Engineering/Architectural Graphics – Part 1 Orthographic Projection

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Lecture – 09
Curves Used in Engineering Practice: Conic Sections

Good morning. Welcome to the 9th lecture of this course and the second last for this week. Today, what we are going to learn is we are going to learn about curves used in engineering practice and in today's lecture we are only going to look at the conic sections. So, you might have used some of these curves which I am going to discuss about today, but you might not have understood fully the origin of this particular curve.

So, the conic sections or the curves which are derived from the sections of the cones are basically derived by cutting this cone. So, the cone would be cut in different angles at different angles by different planes and depending upon how the cone is cut, we will receive, we will be able to observe these different sections. Just imagine any of the cone, a regular cone like this.

If you cut this; cone with a section with a plane which is going parallel to its base. So, this is the base of the cone, if we cut this cone by a plane which is parallel to its base. So, just imagine that this is the plane and if it cuts the cone parallel to its base we will see a circle. So, at all points of time wherever it will cut we will see a circle emerging which is what you are seeing on your screen.

So, the first shape that we derive out of cutting the cone by a plane parallel to its base is this circle. Next, if we cut this cone by a plane in such a manner that the invert of this cone so always the cone can be assumed as a double cone. So, it is not the same shape exactly, but it is approximately the same height of the cone and assuming that this is a circle and not an octagon.

So, a cone can actually be assumed to be an inverted it is a double cone. So, if we cut a plane in such a manner that the access is split and the base is not cut. If the base is not cut and we just cut this cone by a plane say inclined at this angle what we will achieve in between. So, for example, I am taking this assume this be a circular cone and I cut it with a plane like this. So, if we cut it with a plane like this the plane goes all the way this.

And the shape that we get is an elliptical shape this is an ellipse. So, that is how we achieve ellipse. Now, suppose we cut the cone in such a manner that its base is also cut and the plane which is cutting this cone and also cutting its base is parallel to one of its generators. So, just see that this plane is parallel to its generator and this plane cuts the cone. So, we had a cone which was parallel to one of its generators.

And we cut this cone with this plane what we get out of this is a parabola. Now this parabola could be of different shapes depending upon where the cone is being cut. So, depending upon where the cone is cut we will get a parabola, but whenever a plane which is parallel to its generators cuts the cone, the double cone the cone which is on top of this cone it will not be cut.

Suppose, we have a cone like this now just imagine that we have a plane which is parallel to this generator. So, what will happen that it will pass parallel to the upper cone and it will not cut the upper cone.

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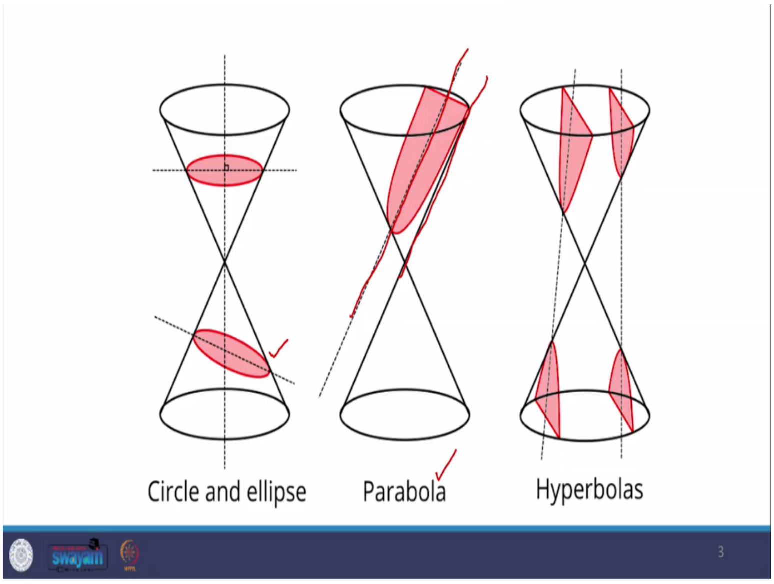

So, when we cut a cone with a plane parallel to its generators we will actually get only one parabola which is what you can see in this image where the plane which is cutting the cone is parallel to one of its generators and what we get out of this is a parabola. So, in this case we will only get one parabola. However, if the plane is not parallel to its generator then in that case and the base is also being cut, the shape that we will cut in the section is a hyperbola.

