Engineering/Architectural Graphics - Part 1
Orthographic Projection
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Lecture - 29
Projection of Solids with Axis Inclined to Both the Reference Planes

Good morning, welcome to the 4th lecture of this week where we are learning about how to draw orthographic projections of regular solids. This is your online ongoing course on architectural graphics or engineering graphics. And I am your instructor, Doctor Avlokita from IIT, Roorkee, Department of Architecture and Planning. So, what we have done so far is that we have understood how to draw orthographic projections for solids which are either kept in simple positions which is when their axis is either perpendicular or parallel to the reference planes.

Next, we understood how to draw orthographic projections when the solid has its axis parallel to one of the reference planes and inclined to the other reference planes. And we have seen the examples. Today what we are going to do is we are going to draw orthographic projections of solids which have its axis inclined to both the planes. So, that is what we are going to take as an example.

But before I start to draw let us explain how the movement of the solid is going to happen by taking these solid examples. **(Video Starts: 01:45)** So, we have this reference plane and let us assume that we have a cube which is kept with us. Now, what happens in simple position is that this cube is kept like this and we are assuming that the axis of this cube is perpendicular to HP and parallel to VP which is what we had seen yesterday.

Now, if I want to incline this suppose this cube is assumed to be kept in a position such that its faces or the side of its base is already making an angle of 45 degrees. The cube is still considered to be in a simple position because we will still be seeing these faces being few of the faces being parallel to one of the reference planes and that is what is happening. So, what we have here is the axis still remains perpendicular to HP and parallel to VP.

Now, inclining it such that the; axis now makes a certain angle with HP. So, what we see? That the; cube is inclined by one of its points like this. So, from the top that is what I had asked you to do yesterday. That the top when we see it from the top we will actually be seeing 3 faces. And when you see it from the front then also you are seeing these 3 faces being visible. Now, this is when the axis is inclined only to HP.

What if the axis is further inclined by another angle to VP as well? In that case also we see 3 faces only but these are slightly different and we get a slightly rotated view. Now, what is happening when we draw the projections? We have to mainly concern ourselves or start with the axis. So, what happens when we move the axis assuming that the axis is only from the bottom of the base to the top of the base of this cube?

So, when you incline 30 degrees we assume that this line which is the axis is inclined at 30 degrees. So, in the front elevation we will be seeing the axis being inclined at 30 degree or whatever angle to HP. And we will rest of the things will remain as they were if it was kept like this perpendicular to HP. Now, when we tilt it like this and then we further tilt it, it is the same phenomena happening as we learnt in doubly inclined lines.

So, what happens when this axis moves when this axis is moving about the reference plane? So, when we move it like this if it was parallel and then when we move it like this in that case what is happening that the point is changing. But when we further move it like this the vertical trace the height of this point from the base remains the same. So, we draw a locus a straight path along which it will be moving.

Or, if we were taking it to be inclined to VP by say 30 degrees and then we raise it like this. In that case also what we see that the vertical trace remains the same. So, this is the trace of line which is going to remain the same when we move it like this. It is like a cone since the solid is comprised of many such lines and there are several points, it will be very difficult for us to know how and where to draw these points and lines.

So, what we do is we follow the same process assuming that this axis is the line which is moving. And that is how even the rotation of solid is also defined. So, we define the rotation of solid by the angles its axis is making with the reference planes. Or, sometimes we also have some of the edges or planes defining the position of the solid which is what we are going to be seeing in today's lecture. And we will be learning how to draw them. **(Video Ends: 06:03)**

(Video Starts: 06:04) So, to begin with, I will start with the example of this cube only because it is simple. But we will be able to grasp the fundamental, the basic of movement or rotation of solid. So, let us assume that there is a cube. We are assuming that there is a cube which is kept in simple position we are not it is it has its 2 faces parallel to VP. So, we will be having the cube having its initial position where 2 of its faces are parallel to VP.

Now, the axis of the cube makes an angle of 30 degree with HP and it also makes an angle of 30 degree with VP. That is what we are going to start drawing. So, let us start by assuming the cube to be kept in a simple position first. So, we will always begin with simple positions. So, assuming that this is a 5 centimeter by 5 centimeter cube and it is kept on the HP. We will draw. So, this is how the cube is going to look like, the front elevation of the cube.

And we will locate the axis of the cube which here we are assuming that this is this line which is perpendicular to the cube. Now, what do we see from the top is again a simple square like this. This is our cube in simple position. Assuming this is the horizontal plane and there is an imaginary vertical plane here. So, this is what the cube is. Now, the given condition, let us assume that the axis makes an angle of 45 degree with HP and 30 degree with the VP.

