Course on Momentum Transfer in Process Engineering
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Lecture 15
Module 3
Fanning friction factor

Hello dear boys and girls, hopefully we have gone through till now that we have done pipe flow both with the help of shell momentum balance and with the help of Navier-Stokes equation. And we have also shown how the Navier-Stokes equation are helpful in solving this type of Problems, right? Rather Navier-Stokes equations are known as the mother of the flow behavior characteristic of fluids. So if you are accustomed with that you can solve many many problems in future wherever you can apply them.

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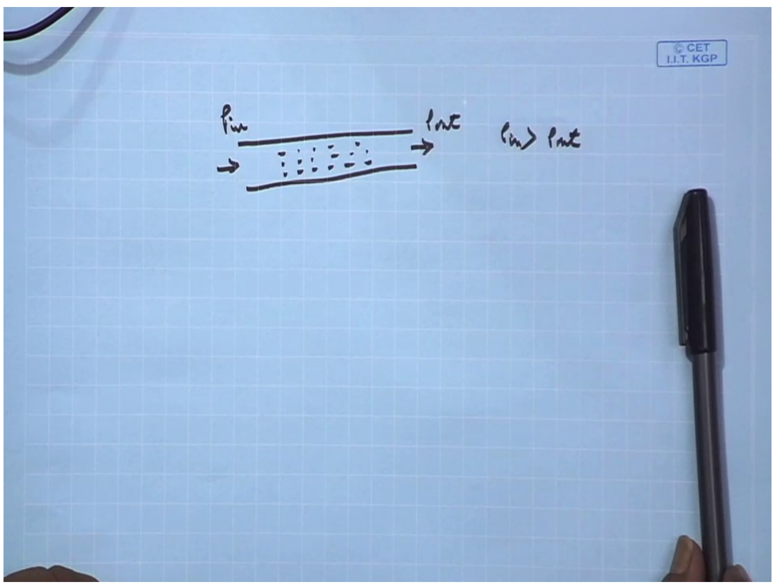

Now we will go to another main thing which I had told you that which I had told you that during the first rather last class I had said that you are looking into the flow through pipe if this is a pipe if this is inlet if this is outlet then the layers of the liquid or the fluids they are while there is a P inlet and P outlet why this P inlet is more than P outlet or where there has to be, right? You know in this connection another thing I tell you which you can obviously look into that whenever you are generating electricity at any point, right? And there you are generating at a definite voltage and when you are transmitting it and when it is reaching you not a voltage getting drop, why?

Because the median through which it is coming that is offering a lot of resistance, so depending on whether your median is offering a distance for the flow or not that will dictate whether the in at the inlet whatever you are given that thing will be same at the outlet or not and in this selection I would like to mention that yes till now people people have developed some or other kind of devices by which you can go even up to minus 100 degree centigrade while there is no resistance of such transit, right?

But here also in the fluid flow a similar thing we will now discuss where why this factor that is inlet to outlet why the pressure is getting dropped, right? That means like when you are running if somebody is holding you then you cannot run, there is what? When you are swimming if somebody is not allowing you to swim over then you would have been swimed faster? The same thing happens here also when the fluid is flowing some or other factors which are pulling the fluid not to go not to go and this pulling known as the friction factor, right?

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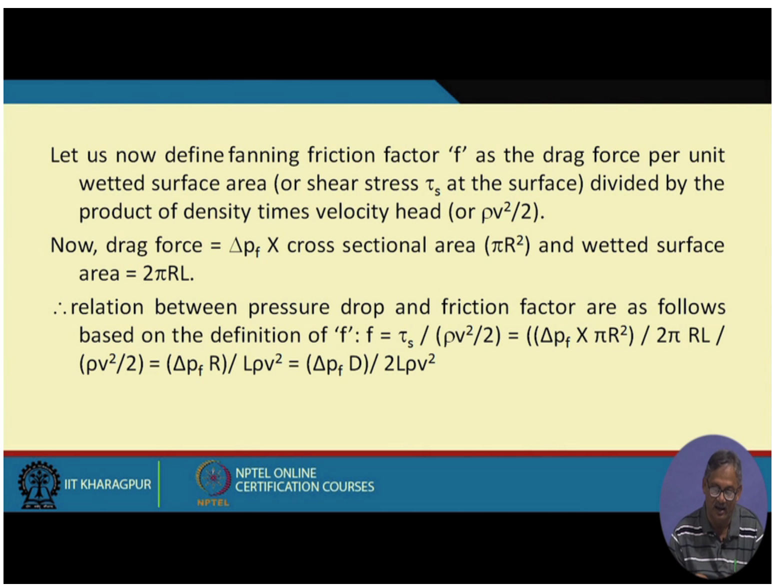

