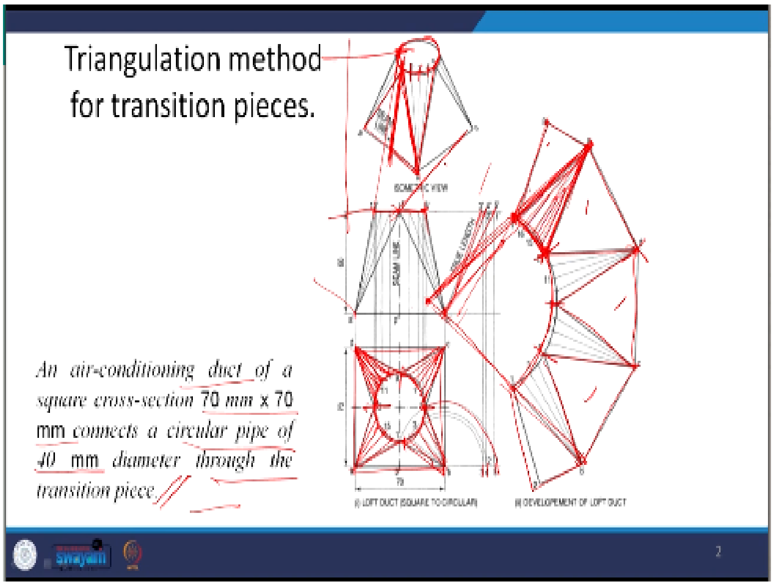
Engineering/Architectural Graphics – Part 1
Orthographic Projection
Prof. Avlokita Agrawal
Department of Architectural and Planning
Indian Institute of Technology – Roorkee

Lecture – 38
Development of Surfaces – II

Good morning. Welcome to the third lecture of this week for the ongoing online course on architectural or engineering graphics. And I am continuing with the yesterday's topic, which was development of surfaces for solids. And in the previous lecture yesterday, we had already looked at how to develop surfaces of various types of regular solids, upright right angled regular solids and I was discussing about various methods.

One thing which was left for me to discuss there, which I am starting with the lecture here is the triangulation method of transition pieces. Now, what these transition pieces are? We will just look at with an example here.

(Refer Slide Time: 01:12)



So, just imagine that there was an air conditioning duct of a square cross section 70 by 70 mm and it connects to a circular pipe of 40 mm diameter through our transition piece. Now, if we have to design that transition piece and I mean, just before we start to draw, if you have seen those air conditioning ducts. What are those air conditioning ducts made of? They are made of thin metallic sheet. So, this sheet is folded, okay.

Now, this duct, they come in standard sizes, which is what seeing here. So, this duct of 70 mm by 70 mm square cross section would be a standard duct. This pipe of 40 mm diameter would also be a standard pipe. How do we join them together? So, this transition piece will be made by using a plate which will be folded. This has to be designed. This is where the development of surfaces is actually required.

So, what do you do so? We know that we actually have a bottom square, which is where it connects to the square part of the duct and then we have a circular piece which is 70 mm. So, how will it be? So, we will actually have 4 triangular pieces, which will be joining like this and in between, we will have these conical parts of cone, conical pieces, which are going to be drawn here.

Now, this method, this triangulation method is actually a combination of the radial line method, which we have seen earlier and the so, this is basically a combination where we are doing for cone and pyramids, different polygonal pyramids together using the radial line method. So, what we do here is? We will start by drawing. So, depending upon where the stitching is going to happen, where the sheet is going to be put together.

So, we've assumed that here is this seam line where the sheets are going to be stitched. The seam is going to come. So, if we look at this, we will have this, the half of this triangle. Now, for this, we have to look at the actual height actual slant height of this. So, which is what we are going to get in the elevation. This is slant. So, you have to determine the true shape of this triangle.

So, the true shape of this triangle will be determined. So, which is how they have taken the true length. So what are we doing here is; we are very simply, taking the radial base and then projecting it above to get the true length or what you could also do is you could just project and you could make a face parallel on an auxiliary plane parallel to this face to get the actual shape of the triangle. So, if I projected here and I want to get the actual shape of this triangle, we can get it here.

