**Course on Momentum Transfer in Process Engineering**
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**Lecture 18**
**Module 4**
**Flow through flat and parallel plates**

Okay if we remember that in the previous class we had given a problem and we asked that you try and if you cannot do this then we will do that in this class so let us see whether you are able to solve that or not, okay.

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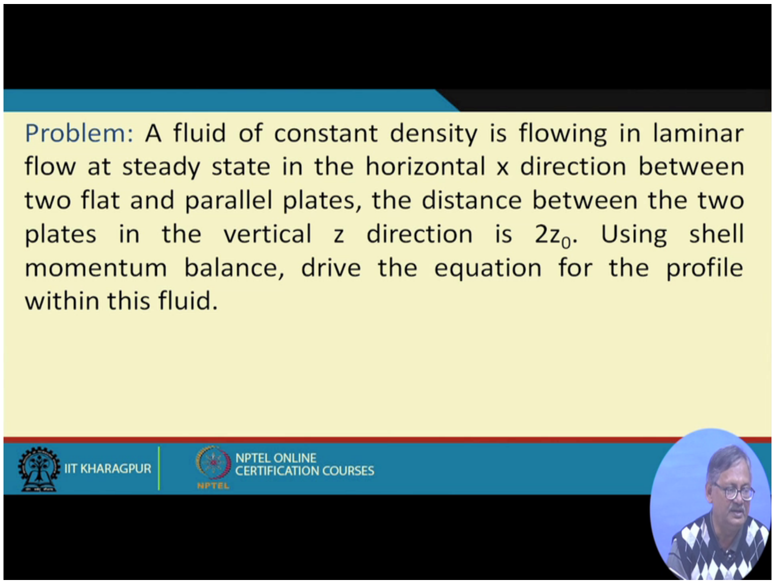


So yeah this was the problem which we had given you that A fluid of constant density is flowing in laminar flow at steady state in the horizontal x direction and between two flat and parallel plates, the distance between the two plates in the vertical z direction is said 2 z0. Using shell momentum balance, derive the equation for the profile within this fluid, right?

So that means say a fluid of constant density is flowing in laminar flow at steady state in the horizontal x direction, so if we have the plates like this if we have the plates like this, right? 2 plates like this if we have, then these two plates, right? One over the other these two plates the the fluid is flowing within this and we have to find out the profile, right? Derive the equation for the profile within this fluid, this we have to get it.

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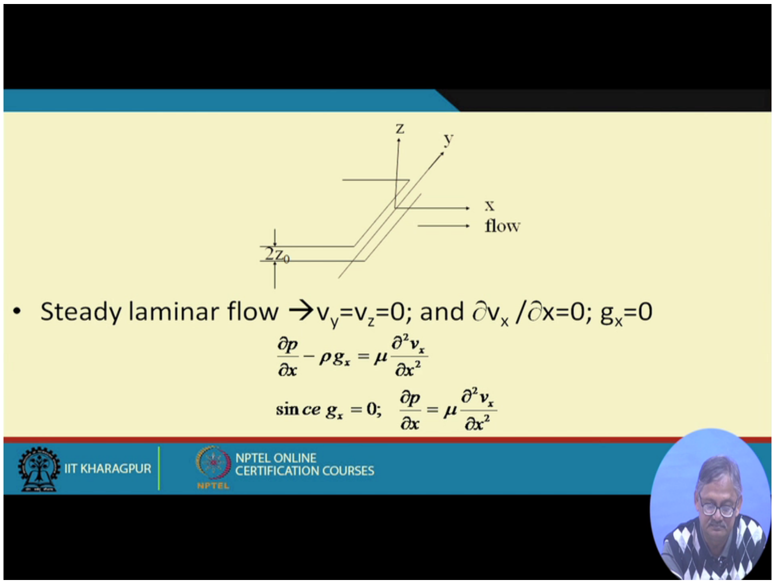


Now this if we (plo) if we if we draw if we draw this that as we showed you one one paper like this, right?

