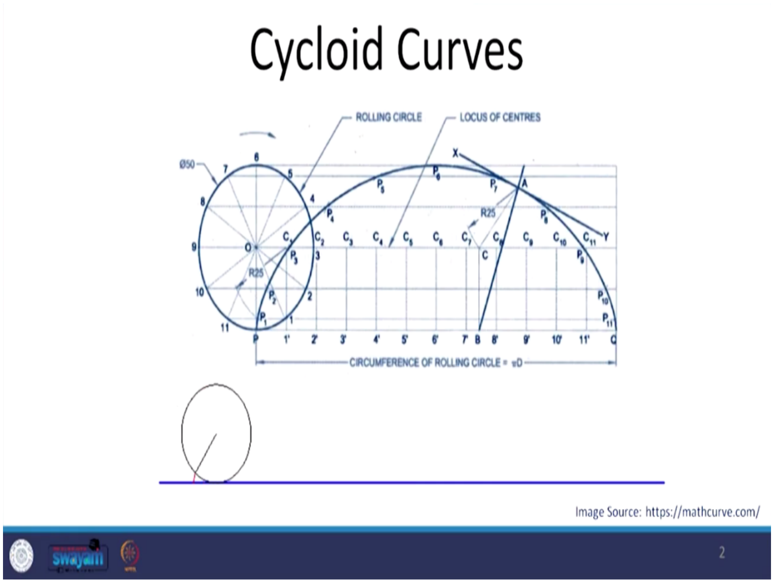
Engineering/Architectural Graphics – Part 1 Orthographic Projection
Prof. Avlokita Agrawal
Department of Architecture and Planning
Indian Institute of Technology – Roorkee

Lecture – 10
Curves Used in Engineering Practice: Cycloids, Trochoids and Involutes

Good morning. Welcome to the last lecture of week 2 where we are continuing with the curves used in engineering practice and today we are going to look at some very interesting curves which are generated because of the movement of circle so it is like a locus. So, these curves if you go to the ancient practice and we are also going to talk about the involutes. So, we will come to each one of these curves, but if you look at the ancient practice of engineering and architecture.

You would find that some of the best geometries in the world which are used in the wonders of the world, the most proportionate of the forms are created using some of these shapes. So, we will understand what is cycloid, what are trochoids and we will also understand what is an involute?

(Refer Slide Time: 01:22)

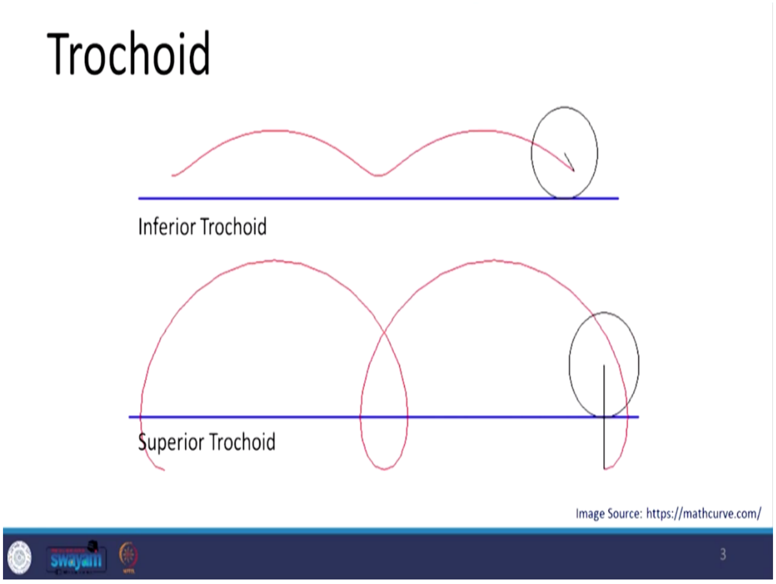


So, let us start with cycloids first. So, what is a cycloid? Cycloid is actually the path traced by a point on the circumference of the circle as the circles rolls along a fixed straight path. So, what we are seeing here on screen is that there is a circle which moves in such a manner that it is moving along a straight line and the path which is in red that we see is the path which is traced by a point which is on its circumference.

This is a cycloid curve, it is a periodic curve, it repeats. Most of the cycloid they could be epicycloids, hypocycloid, trochoids they are repetitive in nature they will repeat and I do not know if you have ever come across this very beautiful tool it is very simple to use where you get these two different circles and they move on each other. These are other forms of cycloids, epicycloids or hypocycloids, but this one is a simple cycloid curve.