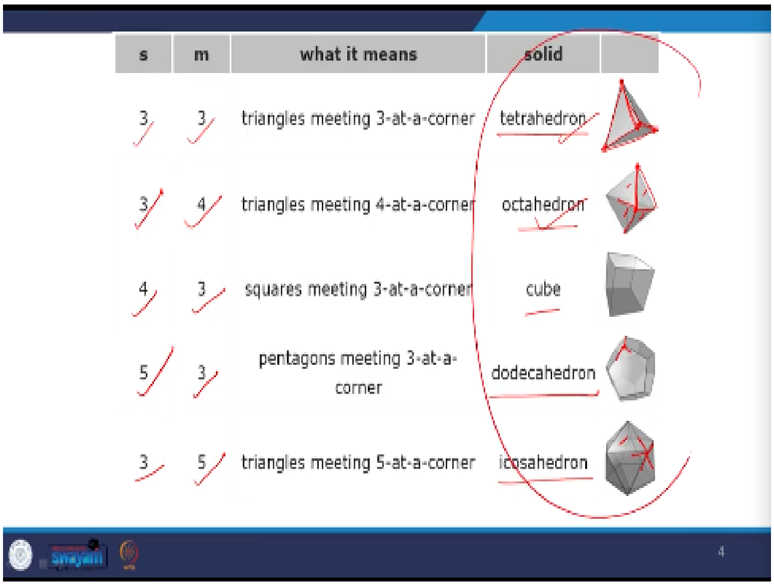
So, either ways, whether you arrive at the true length like this by taking the radial length and then projecting it along the same height or if you take it on an auxiliary plane either ways is a correct method. So, we draw half of this triangle here. Now, what we next have is; we have this so, we actually have this piece which is coming here. The fold this piece actually what we have is if this cone was extended. So, the cone which is coming here, if this cone was extended up, so, this apex point was what could have created a circular cone.

So, what will actually happen is that this part of this sheet development, well actually we developed like a radial line. So, we will develop this. This is the part which is actually getting cut, which we get from here and this is the rest of the part and this circle, the circumference of this circle is going to come here. So, we are then divided into 12 equal parts. Only the top the circle, we divided into 12 equal parts, 4 parts, for 4 full of parts.

So, we just connect to the 4th part, this is the part we get and now, we have we know that this is where the triangle is going to come. We already have one of its sides on taking this, we draw the rest of the triangle. Again, you connect, this you get the rest of the triangle. You connect this and you get rest of the triangle. This is the triangulation method, where we are arriving at these transition pieces using a combination of these 2 methods.

So, this is how the development of surfaces is fairly important when you are drawing machines and when you are drawing different parts in engineering and architectural structures. I will now move on to surface development for platonic solids. So, if you remember, there were 5 platonic solids that we have discussed about.

(Refer Slide Time: 06:51)



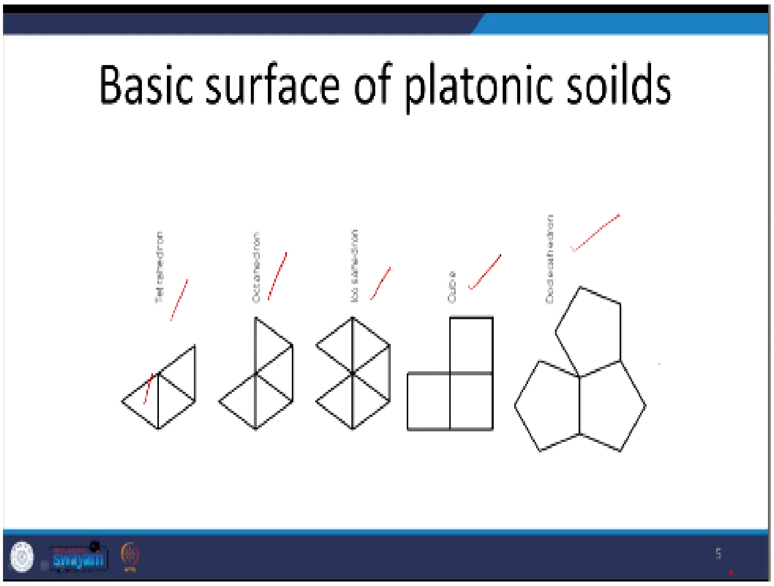
So, the first one is a tetrahedron. What is it the tetrahedron? We have at each point, we have 3 triangles coming together at each of these points. So, here also we have 3 triangles, here also we have 3 triangles, triangular surfaces. So, what we have is for each surface, we have 3 sides and the number of sides, the number of faces is also 3. So, that is tetrahedron. Octahedron is where there are 4 triangular surfaces meeting at each point.

So, at each point, there will be 1, 2, 3, 4; 4 triangles surfaces. The side for each surfaces, 3 number of sides, but the number of triangles meeting is 4 that is your octahedron. It is like 2 tetrahedrons, not exactly 2 tetrahedrons but it is like repeating this coming together, then we have cube, of course, simple 4 sides and 3 surfaces coming together. Next, we have dodecahedron where the base shape is a pentagon.

So, we have 5 sides on each surface. And at each point, we have 3 of these surfaces meeting. So, you just have to keep all these in your mind before you start to draw. And next, we have an icosahedron which has at each point, we have a triangle meeting. So the number of sites for the surfaces 3 and at each point if you look at it here, we basically have 5 triangles meeting.