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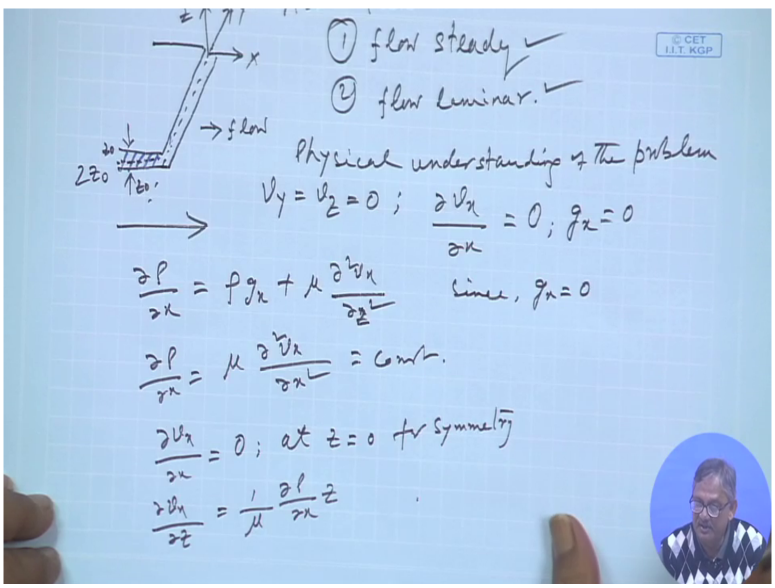


And the other one as say like this okay and if we have the say this is y direction this is x direction and this is z direction, right? So x, y and z and the flow is taking place like this the flow is taking place like this the distance between these two plates are say 2 z0, so this was our problem which we had asked to solve. So it is flowing along like this, right? Between these two plates between these two plates, now these two plates as small as we can think of. That means it will two plates we there is there are the two plates and say one over other like this and the fluid is flowing below this in this direction so this we had taken x as this y as this direction and z is the gap between the two, right?

So if we take this kind of solution, then we assume here again number 1, the flow is steady, number 2, the flow is laminar, right? So this assumption we make that flow is steady flow is laminar if that be true, then from the physical understanding of the problem so from the physical understanding of the problem we can then say that vy is equals to vz is equals to 0 because flow is taking only in the x direction, right? So there is no flow in the y direction, there is no flow in the vertical z direction.

If that be true if there is no flow in the vertical y and z direction horizontal this and vertical z direction only in the x direction because it is steady because it is laminar, right? So if that be true, then vy is equal to vz is equals to 0 and there is no del vx del x present, that is del vx del x is equals to 0 and we simply for all practical purposes say that gx is equals to 0 because that is in the x direction is the flow. So gx also we can assume to be equals to 0, so we can write delta P over del del P del x that is is equals to rho gx plus mu del 2 vx del x square, right?

And since gx is equals to 0 we can write del P del x is equals to mu del 2 vx del x square, right? Because gx we have assumed to be equals to 0, right? And we said in the beginning that this distance as small as we can think if that be true then also then also no is a proper definition of the physical problem is there that this distance is as small as we can think of, right? If being very small is 2 z0 we can say that it is so. So then we can say del P del x is mu del 2 vx del x square which is nothing but a constant, right?

So we can we can now integrate it and say del vx del x this is equals to 0, right? Because del P del x is constant so del vx del x is equals to 0 and this is true at z is equals to 0 for the symmetry del vx del x is 0 at z is equals to 0 because this is 2 z0 so 1 you can tell that it is 1 z0 and another is another z0 from the center, right? So this we can that 1 is z0 and the other is another z0 from the symmetry, so del vx del x is equals to 0 at z is equals to 0, right? So then we can also write del vx del z it was del v del 2v x del z square, right? del 2 vx del z square, then del vx del z this is equals to 1 by mu del P del x into z, right?

