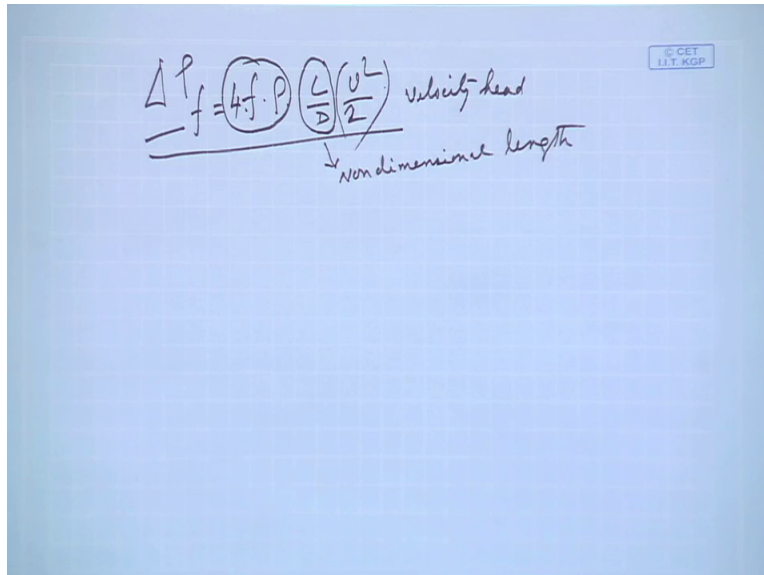


**Course on Momentum Transfer in Process Engineering**  
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**Lecture 16**  
**Module 4**  
**Moody's Chart**

Hello, so you remember in the last class we have done what is the definition of fanning friction factor and how we can calculate this fanning friction factor with the help of Hagen–Poiseuille equation, right? For laminar flow and one thing of course you have to keep in mind and remember that that this fanning friction factor relation  $f$  with the  $\Delta P_f$ , right?

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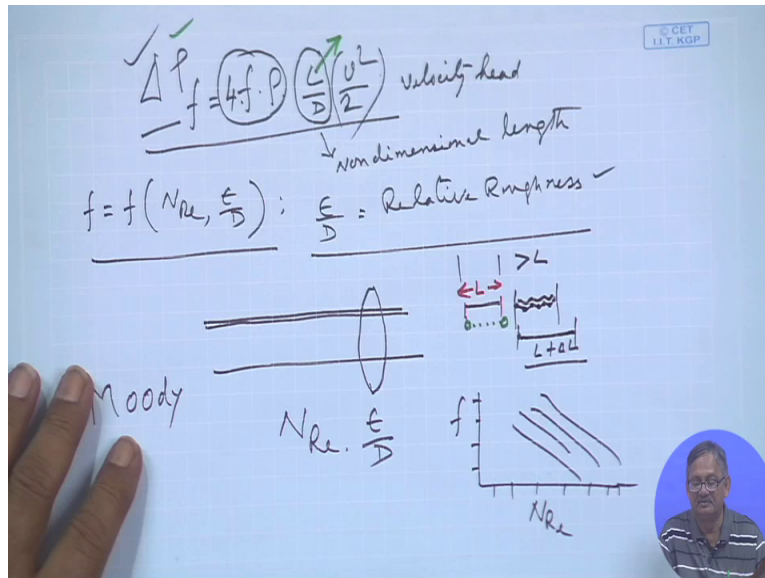


$$\frac{\Delta P_f}{\rho} = 4f \cdot \rho \left( \frac{L}{D} \right) \left( \frac{v^2}{2} \right)$$
  
velocity head  
non-dimensional length

So let us write it to be  $\Delta P_f$  is equals to  $f$  into  $\rho$  into  $L$  by  $D$  into  $v$  square by 2, right? And it was 4 if you remember  $4 f \rho L$  by  $D v$  square by 2 this way is easy to remember the relation between fanning friction factor and pressure drop, right?