And we will come to the construction of this cycloid curve how do we draw it. I am sure in mathematics when you were reading, when you were going through your schools there you might have come across some of the equations to derive the path of this point on the circumference of the circle which is actually the cycloid. We will actually see how to draw it geometrically here.

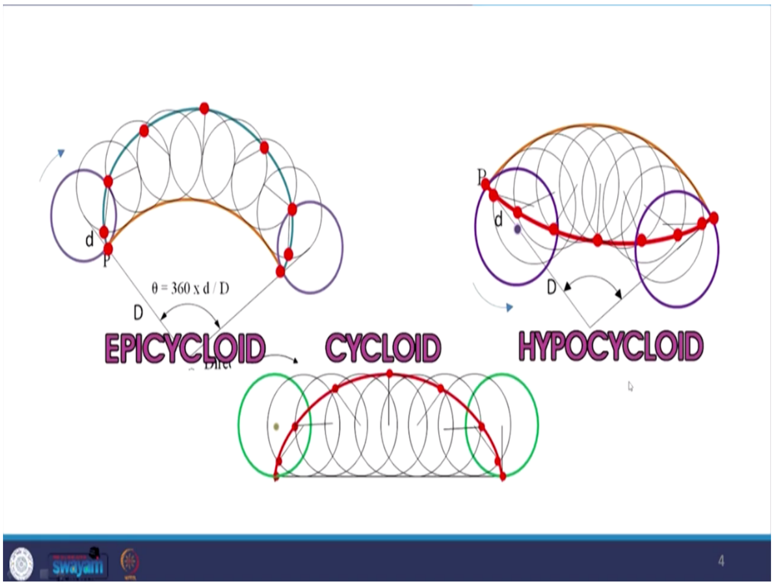
(Refer Slide Time: 03:00)



So, this is a simple curve cycloid we will then see trochoids. Now suppose this point P for which the path was being traced as cycloid is not on its circumference, but inside the circle or outside the circle. So, the path which will then be traced will be called a trochoid. So, if you look at the screen, the inferior trochoid is the path traced if the point is inside the circle. It need not be on the circumference this is called inferior trochoid.

If the point is outside then the path which is traced by that point as the circle rolls along this straight line is called a superior trochoid this is similar to cycloid, but this is by a point which is not on the circumference. The prime process of construction remaining the same. Another type of curve is epicycloids and hypocycloid.

(Refer Slide Time: 04:08)

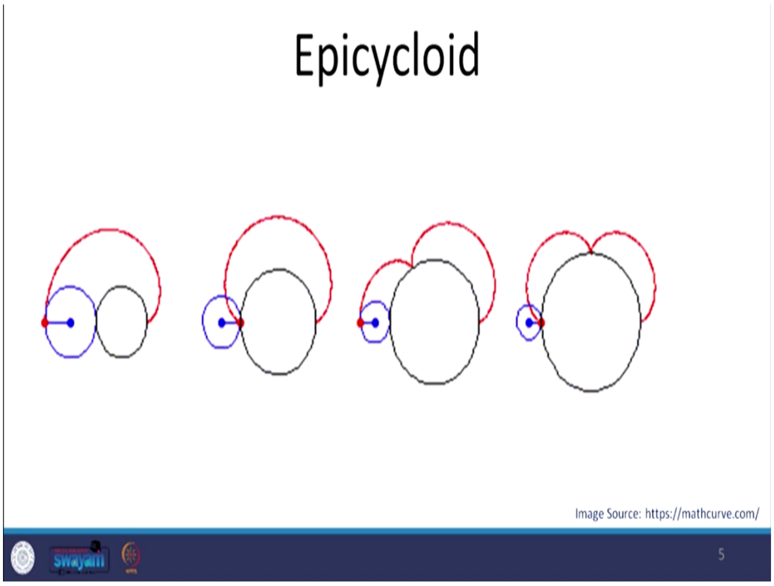


Now cycloid we have seen as the circle moves along a straight line. Now, if the circle moves along another circle. So, if it is moving at the outer edge of the circle such that it is the circle the path on which is moving is inside it and the circle moves on the outside the path that it traces and the point we are considering here to be on the circumference of the moving circle. So, the path which will then be traced is called an epicycloid.