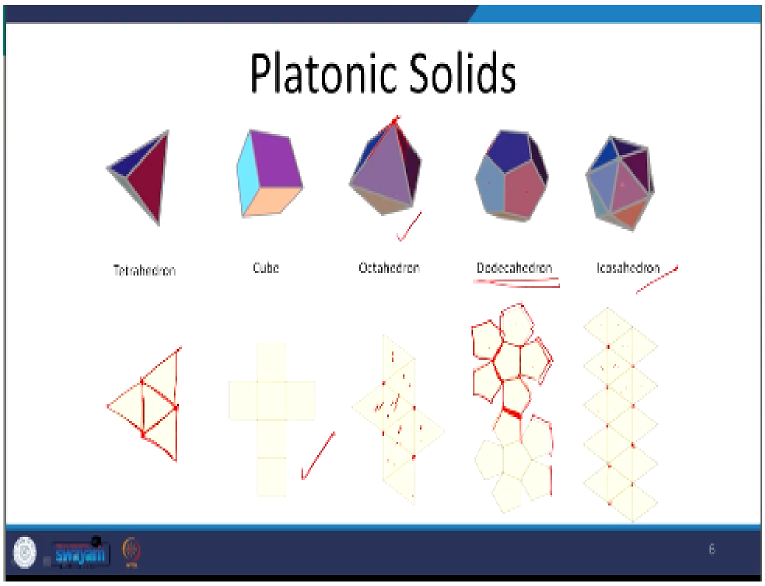
So, we have 5 surfaces meeting, 5 triangular surfaces meeting at each point. These are the 5 platonic solids that we have.

(Refer Slide Time: 08:38)



Now, when we go on to develop the surfaces for these platonic solids, we know very clearly what each what is the generating surface. So, in case of tetrahedron, we basically have triangle equilateral triangle octahedron, also an icosahedron. Also for cubic square and for a dodecahedron, it is a pentagon.

(Refer Slide Time: 09:00)



Let us see how this unfold. So, if it is tetrahedron, we basically have a triangular surface. It is connected to another triangular surface, another and another. There are 4 triangular surfaces, each connected. So, at each point, if I look at this, so we basically have these 3. So, if we fold it like this, at each of these points, we have basically 3 triangles coming together. For cube, we have already seen. It is done. It is one of the simplest platonic solids that we know of.

For octahedron, what we know that at each point, we basically have 4 triangles coming. So, at this point, I have these 4. At this point, I have these 4. At this point, I have these 4. At this point, I have these 4 and when I fold it together, if this one goes to the back. We will have at each point, we will have 4 of these triangles meeting and that is what an octahedron would be. So, these platonic solids are very simple to draw unlike Archimedean solids.

When we come to Archimedean solids, then we will see. So, dodecahedron, we basically have a pentagon. And we will just draw another pentagon at each of the side. So, that is how that is simple drawing procedure, we do not have much difficulty in that. And once we have this, so, if we are looking at it from the top, so we basically have 5, 5 petals and one top and the similar thing is repeated at the bottom.

So, we have exactly the same shape just that it is connected, it would be anywhere. It is connected and then we have the same shape repeated. So, basically in a dodecahedron, we have 12 pentagons coming together. If you look at this solid **(Video Starts: 10:57)**, which is here in my hands, we have developed this dodecahedron using the pentagons. So, the base surface is pentagon.

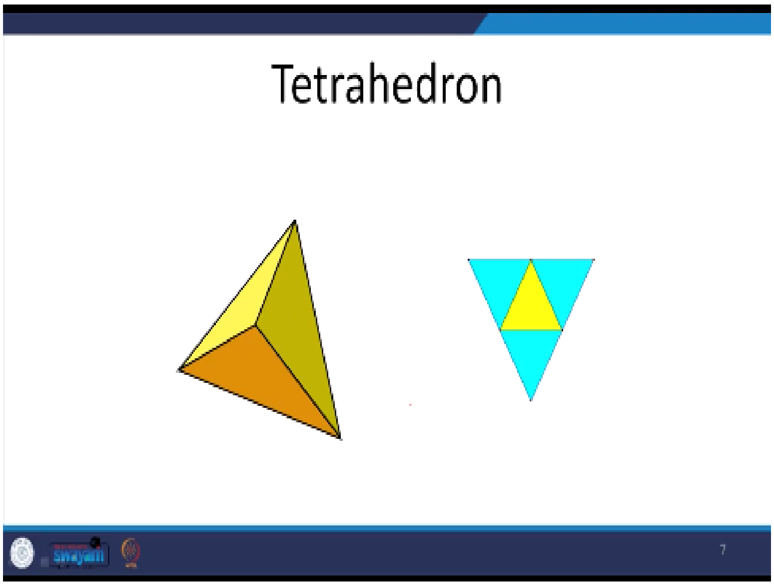
So, we started drawing this pentagon and you keep developing the pentagonal surfaces and once you put them all together in case you have to develop them right now, it is only what will be seen on this surface. If you have to make space for sticking it together or welding it together, you have to have some little extra pieces, which is what we will draw along. So, we know how these surfaces are going to be joined.