So we will now go to that friction factor and that we will do yeah, that we will do like this, right? This frictional factor let us define it to be f, right? And this is nothing but the drag force per unit wetted surface area which is known as fanning friction factor fanning friction factor f which is nothing but drag force per unit wetted surface area or shear stress tau s at the surface divided by the product of density times velocity head which is known as rho v square by 2, right?

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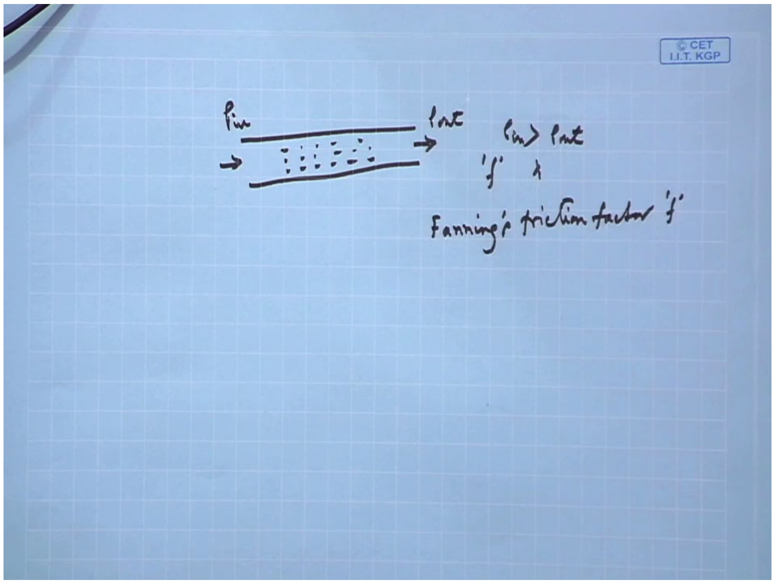

So let me repeat, we define a new term called fanning friction factor or simply this denoted by f fanning friction factor or simply denoted by f this in many books you may get as lambda or some other whatever at least this word should be there that is called fanning, right? Fanning's friction factor, right? So this fanning friction factor commonly or generally it is denoted by f small f, right?

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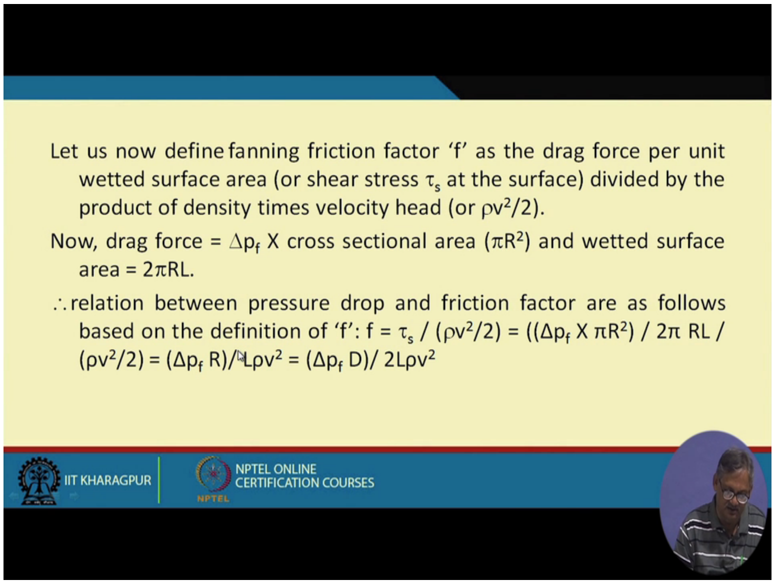

And this is defined as the drag force per unit surface wetted surface area or shear stress tau s at the surface divided by the product of density times velocity head or density is rho and velocity as is v square by 2.

