# Engineering/Architectural Graphics – Part 1
## Orthographic Projection

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## Lecture – 37
## Development of Surfaces – I

Good morning, welcome to the 37th lecture of this course, here we are running in the last week of the course and this is the second lecture of this week. So, so far, we have completed almost all various types of solids, planes, lines, points and their orthographic projections. We have also looked at the sections of various regular solids. So, in this entire course, we have mainly focused on orthographic projections of regular solids.

Now, once we have completed the orthographic projections of these different solids, one thing which is still left to do and which is what we are going to cover in this week is how to develop the surfaces. Now, first, why all do we require to develop the surfaces of these solids? So, just imagine that there is a machine being made. So, the design of the machine is now done and now, this machine has to be made, not all the time, but this machine is going to be a solid body, a solid metallic thing which will be cut and chiselled like wood.

Majority of the time, it will be sheets connected together, for example your car. So, the body of the car the outer surface of the car is already designed. Now, we need to produce it. We need to manufacture it. How will you cut this metallic sheet to form the design of the car which the designer has done? It could be anything for example, this the body of the computer.

So, how would the metal sheet be folded and cut to give the shape to a computer and like that any machine, any piece of furniture, anything that is around us. It is designed and then it is manufactured. For that we need to know the development of surfaces. Now, it is not just one solid which is going to be there, which is going to be cast; there will be multiple solids put together.

So, we will also require to understand intersection of surfaces, intersection of these solids, which also we will cover in this week. So, today, I am going to talk about development of surfaces for simple solids. So, here, we are taking prisms and pyramids largely and we will also have some solids the regular solids like spheres. So, let us go ahead with the lecture.

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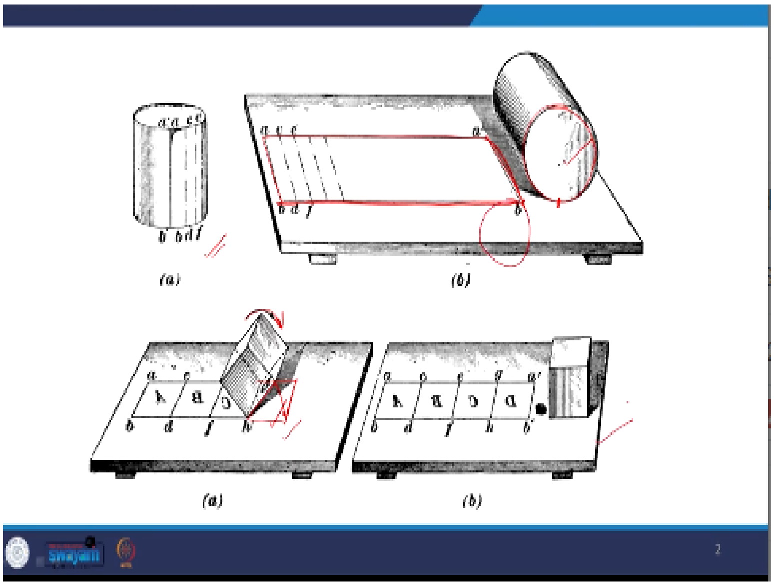

So, how do you develop the surface of the solid? Just imagine that we have a cylinder which is given here and what do you do? If you just roll it along on a flat sheet so, we will continue to get a rectangle. So, if we start from this point and we come back to the same point, this is the entire trace of this prism which is the cylinder in this case. The similar thing could also be done for other prisms. For example, here we are taking a cube. So, if you keep rolling it over, if this keeps rolling over, every time a surface touches the ground, we will have its true shape here and that is what is going to be the surface of it if it is open. The only thing which is left in all these cases is the base. So, the base has to be added which is what we will see. So, for that matter, any solid any prism, prismatic solid can be produced like this (04:12) surface can be produced like this.

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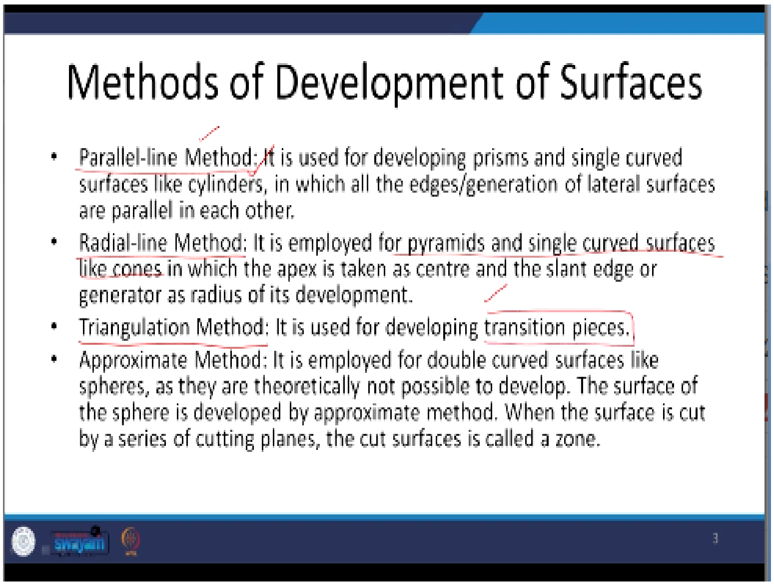


So, there are multiple methods of development of surfaces which can be followed. One is what I have just showed you, which is a parallel line method. So, we have the first one which is the most common; it is a parallel line method and it is primarily used for developing prisms where we are going to get straight lines. So, if you remember, if you recall what we were talking when we were doing the orthographic projections of prisms, we said that we will always see in most likely cases.

We will see the rectangular surfaces even when they are cut or in elevation or depending upon the positions. We will largely see the rectangles because that is what is connecting the 2 bases. The 2 bases could be any shape; it could be pentagon, hexagon, anything, but the surface which is connecting these 2 bases is essentially a rectangle. So, in this method, we actually have parallel lines in the height and if we roll over the prism on a flat sheet.