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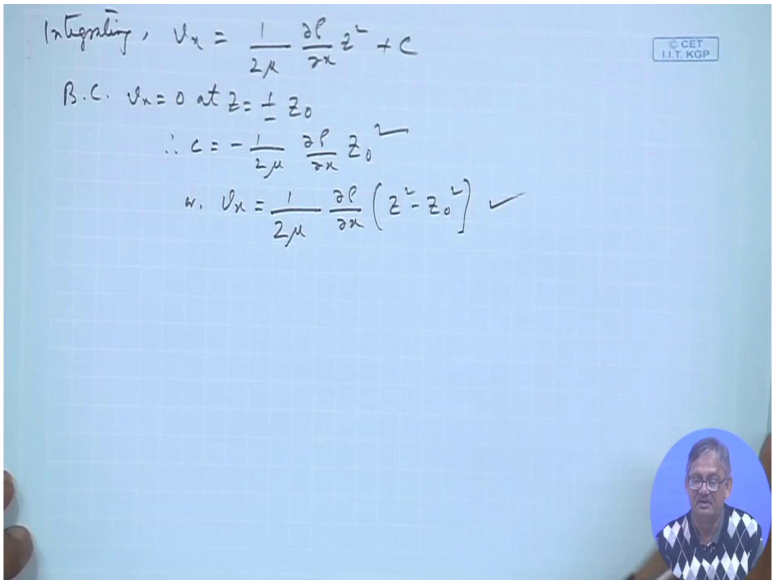


So on integration we can say on integration we can say that vx is equals to 1 by 2 mu del P del x z square plus C that is constant. So we can now put the boundary condition that is vx is equals to 0 at z is equals to plus minus z0, right? So you can write C is equals to minus 1 by 2 mu del p del x z0 square or you can say vx is equals to 1 by 2 mu del P del x z square minus z0 square, right?

So this way we can easily find out some or other problem which we come across and say this is the velocity profile for vx 1 by 2 mu del P del x z square minus z0 square, right? Then now let us move to some other situation other situation in the sense other come problem, in this manner we have problem now we let us say let us say a vertical or horizontal calandria, you know calandria? Calandria is normally said for heat transfer where evaporators or (())(11:59) they are being concentrated, right?

So you you have a concentration of one say food material or a chemical solution whatever, so one solution you have which has one concentration and you want to concentrate it to another concentrated one, right? So say for 60 percent to 80 percent you want to concentrate or 60 percent to 90 percent you want to concentrate in a in a film. So what you will do, you will you you will fall this a one side say say if you take this as a heat exchanger so one side you will make this like this one is the film which is follow and another side is your heating medium.

So if that be done that one side is film and another side is the heating medium, then while this fluid is falling from here to here when this fluid is falling from here to here that time by the time it falls from there to there it will get concentrated, right? Now, for that as as far as heat transfer is concerned you have to consider heat transfer, but for the flow of the fluid how the fluid will behave what is the velocity profile what is the stress profile shear stress profile that if we want to know then you have to consider it right from the beginning it may be a vertical film or vertical vault or vertical heating medium or it can be a inclined one.

So depending on or which can be made depending on the inclination angle it can be made vertical or horizontal, right? So let us now talk about that it may take it may not be possible to complete in one class, but okay if require we can carry over to the next class, right?

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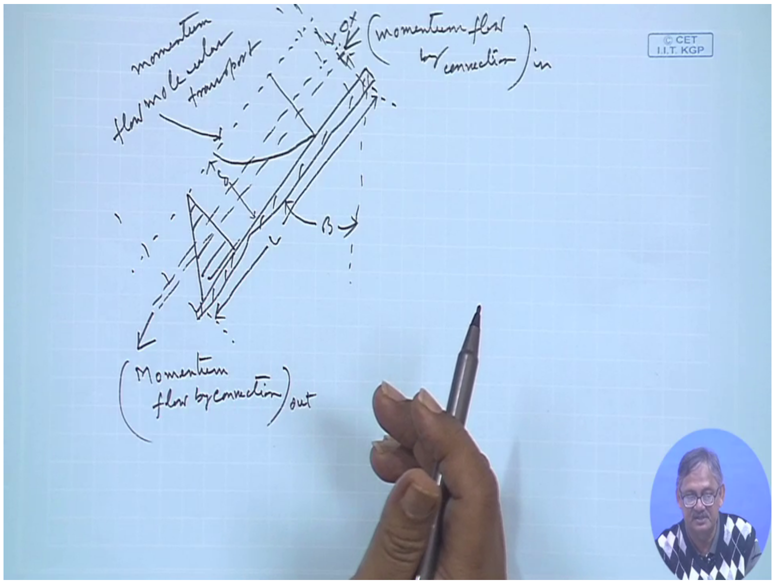