In that case, we will get 2 hyperbolas one in the bottom cone and the other one in the top cone. So always the hyperbola we will get in double. So, if you remember your geometry drawings and what you have read in geometry in your Math class we would always have hyperbolas in twin while parabolas would be single. So, that is how we achieve these different shapes circle, ellipse, parabola and hyperbola and 4 of these are conic sections.

And these are the curves which are derived from the cone. This is what we often use in a designing various structures, designing various different forms for both architecture and engineering. Circle you are all familiar with. Ellipse we use ellipse in designing bridges and dams and we also use them for various different forms in architecture. Manholes are often made using circular or elliptical shapes.

So, these shapes are used for these purposes. If you look at hyperbolas a lot of pipe sections and the flow in them is derived using this parabola. We also design a lot of arches using the parabola design. Similarly, hyperbolas are also used in fluid mechanics and calculate the sections and all. So, that is what the hyperbola is also used for. Now, the next that we come to is how do we draw each one of these.

So, circle of course is very simple, we have a center and we have a given radius using which we will draw a simple circle. Circle is absolutely simple you must have drawn circle umpteen number of times I do not need to explain. Now, the next is this ellipse. Now what are the things which are essential in ellipse? So, the parts of ellipse which are essential one it has two axis.

One is a major axis and the other one is a minor axis. So, ellipse is an oblong circle. In circle, we have both the axis being equal.

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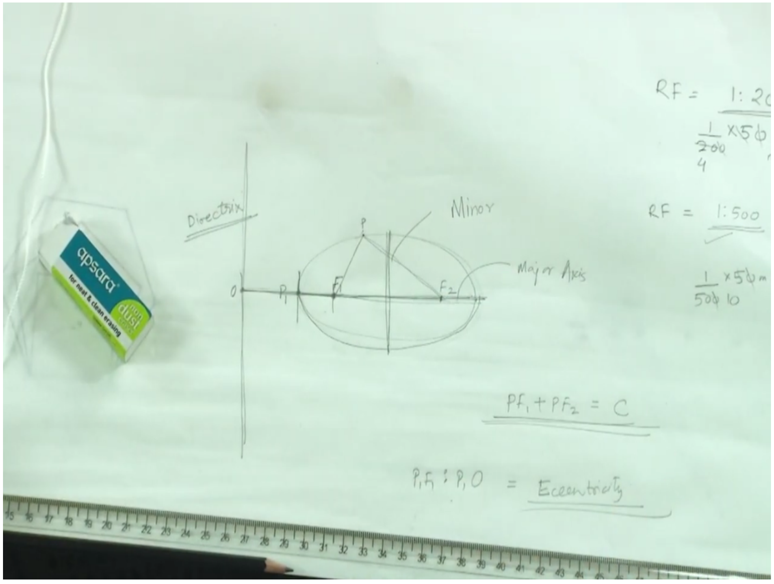

While in ellipse so when we draw an ellipse we have a major axis this is called a major axis and this is called a minor axis and the ellipse is symmetrical about these two axis both by this axis and this axis, but the axis are not equal. The major axis is bigger, minor axis is smaller. In addition to this, we have two points which are called the focal points. So, this curve which is ellipse is generated when any point moving on the ellipse is the sum total of its distance from the two focal points is the same.

So, if you want to tell that what is this ellipse it is actually the path traced by a point P when PF 1 + PF 2 is constant. It is a fixed number, it is a fixed distance. So, we can draw this ellipse if we know this distance given which is this and if we also look at this here suppose the point P was here so P 1 F 1 + P 1 F 2 which is this is equal to its total length of the major axis.

So, this is the constant and we get this entire ellipse drawn here. One more thing which we would need to know when we are drawing this ellipse is a directrix. This line is called directrix and the distance of the nearer focal point from this directrix and the ratio of this distance from this directrix for example this is O so the ratio of P 1 F is to P 1 O is called its eccentricity this is the eccentricity and thus this distance determines what will be shape of this ellipse.

So, we would either know the eccentricity, the eccentricity and the distance of focal point from the directrix either we get that or we get this constant that what is the distance of the point from the two focal points of the ellipse plus the distance between the focal points. If we know either of the two conditions we can actually draw the ellipse often we are given the directrix the eccentricity and the distance of the focal point from the directrix. Often this is the condition that we are given.

