

Course on Momentum Transfer in Process Engineering
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Lecture 20
Module 4
Problems and solution of falling film

Hello, let us now do some problem on the falling film we have done this falling film so let us do some problem on that and in that case we will be able to know and you will be able to find out this much better that this we have done say,



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Example: An oil is flowing down a vertical wall as a film 1.7 mm thick. The oil density is 820 kg/m³ and the viscosity is 0.2 Pa.s. Calculate the mass flow rate per unit width of wall, Reynolds no. and average velocity.

Solution:

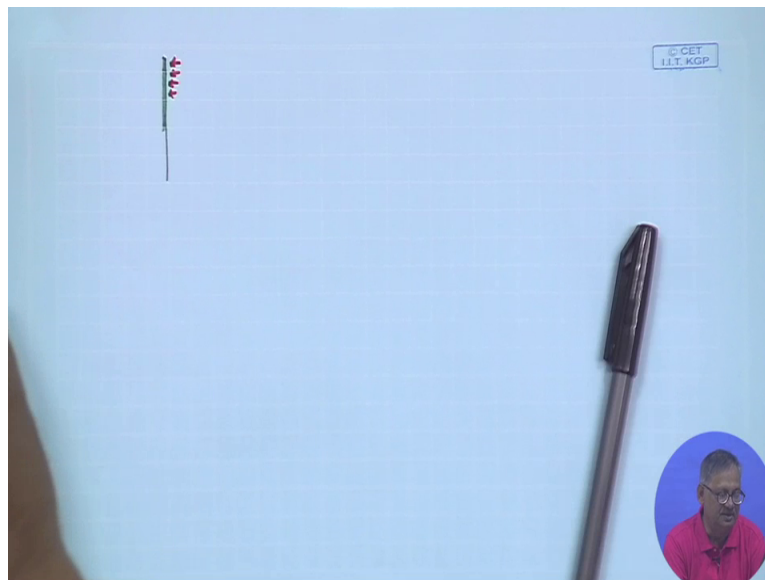
$$\dot{m} = \frac{\delta^3 \rho g}{3\nu} = \frac{(1.7 \times 10^{-3})^3 (820)(9.81)}{3 \left(\frac{0.2}{820} \right)} = 0.054 \text{ kg m}^{-1} \text{ s}^{-1}$$

$$\text{Re} = \frac{4\dot{m}}{\mu} = \frac{4 \times 0.054}{0.2} = 1.08$$

$$v_{av} = \frac{\rho g \delta^2}{3\mu} = \frac{(820)(9.81)(1.7 \times 10^{-3})^2}{3(0.2)} = 0.037 \text{ m s}^{-1}$$



An oil is flowing down a vertical wall as a film 1.7 millimeter thick the oil density is 820 kg per meter cube and the viscosity is 0.2 Pascal seconds. Calculate the mass flow rate per unit width of wall Reynolds number and average velocity. So this is a then pure case of falling film where we are saying that an oil is flowing down a vertical wall as a film 1.7 millimeter thick, so thickness of the film is also given,

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That means we have a vertical wall and down this wall of film is travelling like this a film is travelling like this, right? Obviously as we said earlier that when it is happening then there is some heating source also there so that this concentration of the oil that can go up or film can go up.

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Example: An oil is flowing down a vertical wall as a film 1.7 mm thick . The oil density is 820 kg/m³ and the viscosity is 0.2 Pa.s .calculate the mass flow rate per unit width of wall , reynolds no. and average velocity.

Solution:

$$\dot{m} = \frac{\delta^3 \rho g}{3\nu} = \frac{(1.7 \times 10^{-3})^3 (820)(9.81)}{3 \left(\frac{0.2}{820} \right)} = 0.054 \text{ kg m}^{-1} \text{ s}^{-1}$$
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$$v_{av} = \frac{\rho g \delta^2}{3\mu} = \frac{(820)(9.81)(1.7 \times 10^{-3})^2}{3(0.2)} = 0.037 \text{ m s}^{-1}$$

Density of the oil is 820 kg per meter cube and the viscosity is 0.2 Pascal second, now calculate the mass flow rate per unit width of wall, Reynolds number and average velocity, right?