So let us now take here one such situation say we have one film we we have one solid heating surface or solid heating surface like this on which a film is there. Now this film we are assuming it to be a little assuming it to be a little expanded or in a in a microscopic view or enlarge view rather if we look into an enlarged view, then so if take a volume element here also we will do the mass balance or momentum balance rather that momentum balance on the on the volume element this volume element what we are looking into that we have here say x, y and z three coordinates if we are talking about, right?

And if this is the length, right? And if we say that okay from here from here, right? So this is the end effect without the end effect we have so you remember earlier we had said the end effect, right? We take the liquid where there is no end effect, right? And fluid is flowing this way and we do the momentum balance, right? So here momentum flow flow by convection this is out, similarly momentum flow by convection this is in, right?

So in minus out is like that and (moli) momentum flow by molecular transport so momentum flow by molecular transport that we can say this is working here, okay on the on on the on the fluid, right? Okay and say afterwards when we will look at momentum flux distribution how it is, it will look this, right? It will look like this momentum flux distribution and if we look how the velocity would like so it will look like this this is the solid surface this is the open surface that this is the film this is the film which is there or this is the film which is there, right?

So this film is moving from this angle, right? Let the angle be beta let the angle be beta like this, right? So so we can do and this length we have said and okay now the the volume element which we are taking is this as delta x, right? And this we have taken and if the thickness of the thickness of the film is del thickness of the film is del, right? So we have del x as 1 and the other one del other one is taken as L and third one may be w, right? So in this then some assumptions and some assumptions are there which we must know.

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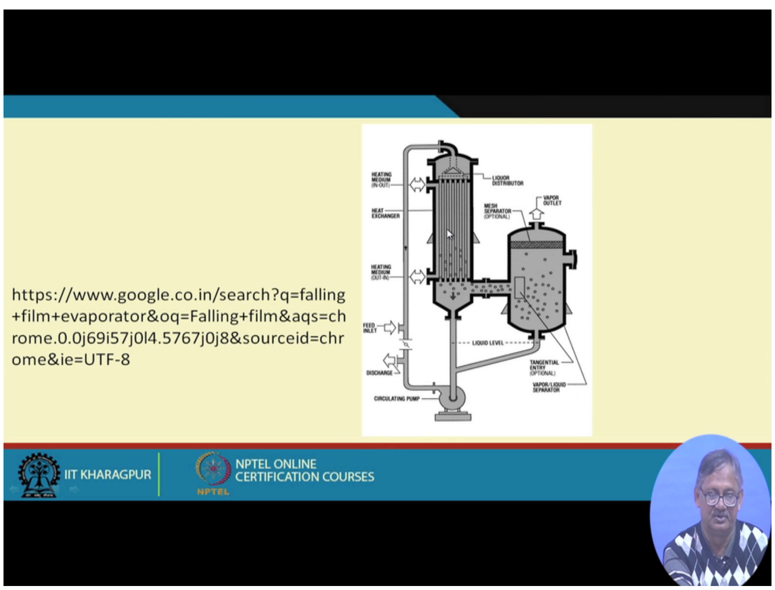


So this is a typical typical such kind of thing actual you see that fluid is flowing this is absolutely of course vertical, so a film is falling here and this film by that time it comes to the other end is getting evaporated and getting concentrated, right? So one is heating medium and other is the film which is to be concentrated, right? So from this this type of things are called calandria and through which we are getting things evaporated, okay.

Now we will do the shell momentum balance and for that we need the assumptions to be also kept in mind. Now these assumptions obviously are very very (perti) pertinent such that we can we we can actually visualize the problem described. Now the problem described is that we have a vertical plane we have a vertical heating medium and this is inclined the inclination is angle beta and we have taken a volume element where it is delta x is the thickness, right? Of the film which we are doing the shell momentum balance that shell thickness is delta x, right? We have shown that the length is L and and the other third z direction that can be as maybe w or width or whatever corresponding, right?