We will actually be getting the entire surface of this prism that is the parallel line method. So, all prisms and single curved surface like cylinder, which is not converging in a point. They will be generated by this parallel line method just as I have shown.

The other is a radial line method, where the solid is converging in the apex. So, just imagine if we have a cone so, just imagine that we have **(Video Starts: 05:51)** a cone like this, now, if I just slanted on a flat surface, a flat sheet of paper and then I start to trace its surface.

Now, what happens? It actually revolves around its apex such that we will just start with any one point and we will come back to the same point. Till that time, we will keep rotating it. Here, again the base has to be drawn separately as a tangent to one of the surfaces, one of the generators and this is how; this is actually radial line method. So, what we are assuming is that this is the radius of the surface which is being generated?

Now, cone that way is a little complicated, but if we have a pyramid like this. Now, I am keeping this. This is the pentagonal pyramid. So, if I keep it like this, now, if I move on to the next triangular surface, what happens? The distance of this surface, this base from the apex is the same. So, if I just move it, this is how it will still keep moving. It is just that instead of infinite lines, in the case of a circular cone, we will get some defined lines and surfaces depending upon the base of this pyramid **(Video Ends: 07:10)**.

So, with this method, this radial line method, we can actually develop the surfaces for pyramids and circular cones, which is what we will be seeing how to develop this. The next is the triangulation method, which is used for developing the transition pieces, for example, the base. **(Video Starts: 07:31)** This is where we will actually have to use this triangulation method, I will show you on how to draw that these pieces will be attached to the rest of the surface which will be generated for example, even in case of a cylinder.

So, in case of a cylinder, while we are rolling it over, this entire surface will be generated by parallel line method but these 2 bases which are here, they will be generated using triangulation method which we will add to the surface which is generated here **(Video Ends: 08:02)**.

And then the last method is this approximate method. Approximate method is used to generate the surface for a double curve.

So, single curved surfaces solids are like cylinders and cones and double curved surfaces are mainly spheres at different types different parts of spheres. So, it is very difficult to generate the exact surface of a sphere. And that is what you must have seen in globes when the globes are made. So, what happens? We actually have these smaller pieces and they are all approximated. So, it is an approximation method which is used here.

There are 2 ways of approximating the surface of a sphere and we will see both those surfaces. So, each of the part the surface for a sphere is assumed to be cut in a series of these cutting planes and each of this curved surface is called zone. So, we will see how these zones are made as we go on to take the examples.

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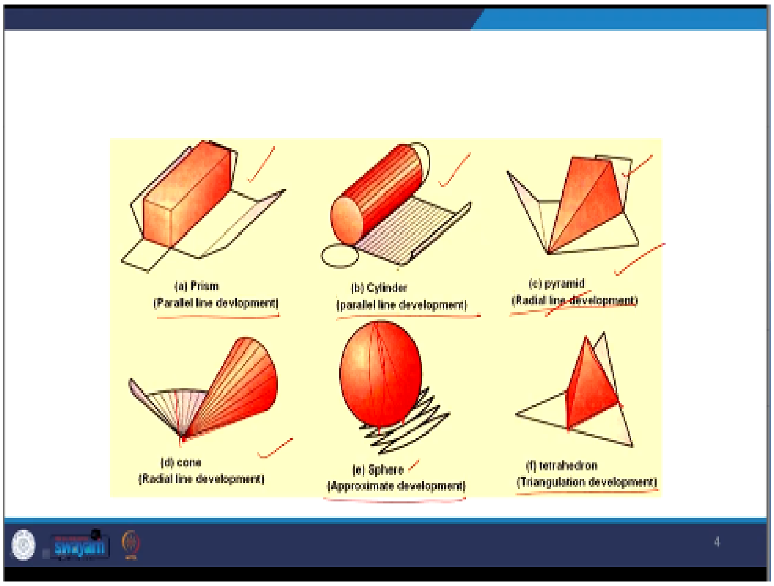


So, these are some of the examples just to explain how different types of solids and their surfaces will be developed. So, as I told for different types of prisms and cylinders, we will have parallel line development which is what we are doing here. So, these are all parallel line methods, the radial development is for your pyramids. So, this is a square based pyramid here.

This is a cone here. So, we just have these different lines which are actually converging at this point and this distance is actually acting like radius that is why it is called a radial line development. Then we have sphere where we have the approximate development. So, each part of the surface, sphere is actually assumed to be cut in smaller surfaces. And then when they are joined, they will approximately give us a shape of the sphere. It will not be accurate.

Triangulation method is used to develop solids like tetrahedrons and other platonic and Archimedean solids as well, in addition to developing the transition pieces like this and in case 2 different types of solids are converging together. So, we will use this triangulation method there. So, let us see each one of this different types of solids and you will know how to develop the surfaces.

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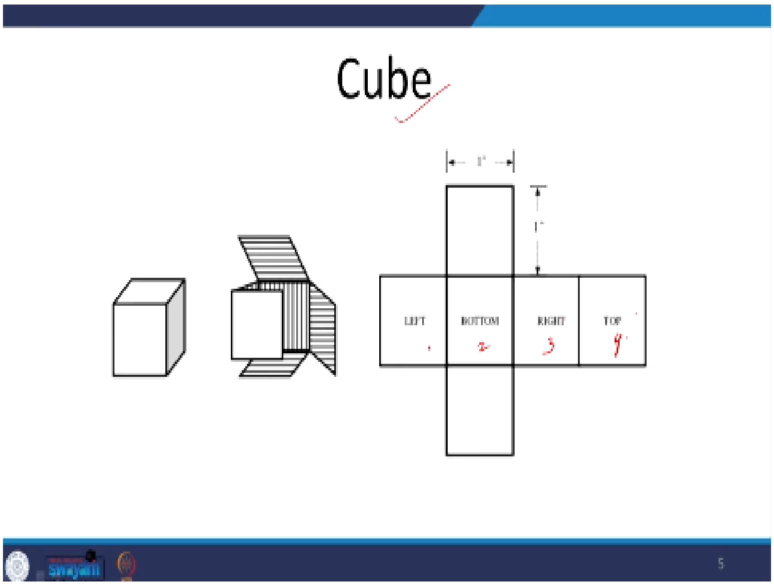


So, cube starting with a very simple one, which is cube. Cube is a very simple solid. All we know very clearly that it has 6 surfaces; all have the same size, which is a square. So, all that you have to do is you roll over the solid, this cube, so, you get 4 squares assuming the cube to be a prism. So, we are using this parallel line development method and any one of these pieces, which is on the surface, we will develop the top and the bottom surface which is again going to be a square.