However, if the moving circle is inside the path which is circular then the path traced by this point on the circumference will be called a hypocycloid probably these are some things which you have already known, we will only see how do we construct it here on sheet geometrically. So, why would these be needed is when you have to create these beautiful forms.

You have to draw dams or you have to design dams or other structures or buildings where some of these very interesting forms could be used that is how you will start to draw it on sheet. So, this epicycloids and hypocycloid, cycloid we have already seen.

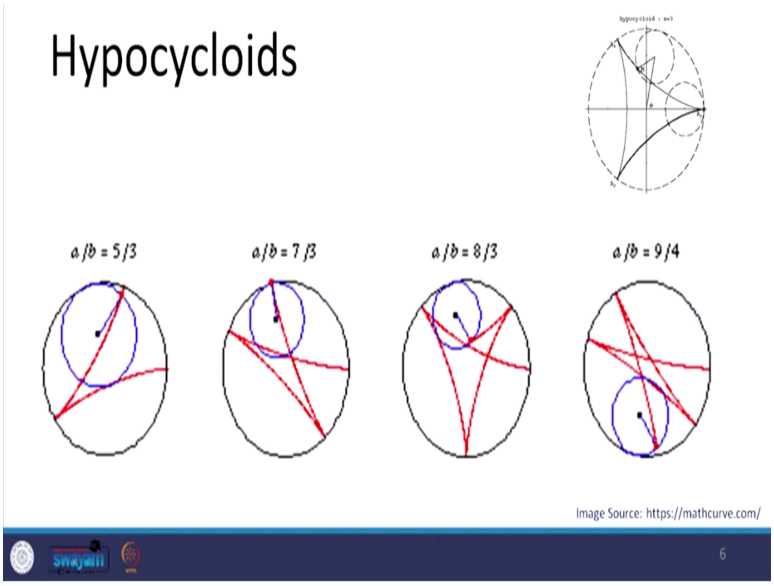
(Refer Slide Time: 05:29)



So, let us look at how an epicycloids will be created. Now, if you see in all these four figures which are on the screen we have an inner circle represented in a black circle which is the path for this moving circle which is shown in blue. Depending upon the size of this moving circle in relation to the path that it traces we will get different types of these epicycloids each one of this if you look at the shape they are very similar shapes.

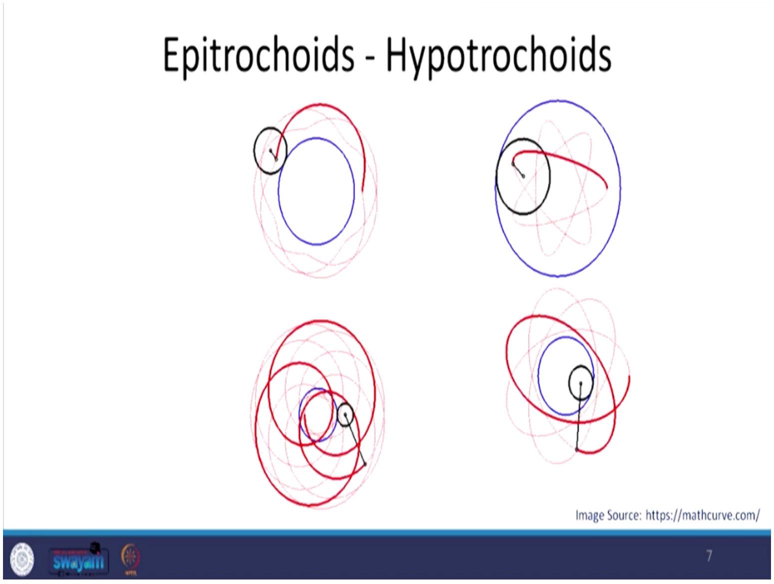
But depending upon the proportion of these two circles the trace would change this is epicycloids.

(Refer Slide Time: 06:15)



Now we will look at hypocycloid. In this case the circle which is moving is moving inside a limiting circle. Again depending upon the proportion between this fixed circle which is the path and the moving circle which is inside we get these different patterns which is what hypocycloid is. Again, there are numerical formula which we can use to trace these path we only have to know how to draw it geometrically that is what we are going to be learning here. So, this is hypocycloids.