So, probably, we will have these small jetting out surfaces which will be drawn along. So, that is what we will draw when we are actually deciding to develop this into a solid form. So, these additional stitching for welding surfaces will be drawn. So, these will be very thin strips. So, you might not be able to see it of course, after joining we will not be able to see it. But then if we open it up, if I open it up, we will actually see that some of these surfaces are going beneath. So, if you if you can see it here now, so, if you see it here, after the pentagon, some small folding surfaces have been provided, because this was a paper, we just had to glue it.

So, if you can see, at each end, you actually have these small thin surfaces which are there and then they will just be put inside and they will be concealed such that on the top, you will only be able to see the pentagon. So, that is how the dodecahedron will be drawn **(Video Ends: 12:59)**. The icosahedrons, again, it has triangular faces. But at each point, we have 5 triangles meeting. So, in this one, we had 4 triangles coming together.

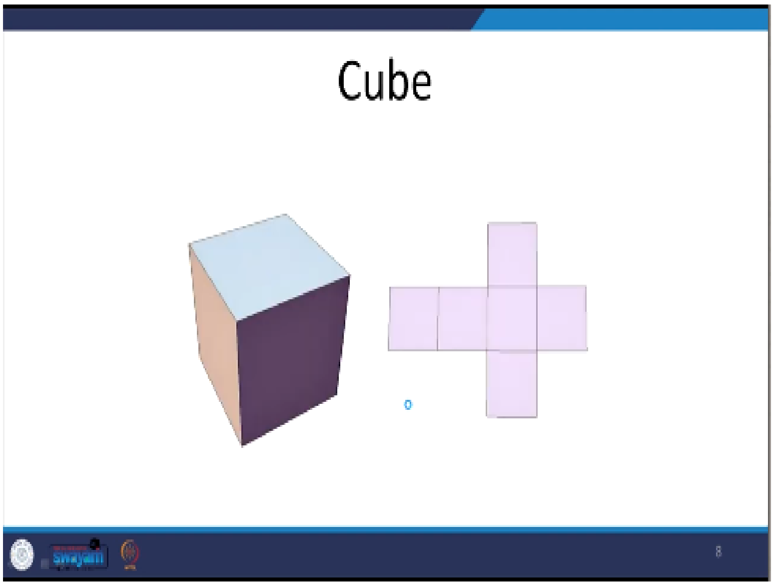
Now here, if you see 1, 2, 3, 4, 5; again, 1, 2, 3, 4, 5; here 1, 2, 3, 4, 5 like that. So, at each of these points, it is 5 triangles coming together. So, if we just start to fold it and join it together, we will get 20 surfaces of icosahedron and all being equilateral triangle.

(Refer Slide Time: 13:34)

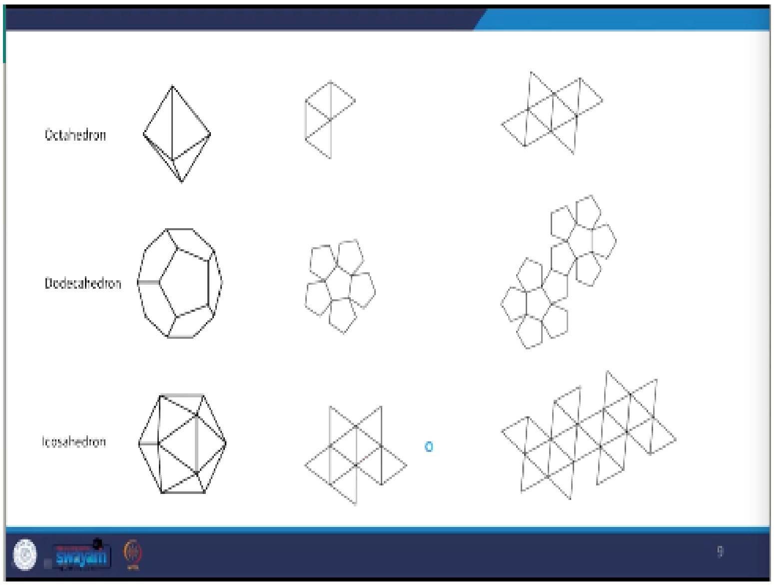


So, drawing for platonic solids is very easy. It is very simple, we just have to make exactly the same size of shape. So, same size of the face that we have. The moment to make it a little more or less, we will not be able to stick it together properly.