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$$\text{mass flow rate } \dot{m} = \frac{\rho g^3}{3\nu} \quad \nu = \frac{\mu}{\rho}$$

$$= \frac{\frac{\text{m}^3 \text{Kg}}{\text{m}^3 \text{s}^2}}{\frac{\text{Kg m}}{\text{s}^2}} = \frac{\text{m}^3 \text{Kg}}{\text{s}^2} \cdot \frac{\text{s}^2}{\text{Kg m}} = \frac{\text{m}^2}{\text{s}}$$

$$S = 1.7 \text{ mm} = 1.7 \times 10^{-3} \text{ m}$$

$$P = 820 \text{ Kg/m}^3$$

$$\mu = 0.2 \text{ Pa.s.}$$

$$n_i = \frac{(1.7 \times 10^{-3})^3 (820) 9.81}{3 (0.2)} = 0.054 \frac{\text{Kg}}{\text{m.s}}$$

$$\text{Reynolds No. } Re = \frac{\rho v S}{\mu} = \frac{4 \times 0.054}{0.2} = 1.08 \text{ Laminar}$$

$$\text{Average velocity } v_{av} = \frac{\rho g^3}{3\mu} = \frac{(820)(9.81)(1.7 \times 10^{-3})^3}{3(0.2)} = 0.037 \text{ m/s}$$

$$25 \leq Re < 25$$

So if this is the case then we can say that mass flow rate mass flow rate as you remember it was \dot{m} and we could find out from two three relations we had, so one such relation if we use then it becomes $\frac{\rho g^3}{3 \nu}$, right? $\frac{\rho g^3}{3 \nu}$.

So this ν is equals to ν by ρ , right? So this ν equals to ν by ρ because whatever we had given earlier or we developed it was like that and if you see the unit then it comes also like that let us see this is millimeter or say meter cube and say this is kg per meter cube, this is meter per second square so this is 3 and ν is viscosity by ρ that is we can say kg per kg meter per second and density is kg per meter cube.

So this density goes out kg per meter cube, right? So meter per second this is meter cube this is meter so so it becomes per unit mass, right? so we can say this second this square goes out and this meter okay okay so it comes this kg should go up, right? This was kg per meter cube, right? So then it does not go out so I made the mistake so meter cube, right? So kg per meter cube and this is meter per second square, then this is kg meter per second and divided by kg per meter cube.

So that means this is meter cube and this is kg per meter cube square, right? Kg per meter cube square and this is meter per second square and this is kg meter per second. So one second goes out this 1 kg square goes out, 1 meter cube this goes out, right? And this meter and this meter goes out so it is kg per second, that is the flow rate, right? So if we then substitute this values we

substitute the given values given or there thickness is 1.7 millimeter, so that we can write 1.7 into 10 to the power minus 3 meter, right?

We are also given density ρ as 820 kg per meter cube we are given viscosity μ is equals to 0.2 Pascal second, so if that is true then we substitute \dot{m} is equals to Δ^3 that is 1.7 into 10 to the power minus 3 whole cube ρ 820 and g 9.81 and 3 into 0.2 Pascal second by 820, right? So this on simplification should give you 0.054, right? Kg per meter per second per unit width, right? Kg per meter per second. So kg meter square by second also this was that kg meter square per second, so 1 this was out so kg per meter second it remains, yes yes.

So 0.054 kg per meter per second is the mass flow rate, then Reynolds number Reynolds number we can say this to be equals to N_{Re} so that is $4 \dot{m}$ by μ so this Reynolds number also we had given in different forms one such is $4 \dot{m}$ by μ , right? So that is 4 into 0.054 divided by μ is 0.2 Pascal second is equals to 1.08, right? So that means it is the only laminar, right? If you remember it we said that with ripple and without ripple, right? We said with ripple and without ripple the Reynolds number if it is without ripple, then that is between less than 25, so far I remember less than 25 was the Reynolds number.

So Reynolds number if it is less than 25, than we get a okay N_{Re} less than 25 it should be and then we can get that this is laminar flow, third one was asked that what is the average velocity what is the average velocity or v average, right? What is the average velocity or v average, if you remember that we had said (11:15) this average velocity we had given into three forms, so 1 was $\rho g \Delta^2$ by 3 μ , right? 1 $g \Delta$ $\rho g \Delta^2$ by 3 μ .

