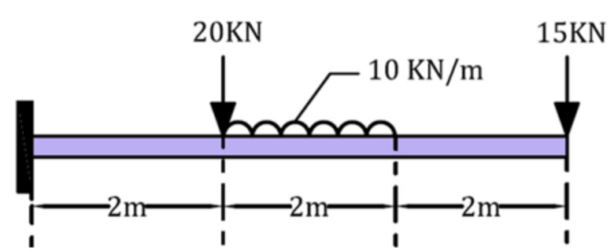


Figure 21 Cantilever beam subjected to UDL and Point Load

Lastly, we have another cantilever beam of span 6m which is subjected to a UDL of 10KN/m across 2m length at the center of the beam. Further, two point loads of 20 KN and 15 KN are also acting on the beam as shown in the Figure 21.

Step-1: Compute the bending moment

Let A, B, C and D are four points on the beam starting from the free end of the beam and marking all the eventful places.

Then,

$M_A = 0$

$M_B = -(15 \times 2) = -30$ KN-m

$M_C = -(15 \times 4) - (10 \times 2) = -80$ KN-m

$M_D = -(15 \times 6) - (10 \times 2 \times 3) - (20 \times 2) = -190$ KN-m

Step-2: Draw the BMD

Plot the above values on the graph to get the resultant BMD as shown in the Figure 22.

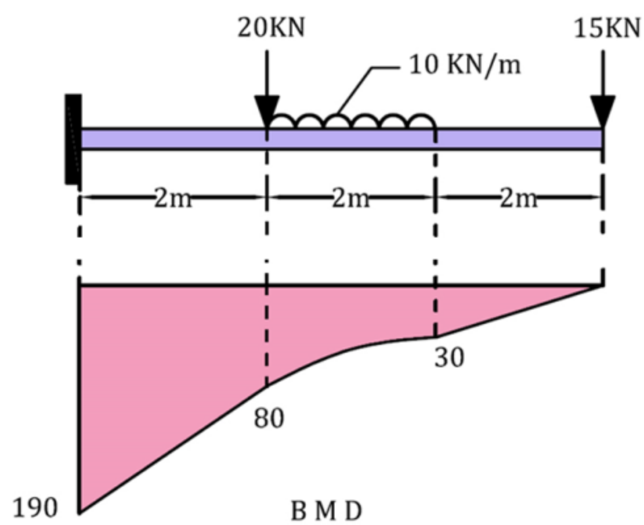


Figure 22 BMD of the cantilever beam subjected to UDL and point loads

References

- Engineering Mechanics by Timishenko and Young McGraw-Hill Publication
- Strength of Materials By B.C. Punmia, Ashok K.Jain & Arun K.Jain Laxmi Publication
- Basic Structures for Engineers and Architects By Philip Garrison, Blackwell Publisher

- Understanding Structures: *An Introduction to Structural Analysis* By Meta A. Sozen & T. Ichinose, CRC Press

Conclusion

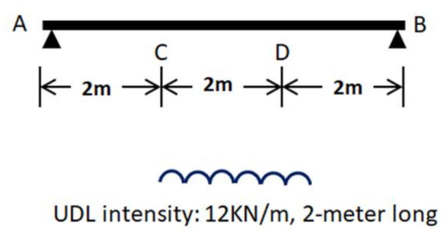
This lecture can be concluded with the following points:

- The beam shows a tendency of bending under the external load and support reaction.
- The graphical representation of variation of bending moment over the length of a structural member is called Bending Moment Diagram.
- It is one of the prime factors to design a structural element.

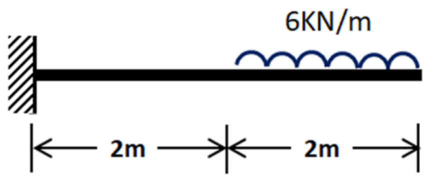
Homework

Q1. A 6 meter long beam AB is divided into three equal parts at points C & D (AC=CD=DB=2m). Place the 2-meter long UDL of intensity 12 KN/m as per the following three cases and develop the BMD.

Case-I: UDL in AC only
Case-II: UDL in CD only
Case-III: UDL in AC & DB



Q2. A 4 meter long cantilever is loaded with 6KN/m intensity UDL as per the figure given below. Develop the BMD.



If two equal pointed loads are placed at free end and mid span (two ends of UDL) then the bending moment at the fixed end become 60KN-m. Find the value of the point load.