So this definition if we if we follow then you will say that this what is this drag force per unit wetted surface area, right? Then what is the drag force? Drag force is delta Pf that is delta P pressure due to the friction factor so delta Pf times cross sectional area that is pi R square and wetted surface area is known as 2 pi R L, right? So drag force is that pressure force times sectional area that is pi R square and wetted surface area is 2 pi R L, if it is known then the relation between the pressure drop and friction factor that can be related as f is equal to f is equal to tau s over rho v square by 2, right?

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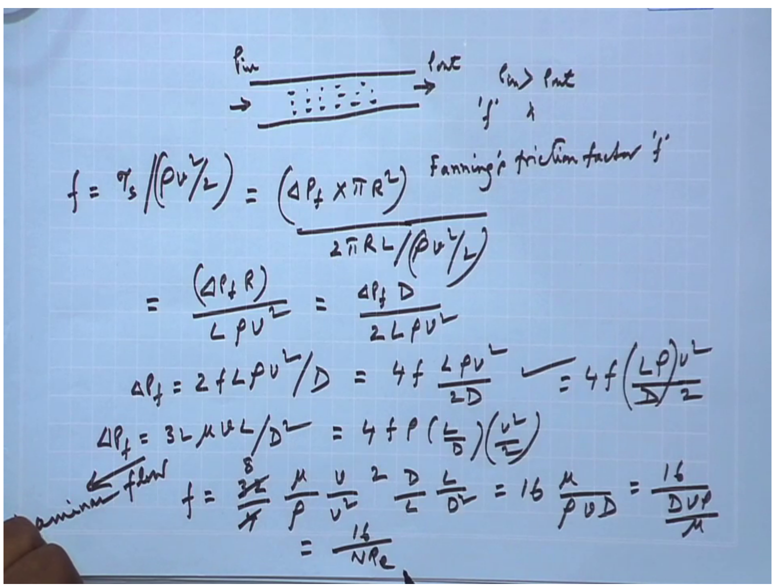

So this we can write as tau s as delta Pf times pi R square, right? Divided by 2 pi R L divided by rho v square by 2, right? So by definition we have said that fanning friction factor is the shear stress tau s, right? And this shear stress is nothing but drag force or pressure drop due to frictional factor times the times the sectional area that is pi R square divided by the wetted surface area that is 2 pi R L divided by the rho times velocity here that is v square by 2, right?

So we then if this is true we also can write this as delta Pf times R, right? Divided by L rho into v square, right? Because this this 2 this 2 goes off, this pi this pi goes off, this R this square goes off, right? Then remains delta P R over L that remains, right? This rho that remains this this square that remains, so delta Pf R by L rho v square which you can also write as delta Pf this R as D divided by 2, right? L rho v square, right? So now this L rho v square delta Pf D this can also be written as, now this we can write now delta Pf fanning friction factor as 2 f L rho v square, right? By D, right?

Which also we can write in terms of easy remembrance 4 f L rho v square by 2 D, right? This we have written in terms of f and delta P. So the relation between delta P and f is that delta Pf is 2 f L rho v square by D or 4 f L rho v square by 2D this why this is 4 f we can remember L is the characteristic length, flow is the density, D is the L by D is this length dimension non dimensional length rho and v square by 2 that is the velocity head, right? That is why in this form it is written the 4 f rho L by D or L by D rho v square by 2, right?

Now this we can also equate we know that delta Pf is 32 mu v L by D square this we obtained from the Previous class that this known as Hagen–Poiseuille's equation. So from that Hagen–Poiseuille's equation if we relate this with 4 f rho L by D v square by 2, right? If we relate them then, from here we can write f is equals to write, from here we can write f is equals to 32 by 4, right? mu okay by rho, right? v by v square, okay 2 that 2 goes there, right?

So this D by L, right? And and here we have L by D square I think we have cover everything f is 32 by 4 4, so mu remains mu and rho comes down mu by rho then v by v square yes this v square this 2 goes up, then D by L which was already there L by D as D by L and this is L by D square, right? Then by by simply eliminating we can say so this makes 8 so 8 times 2 is 16 this is mu, okay. So v v goes out mu by rho so 1 v remains, right? And this 2 also 8 into we have taken 16 mu by rho v and this D and this square this goes out so 1 D remains L and L cancels out.