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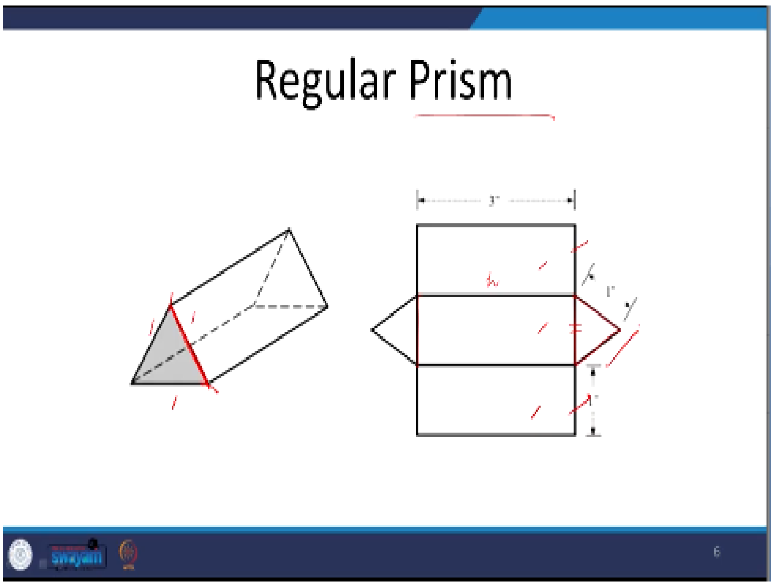

So, a very simple method for developing cube surface of a cube. Now, we have a regular prism. It could be any prison for that matter. So, what we have in this triangular prism, we have 3 surfaces and the size of each is equal to the side of the base. In this case, it is an equilateral triangle. So, we will have this side equal to the side of equilateral triangle and this is the height whatever height is given.

So, we will have 1, 2, 3; 3 rectangles and on anyone rectangle, you could be adding this triangle the equilateral triangle. It is not necessary to have it in the centre, it could be here or here anywhere it could be. So, we will just make this.

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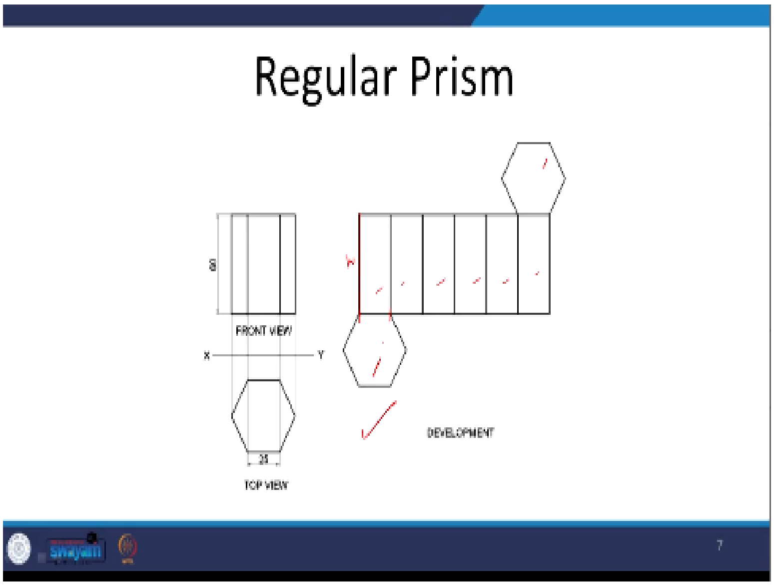


So, for any regular prison for that matter, it is the same process. Here, it is a hexagonal prism. So, what we are doing, the side of each rectangle is the same as that of the side of this hexagon and the height is equal to the height of the prism. So, as we roll over, so, we will have 6 surfaces rectangular surfaces and at the corners, we will have 2 base shapes, which is hexagon in this case. And you just have to remember that the sizes of this is exactly the same as we have drawn in the orthographic projections.

So, this is what we are going to be seeing from the top if the prism is standing in HP. So, this is what you see from the top and which is exactly the basis that we will be drawing here. This is the front of if any one face is parallel, which is what you will be drawing. So, it is going to be equal to the same surface which is the true shape. So, every time, we draw the surface, it is drawing the true shape of each face and putting all of them together.

So, we are just making all these true shapes of the faces of a solid continuously such that they are all joined to form a regular solid or proper solid shape.