$4 f \rho$  this is one  $L$  by  $D$  is the non-dimensional length this is the velocity head, so non-dimensional length velocity  $A$  so 4 times friction factor into density into non-dimensional length  $L$  by  $D$  and velocity head  $v$  square by 2 is the pressure drop, right? This if we remember and if we are able to remember this way is very easy to find out fanning friction factor easily, right? Now we can find out fanning friction factor for the turbulent flow, right?

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Fanning friction factor for the turbulent flow if we can determine it for that for what we need to know that some more relations that fanning friction factor is a function of  $N_{Re}$  epsilon by  $D$ , right? This epsilon by  $D$  is known as relative roughness, right?

This epsilon by  $D$  is known as relative roughness, so once we know this fanning friction factor relation, then we can also find out what the turbulent flow what the fanning friction factor how it is related to the pressure drop, right? Now, for that what is relative roughness we understand that also we should tell a little detail, otherwise it will be difficult to understand, right? Now this is a pipeline, right? This is a pipeline, now this pipeline if I see with naked eye then I will see it this pipeline is straight, right?

But suppose you take a microscope and take this sectional view, right? In that microscope this sectional view if you look at instead of it being this straight you will look into the microscope like this, so it is no longer straight, right? Now say a liquid droplet say a liquid droplet which was travelling this distance  $L$  so if this is the droplet if this is the droplet so that had traveled all along this length  $L$  which here in  $\Delta P = 4 f \rho L \frac{U^2}{D}$  we have considered to be  $L$ , right?

So this  $L$  when we considered this was considered okay whether it is laminar or turbulent does not matter but this was considered. Now, when you saw under microscope this this molecule of water say 1 molecule if we talk about, so this molecule of water that travel from there to here

from here to there from there to again here from here to there and there to here and then here, right? As if the molecule had to travel like this like this like this like this like this and like this, right?

Which is definitely greater than  $L$ , now if this be definitely greater than  $L$  how much greater? 1 time, 2 time, 10 time, 100 time or 1000 time or 10<sup>th</sup> time or what that we do not know, right? So first we have to understand that the liquid really traveled more than the length we considered that is  $L$ , right? It is no longer  $L$  so this as traveled as if this if we extend like this if we extend from this end to this end, then it will be (re) straight and that will be more than that maybe like this, right?

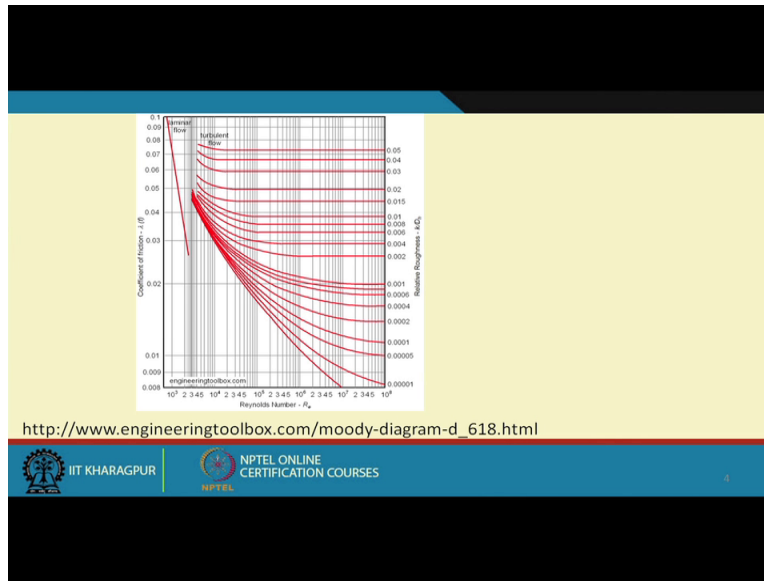
So this is definitely  $L$  plus something say  $\Delta L$  so this new  $(\Delta) L$  plus  $\Delta L$  that was not considered in this  $L$ , so our  $L$  calculation was not correct for which the  $\Delta P_f$  which we had predicted through this relation of  $\Delta P_f$  and  $f$  that definitely will tell a something erroneously. So this we have to consider and this was considered by a renowned scientist called Moody and he subsequently prepared a chart that is called Moody's chart for which we have to first know the Reynolds number  $N_{Re}$  and we have also to find out  $\epsilon/D$  that is relative roughness, right?

