Course on Momentum Transfer in Process Engineering
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Lecture 10
Module 2
Application of Navier Stokes equation for finding out viscosity (Part 1)

Good morning you remember that we had derived equation of motion that is Navier Stokes Equations and we had given you a problem not we had given a we had given the problem and solved also right. We said that the Navier Stokes Equation so useful that in many cases they are used and used in the sense you see that there is a velocity term, there is a there is a viscosity term, so shear stress term all these when whenever it was required or it is required you can use them.

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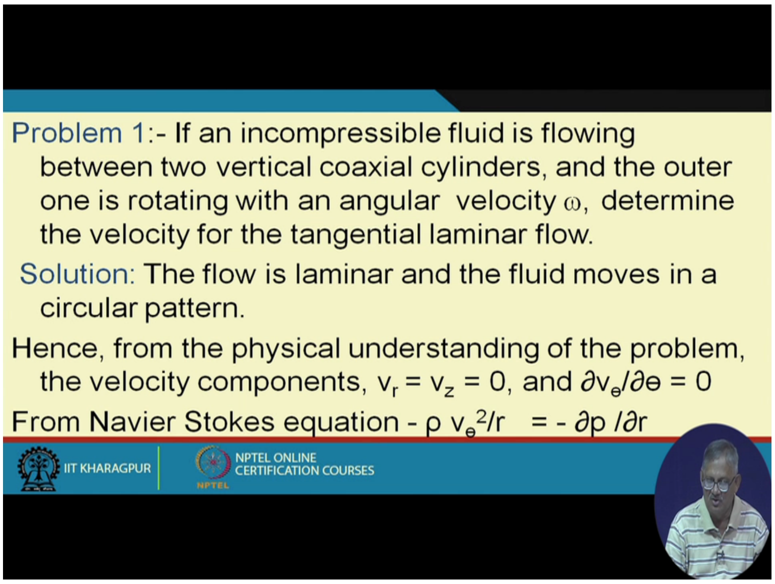

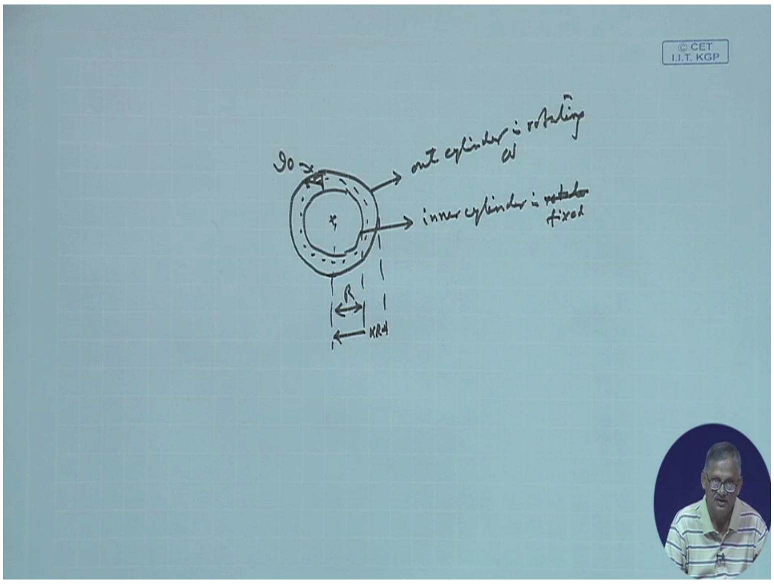

And you remember we had solved the problem like this that cylinder two coaxial cylinders right two coaxial cylinders one is like that and another one is like this they were there and a fluid within this is there and we said that inner cylinder this is the inner cylinder is rotating no is fixed and the outer cylinder is rotating outer cylinder is rotating with an angular velocity of omega right and we said the velocity profile is like this v theta right is the velocity profile. So for this we had done the solution right.

Now if the same problem if the same problem if we say in a different way in the problem remains explicitly same the language wise that if an incompressible fluid is flowing between two vertical coaxial cylinders, and the outer one is rotating with an angular velocity of omega, determine the velocity for the tangential laminar flow.

So there is no change in the language, no change in the problem only if we were asked now that you if you remember we have said that this was R and this was kR, right that was our problem given earlier.