So 16 mu by rho v D which we easily say 16 over D v rho by mu, now D v rho by mu is nothing but NRe, right? Reynolds number, so we can simply say that this fanning friction factor is this is true because when we have taken this, this was for laminar flow, right? So for laminar flow the fanning friction factor that can be said to be equal to 16 by NRe because NRe is nothing that is Reynolds number is nothing D v rho by mu, right?

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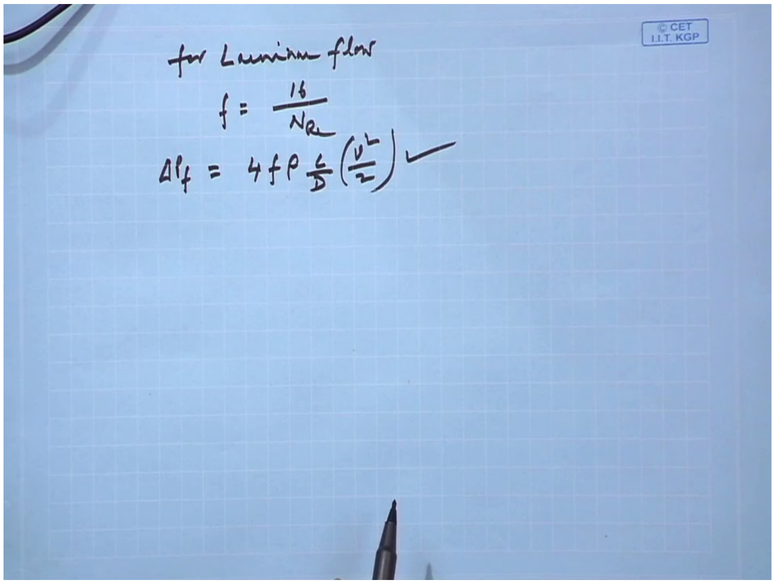

So if this is known, then we can say for laminar flow this f fanning friction factor is nothing but 16 over NRe, right? Done. Whereas, the relation between the fanning friction factor and delta P we can write this is 4 f rho L by D v square by 2, right? This is the fanning friction factor and this is true whether it is flow is laminar or turbulent this is delta Pf for turbulent we can say this can be utilized, right? So once we know the fanning friction factor then we can find out what is the what is the relation between fanning friction factor and your pressure drop, right?

So pressure drop and fanning friction factor this relation we can say for laminar flow is very easy 16 by NRe.

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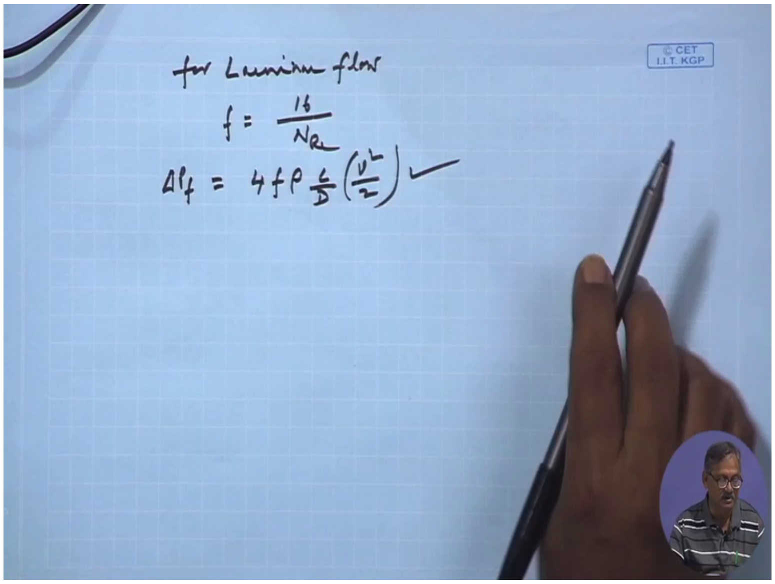

Whereas the relation general relation for delta Pf is 4 f rho L by D v square by 2, right?