So, if this makes an angle of 45 degree with HP. So, what happens? We will be seeing this rotation happening here. So, what do we see? That I am making a separate picture, separate figure so that you do not confuse. But you can always tilt it right here. So, what will we take? We will have the axis being inclined at 45 degrees to HP which will be seen here in the front elevation. So, this is the axis.

So, this is the same square is now this face which will be seen from the front is now inclined at 45 degrees to HP. Because this face was also parallel to the axis. And now when the axis is inclined at 45 degrees this is what we see. Now, what happens in the top view? We see two of the; faces being visible now. So, what we have here is 2 of these faces. The axis would look like or it will appear as the true length in VP because it is still parallel to VP.

And here what we have is, if, we locate the position of the axis then this is our axis. So, the axis has now diminished. And also remember that the point of rotation is this point. And it is actually an edge. This entire cube is going to rotate about this edge. And now when we further say that this is rotating about another 30 degrees it is making an angle of 30 degrees with the VP. We are talking about the axis.

But it will be rotating about say this point which is the midpoint of this. So, we this entire thing will be rotating. Now, assume with this axis. So, what happens is that this axis is now rotated such that it makes an angle of 30 degrees with the VP. So, what we have is we will draw a line which is say 30 degrees about inclined about the VP. Now, what is happening? If you look at this, this is the midpoint of the axis in elevation.

So, this is our reference line. So, what is happening here is this axis half of the axis is inclined about this point or probably this entire axis is inclined about this. Further, when we incline it this is going to be the point. And this is the 30 degree line. Now, what is happening? That the trace of this is going to be inclined at this, 30 degrees. So, what we have is that the original axis if you look at this original axis is going to incline at 30 degrees.

So, which was originally parallel is now inclined at 30 degrees. And we will be having a line this. And the trace of this will actually be taken onto this line which is passing through 30 degree point which is exactly what we had done earlier. So, what we have now and similarly we can do it here on the reverse side. So, what we actually have is we have an axis passing through this point and having its trace there.

So, what we have is that we have this entire cube rotated in such a manner that this axis now occupies this position. So, rest of the drawing will move about this axis. And we will get a cube which is rotated in such a manner that the axis makes an angle of 30 degree with the VP.

So, this is the new. This is the revised axis which is what we have got. So, what we get here is this. And now we will rotate this entire cube about this axis.

So, since if this was the axis and there was this perpendicular let us set our set square to match this axis and draw perpendiculars and trace all the points onto these lines. This is the axis which was seen here. This is the line perpendicular which is now come here. And if we trace this point further which is what we are going to take on the axis. We will get the same cube which is now rotated about 30 degree.

We are seeing the same traces but the angle is not exactly 30 degree which is what we saw here. So, following the same process which we followed in doubly inclined lines we will get the final shape of this cube which is now inclined like this. So, if you look at this we are seeing this cube like this. And now we will go back to draw the front view. So, let us take the projections of these points.

So, what we have here? Now, we will have to properly number it. So, let us number the cube from the beginning. So, if we had say on top let us write this from the top view. So, A, B, C, D and below it may be we can have P, Q, R and S. And when we see it from the front, we will have A, B P, Q here and then D, C and S, R projected here. Now, when we rotated it, what do we get?

We get the same points moving here A, B D, C and here we had S, R and P, Q. That is how the entire thing has moved. And if we look at this the picture here, what happened? B and Q remained here. So, just moved it like this. So, this was B. This was A. And now in the new one this is B and this is A. This is C and D. And this point which was P is here which is below this A.

P, Q and then you have another 2 points which have actually gone beneath this. So, this R and S have actually moved. So, if you look at in the original picture which was before we rotated it further, what did we have? We had this A, B, C, D. And we had P, Q, R and S. The same thing has happened here. So, we have R and S here. So, now we will take it back. The P goes and meets P. So, the horizontal projections have remained the same.

So, we have the P here. The Q, the S and then the R comes here. That is what the P, Q, R, S is. And let us look at the A, B, C and D as well. So, the A goes up A, B, C and D. Let us now join these points. And you will know what do we actually see? So, what we have here is this A, B. This is the C. This is where you get D. This is the top one. And then P, S, Q and your R. R is here. Now, A and P was joined. So, let us join A and P. So, we get this D and S.

And then we have R and C. Now if I darken this probably you will be able to figure out how by following a method we get the exact view of this cube which is inclined to both the reference planes which is precisely what we should have been seeing. So, if you have simple logic here this was the cube which was kept originally like this here A, B, C, D, P, Q, R, S and then we inclined it like this. So, what we saw?