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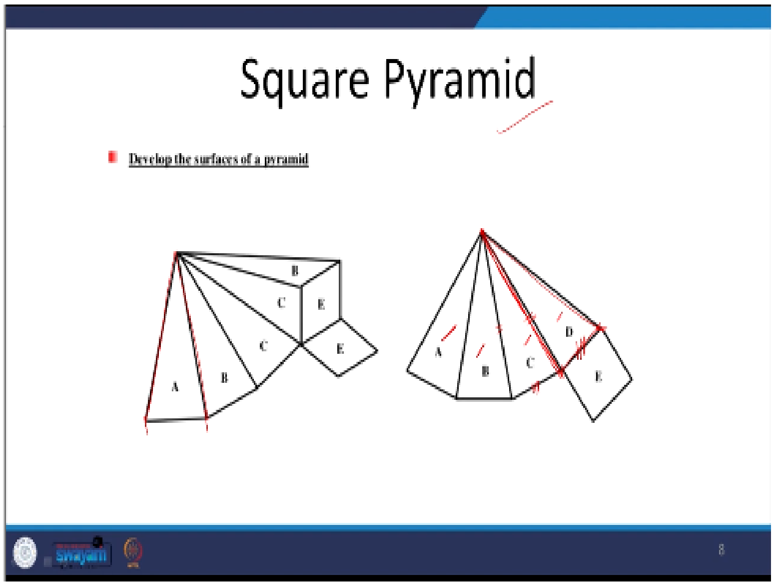


This is the example of a square pyramid. So, what we have done? This is just shown in 3D, but what we get here is 4 triangles. So, now, this is the radial line method. So, we have this one single point. Now, how do you draw it? In case of prisms, it was very simple. In case of pyramid, what we do? We have to know the true shape of the face. So, we have to know the slant height and for a regular pyramid, this slant height is going to be the same and we would know the base, the side dimension of the base side.

So, how do we start? We will draw the side and it is definitely going to be an isosceles triangle. So, we know the slant height. So, we draw an isosceles triangle here. The next triangle which we are going to make for that the apex remains the same. This is the side which we have here, we already know. This side is equal to this and this side is equal to this base side. So, all we have to do is; we already have the side.

Now, we will take this distance, make an arc and we will take this distance which is this, make an arc. So, we get another line and we get another surface. So, this surface is now exactly same as the surface and likewise, you will keep drawing these triangles. So, if it is a square, we will make 4 surfaces and at the end of one triangle, we will make a square because this edge is common.

So, keeping this edge common, we will just make 90 degrees and this square. And if you fold it, we will actually get a square pyramid. So, it is the same method that we are going to use for any pyramid.

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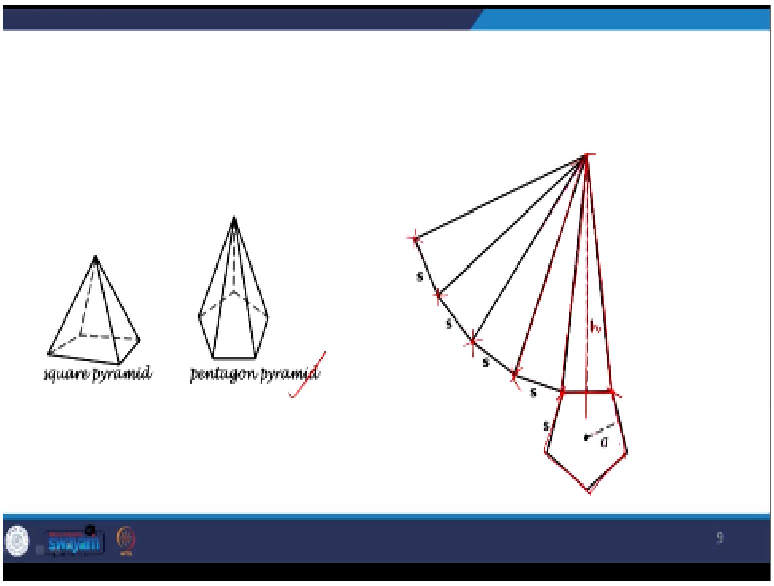


Let us look at another example. Here, I have taken a pentagonal pyramid. So, the pentagonal pyramid, again, what we are doing? We have the base. If you do not have the slant height, that is okay, you could also take the height. So, we draw the height perpendicular to through the base perpendicular bisector, take the height 'h' and join this. This is the true shape of the triangular face of the pyramid.

And then you do the same thing, you just repeat making the same triangle and so, we will keep getting. So, we will make 5 triangles like this and at one edge, we will make this pentagon which was already given to us. So, we now have this pentagonal pyramid, the surface of pentagonal pyramid. So, we can put it all together, we will actually get a pentagonal pyramid here.

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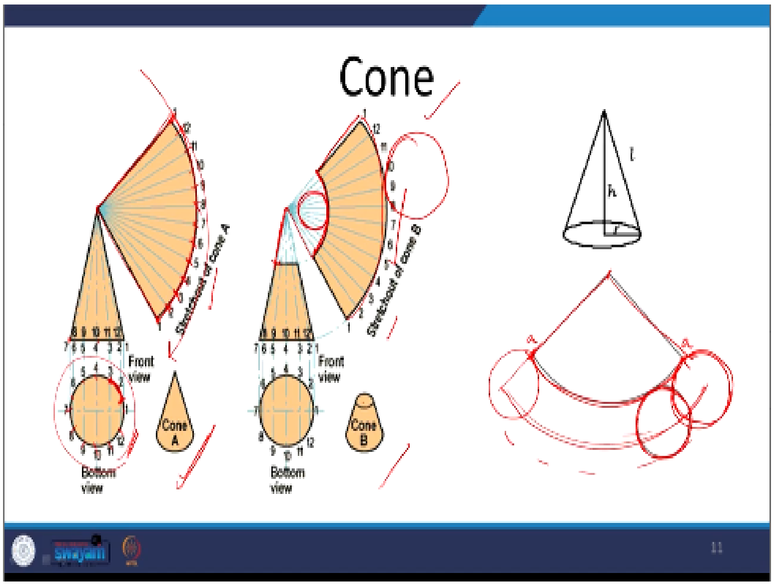


This is for a cone. Now, in case of a cone, we do not have these defined edges. We just have infinite generators, because it is on a circle, they could be infinite generators. So, what we do? We will follow exactly the same method as we have used for drawing cylinders and cones for the orthographic projections for solids as well as planes. We will divide it in 12 equal parts. So, what we will do?

We will just mark these points which are here. So, these are the 12 equal parts of the circle that we have. We will take the projections and then join them. So, instead of these very fine edges, which we will get in case of pyramids in cone, we will have to mark these generators. So, there are 12 generators. Now, what we have actually is this slant length, the slant length of the cone, which is what we will get from the front view when we draw the cone.