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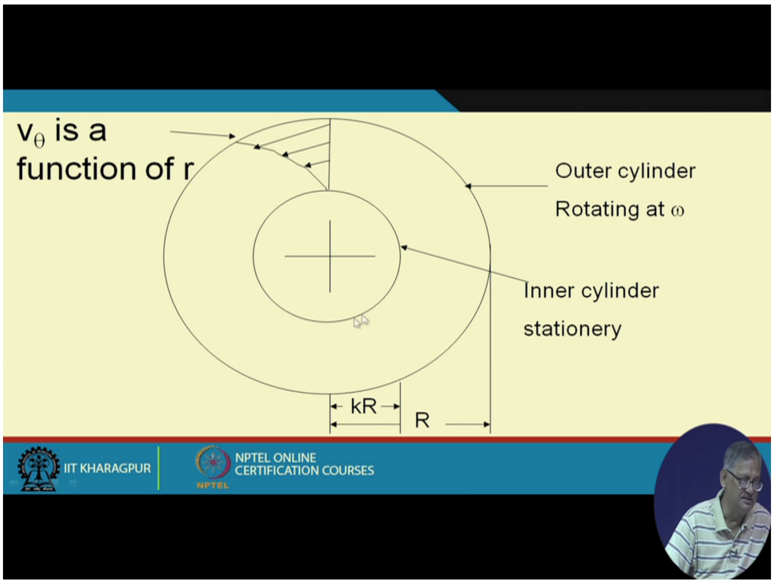

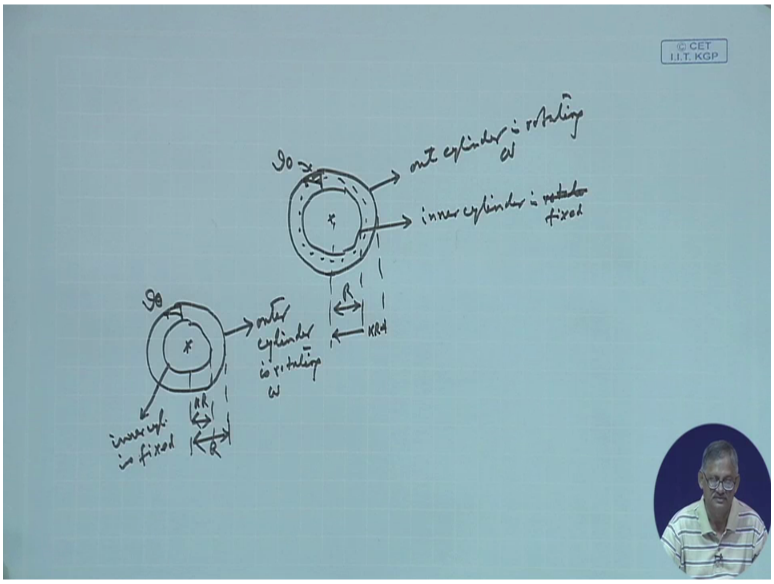

Now if we do the same problem like this right if we do the same problem like this right where you see here instead of instead of R at the boundary at the inner one and kR at the outer one if we just interchange that kR is in the inner one and R is in the outer one, right.

So we say now that we have these two coaxial cylinders right and the radii R like this this is kR and this is R, right all other remains same that velocity profile is like this v theta right here we have seen that outer cylinder is rotating with an angular velocity of omega, inner cylinder is fixed right so the same problem which just changed the radii.

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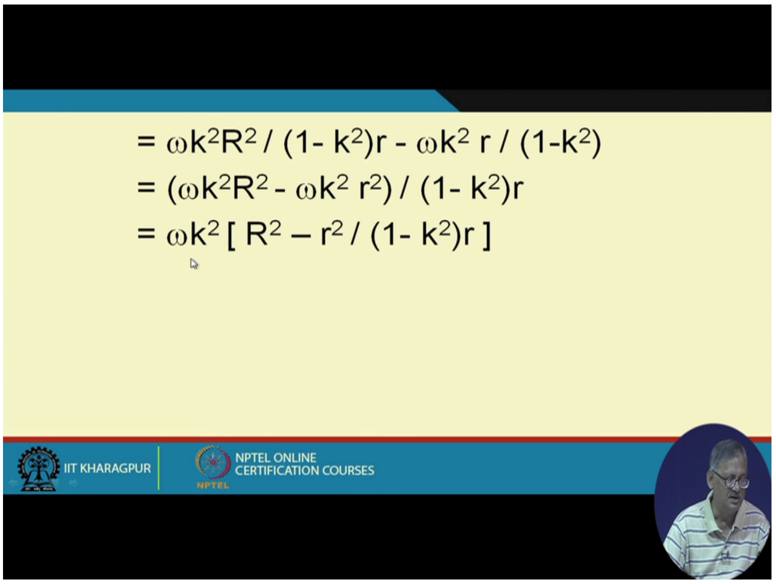

Then if you remember what was our earlier solution that let me show you first our earlier solution was like this v theta was omega k square into R square minus r square divided by 1 minus k square into r, this was our earlier solution that is under this situation, this was our solution, right.