A, B, C, D and P and Q which is what we had seen here from the top and now when we inclined it further 30 degrees. So, what we are seeing is we are seeing these 3 faces when we are seeing it from the front. So, if you look at it we are actually seeing this. So, we are seeing A, B, C, D and P and R and P, S and R here. So, Q goes back. We will not be seeing this Q but you can always draw the dotted lines to depict that this is hidden.

So, what we have is we have a P, Q. So, if we had A, B, C, D, P, Q, R and S and then we will only be connecting the lines which originally exist in the solid. So, you have to be very careful when you connect these lines because if you connect the wrong points and for that you have to have a clear understanding of what this solid which is being discussed here is. And then we will have to darken the top view. This is the top view of the same cube.

So, we have A, B, C, D, P, Q, R and S. And then we get the same points here A, B, C, D, P, Q, R and S. And in this manner by following the very simple rule which was the methodology of projecting lines which are inclined to both the reference planes. We will get this cube the projections of this cube which we did not start by assuming that this cube is kept in a very difficult position.

We started with the simple assumption of the axis being perpendicular to HP and parallel to VP. And then we moved on step by step to finally arrive at this particular solution. It sometimes appears tricky. It is tricky. And some of the examples might be trickier but we will take another example to make it little more understandable to you. **(Video Ends: 25:42) (Video Starts: 25:43)** So, this time let us take this example of this pyramid.

So, it is a pentagonal pyramid which is what we are going to draw. So, assuming that this pentagonal pyramid is having one of its faces in VP, so what happens if we have just assume that this is a vertical plane and this is the solid that we have and this is horizontal plane, so what happens if we have a face of the pyramid here? So, what will be the initial original position which we can start with?

So, we can start with having either the pyramid being perpendicular to VP where we will see the true pentagon or the true shape of the base. And then we incline it in such a manner that this base this face is now in VP. So, we will have to incline it in such a manner that this axis now makes certain angle with this. So, this is how we are going to incline. So, if you look at this, this axis is actually making a reverse angle equal to the angle of the cone here.

And then we will draw the front view. So, that is one inclination. And the axis is still parallel to HP. So, we will still be seeing the true shape. And now if it further makes an angle with HP we will be inclining it further. That is how the movement of it would be. So, let us assume a pentagonal pyramid and then let us see. So, assuming that this pentagonal pyramid has a base of 3 centimeter side, let us quickly draw this pentagonal pyramid first.

So, I hope you have you just remember how to draw pentagon. Hexagons are easier. Squares are easier. Pentagons are sometimes more difficult. So, that is the 3 centimeter side that we are going to take as reference. This is the center point for a square. This is the center point for a hexagon. So, you bisect it here. So, what you get here is this center point for a pentagon and taking this radius. This is the pentagon which was the first step.

And I would suggest that you make a pentagon every time that you have to make it. It might take time but then the construction should be firm. So, this is the pentagon which is the base of this. Now we are assuming in the original position that it is kept in such a manner that this side one of its side is perpendicular to VP. And it is it pyramid is kept in such a manner that the axis is perpendicular to VP.

So, we will actually be seeing the apex here. And that is what the original position of the cone is assumed to be. And what do we see from the top is the true length of the pyramid because the axis is assumed to be parallel to HP. So, we will be seeing the true. So, let us assume that this cone has a height of say 7 centimeters which is this. So, what do we see from the top is we see this face which is flushing.

So, both the points if I start numbering it from here A, B, C, D and E and this is the apex O. So, we have the apex O here. We have C and D both flushing. And then we have point B and E. B covering E and the point A. This is the top projection. Now, what happens? And I will also simultaneously make this axis here. So, we have A, B, C, D and E beneath it. Now, what we are doing is we are making this side which is represented by this line which is connecting CO and DO.

We are rotating this entire figure in such a manner that this face is now parallel to VP. So, what happens? That this entire slant line the cone rotates about this side this edge CD in such a manner that this cone is entirely resting almost in VP. So, that is what we are going to do. So, what do we do? We will have to measure and then start cutting it here. So, assuming that this is the point of rotation or this is the side of rotation.

All the points will now be taken. And that will happen is if you were seeing the cone like this suppose this the entire thing is now rotated. The entire thing is now rotated in such a manner that we have this axis inclined. This axis which is parallel to HP which was kept like this is now inclined to VP. So, what do we see here? We will start to incline. since CD is the edge about which we are inclining. So, we first got our apex O here.

Now, we know that about this line which is connecting CO. We have this line AC making a certain angle which is what we will repeat. So, what we have here is if I set my set square about the parallel which is here by a certain degrees. So, this is the angle which this line is making with the horizontal which is this line. So, when we incline this in such a manner that this OC now is has become parallel. OC and OD both have become parallel to VP.