So, this is the radius of this surface. Now, what is the total length of this arc? It is equal to the circumference of the circle. So, what we will do? We will have to calculate. We will draw this circumference. The radius is this and you will get this curve here. Or, what we could do? We could just make an arc without calculating and we can just start cutting these 12 equal parts which is equal to this distance.

So, the length of this distance, which is 1/12 of the circumference for the circle, we will just mark these points 12 points and wherever it ends, we will just join this. This is for the cone and when we fold it, we will actually get the cone which is given in the question here. Now, if we have to draw and then of course, which is something which is missing here is a circle. So, at one end, it could be anywhere, we will draw the base circle, which is going to come here.

So, that is what we will draw. Now, one simple method, which is what we are doing here is also that this part is the locus of this circle. So, if I start, if I draw a circle here and if I rolled a circle along this path and we have we have already seen how to draw the locus. So, if I just roll the circle along this path such that this point say A comes back here as A. I will actually know that what is this arc which is going to be formed, which is what we can draw again.

So, this is equal to the circumference of this and we will draw a circle at the end of the span of cone here. Now, one more thing, which we are going to see here is what if it is a frustum. It is a cut cone. So, in that case, what we have to do? Nothing. We will start with the same process. We will draw the same and the only thing that we have to remove out of this is this part which is this part of the slant length and this is where we will draw.

So, if you have to actually make a cone out of it, this is where we will draw the base circle and somewhere along this, we will make the top circle. So, this top circle will be here and this bottom circle will come here. So, this is how you will make the frustum develop the surface for the frustum of a cone.

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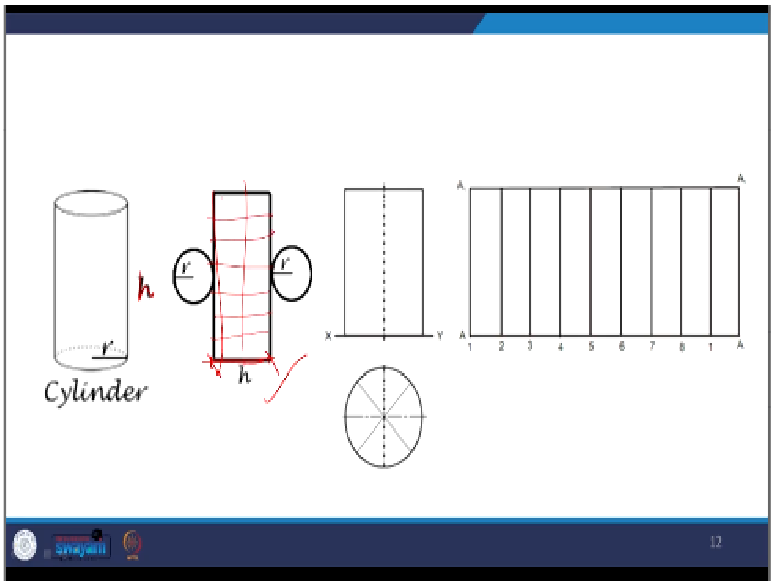

As we have already discussed, this is for cylinder. So, what we have done here is this is h so, do not get confused. This is not the height. This is the height of the cylinder and we are just rolling it out. So, it is actually again infinite parallel lines in case of a cylinder and at the 2 ends, we will have the 2 circles for the bases.

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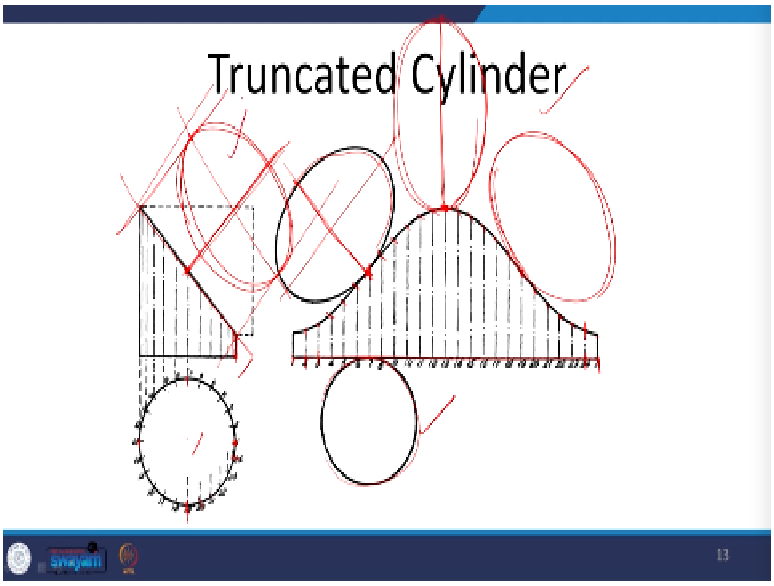


Now, what if we have to draw truncated solids, so, we have already seen sections of solids. So, here what happens if a solid is cut and we have to draw the surface of that solid as well, because majority of the machinery parts are truncated solids only. They are cut by planes or they are intersected with the front solids. So, how do we draw them? So, let us quickly look at some of the examples here and the fundamental remains the same.

And the methodology for arriving the true shape is coming from what we have already seen in the true shape of this section. So, we will borrow from there and we will develop what we have just discussed here. So, what we do? Suppose, there is a cylinder which is cut by a plane like this. Now, again, what we are doing is we will divide the circle into multiple parts. Here, instead of 12, this circle has been divided into 24 equal parts and all we are going to do is just open the surface up.

So, what we are doing? This is again equal to the circumference of the circle. And we are dividing it into 24 equal parts. Now, the height for each part. So, starting with this, so, this is for one and then 2 and 24 like this. So, for each part each of this parallel line, the varying heights will be marked. And when we mark these varying heights, once we join, this is the vertical part of the truncated cylinder and what comes in the bottom is a base circle, which is exactly the same as this, which we will mark here.