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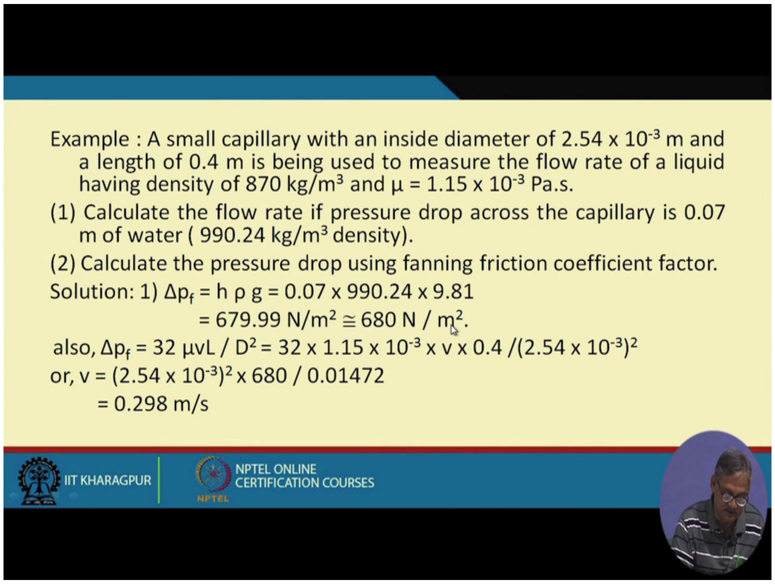

Now if we look into a problem that a small capillary with an inside diameter of 2.54 into 10 to the power minus 3 meter and a length of 0.4 meter is being used to measure the flow rate of a liquid having density of 870 kg per meter cube and viscosity of 1.15 into 10 to the power minus 3 Pascal seconds, then calculate the flow rate if pressure drop across the capillary is 0.07 meter of water having a density of 990.24 kg per meter cube and calculate the pressure drop using fanning friction factor or frictional coefficient or fanning friction factor whichever you call (ra), right?

So these two are asked to be done, so (thi) though this is easy problem but we can check how much the difference is if we use fanning friction factor and if we do not use that straight way if you find out then what is the delta P, right? So I repeat the problem, A small capillary with an inside diameter of 2.54 into 10 to the power minus 3 meter and a length of 0.4 meter is being used to measure the flow rate of a liquid having density of 870 kg per meter cube and viscosity of 1.15 into 10 to the power minus 3 Pascal seconds then calculate the flow rate if pressure drop across the capillary is 0.07 meter of water having density of 990.24 kg per meter cube and also calculate the pressure drop using fanning friction factor, right?

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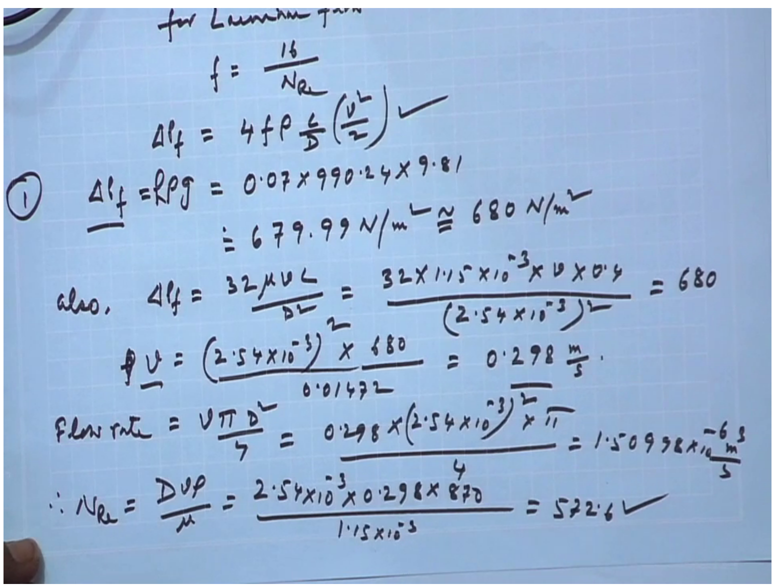

So in the first case in the first case when we are asked that calculate the flow rate if the pressure drop across capillary is 0.07 meter of water having density of 990.24 kg per meter cube, right? In that case delta Pf this can be written as h rho into g, right? So h rho g if we know h is 0.07, rho is 990.24 and g is 9.81, right? So if you multiply them, then it becomes roughly equal to say 679.99 Newton per meter square or roughly we can say 680 Newton per meter square, so this is the normal h rho g pressure drop, right?