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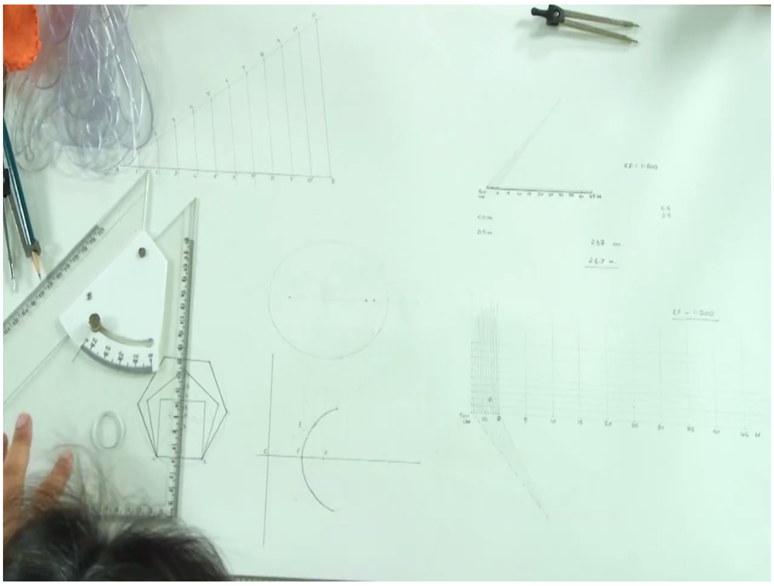

However, we would see how do, we draw using both the given conditions. So, suppose; what we know is the distance between the two focal points. So, these are the two focal points where I have pegged my pins here and I know the sum total of the distance is the P F 1 + P F 2. So, suppose I know the total distance so all I would need to do is I would need to fix this point here such that it does not move.

And then you can fix your pencil anywhere onto this and gentle you can trace this and then I take it to the other side. The thinner the thread the better the ellipse that you get because it sometimes sticks, but what you get is a perfect ellipse which is traced by the simple method where we know that this thread, the length of this thread between the two points has a sum total of this distance which we know as this constant.

So, we know the distance between the two focal points, we know the total constant which is the distance of a point from these two and we simply trace these parts this is our ellipse and you can see that we get a very firm regular ellipse with this method. However, we often do not draw using this method in engineering drawing, in engineering graphics and what we need to do is we need to know the directrix, the distance of focus, the eccentricity and we will draw using that method.

Now, if we have to draw using directrix and that method. So what we have here is a directrix which is given. We will draw an axis of the ellipse perpendicular to it at any point it is not fixed these are just generators. What we would know is we would know the distance the focal point from the directrix. So, for example, our focal point is at a distance of 50 mm that is 5 centimeter from the directrix.

So we know that this is focal point and say the eccentricity of this ellipse is 2 is to 3. So, what we would do we would divide this line into 5 equal parts and dividing this line. Here it is only 5 centimeters so it is easy for us to divide, but it was some odd numbers say 7 centimeters and then we have to divide the eccentricity was given us 2 is to 3. So, we would still divide it into 5 equal parts.

And so this is the point P so this is focal point F and this is the point P and say this is the point C where the axis meets the directrix. Once we have done that, we will draw a perpendicular at this point P and we will measure the distance equal to P F and cut mark a point say E on this perpendicular. So, this is equal to this and now we will draw a line which is passing through C and E.

So, we draw a point passing C and E. Now, what we will draw here is we will just mark some points randomly on the axis and we will draw very thin perpendiculars. Now, what we have to take is see the concept here is that we have a distance which is from P E equal to the focal point here. Now, next we take this point and we cut the same distance from the point F on this given perpendicular line.

This is the point here and then we measure this distance of this a point passing on this line which is the tangent C, E and then cut the same distance on the perpendicular line taking the distance from the focal point F and we can take the same distance on the other side because ellipse is symmetrical. So, we will just keep cutting so if I draw very lightly I would get an ellipse like this.

This is what I am getting, but how do I fix it? You can draw a complete ellipse using the same method. However, I am only showing you how do you fix the curves. For this what we will need here is a French curve I had already shown you the French curves. If you notice keenly there is a number written on each of these curves. So, what we need to do is we need to fix the curve in such a manner that it passes through any three given points.

And then we can keep fixing the curves such that the three points are being matched. So, if you look at it here I will draw it very lightly, the curves have to be drawn very lightly. The same curve will be used on the other side always of the ellipse. So, you do not need to find another curve to fit this so this will be the same curve which will be used here fitting and you can see which is the; other curve which fits the next three points.