Now we have changed we have said only all other things remains same only we r changing the radii inner radii and outer radii, so we made inner radii kr and outer radii R, right. In in this case outer radii was multiple of k which is more than 1, in this case inner radii is a multiple of k R rather which is less than 1, right because here in this case as it is appearing R is greater than kR in this case it is appearing R is less than kR. So it is multiple of kR where the constant is greater than R or 1 rather in this case it is less than 1, right all other things remains same.

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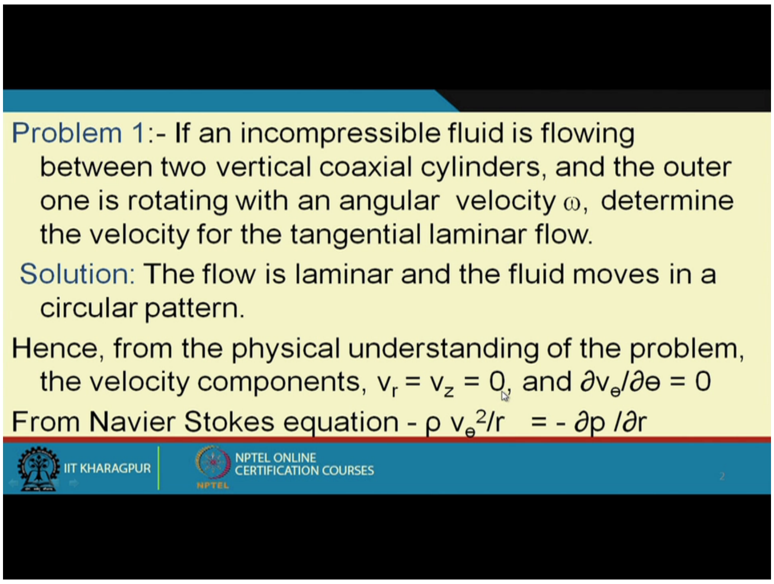

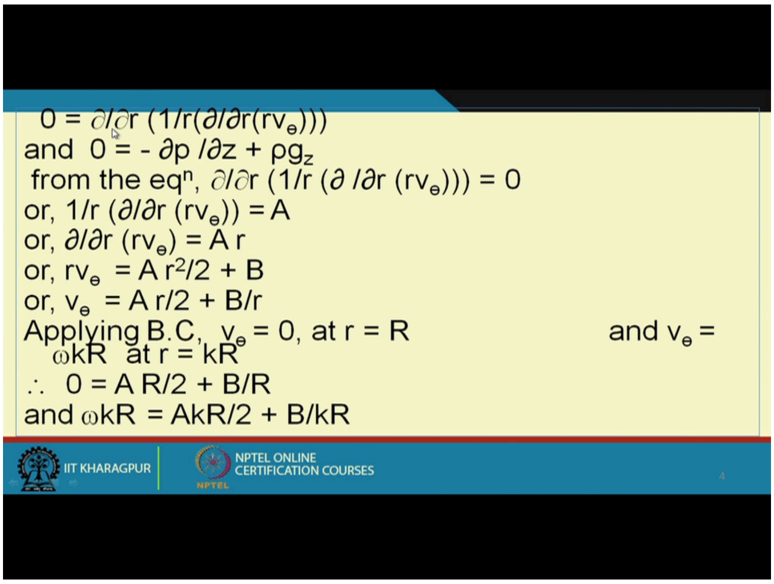

If this be the case then let us see how we can proceed right, you see we have studied here with that from the given problem vr is equals to vz is equals to 0, del v theta del theta is 0 same.

From Navier Stokes Equation we got the first equation rho v theta square by r is equals to minus del P del r minus rho v theta square r is equals to minus del P del r this was (())(7:55) from the first equation, from the second equation we got 0 is equals to del del r of 1 by r del del r of r v theta, this was from the second equation. And from the third equation we got 0 is equals to minus del P del z plus rho gz.