Now, at the top, we also have the true shape of this section, right. So, if you look at this from the side, so we know that this is actually we seen as an ellipse, but where will you draw this ellipse? If you have to fold this cylinder, where will you draw this ellipse? Can you draw this ellipse anywhere? Or how will it look like? So, wherever you can write anywhere, but then if you want to draw it simply, we will take it where this axis is passing through, we can take it through any one axis.

So, here what they have done is they have actually taken it through where the smaller axis of this ellipse is meeting. So, this is the point, this is 7, this is 7 here. And here, this ellipse has been drawn. It could be at the 19 also. We could have drawn the ellipse here as well. It would be exactly the same thing. And when we fold the cylinder, it will actually give us a truncated cylinder. What happens if I draw an ellipse here? How will I draw it?

If I have to draw it at either 13 or one, so I am taking it at 13. What happens? We are drawing about the longer axis, so we can still try it. It is just that the ellipse will now be rotated. So, we will actually have an ellipse the true shape of the ellipse drawn here. If we are drawing it here, it will be like this and then we can fold the cylinder about the ellipse, we can just stick it and that is what the final shape of this truncated cylinder would be.

So, I am not repeating how do you draw the true shape. You already know how to draw the true shape and the same true shape will come here, whichever way you want to put it.

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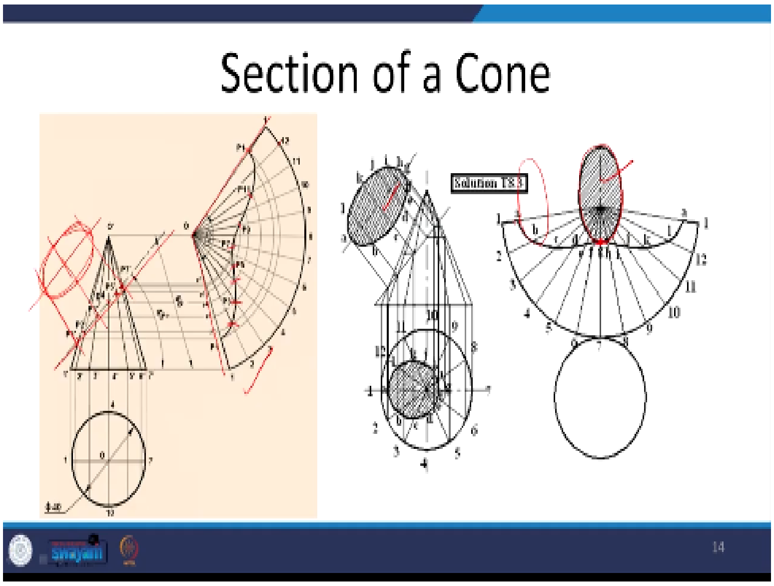


Similarly, if we have to draw develop the surface for a cut cone or section cone, so, exactly the same process will be followed here. Let us quickly look at this. So, what is happening? This cone is cut by a plane like this. Now, we have to develop. So, what we are doing? We will first develop the entire surface of the cone like we have just seen the original one, dividing it into 12 parts and now, along each generator what we have is; we just have to mark what is the slant length.

So, what we have is the slant height, which is what we are going to take here and draw at each point we are getting this slant height of all these various points, which when joined together, we will get this surface the bottom surface. But again, what happens to the top surface? If we look at this, we actually will see an ellipse coming here. This is what the conic section is going to be.

So, this true ellipse again, we will mark where the smaller or the longer axis for the ellipse is and you can see it here if it was drawn. Here, it is this ellipse. If it was drawn here, it would probably be this ellipse, but it will be difficult to make it here because we will not have sufficient sheet the paper or metal or whatever. So, it is put here. So, depending upon, it is the convenience which you have to decide, but then how the ellipse is going to be placed has to be seen.

So, if it is here, this is going to be the true shape of this cone is exactly the same. It is just the location which you have to decide and accordingly the orientation of the true shape of the section and in the bottom of course, you have the circle.

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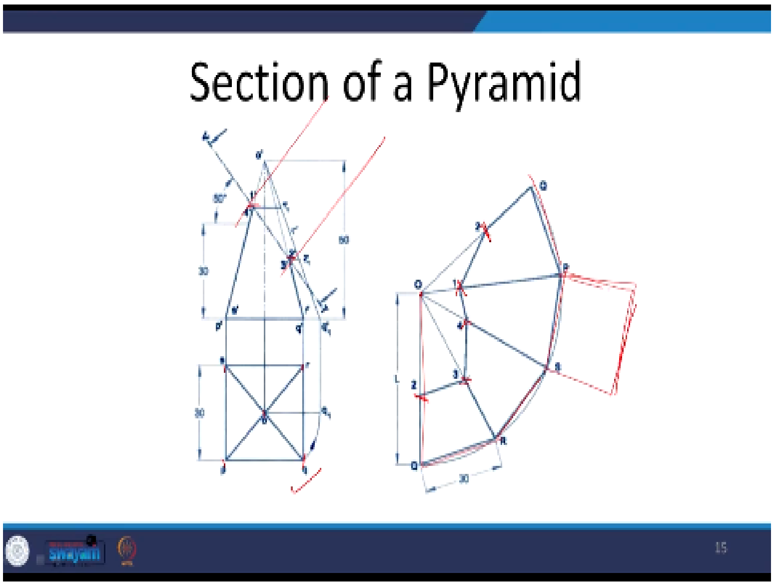


So, same process exactly the same process, which is a little complicated little difficult in case of a cylinder or a cone, but in case, it is a polygonal pyramid with a polygonal base. We will actually be having it much simpler. So, suppose this is a square based pyramid. So, what we have? We very clearly know that we just have these forced slant edges and the height of these edges is now marked here, 4 of these edges.

So, what we are doing? We are making the original pyramid and then cutting and marking these. This is what our bottom surfaces and exactly in the same manner, we will develop the true surface here somewhere and in the bottom also, we will have the square. So, we will have the square surface developed and in the top, we will have the remaining surface which is the true shape of this section developed and it will all be joined.