Such that the curve fits tangentially otherwise it will not be a continuous curve that is how we fix the curves on an ellipse using these French curves. It could be any of the curves which could fit, but then you have to remember the number which has to be repeated on the other side and just like this if you continue to develop more points like this you will get other points on this ellipse and it will become a closed curve.

So, these are the two methods by which you can very conveniently draw the ellipse and it will come out to be a closed curve similar exactly like this. So, the eccentricity and the distance between the focal points are different from this ellipse. So these two will be different shaped ellipse, but we can get the ellipse like this. So, this is how we draw an ellipse in engineering drawing.

The next one is a parabola. So, now we will draw a parabola drawing parabola is very similar to how we draw ellipse, the only difference being that the eccentricity of a parabola is always one. So, we know that this distance which we had taken between a PC and the PF that is the ratio is always going to be one so these two are going to be equal.

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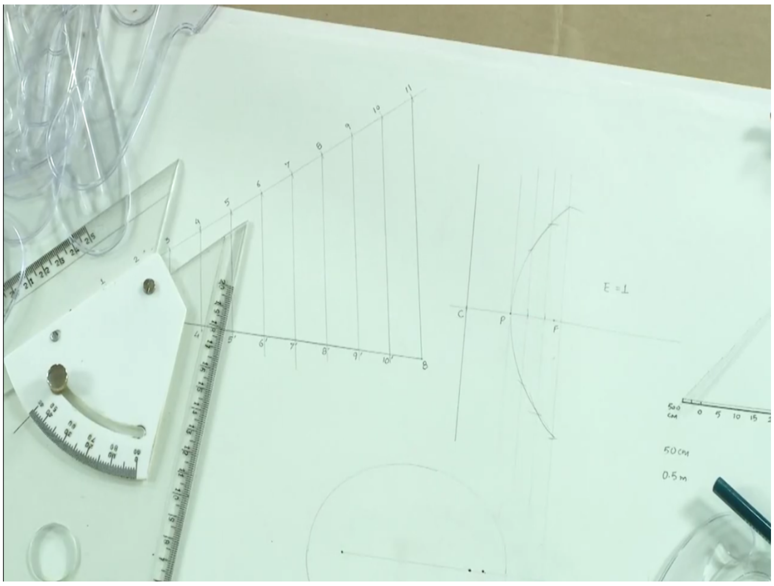

So, I will quickly draw a parabola here. Suppose, we have a directrix given here we will just draw a perpendicular axis for a given distance of focal points say again 5 centimeter what we had taken earlier and the midpoint of that is going to be the point where the parabola will start or meet the axis this is the focal point. Now, what we will draw here is we will draw these perpendiculars passing through these points.

And now just see that this point P has certain distance from this point C which we took as C earlier and which is the same distance which P has from F so that is what we are taking. So, the next point on this vertical line the distance that this point has from the point C is the distance that the next point will have from F. So, every time we get this vertical and this is not going to close because whatever we measure on any given perpendicular line here it is going to increase only and we will not get a closed curve.

So, it is simple we get another distance here and that distance as the distance of this point from the directrix increases it is the same distance that we take from this F. The change is how far is this focal point from this directrix that will govern the shape of the parabola. So, if you see again we get certain points and they will be fitting using a curve. So, if; you can match and of course matching these curves comes by practice.

You should not hurry up you should try patiently and then see which curves fit these three points. So you may have to do these multiple trials before you come to a fixed point, but definitely know this for sure that whatever curve for a parabola or an ellipse fits on the upper side of the curve will be the same curve which will fit in the lower side of the curve as well. So, we have got this curve and the next three points.

So, not next three points, but as you go ahead, as you move further you have to take it such that the previous points are also joined. So, this is what we are seeing here so that is how we arrive at a parabola. So the method of construction is very similar for ellipse and parabola, but in this case the whole difference is generated because of this eccentricity. If we have this eccentricity E = 1 here while here this E = 2 / 3.

The moment we have this eccentricity equal to 1 we will always get a parabola while in other cases we will get an ellipse. So, we will close it here today. These are the conic sections. These are the curves which are derived from conic sections and what we use in our engineering drawing, engineering practice most often. In the next class, we will understand about cycloids and involutes and how do we draw them. So, that is all for the lecture today. Thank you and see you again for the next lecture. Thank you.