(Refer Slide Time: 13:53)

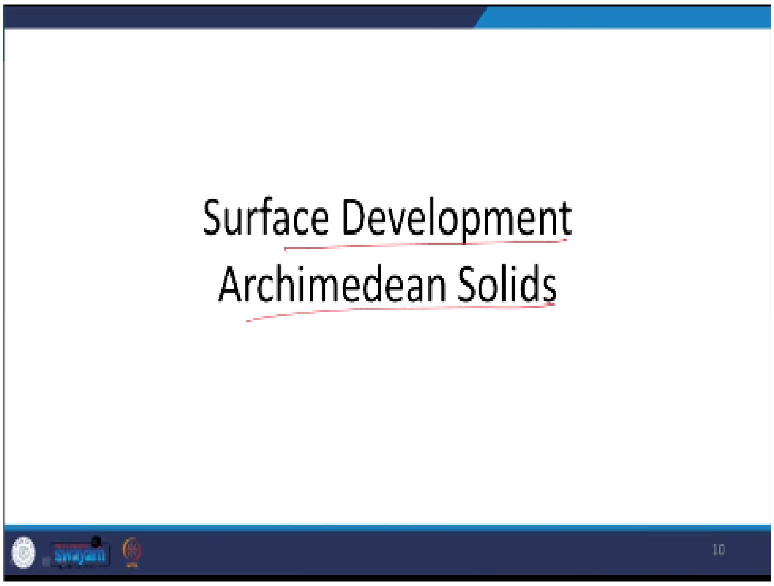


(Refer Slide Time: 13:54)

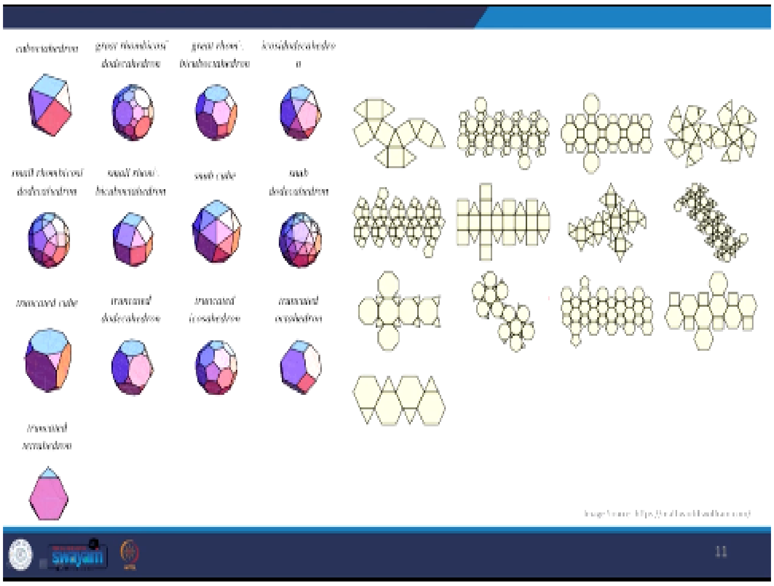


So, cube is simple. I have already shown you how to draw the octahedron, dodecahedron and icosahedron. So, we can arrange them whichever way you want. But all of them have to meet together and you have to keep in your mind the original shape, the final shape of the solid that you are going to develop. Now, we will come to surface development for Archimedean solids.

(Refer Slide Time: 14:18)



(Refer Slide Time: 14:22)

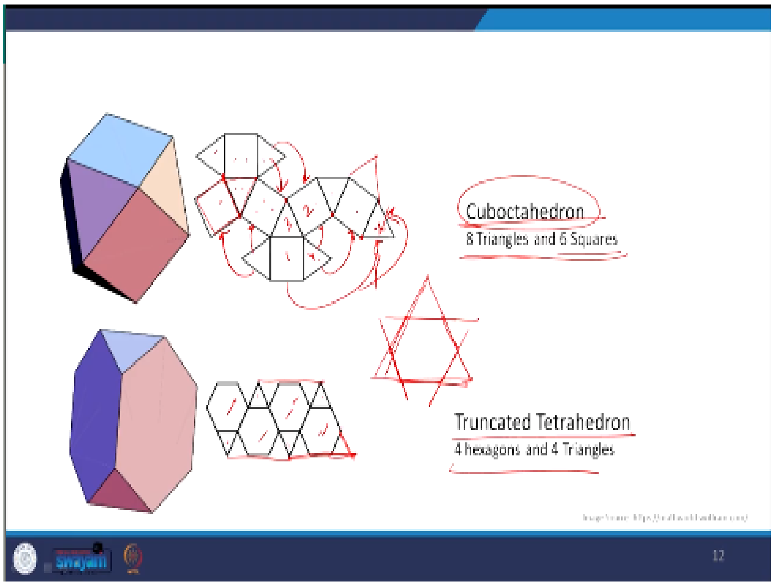


Now, if you remember, there are 13 Archimedean solids that we know of. And these solids, they again follow some order. So, there is an order just that there is not just one single shape. In platonic solids, we just have one single shape which is repeated and it follows some rule that at each edge, there will be 3 surfaces meeting or 4 surfaces meeting or 5 surfaces meeting. So, that is what gives us these 5 platonic solids.

Now, what happens in Archimedean solids? If you remember, we have 2 different shapes, 2 or more different shapes which are coming together but the rule remains. So, either if we look at each of these Archimedean solids, we can figure out a rule, how these solids will be formed. So, we have this 13 Archimedean solids, let us go over each one of these. If you go to read about these Archimedean solids and you remember what are these Archimedean solids made of like what shapes, then you can draw them conveniently.