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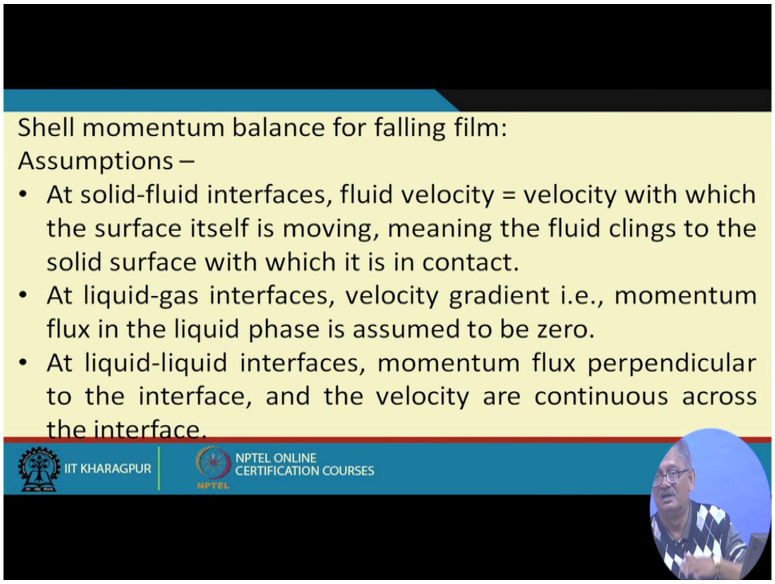


So we need certain assumptions to be kept in mind and those assumptions are like this that a solid fluid interface and fluid velocity is at the solid fluid interface fluid velocity is the velocity with which the surface itself is moving, right? This we have said earlier also that the surface on which the fluid is moving if the surface is the surface is stick or surface is fixed, then this fluid surface is also sticking or clinging to the surface of the solid, so that solid fluid interface this fluid is clinging and this is called that it is attaining the velocity of the solid since the solid is not moving so we can say that the fluid is clinging to the solid and that is what we have assumed first, right?

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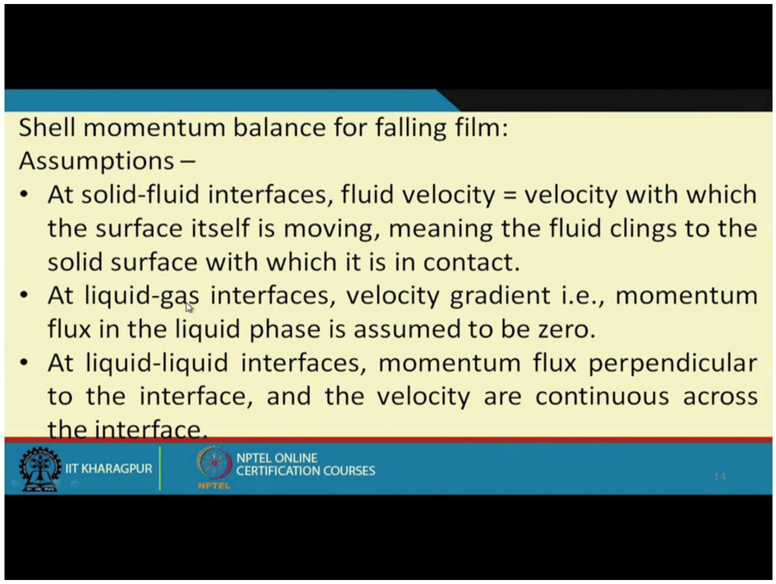