Now let us look into what it happens when we are taking the other one, also we know that delta Pf is equals to 32 mu vL over D square from the Hagen–Poiseuille's equation. So 32 into mu that is given as 1.15, right? 1.15 10 to the power minus 3 Pascal seconds into v into 0.4 over D square D is given 2.54 into 10 to the power minus 3 so much meter, right? In whole square. So this from there already delta P we know 680, right? So from there we can go this is equals to 680.

So from there we can find out the value of v velocity as 2.54 into 10 to the power minus 3 this square times 680 over 2.54 and 10 to the power minus 3 whole square this becomes equals to 0.01472, so this is 0.298 so much of meter per second, right? So v becomes equals to 0.298 meter per second, right? So then the flow rate can be written equals to flow rate is equals to v into pi D square by 4 that is equals to 0.298 into 2.54 into 10 to the power minus 3 whole square into pi divided by 4 so this is equals to 1.50998 10 to the power minus 6 so much of meter cube per second, right?

So much of meter cube per second, therefore we can write NRe is equals to D v rho by mu is equals to 2.54 into 10 to the power minus 3 into 0.298 into 870 divided 1.15 10 to the power minus 3, so that becomes equals to 572.6, right? So NRe is so much. Now next it comes that how much fanning friction factor, right? So NRe we got 572.6, why did you do NRe? Because from there what we were asked to find out in that calculate the flow rate if pressure drop across the capillary so you have to find out the flow rate, now to find out the flow rate now to find out the flow rate we had used the Hagen–Poiseuille's equation.

Now Hagen-Poiseuille's equation is true for laminar flow, so since again Hagen–Poiseuille's equation is true for laminar flow so then you have to also show that the flow is laminar that is why we have found out the Reynolds number and this has become equal to 572.6 which is in the laminar range, right?

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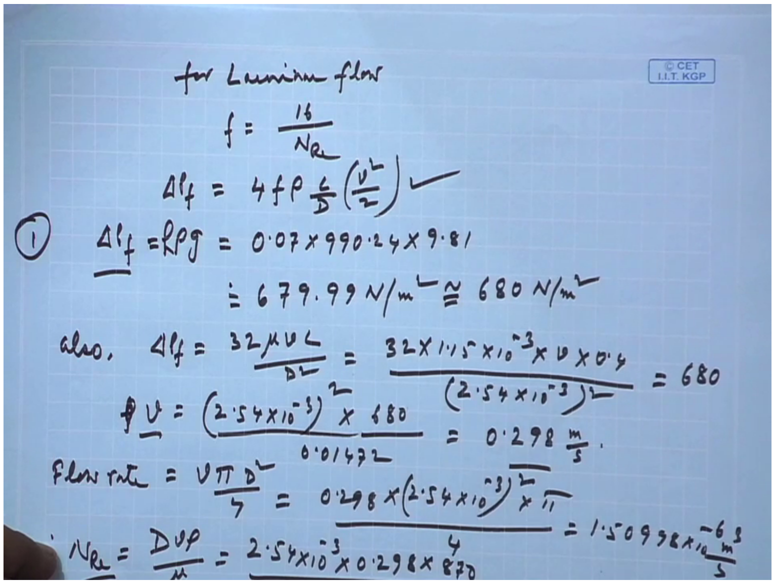

So we know that laminar range is 200 to 2100 up to that normal fluid say water it is up to 2100 and if it is a Newtonian fluid, then 2100 as well this is the lower and and the turbulence is more than 4000 so within this range that it is intermediate so if it is less than 2 2100, then it is under laminar flow, so you have established that it is under laminar flow.

So using Hagen–Poiseuille's equation was okay, now second part we have to do that is the pressure drop using the fanning friction factor, right?