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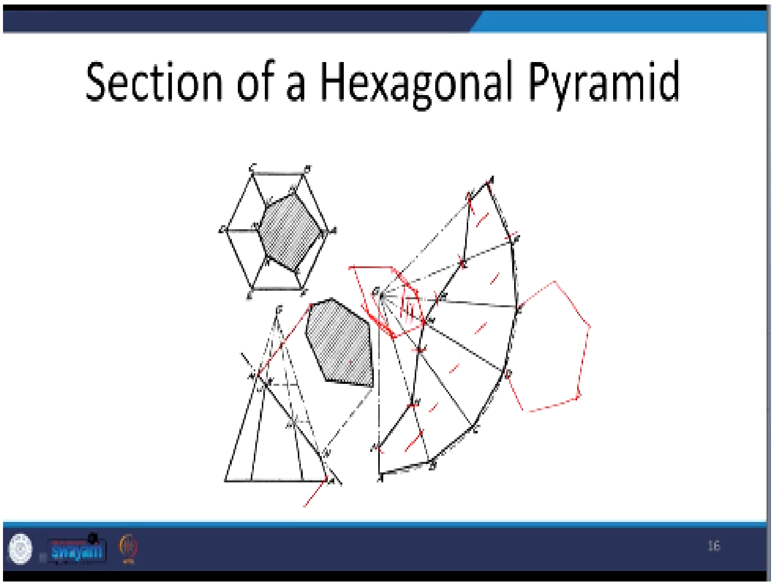


Very simple if you have already understood the process for cylinder and cone, making it for these different shaped pyramids is definitely easy. Same thing here, hexagonal pyramid, we have the hexagon in the base, same 6 edges, 6 equal surfaces, triangular surfaces, different heights which you will get here and in the bottom, we will have the hexagon and in the top, we will have this true shape.

Now, what we you have to remember? If you making it at M, this is going to be proportionally. So, this is what you will actually see or distorted hexagon, but then how and where are you placing it will determine how this true shape of this section is going to be drawn. And this is the base hexagon which will still remain the regular hexagon.

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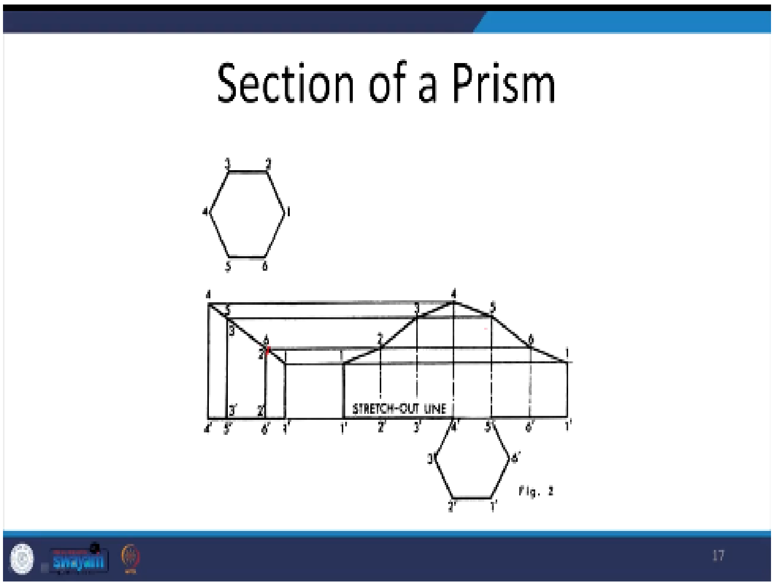

So, this is section of a hexagonal pyramid. This is again, this is for section of a prism which is relatively simpler because in this case again, process is the same. It is just that we are going to get the parallel lines here. And in the previous case, in case of a pyramid, we were getting the radial lines.

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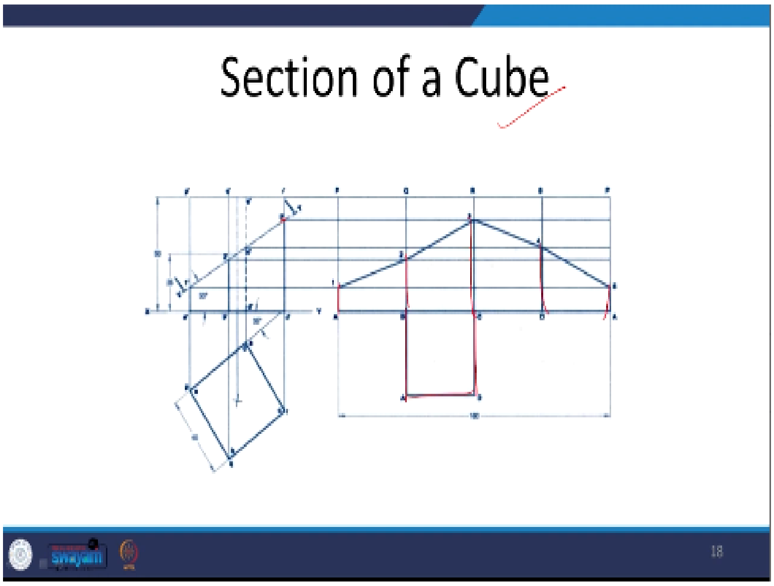


The process will remain the same. Same is for this cube, you cut it and then you get the different heights which you are going to get unfolded have these different heights coming and you have the base. Again in the top, you will actually be taking the true shape of the section and we join the true shape of this section however, it comes.