This if we can find out, then we can find out the value of  $f$  from a relation of  $f$  versus  $N_{Re}$  as given by Moody and this is in log scale and this is normal scale, right? So if that is known, then from there by knowing the  $\epsilon/D$  we can determine what is the friction factor value for different. Now here one thing definitely comes into mind that if as we have said that the liquid droplet will travel from one place to the other for a definite length, right? More than the length we considered.

And we also said this is true when we are looking under say microscope or in a magnified way, microscope means in a magnified way in naked eye we cannot determine we cannot visualize we cannot see the or differentiate the the the  $(\epsilon/D)$ (10:09) but if we see under microscope or in extended or enlarged view, then we can see really it was not so it was more than  $L$ . Now next the question comes this pipe can be made of now it is polythene polythene pipes stainless steel pipe or copper pipe or mild steel pipe so any and every pipe will have the same, no.

So that is what for different for different pipes it will be different values and that was first found out or it was brought to the notice by Moody and prepared that Moody's chart, right? So if we look at I I let me check whether it is here or not perhaps it is not here at some other point I have made it just just a second at some other point I have made it.

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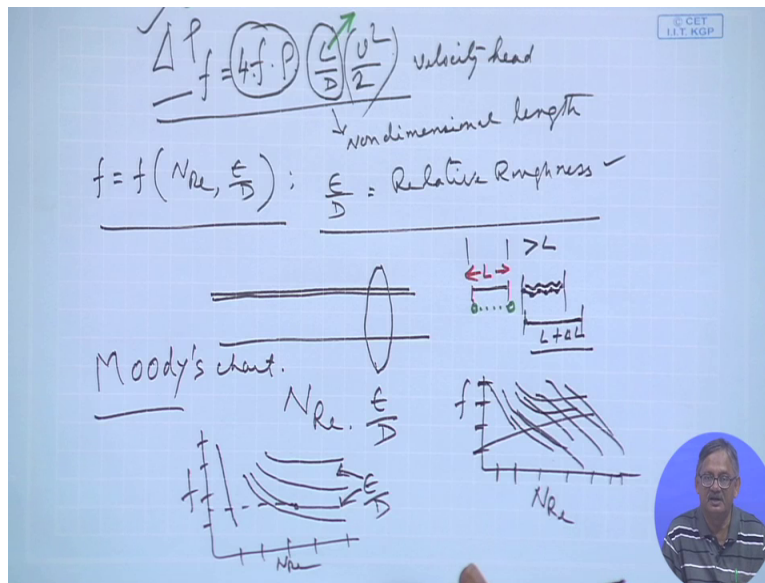
So this is that chart we will come back to this again this is the chart which we are referring to as Moody's chart this is available in any network or in any book any book any network you just give Moody's chart it will come like that, right? Here you see this is that coefficient of friction either as lambda or f which we said normally we write it in terms of lambda or f, right?

And this is the Reynolds number both are in log, okay this was mistake both were in log log scale, right? So in log log scale both are there and this is that relative roughness which we said epsilon by D, right? Here it is written k by D or it can be epsilon by D. So this is for laminar flow this curve for laminar flow you see this is f versus Reynolds number which are in log log scale, right?

And the laminar region is given like this and for turbulent region they are given like that and if you look into some other if you look into some other this Moody's chart or some other referred to there also you will find out this is true when you are going with when you are going with this for different materials that if it is stainless steel one, if it is if it is mild steel rather so because everybody will have different construction different construction and level.

So that that  $(\Delta P)_f$  is different for different material, so that we have to think of and the moment we know that we can say yes we know the relative roughness,  $\epsilon/D$  for that material we can find out the Reynolds number of the flow we can find out from there the relative friction factor, right?