So, the first and the only important thing is to know what each of this Archimedean solid is made of. So, we have this cube octahedron which is made of 8 triangles and 6 squares.

(Refer Slide Time: 15:44)



So, what we have is that at each point, we have 4 surfaces meeting and these first out of these 4 surfaces, we have 2 squares and 2 triangles equilateral triangles. So, when all of these come together, what we get is actually a cuboctahedron. So, overall, we have 14 surfaces only here. The development process is again very simple. You can start from anywhere, maybe, we have started from this square and then you just keep drawing the remaining surfaces.

The only thing that you have to remember is at each point 4 surfaces should be coming. So, here and this surface will come and join. So, we will have this one going here and this one coming here. Again, here we have 1, 2, 3 for this point, this one goes here and this surface goes here and joins. For this one, we have 1, 2, 3 and this one will come and join not here, but here like that.

So, we will actually have the overall system now. Sometimes what you may actually get confused with is where to draw the next triangle. So, instead of this, if you had drawn this triangle, probably this would go wrong. So, every time we make it, we have to remember where this is going to go and meet. So, we know that this is going to go and meet here and we have to keep imagining how the solid is finally coming up.

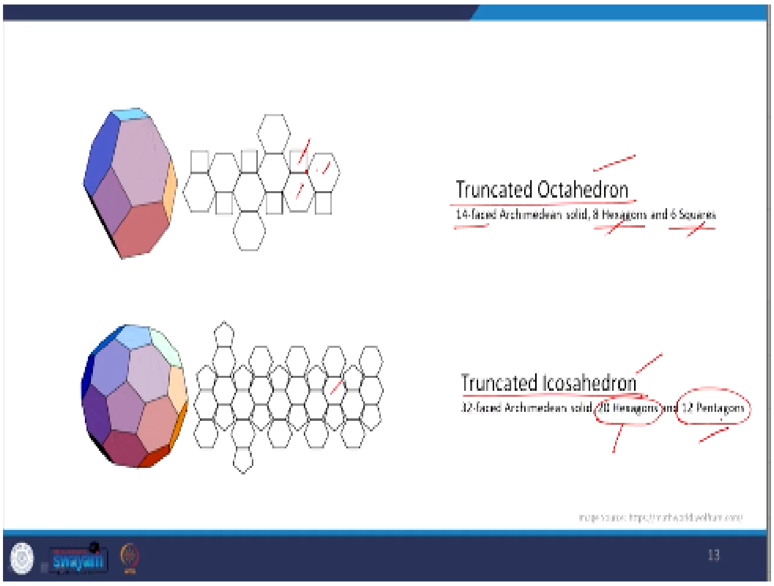
So, this is for cuboctahedron. Then we have a truncated tetrahedron. What a truncated tetrahedron is? You remember tetrahedron, it has 4 triangular surfaces. Now, if I chop off the triangular surface, each of the pointed edge of the tetrahedron, I will get a hexagon there. So, what I will get? I will actually get 4 hexagons, because it has these 4 surfaces, 4 triangular points.

And so, what we are getting here is these 4 hexagons and 4 of these triangles. What size? Totally depends upon how you are cutting it. Right. So, for that, what you really have to do is, suppose I imagine that this is the tetrahedron that I have. And now, if I want to cut it like this, so this is where I am going to cut it. So, I will get the shape that I am going to see as the hexagons.

And I will also get the remaining portion which I will be seeing as the triangle which is the remainder of it. So, we have to first calculate approximate what is the size of this that we are going to get and then we will be able to join it. So, this is for truncated tetrahedron. It is simple; it is not very difficult. Now, if you look at this, you can extend this and you will get the same triangular faces as we were getting right.

So, it is exactly the same. If you look at how tetrahedron was drawn and now, this has just been truncated. So, that is what we are seeing here.

(Refer Slide Time: 19:11)



We have a truncated octahedron. So, again, if you remember how the octahedron was being drawn and what the shape of an octahedron was and at each face, if it is again a triangular surface that is coming together. So, if we chop it off, now in an octahedron, we actually have 4 triangles coming together. So, if I cut it, I will actually get a square there. And the remaining portion of this triangle of face has become a hexagon. So, which is what we are going to see again.