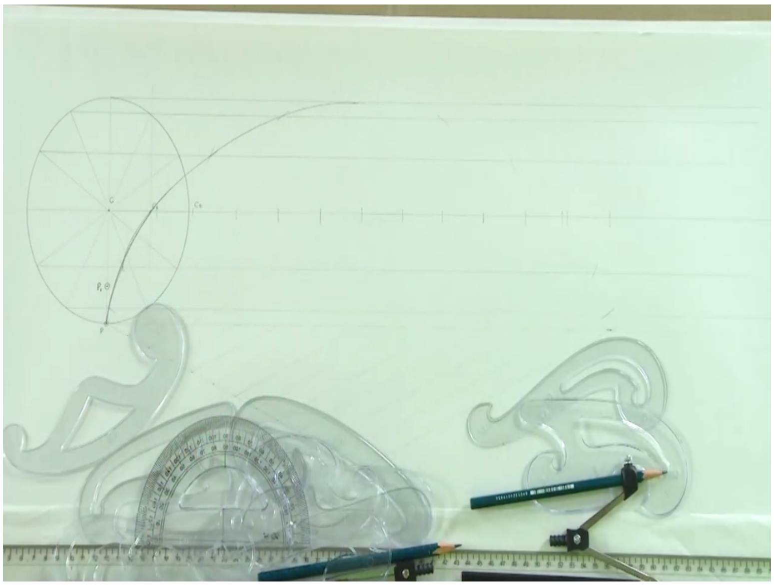
(Refer Slide Time: 06:53)



Then we will move on to the next curve which is epitrochoid or hypotrochoid. In this case just as we had trochoids, the inferior trochoid or superior trochoid this is the path traced if the point is not on the circumference of the circle, but inside or outside when the point is on the circle inside the circumference then the path which the moving circle traces when it is moving along a circle is epitrochoid.

And if the point is outside then what we know is hypotrochoid. So, that is what we are seeing is epitrochoids and hypotrochoids. These are some very interesting patterns, these are some very interesting forms which can be used to make very interesting design. Now, what we will do here we will quickly learn how to draw a cycloid.

(Refer Slide Time: 07:58)



So, we are going to draw the cycloid for a given circle. To start with what we need to do is we need to draw the circle. I have considered a circle of radius 7 centimeter which is drawn here. We have the circle the next thing we need to do is we need to divide the circle into some equal number of parts 12 is very simple because we can very simply take the 30,60 set square and draw it and we divide this circle into 12 equal parts here.

Now we will draw thin horizontal lines passing through these points. So, from each of the point we will draw these horizontal lines. This is the line which is passing through the center and also through two of the points. So every time the circle roles the center of the circle actually remains on this line which is passing through the center this is for cycloid. So, what we have done is we have drawn parallel lines passing through horizontal lines passing through all the points which were marked by these 12 divisions.

Now, next what we need to do is we need to draw or we need to identify the length of this length equal to the circumference of this circle. So, for a circle with radius R the circumference for the circle is approximately equal to 44 centimeters so that is what we will mark here. So, this is 44 centimeters and now we will divide this line which we have drawn here which is equal to the circumference of the circle into the same number of paths into which the circle has been divided.

So, here we are taking 12 parts let us draw an acute angle line a line at any angle and divide this line into 12 equal parts. So, I am just taking a 24 centimeter line which will be easy for us to draw and then mark 12 equal divisions on that. Next, what we do is connect this and divide this original line into 12 equal parts because we have taken 12 we are taking because that is the number we are using to divide the circle.

We are dividing the circle in 12 equal parts so we are dividing the line into 12 equal parts. You could choose 8 there is no restriction on that you have to take 12 only, but 12 just becomes convenient. So, we will draw parallel lines and we will divide. So, here we have divided this line into 12 equal parts. Now since this was equal to the circumference, the length of this line was equal to the circumference of the circle.

Each parts actually represents one part of this circumference. Now assume that there is this point so what we are going to do is we are taking the path traced cycloid for the path traced by this point P which is here and there is this circle which is center of the circle which is C and this line is equal to the circumference. Now what we are doing is as the circle rolls so just assume that as the circle rolls from this point C which is the original position to C 1 which is when one part the circle would have completed along its circumference one part.

So, this new position which is C 1 so when the point C moves to the point C 1 the point P would have moved in such a manner that its distance from the center remains the same, but it would have covered some horizontal distance now how much would that be. So that is along this horizontal line. So, this P would have come to the position in horizontal the position which is taken up by this next point.