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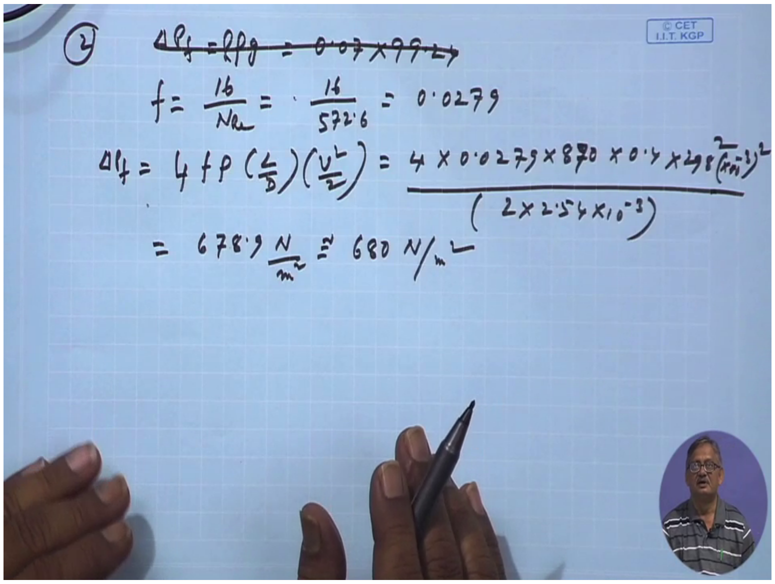

Pressure drop using fanning friction factor delta Pf is h rho g, right? Is equals to 0.07 into 990.24 into into yeah so we have to find out this okay this we have already found out, now the second the fanning friction factor is 16 by NRe so 16 by NRe means 16 by we have already found out NRe as 572.6 so this is nothing but 0.0279, right? If it is 0.0279, then delta Pf fanning friction factor using we know this is 4 f rho L by D into v square by 2, right? v square by 2 that is equals to 4 times point 0.0279 into 890 870 sorry 870 into 0.4 into 298 square over it was 298 square into 10 to the power minus 3 square, right? Over 2 times 2.54 into 10 to the power minus 3, right?

So L by D so this was to this, now this is if you calculate that becomes 678.9 Newton per meter square that is roughly again we can say closer to 680 Newton per meter square, right? That means if we look at with the help of h rho g what we are gotten for the laminar flow and with the help of fanning friction friction factor what that relation delta Pf is 4 f rho L by D v square by 2 that relation we also got almost similar of course earlier one was a little more if you remember it was it was around 680 that is 679.99 is very very closer to 680

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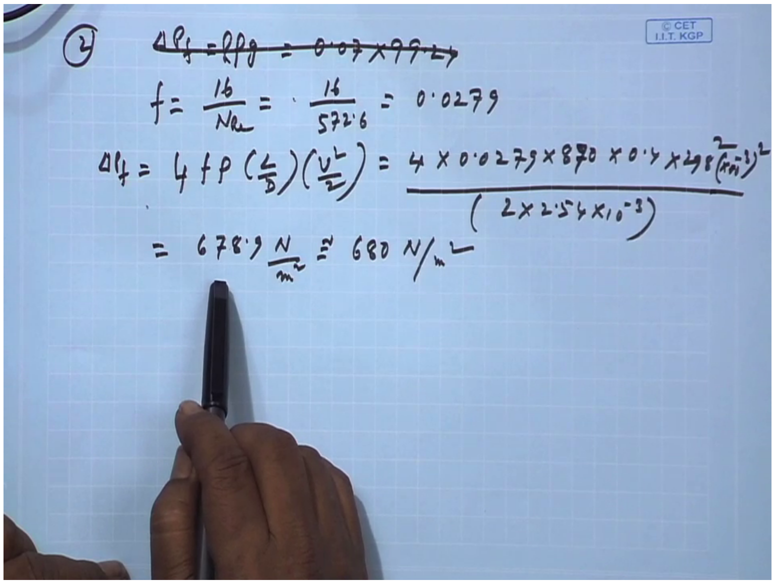

Whereas it is 6 Point 678.9 so it a little less than the earlier but or this is also closer to that, right? So with this we close we end today's lecture and next we will try to see what is the same for if the flow is turbulent, right? Thank you.