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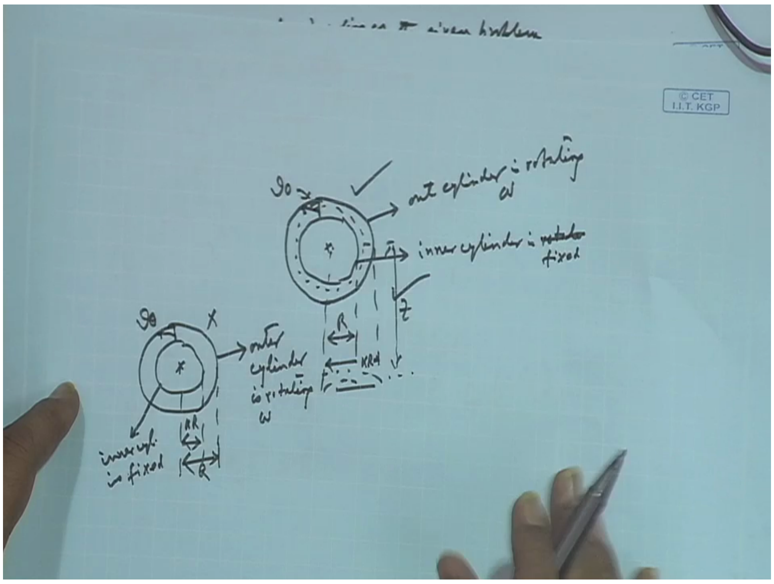

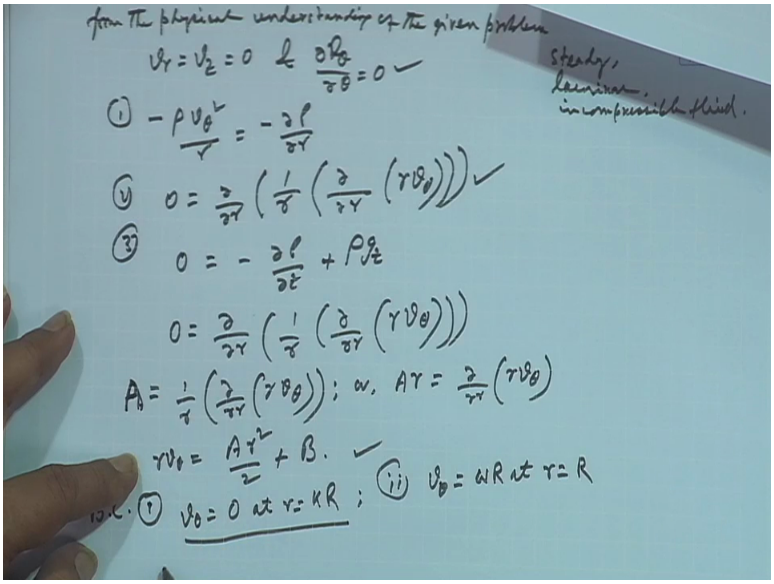

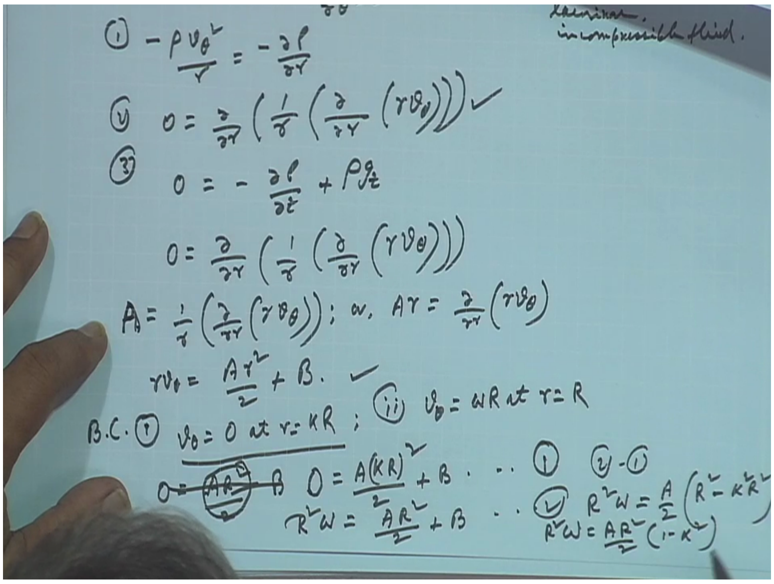

So all these three equations remain identical so we can write from the physical understanding of the given problem we can write vr is equals to vz is equals to 0 and del v theta del theta is equals to 0, it is a steady, laminar and incompressible fluid right this was all given. So this two remain same so then from the Navier Stokes Equations we can write number 1 equation minus rho v theta square by r is equals to minus of del P del r right, than we also can write that second equation we can write 0 is equal to del del r of 1 by r of del del r of r v theta, right (2, 3) right.