So what we will do we will keep our compass at 7 centimeters which was the radius of the circle and from C 1 we will now mark a point in this line which is the second line. So, this is where the point P would have moved. We will further keep moving ahead so the point C 2 and the next line is this. So, we will cut we will mark a point on this horizontal line. So, this is the point 2.

Likewise, we keep marking the perpendicular lines on to the central line which is where the center of the circle is going to come and we will keep cutting the point on horizontal taking the center as the new moved position. The radius remains the same because that is fixed and if you can see where we can very clearly note that the path of the cycloid is almost traced you can complete it like that and then we will darken using the French curves.

So, we can see that the path the cycloid path of the point P is almost traced and what we have to do next is a little difficult task which is to fix these curves. So, we just have to fix a curve which fix the three points any three points and then we keep moving. So, just see which one would fit your curve here. Every time it will be a different curve that fits. One or the other curve would surely fit you just have to be patient while drawing these curves.

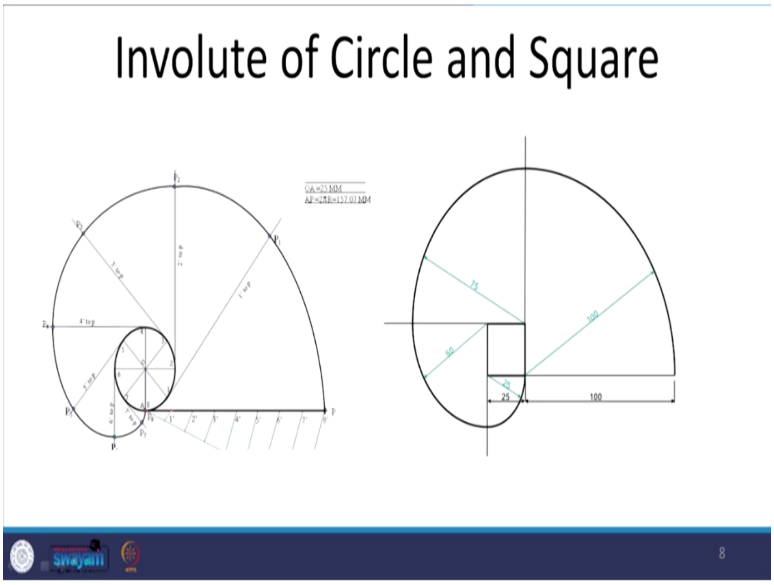
So, I am drawing it very simply so if you can see we have almost trace the half of this cycloid and it is symmetrical this is how it will move. This is the process to draw cycloid very similar is the process to draw and epicycloids or a hypocycloid or even a trochoid. So, if we were to draw a trochoid the only thing we would do is for example we had a point which was not this, but there was a point P 1 here.

So, if we were to draw a trochoid so what we would have done instead of this line or instead of this point and the distance CP we have taken the distance CP 1 and we would mark the movements along these horizontal lines. So, the only thing that happens is that this distance changes. So, it keeps changing its position and within the same process we would actually get the trochoid.

This is for inferior trochoid and if we take a point outside we would get a superior trochoid exactly the same process. If we are to draw epicycloids in that case the only change is that this length of the circumference which we have drawn as a straight line would actually be taken on a circle. So, whatever circular path the circle is going to trace so we would take the curve, the arc of that circle where this smaller circle would be moving.

And that circumference would be divided into equal number of parts and exactly in the same manner instead of these parallel straight lines we would be drawing concentric arcs at the same distance from the center. So, this is exactly the same process the only thing is instead of tracing the points on these horizontal lines we would then be tracing the points on to those parallel curves that is how we would draw the epicycloids and hypocycloids. So, this is the process to draw cycloids.

(Refer Slide Time: 20:18)



We will now move on to the next curve which is the involute. So, what is an involute? Involute is actually the path which is traced when a thread which is wound on a circle or any other shape is opened gradually in such a manner that the threads remain in tension. So, what is an involute? It is the path which is traced if a thread which is wound on a circle or any other shape is gradually opened.