We will rotate this entire thing this point this line CD. So, what we did? We will set our set square in such a manner that we get this angle. And then we will have to make a line which is perpendicular to line which is OE. So, we will have a line passing through C and perpendicular to OE which is what we are taking here. So, this is O. This is the axis. And then we will take the same distances say OB and mark OB.

And then we will take OA which is what we are going to mark here. So, what we have done essentially is we have rotated this cone in such a manner that the condition that one of its faces rest or is parallel to VP is now fulfilled. So, we rotated this entire thing. Let me mark it again. So, this is if at the back this was O 1. This is B and E. And we have A here. Let us take the projections up. The horizontal projections remain the same because that is what we kept.

And let us match the projections again. So, we have the C and D which remains the same. We have this O 1 beneath it. So, what we get here is this O 1. We will get this apex here which is O. We have B and E. So, we get E and B here. And we get an A which is here. So, now if I connect these points, so what I have is A, B, C. This was D and this is E here and now this is O. So, what we have here is OA. This is what we are going to be seeing from the front.

So, if this was rotated like this. So, what we see from the front is these 2 faces. While 2 faces go at the back and one of the faces which is if I draw the hidden line here this face which is a triangular face named OCD is going to be the face which will actually be seen in its true dimensions because it is absolutely parallel VP. So, what we have here is A, B, C, D, E and O. And we also have an O 1 which is the axis.

So, now if you see this axis is still parallel to HP. But it is making a certain angle VP which is the angle of the cone. So, the angle of cone is the angle which this is making. And so, we see it slightly diminished here but parallel to HP. So, the true this is the current shape in front elevation and in the top view. Now, the condition of the problem says that this is further incline.

So, if this was initially kept like this, this is further inclined by say 30 degrees to the HP. So, what happens? This axis which is currently parallel to HP will now be making an angle with HP. So, what do we do here? We will incline this entire cone by 30 degree such that this O 1 O your OO 1 it makes an angle of 30 degree HP. So, what we have to take now is one step further. So, what we have here is this.

Now, we can decide about what point are we rotating it? So, it could be sometimes defined. So, suppose this is like this. We can also move it about the point. So, that the point the apex remains fixed. And we move it or we could keep the point C fixed and then move the rest of the things. Either ways you have to decide how we are going to do. So, what we have here is suppose we are moving it about O.

So, we will have to recreate this entire cone in such a manner that it is inclined at 30 degree to HP. So, this is the new position. So, suppose we take that we have this point O. So, if this was the point O. And correspondingly we will mark all the points. So, what we have is this distance OO 1. And we will also have this distance from the line CD of the point. And this face CD is perpendicular to OO 1 and which is what we will mark here.

And it is seen in its true dimension which we took as 4 centimeters. So, this is what is CD. This is the point O 1. And now correspondingly we will mark. So, this was the line OC, OD. And we will now mark the points E and B. So, to do that what we do? We have OB equal to OE here. OB and OE and then we have the distance of CB and DE. So, these are the points and E. And now we connect them.

Keeping the point O as fixed, We rotated the entire cone about this point O. So, now I am just marking the last point A. And we get the final front view which is this which is exactly the same as the previous one. It is just that it has been tilted. And we all we have to do now is we have to get back top view or the plan for this cone. So, we will only start to project all these back. And following the same rules of orthographic projection we will just match up the points.

So, we have an A here. B and E and we have C and D here. So, this is the D. And this is the C. And we have O in the same point here. We can also mark for O 1. So, if you bring it back from the top you will get this O 1. And if I now darken it you will be able to get the final shape of this cone which has its face resting in or parallel to VP. And it is inclined at 30 degrees to the HP. And you can make the hidden lines also here.

And this is the final axes of this cone which is seen here and given the conditions that one of its faces is going to be parallel to VP. So, this is the final shape of the cone that we are going to get. Let me quickly label it for your convenience. So, this is what it is. This is O. And then we have B, A, C, D and E. Now, one thing which you would have noticed by now is that the order in which the solid is numbered or labeled right in the beginning will be the one that you will ultimately get.

And in case you are getting a wrong order it means something has gone wrong in your drawing while you were drawing it. So, that is another indication if the order is correct that you are on the right path. And this is the final correct product. So, for the given condition of this pyramid this is the kind of solution that you are going to get. **(Video Ends: 48:14)** I hope you are able to understand the projection orthographic projection of solids with axes inclined to both the planes fairly well.

You will become more confident in attempting orthographic projections once you start practicing it, once you start drawing it on the sheet. So, while I am explaining, please pick up any book on engineering graphics or engineering drawing and start to practice all these orthographic projections and the problems that are given in the book to firm up the understanding which you are gaining from here.

So, I hope you are following up with me. So, thank you very much for being with me in this lecture today, bye, bye.