And third one from the Navier Stokes Equations we can write 0 is equals to minus del P del z plus rho gz right. So obviously that this is R and this is theta and z will be this right z will be this one is there, another is this so this is the z, right the same is here true, ok. If that be true than here we have one unknown v theta because we have been asked what is the velocity right so that tangential velocity if we want to find out that is v theta than this solution is good enough 0 is equals to we can take del del r because here v theta is there, here also v theta is there, but here atleast the equation is much from here also we can solve, from here it is difficult because obviously you will take the one where your solution is feasible.

So if we take this 0 del del r 1 by r del del r of r v theta, right if we take this as solving equation then on first integration as we found out earlier 0 is equals to or this was ok constant integral constant let us write here A is equals to 1 by r of del del r of r v theta, right this was on first integration so which we could write on on simplification as Ar is equals to del del r of r v theta, right right.

So on second integration we can write than r v theta is equals to Ar square by 2 plus B right Ar square by 2 plus B, upto this the solution is exactly identical. Now where the change from the two problems that is again I am showing that this was already solved problem and this is the problem which we r now handling, right.

So here it was rotating if it is constant here also it is rotating this is constant, here the outer radius was kr, inner radius was R, here outer radius is R and inner radius is kr, this is the difference, right. So upto this we have the same solution now when we apply the boundary condition the boundary condition is what first boundary condition we can write boundary condition one is that here right.

So this is constant and this is rotating. So this v theta is equals to 0 at r is equals to kR right, this is boundary one and boundary condition two we can write v theta is equals to omega R right at r is equals to capital R, right so omega R at r is equals to capital R. So if that be true by applying the first boundary v theta is equals to 0 at r is equals to kR right, we have this two one A and B, ok this was A and this is B.

Now if we put first boundary here that r is equals to kR v theta is equals to 0 that means this is 0 0 is equals to Ar square by 2 or in this case r is equals to 0 so this also becomes 0 then B becomes equals to 0 at r v theta is equals to 0 r is equals to kR sorry sorry v theta is equals to 0 at r is equals to kR. So v theta is 0 so this becomes 0 at r is equals to kR.

So this is becoming A K square R square by 2 plus B this is say equation number 1, right and the second equation is v theta is equals to omega R at r is equals to capital R. So instead of instead of this we write v theta is equals to omega R at r is equals to capital R. So second one we can write R square right omega or omega R square is equals to A into R square divided by 2 plus B right.

So from these two equations by solving we can say that B becomes equals to 0 right so if we if we subtract this 2 minus 1, if we do 2 minus 1 then B becomes equals to 0, then R square omega this is equals to A by 2 if we take common then it becomes R square into R square minus k square R square, or if we take R square common then omega is equals to or R square omega is equals to AR square by 2 1 minus k square, right.

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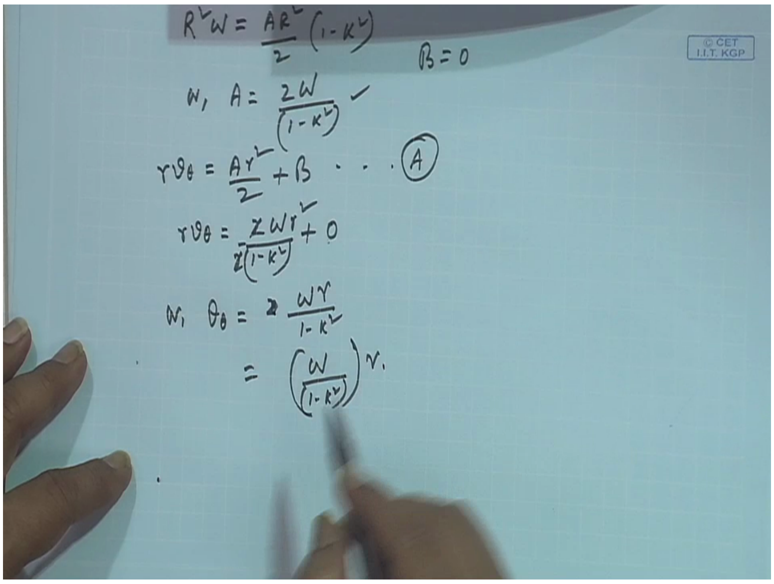

So from there we can also write this is that ok let us write here once again R square omega is equals to AR square by 2 into 1 minus k square or A is equals to this R square goes out so 2 omega over 1 minus k square right. So we had B is equals to 0 and we had the original equation v theta rv theta rv theta is equals to AR square by 2 plus B in this equation if we if we if we if we substitute the value of A and B right if we substitute the value of A and B, then rv theta is equals to A is 2 omega over 1 minus k square divided by 2 right plus B is equals to 0 into r square right or we can write v theta is equals to this r square and r goes out so 2 omega this 2 and this 2 goes out omega by omega r by 1 minus k square. So so v theta is rather is equals to omega by 1 minus k square into r right so v theta is that.