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So that is what we said, so both are in the log scale this is also in log scale, this is also in log scale and the curves are like that as we saw this for laminar region one and for the turbulent region they are different, right?

The nature of the curve is little like this this was for laminar and for turbulent it is like that, okay so right? So if know for different  $\epsilon/D$  this is 1  $\epsilon/D$  this is another  $\epsilon/D$  so different  $\epsilon/D$  if we know that you can locate this point and then find out what is the value of frictional factor, right? So this way Moody's this is called Moody's chart. So once you know the from the Moody's chart the value of frictional factor then you can calculate what is the pressure drop, right?

So if you now go back to that original which we left with, right? So okay this was over now if we look at this let us then look into a problem this problem tells that,

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**Problem 3:-** A liquid is flowing through a steel pipe of  $\epsilon = 4.6 \times 10^{-5}$  m and  $D = 0.0508$  m at a velocity of 5.0 m/s. The viscosity of the liquid is 6.0 cp and the density is 800 kg/m<sup>3</sup>. Calculate the pressure drop in a 40 m section of pipe and friction loss.

**Solution:**  $\mu = 6 \times 10^{-3}$  Pa.sec

$$N_{Re} = \frac{Dv\rho}{\mu} = 0.0508 \times 5 \times \frac{800}{6 \times 10^{-3}} = 3.39 \times 10^4$$



A liquid is flowing through a steel pipe having epsilon that is called equivalent roughness epsilon 4.6 into 10 to power minus 5 meter and density sorry diameter D 0.0508 meter at a velocity of 5.0 meter per second. The viscosity of the liquid is 6.0 centipoise and the density is 800 kg per meter cube, so calculate the pressure drop in a 40 meter section of pipe and the frictional loss, right?

Frictional loss means that frictional head, right? How much frictional loss it happened so that if you can find out, then we can see from the Moody's chart this is very helpful. So for that let me repeat, A liquid is flowing through a steel pipe of epsilon having the value of 4.6 into 10 to power minus 5 meter, right? This is called equivalent roughness and D as diameter 0.0508 meter at a relative velocity of 5.0 meter per second. The viscosity of the liquid is 6.0 centipoise and the density is 800 kg per meter cube, calculate the pressure drop in a 40 meter section of pipe and the frictional loss, right?

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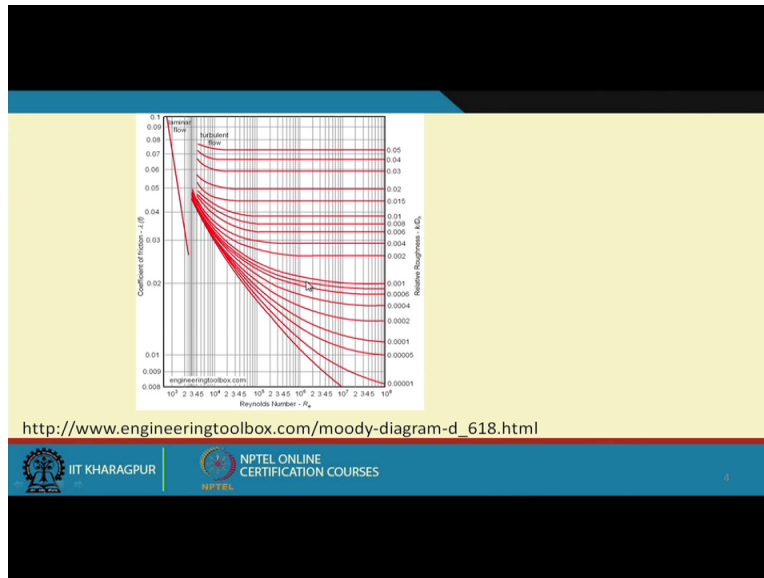
$\mu = 6 \times 10^{-3} \text{ Pa.s.}$

$$N_{Re} = \frac{Dv\rho}{\mu} = \frac{0.0508 \times 800 \times 5}{6 \times 10^{-3}} = 3.39 \times 10^4 = 33900$$
$$\frac{\epsilon}{D} = \frac{4.6 \times 10^{-5}}{0.0508} = 0.0009$$