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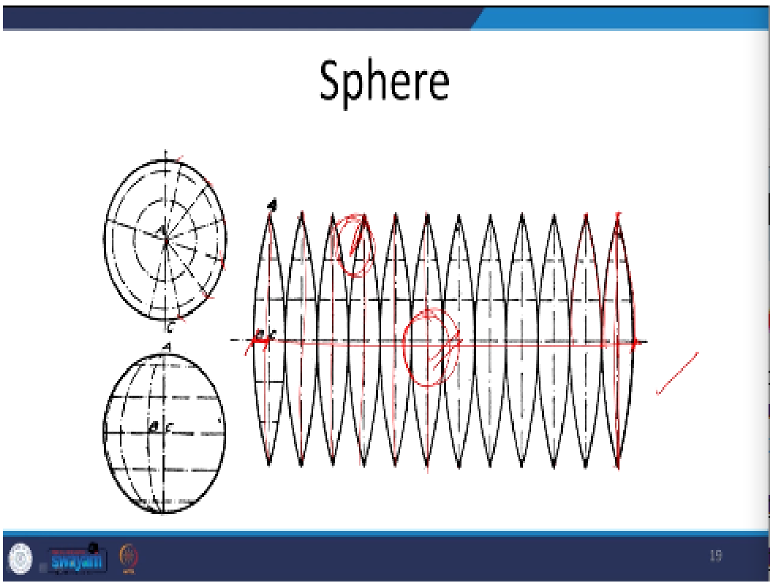

Now, we come to the sphere, which is where we will use the approximate method. There are 2 ways of drawing the surface of the sphere. In this one, we are cutting it longitudinally. So, you all know about longitudes and latitudes. So, longitudes are what are connecting, they are imaginary lines which are connecting the north pole to the south Pole. So, we just imagine that this entire sphere and we will again divide the sphere into whatever number of equal parts 12, 24 depending upon the convenience, depending upon the size of this sphere.

So, when we roll it over; when we open it, so, what we have? We have this diameter, diameter of the sphere. So, this line is equal to diameter of the sphere and at each end, what we are actually going to get is this single point. So, all these points are going to converge at this single point and if we draw perpendiculars at each regular interval, so, if this sphere is divided into 12 parts, we will divide it into 12 parts such that the last 2 parts are the half of each part.

So, we have these 12 parts, which are the perpendiculars here and the total height of this is going to be equal to the slant height of the sphere and then we will just join them. So, this is approximate method. So, this when we join will all converge in one point on the top and another point in the bottom and it will all come together as a sphere. So, when we make maps, when you have a world map and you see it as a flat, it is actually not a flat thing, it is actually this approximation method only.

The earth cannot be represented as a rectangle, which is what we see in a map. So, that is why if you have read the countries which are closer to equator and the area which is shown in the world map is actually larger than a country which is represented here on the top and if it is shown as the same area. In reality, in actual on ground, this area is much lesser than the area here on equator, because this is actually the real area, this is the actual area.

While this is just a visualisation, the country is actually much smaller and you do not see those lines being marked on a world map. So, which is this is the method which they follow when you are making the world map.

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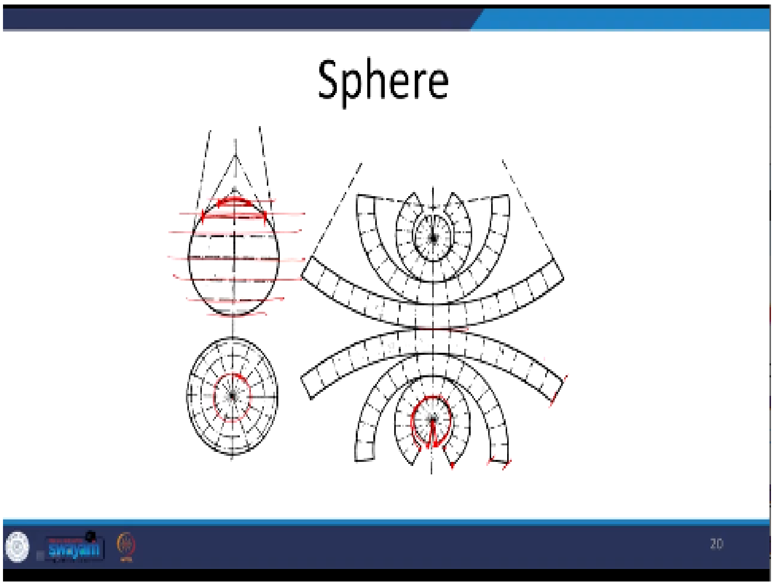


The another method for drawing sphere, this is also an approximation method is when you draw when we take out the horizontal strips of the sphere out. So, what we are doing here is, we divide the sphere into these little horizontal strips and these strips are opened up here. So, we have one part if you look at it from the top, we have this one part which is the circle and the circumference here if you look at this, this is a reduced diameter.

So, what we will take? We will actually be taking this is here, this is the top of the circle. So, this reduced diameter is what we will use to calculate the actual circumference. So, this circumference is different from the circumference, which we will actually be seeing from the top. So, some portion will be cut and left. Similarly, others so, there will be different circumferences.

So, this circumference is equal to this part, this one is arrived at using this diameter and likewise, so, we keep opening up the horizontal strips of the sphere and then we can put them all together. We can join all of them together to give you another approximate shape of the sphere. In this one, we are cutting it horizontally like with latitudes and in the previous one, we were cutting it along the longitude.

So, these are the 2 methods, approximate methods, in which we can develop the surfaces of the sphere. None of these 2 methods will give you the actual surface of the sphere. It will be approximate. It will be close to the real one, but it will not be the actual. So, you have to keep that in mind. While for all other solids, you would get almost the same surface developed as the solid itself.

So, it will be actual one except for the sphere here. So, that is all in this lecture today about development of surfaces. In the second lecture, which is continuation of the same topic development of surfaces, we will be looking at how to develop the surfaces of platonic solids and Archimedean solids which is what we have introduced or I have explained to you what these different solids are right in the beginning of our lecture where I was introducing different types of solids.

So, there are several places in engineering and architectural practice where you would be using these platonic and Archimedean solids. So, we will see how to develop the surfaces for those as well tomorrow. So, thank you very much for being with me here today. See you again tomorrow. Bye, bye.