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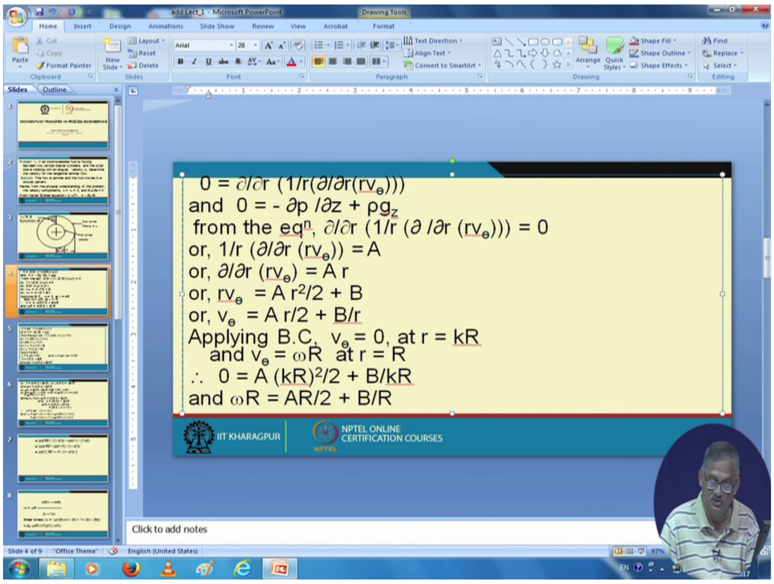

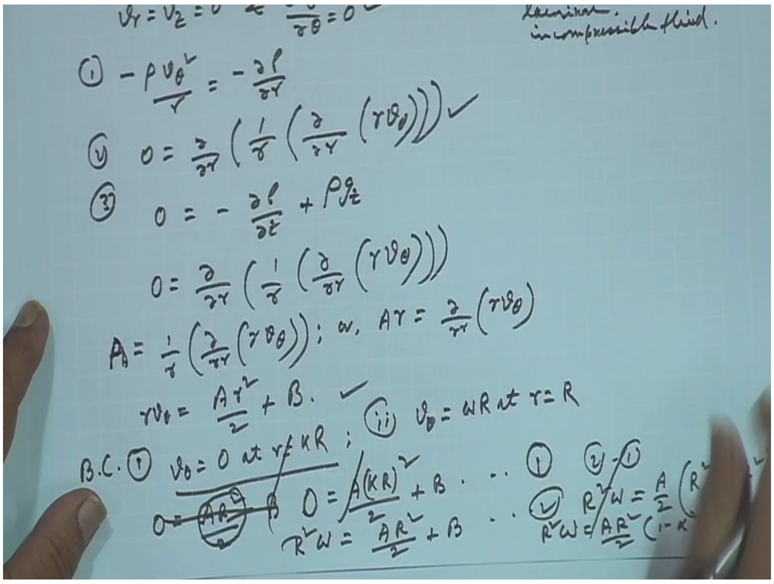

So let us also check whether we have the same thing ok here in this of course we will try to because this will be with you. So boundary condition we change v theta against 0 if you remember that v theta is equals to 0 right at r is equals to kR.

So let us substitute here v theta is equals to 0 at r is equals to kR, this is one and v theta is equals to let us let us take it to that site and v theta is equals to second omega R at r is equals to R, not omega kR, omega R at r is equals to R, right. So if that be true than by putting the first boundary that is v theta is equals to 0 at r is equals to kR, we can write this was rv theta Ar square by 2 plus B that was our integration, right.

So we write here that in this case we do not need this right so we can write then that (()) (21:42) v theta is 0 at r is equals to kR, and v theta is equals to omega R at r is equals to R. So if we put this boundary that is v theta is 0 at r is equals to kR, so v theta is equals to 0 at r is equals to kR, then we get here this is kR square right kR square. (())(22:21) we can we can write it here if we have that thing ok, it will be 2 square, right and obviously this will bring down to normal ok kR square this also has to come to normal only so this is (())(23:00) is out.