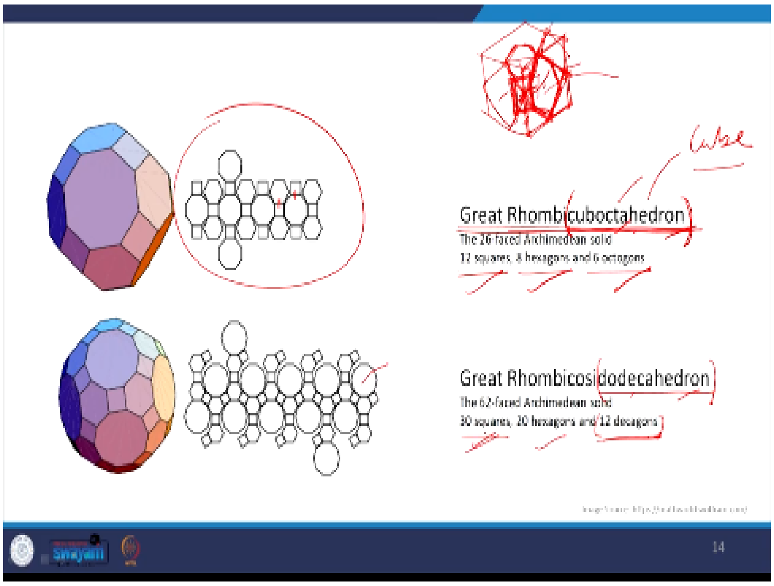
So, again at each point what we are getting? There are total 14 faces for this truncated octahedron, 8 as hexagons, 6 as squares and at each point, we are going to get the 4 surfaces again. They will be meeting. They will be meeting together such that 8 hexagons and 6 squares come together. The truncated icosahedrons: What was icosahedrons? We again had, we had 20 triangular surfaces equilateral triangular surfaces such that at each point, we have 5 triangles coming together.

Now, if we chop off, if we truncate each of the point on icosahedron, we are going to get a pentagon. So, instead of the triangles now, we have the pentagon, which is joining all these 6 surfaces. And the triangle has now been truncated to result in a hexagon. So, it is doubled up because we have chopped from 3 sides, we have truncated from 3 sides, and the filler surface is a pentagon.

So, originally it was icosahedron is 20 faced solid. So, finally, what we get is these 20 hexagon, which is resulting from the 20 equilateral triangles, which we were drawing and the rest of the filler ones are these 12 pentagons. So, if you remember how to draw icosahedrons, from the same thing, we will be deriving how to draw the truncated icosahedron. **(Video Starts: 21:32)** So, if you look at this, this is the truncated octahedron, which is what we are going to get. So, we see the hexagon shapes and the 6 squares.

And if you open it up in this fashion, which is shown on the screen here, we will actually get a solid like this. So, this truncated octahedron looks like it has been derived from a square but it has actually been derived from an octahedron **(Video Ends: 21:58)**.

(Refer Slide Time: 21:58)



So, what is this great rhombicuboctahedron? This is further truncation of cuboctahedron. So, cuboctahedron was derived from a cube when we were actually cutting a cube when we were truncating a cube. So, we were truncating it like this. So, when we cut the cube like that we were getting triangular surfaces and here we were getting these square surfaces on the faces that is what a cuboctahedron was. Now, once we have this cuboctahedron, further these points that we were getting, which had 2 triangular surfaces and 2 square surfaces.

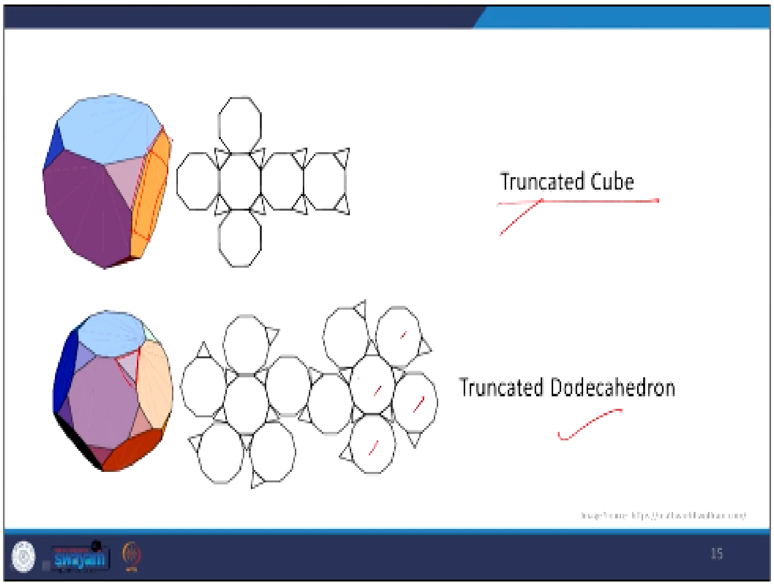
If we further chop them off, so, what we have is we have the square which is filling it up, then we have these octagons and this place which was the triangle, we will get a hexagon. So, we have rhombicuboctahedron. We have 12 squares, 8 hexagons and 6 octagons. So, if you derive it from the cube, the square which we originally had, so, the square face another square inscribed in that square was what the square size for cuboctahedron was.

And the triangle made with the side of the equilateral triangle made with the side of that inscribed square was for cuboctahedron. Now, when we are talking about this rhombicuboctahedron, we are looking at another square which is further truncated in such a manner that we derive or hexagon we inscribe a hexagon within the square of the cuboctahedron and this is the side for square which is the same.