Now we are given  $\mu$  is equals to  $6 \times 10^{-3}$  Pascal second, right? Now we are also given  $D$  and we are given the density, so from there let us find out what is the value of Reynolds number that is  $D v \rho$  by  $\mu$ , so these values are known  $D$  is  $0.0508$  meter  $v$  is given as  $800$  density is given as  $800$  kg per cube, right? And velocity is given as  $5$  meter per second, right? And viscosity is given  $6 \times 10^{-3}$  so this if we calculate, then it comes to be  $3.39 \times 10^4$ , right?

So it is  $3.39 \times 10^4$ , that means it is  $33900$   $N_{Re}$ , right? So now let us find out relative roughness that is  $\epsilon/D$ , now  $\epsilon/D$  will be equals to  $4.6 \times 10^{-5}$  by  $0.0508$ , right? So  $\epsilon/D$  was given  $4.6 \times 10^{-5}$  meter so it is  $D$  is  $0.0508$  so this is equals to  $0.0009$ , right? And if we now look at the Moody's chart which we had shown you earlier like this that if we now look at this chart that we have the value of  $N_{Re}$  as  $3.39$  or say  $3.4 \times 10^4$ .

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So somewhere here  $3.4 \times 10^4$  somewhere here and we will go according to epsilon by D 0.0009309 so somewhere here, right? Within this so we will go there and then we will wherever it is clicking for that particular we will find out from there the value of f, right?

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$$\mu = 6 \times 10^{-3} \text{ Pa.s.}$$

$$N_{Re} = \frac{DVP}{\mu} = \frac{0.0508 \times 800 \times 5}{6 \times 10^{-3}} = 3.39 \times 10^4$$

$$\frac{\epsilon}{D} = \frac{4.6 \times 10^{-5}}{0.0508} = 0.0009$$

$$f = 0.006 \text{ (from Moody's chart)}$$

$$\Delta P_f = 4f\rho \left(\frac{L}{D}\right) \left(\frac{V^2}{2}\right) = 4 \times 0.006 \times (800) \left(\frac{5^2}{2}\right) \left(\frac{40}{0.0508}\right)$$

$$= 1.89 \times 10^5 \text{ Pa}$$

$$F_f = \frac{\Delta P_f}{\rho} = \frac{1.89 \times 10^5}{800} = 236.25 \frac{\text{Pa} \cdot \text{m}^3}{\text{kg}}$$

So this way if we look into the Moody's chart we can say for this given condition of epsilon by D to be equals to 0.0009 and NRe corresponding to  $3.39 \times 10^4$ , then the value of f that can be seen from Moody's chart is equals to 0.006, right? From Moody's chart if we see then it can be like this.



Now this is nothing but is equals to then delta Pf we can find out delta Pf is  $4 f \rho L v^5 / D$ , right?  $v^2$  and if we look at  $L / D v^5$  all these values are known. Then  $4 f$  this is 0.006, right?  $v$  was okay  $\rho$  was 800, right?  $L / D v^5$  was 5 meter per second by 2, right? So  $v^5$  so it is 5 square by 2 and  $L / D$  was  $L$  was 40 meter and  $D$  was 0.0508 meter, right? So if we know calculate it comes to equal to 1.89 into 10 to the power 5 Pascal, right?

So we found out delta Pf to be equals to 1.89 10 to the power 5 Pascal. Now, if it is delta Pf found out then we are asked to find out the what is the frictional loss. Now frictional loss that can be found out that is called  $F_f$  is equals to delta Pf over  $\rho$  that is frictional head or frictional loss. So delta Pf is already found out 1.89 10 to the power 5 divided by  $\rho$  is 800 so this comes to equals to 236.25 this unit will be Pascal into meter cube over kg, right? This was Pascal this is kg per meter cube, so Pascal into meter cube by kg, right?