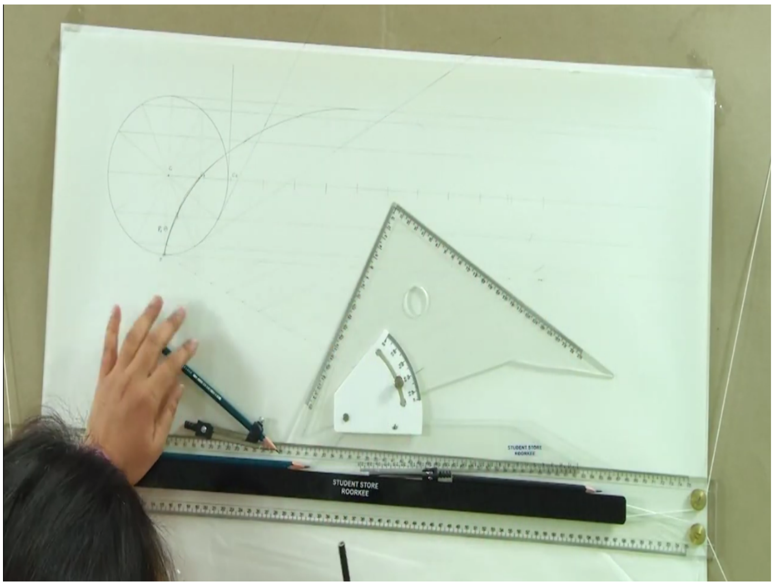
In such a manner that one part of the thread is opened at a time. So, just assume that there is a thread which is wound on a circle I am just winding it up on the circle of this sphere and we start to open it in a gradual manner such that the thread remains like tangential to the circle. So, every time we open it this is the full circumference and then every time we reduce as it get wound on this so as it comes back to the center.

So, just imagine that this is fixed here and this is the total circumference. If we keep moving this path here and the thread gets wound and once it comes back it is back to the original point, but that path that it has traced is the involute of this thread. The only thing that we have to assume here is that it remains in tension all the time and the length of this thread is equal to the circumference or the perimeter.

So, this is how the involute would be drawn. It could be for a square, it could be for a circle, it could be for any shape. The only thing that we have to see here is that how we increment it. If you look at the screen now we can see that when we are talking about a circle we divide the circle or assume the thread to be in equal parts as it unwinds or as it winds up and the total length is equal to the circumference of this circle again.

The same thing is with square so it has four parts that we see that the final perimeter of the square is achieved.

(Refer Slide Time: 22:44)



Let us quickly see within this sheet that how to draw an involute of the circle because we would need to follow almost the same process here. So, what we do for this one is? So, for drawing the involute of a circle we will again take a line equal to the length of the circumference of the circle and we divide it into 12 equal parts any number of parts so I am again taking 12 here that is convenient as I say.

And what we have to do here actually is draw tangents to these lines. All of these lines we have to draw tangents. So, the tangent this point here is a vertical line which is what we have here and now at this point the total length of the thread which is equal to circumference will be opened. If it was a tangent here so if I make this one so if I draw a tangent here this tangent would be something like this.

So, the distance on this particular tangent is one part less on this circumference. So, what we have here is we can either measure so this is the next point that we would have. Again, we would draw the tangent to this one, measure the distance probably we would be running out of this and then so like this we would keep drawing, marking the points on these tangent equal to one length less.

And as we come here as we again come back to this point of course we have exhausted all the lengths so we will be at the same point and what we will get here is the involute of the circle that is how the involute of the circle has to be drawn. For drawing involute of the square assuming there was a square here we would draw 4 equal parts with the final length being equal to the perimeter of the square.

And we would keep drawing tangents and start opening one part one by one. So, in circle it is easier to know what is the tangent, but in square we would just be extending the side of the square on which we would cut the length equal to the perimeter the left over perimeter and that is how we would get the involute. So, I hope it is clear how to draw the involute of the circle and square and following the same process you can draw the involute of any 2D object.

If you are interested you can actually look at a lot of traditional text especially Vastu Shastra where the involute of circle and involute of square is very commonly used. This is a rampant way of using. So, in prototypes while designing cities how the quadrants were joining together to give a bigger shape, bigger plan is exactly the process of drawing involute or how the areas of these involutes would come into picture that is how the planning as per Vastu Shastra was done for cities even for residence and smaller buildings.

So I hope you would be able to draw cycloids and involutes using this method and we would move on to orthographic projection from the next week onwards. Till this point the first two weeks of this course we were actually looking at the basic construction techniques and getting right in place how to use your different tools and equipment. I hope you are familiar with that.

Please practice more and more so that you become very familiar and well versed with all these equipments so that you do not have any problem in picking up orthographic projections later on. So, thank you very much for being with me today and from next week onwards we will be starting with orthographic projections so see you then. Thank you.