If you look at it here, this is the same side for each one of these. So, we get 8 hexagon, 6 octagons and 12 squares to develop this cuboctahedron. Similarly, if you look at this, this is the dodecahedron. So, we know how to draw dodecahedron which is coming out of the pentagon. So, now this pentagon is again chopped. So, what we get? We get 12 decagon where this is actually a 10 sided polygon.

The triangles are now turning into hexagons and the remaining surfaces which are like these are again giving us squares. So, this rhombicosioctahedron is from the truncation of dodecahedron.

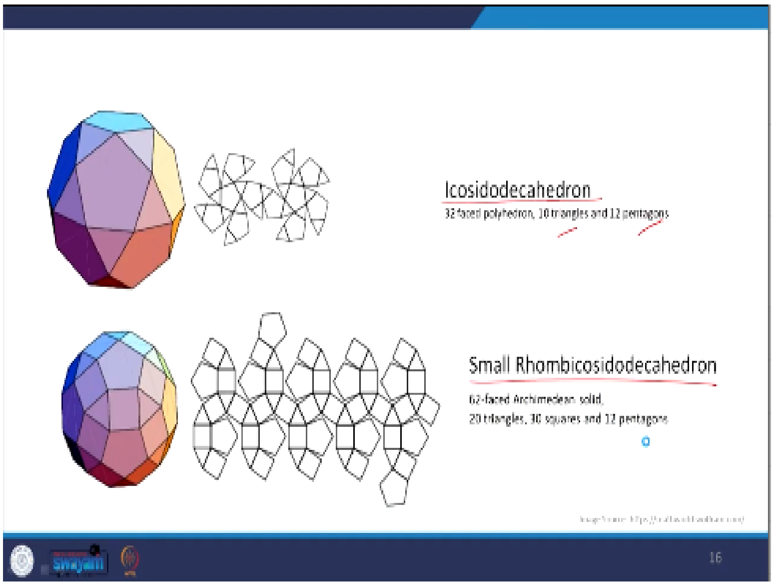
(Refer Slide Time: 24:54)



Truncated cube is simple. So, we are just truncating the cube in such a manner that it is not. In cuboctahedron, we were truncating the cube such that it was giving us a square. Here, it is giving us an octagon. So, if you look at this so, this is actually a truncated cube. In this, we are truncating the cube in such a manner with these planes, 8 planes in such a manner **(Video Starts: 25:25)** that at each surface we are now getting an octagon.

So, what we will have? We will actually have 6 octagons and we will have 8 triangular faces. So, that is what truncated cube is **(Video Ends: 25:43)**. When we will have truncated dodecahedron, so, we will truncate each edge in such a manner that we get only a triangular surface. So, this is a smaller truncation and at each of the pentagonal surface, we get a decagon. So, this is what your truncated dodecahedron is.

(Refer Slide Time: 26:03)



Similarly, we have other Archimedean solids as well, where we have the truncations happening in defined manner. For example, icosidodecahedron has triangles and pentagons only unlike rhombicosidodecahedron where we had scores hexagons and decagons. Now, if you look at how each of these are derived, it is dependent upon how the surface is being cut. So, this is the dodecahedron. And if I cut it in such a manner that **(Video Starts: 26:45)** after a truncation of this pentagon, we will still get a pentagon.

So, if we cut it like this, that we still get a pentagon, so, half is cut. So, you will still get a pentagon and triangles here. However, if we cut it in such a manner, if the truncation is so small that we get a 10 sided, so, if we truncate it like this, if this is how the solid is going to be truncated. We will get truncated dodecahedron. Further if we truncate it, we will get great rhombicosidodecahedron.

So, there are basic families of Archimedean solids which we get. So, we are deriving these out of cubes. We are deriving these out of tetrahedrons, octahedrons. So, from the same platonic solids, we are deriving the families of Archimedean solids. We just have to know how these solids are going to be derived, how the truncation is happening; is the truncation happening in such a manner that we follow a pentagon here, if I truncate it in such a manner that I get another pentagon.

This is also one way of truncation or I could get a decagon. This is another way of truncation. So, depending upon how the platonic solids are being **(Video Ends: 28:16)** truncated will tell us what kind of Archimedes solid are we driving and we have to be very clear about how each of these solids is coming out. Read more on Archimedean solids. This is just an introduction and if you are clear which family is this.

So, either it is a cube family or a tetrahedron family or a dodecahedron family or an icosahedron family. We will be getting 3 or 4 solids depending upon further truncation. So, this is how the fundamental of drawing developing the surfaces it remains the same. We just have to draw these in such a manner that the number of surfaces coming together at each point, it remains the same as per the rule.

The sizes of these are same throughout the surface which has been developed. And in this manner, we will probably be able to draw for any Archimedean solid. So, that is all in the lecture today. I hope that you will be able to develop surfaces and have fun drawing and developing some of these solids. So, thank you very much for joining me here today. See you again tomorrow for the last second last lecture of this entire course. Thank you and bye, bye.