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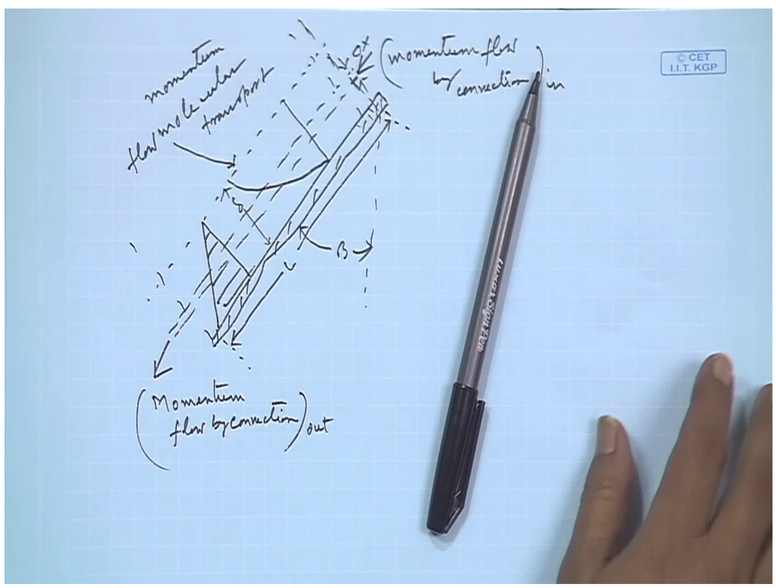


Then at liquid gas interface, now liquid gas interface is this one, right? Liquid gas interface is this one this is the liquid and above which is the gas this is the solid we said, right? This is the fluid or liquid which is getting getting heated or or concentrated.

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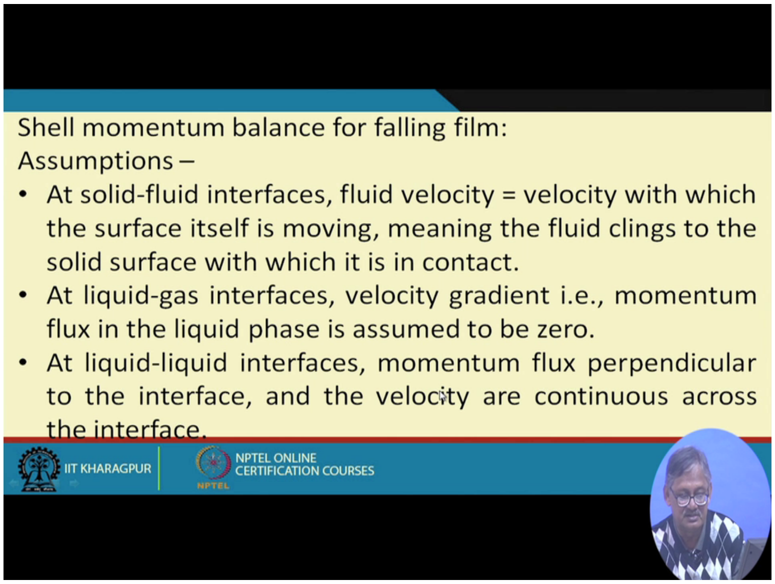


So at liquid gas interface velocity gradient that is the momentum flux in the liquid phase is assumed to be 0 at this point the velocity the velocity gradient at this point that is momentum flux in this phase is liquid phase is assumed to be 0. And at the liquid liquid interface momentum flux perpendicular to the interface and the velocity are continuous across the interface. So these three assumptions we are we are setting in the beginning so that subsequently when we will do the analysis that time we have no problem.

So I I repeat the assumptions once more that for the shell momentum balance for falling film we assume at solid fluid interface fluid velocity is the velocity with which the surface itself is moving this means the fluid clings to the surface solid surface with which it is in contact. At liquid gas interface velocity gradient that is momentum flux in the liquid phase is assumed to be 0 and at liquid liquid interfaces momentum flux perpendicular to the interface and the velocity are continuous across the interface, right?