Now normally the frictional loss is not expressed in terms of this (fric) Pascal meter cube by per kg, then if you are asked that find out what is the frictional loss in terms of joules or in terms of Joules per kg, then how can you determine that?

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$$\mu = 6 \times 10^{-3} \text{ Pa.s.}$$

$$Re = \frac{DVP}{\mu} = \frac{0.0508 \times 800 \times 5}{6 \times 10^{-3}} = 3.39 \times 10^4$$

$$\frac{\epsilon}{D} = \frac{4.6 \times 10^{-5}}{0.0508} = 0.0009$$

$$f = 0.006 \text{ (from Moody's chart)}$$

$$\Delta P_f = 4f\rho\left(\frac{L}{D}\right)\left(\frac{V^3}{2}\right) = 4 \times 0.006 \times (800) \times \left(\frac{5^3}{2}\right) \times \left(\frac{40}{0.0508}\right)$$

$$= 1.89 \times 10^5 \text{ Pa}$$

$$F_f = \frac{\Delta P_f}{\rho} = \frac{1.89 \times 10^5}{800} = 236.25 \frac{\text{Pa} \cdot \text{m}^3}{\text{kg}}$$

You have seen delta Pf from here is Joules per in Pascal meter cube per kg, right? Because it is delta Pf Pascal and density rho kg per meter cube. So it becomes Pascal meter cube per kg, then how can we determine that two Joules per kg this is very simple if you know the definition of Pascal in terms of Newton and then you convert all them you in from there you can determine the value the the unit which can be in joules per kg.

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$$(236.25) \frac{\text{Pa} \cdot \text{m}^3}{\text{kg}} = \frac{\text{Pa}}{\text{kg}} = \frac{\text{kg} \cdot \text{m}}{\text{s}^2 \cdot \text{m}^2} \times \frac{\text{m}^3}{\text{kg}}$$

$$= \frac{\text{m}^2}{\text{s}^2} = \frac{\text{J}}{\text{kg}}$$

For example, this whatever the value we had got we have gotten 236.25 Pascal meter cube per kg, right? Now the value remains same 236.25 but this Pascal meter cube per kg that we can

write in terms of this Pascal as okay Pascal per kg per meter cube this was there, so Pascal is if this is equals to kg meter per second square, right? And this per unit meter square, right? This into meter cube per kg, right? So this is equals to if we take that meter cube meter square so this goes out, right? Meter cube meter square and this meter then it becomes 1 meter and this kg this kg that goes out.

So 1 meter square and 1 second square remains so we can write meter square per second square, now this meter square per second square is nothing but Joules per kg. So we can instead of Pascal meter cube per kg we can write Joules per kg and the way it has been it has the way it has been derived is through the definition of Pascal, right? So much kg meter per second square per unit meter square, right? Times meter cube per kg, so on simplification of this this becomes meter square per second square.

Now this meter square per second square is nothing but Joules per kg, right? So this way if we remember that how we have developed this Pascal seconds rather Pascal meter cube per kg square, right? If we remember then easily then we can find out how this is to be done, okay. Now this perhaps is easy that meter square per second square and we get it that okay in terms of Joules per kg.

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$$\begin{aligned}
 (236.2) \frac{\text{Pa} \cdot \text{m}^3}{\text{kg}} &= \frac{\text{Pa}}{\text{kg}} = \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \times \frac{\text{m}^3}{\text{m}^2} \times \frac{1}{\text{kg}} \\
 &= \frac{\text{m}^2}{\text{s}^2} = \frac{\text{J}}{\text{kg}}
 \end{aligned}$$

moody's chart.

So once this friction factor that can be obtained from the Moody's chart Moody's chart we can find out the pressure drop in the in the pipe also we could find out the frictional head or frictional loss, right?

Frictional head or frictional loss this can be determine in terms of joules per kg or Pascal per kg per meter cube, right? Pascal per kg per meter cube this also is convertible, right? So today we finish it here thank you.