So kR square by 2 and this was B plus B only right plus B only right. Boundary condition first one if we have put here so this is 0 right and then this r is equals to kR is kR square and no no no no no let us let us let us skip let us skip that thing this one v theta then then only because we cannot put r0 so it is Ar by 2 plus Br here we have to apply exactly here we have to apply that v theta is equals to omega R at r is equals to R.

So from the first boundary v theta is equals to 0, at r is equals to kR, if we put that it is equals to 0 so here it is kR square, here it is then kR square, ok so it is kR square by 2, right plus here it is (k) B by kR, right because r we have first v theta is equals to 0 at r is equals to kR, so (())(25:02) plus B by kR and the second one was boundary condition was v theta is equals to omega R at r is equals to R, so v theta is omega R at r is equals to R. So in that case this will be AR by 2 plus this is B by R, right.

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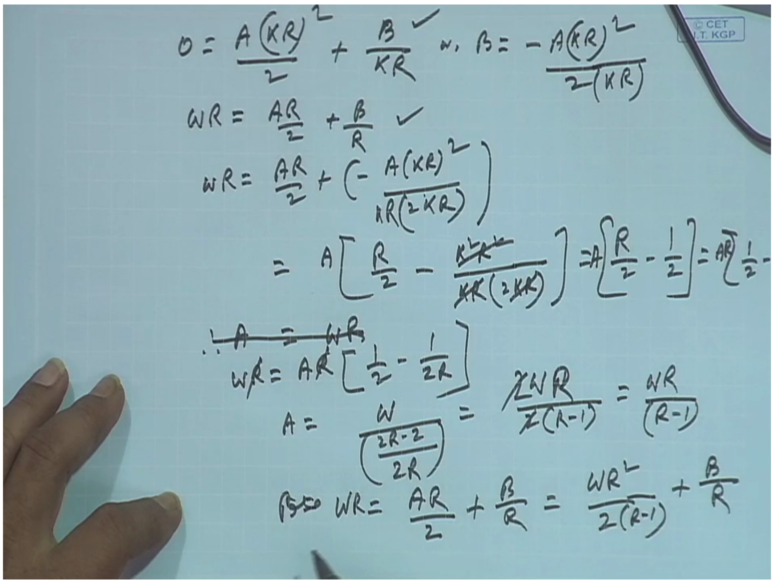

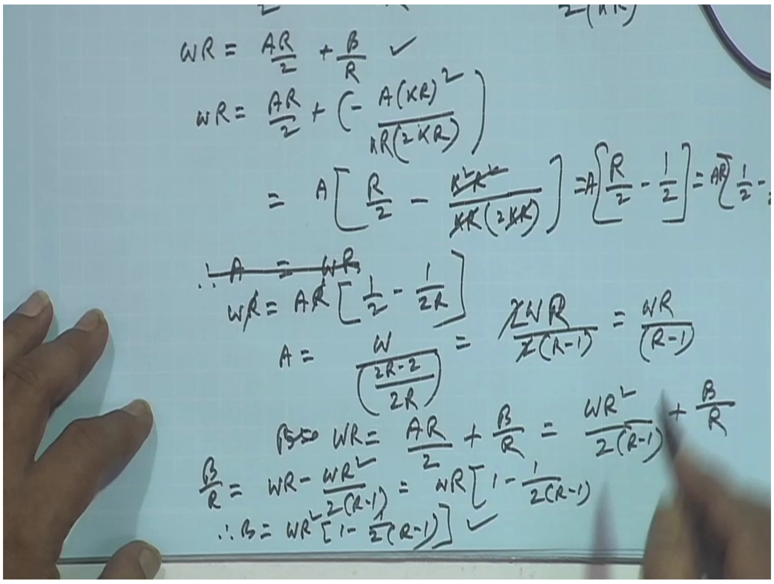

So that which we did a little here something difficult here now let us rewrite here so we have 0 is equals to AkR square by 2 plus B by kR and we also have omega R is equals to AR by 2 plus B by R right. So if we if we take instead of B so B is equals to from here we can write B is equals to minus AkR whole square by 2 right divided by kR so this is kR minus AkR square by 2 by 2 kR that is B if we take this equation and if we substitute here then omega R is equals to AR by 2 plus B is minus minus A kR square by 2 kR right 2 kR and we also have 1 kR here right.