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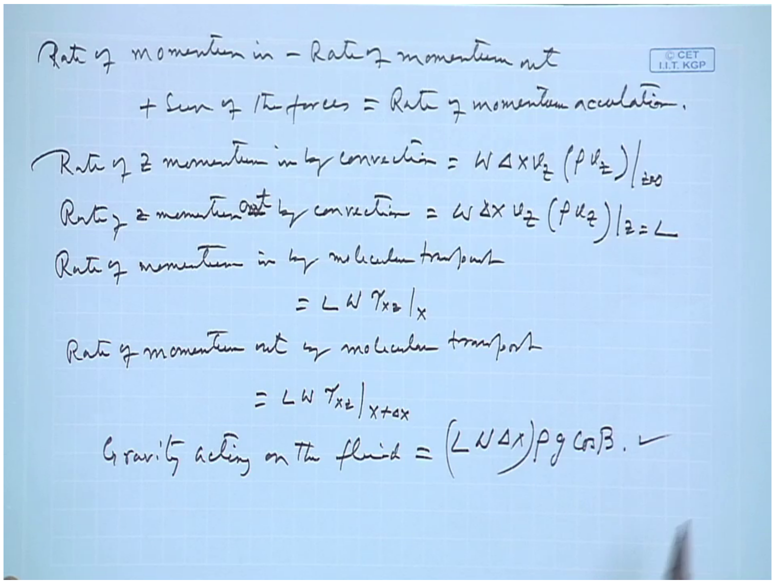


Now as earlier we have seen we have done the shell momentum balance for doing such kind of momentum analysis and here also we will do the shell momentum balance and in that we can say the governing equation is rate of momentum in minus rate of momentum out plus sum of the forces acting on the volume element this is equals to rate of momentum accumulation, right?

So this general governing equation is applicable for momentum balance applications, right? So in that case we can say that rate of z momentum in by convection is equals to w times delta x times vz into rho vz at z is equals to 0 rate of z momentum in by convection is equals to w times delta x times vz into rho vz at z is equals to L, right? This is out not in in was there this is momentum out and rate of momentum in by molecular transport this is equals to L into w into tau xz at the phase x and rate of momentum out by molecular transport that is equals to L w tau xz at the phase x plus delta x, right?

And some more gravity acting on the fluid is equals to L w delta x times rho g cos beta, right? L w delta x that is the volume element into rho whose volume is g cos beta, right? Is the gravity acting on this, right?

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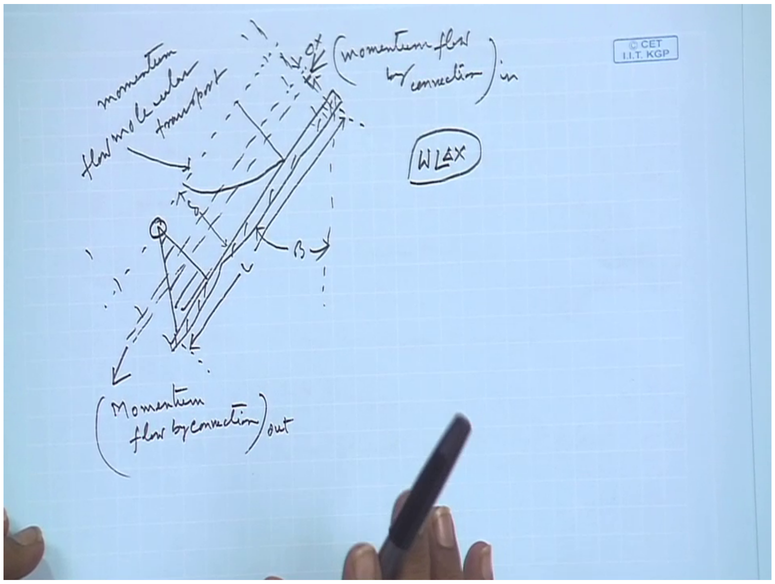


Now if this is understood I hope with this beginning we will move for what to the next but before that let us also have a look so that in the next time we do not have to so this is the actual which we have taken the volume element like this, thickness we have taken delta xz direction is this perpendicular to this direction is y which we have taken as w, right?

So our volume element has become w into L into delta x, right? So this our volume element has become and and we are doing shell momentum balance across this volume element, right? So we stop it today here, next class we will do the but keep in mind that this thing we are not going to do again, right? So next time we will do the subsequent we have already done what is momentum out molecular momentum and convective momentum in and out and the gravitational force acting on it that we have shown, right? And we have also assumed it to be a steady flow steady state so that part we will keep in mind, thank you.