So on simplification we can write that this is equals to if A is common R by 2 minus k square R square divided by kR into 2kR, right. So this kR kR this k square R square k square R square goes out then this becomes R by 2 minus half right. So this is equals to into A of course right. So omega R is there therefore A is equals to omega R of course this we can further simplify as if we take R common right 1 by 2 minus 1 by 2 R, right from there if we take out this R if we write again omega R is equals to AR into 1 by 2 minus 1 by 2R, right.

So this R goes out, so A is equals to omega by by 2R and here it is 2R minus 2 right is equals to 2 omega R divided by 2 R minus 1, so this goes out this omega R by R minus 1, so if it is A then B can be written as form here we can write that omega R is equals to AR by 2 plus B by R from there we can write substituting A omega R square by 2 into R minus 1 plus B by R, right.

So from there if we if we take this omega R so B by R is equals to B by R is equal to omega R minus omega R square by 2 R minus 1, so if we take omega R common than 1 minus 1 by 2 R minus 1, right so B is equals to omega R square into 1 minus 1 by 2 R minus 1, right. So we know A, we know B.

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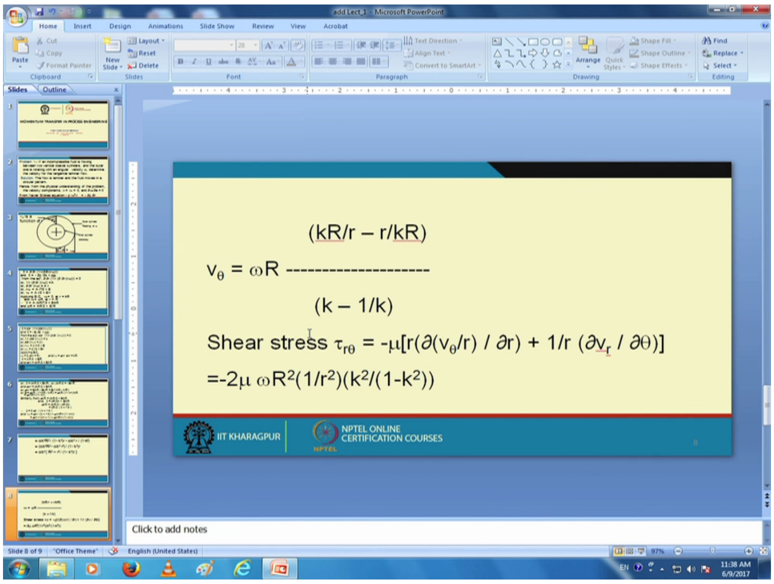

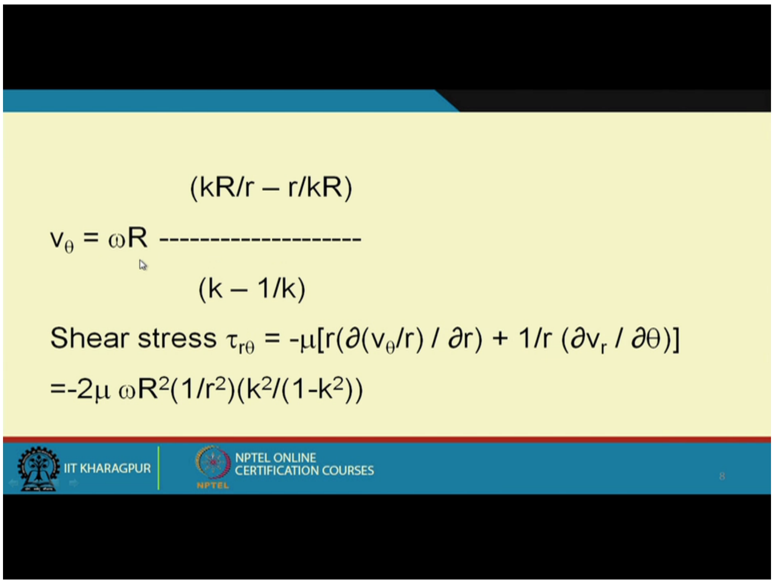

Then we can substitute and by doing that we get this ok we get this this is the solution ok v theta is omega R into kR by R to R minus kR by k minus 1 by k and and and of course in all the cases we have we we have missed that one k yes we have missed that one k in this cases. So this is the ultimate solution which we get that v theta is equals to omega R into kR by r r by kR by k minus 1 by k and shear stress (())(31:28) theta is minus mu r del v theta del r del v theta by r del del r v theta by r plus 1 by r del vr del theta. So that is minus mu 2 omega square into 1 minus r square into k square by 1 minus k square. So you do the changes in the slide accordingly and converting into this, ok thank you.