So it was 820, right? Into (11:50) is 9.81, right? And Δ is 1.7 into 10 to the power minus 3 meter whole square divided by 3 into 0.2 Pascal second. So this on simplification gives 0.037 meter per second, right? So this way we can find out what is the Reynolds number, what is the velocity, what is the mass flow rate given the different things, right? So and if you remember with this we had given in all in different forms that mass flow rate also we we could can that was volumetric flow rate that was also determine then mass flow rate that was also determine. So if we look at this, then we can say that this was like that, okay.

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$$\text{mass flow rate } \dot{m} = \frac{\delta^3 \rho g}{3 \nu} \quad v = \frac{\mu}{\rho}$$

$$= \frac{\delta^3 \frac{\text{kg}}{\text{m}^3} \frac{\text{m}}{\text{s}^2}}{\frac{\text{kg m}}{\text{s}} / \text{m}^2} = \frac{\delta^3 \frac{\text{kg}}{\text{m}^3} \frac{\text{m}}{\text{s}^2}}{\frac{\text{kg m}}{\text{s}} / \text{m}^2} = \frac{\delta^3 \frac{\text{kg}}{\text{m}^3} \frac{\text{m}}{\text{s}^2}}{\frac{\text{kg m}}{\text{s}} / \text{m}^2} = \frac{\text{kg}}{\text{m s}}$$

$$\delta = 1.7 \text{ mm} = 1.7 \times 10^{-3} \text{ m}$$

$$\rho = 820 \text{ kg/m}^3$$

$$\mu = 0.2 \text{ Pa.s.}$$

$$\dot{m} = \frac{(1.7 \times 10^{-3})^3 (820) 9.81}{3 (0.2)} = 0.054 \frac{\text{kg}}{\text{m.s}}$$

$$\text{Reynolds NO. } Re = \frac{4 \delta v}{\mu} = \frac{4 \times 0.054}{0.2} = 1.08 \text{ Laminar}$$

$$\text{Average velocity } v_{av} = \frac{\rho g \delta^2}{3 \mu} = \frac{(820) (9.81) (1.7 \times 10^{-3})^2}{3 (0.2)} = 0.037 \text{ m/s}$$

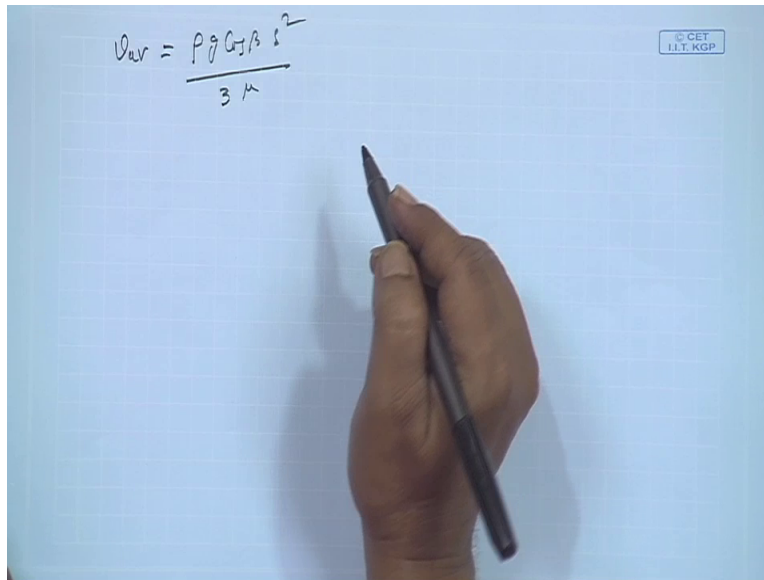
$$Re = \frac{4 \delta v_{av}}{\mu}$$

$$\delta = \sqrt{\frac{3 \mu v_{av}}{\rho g \cos \beta}} = \sqrt{\frac{3 \mu \dot{m}}{\rho^2 g \cos \beta}} = \sqrt{\frac{3 \mu \dot{m}}{\rho^2 g \cos \beta}}$$

And Reynolds number also we had given in different forms and different forms like Reynolds number we had also given as $4 \delta v_{av} \rho / \mu$ this was also one Reynolds number, right? And we also gave that in that thickness δ is equals to in different forms one was under root $3 \mu v_{av} / \rho g \cos \beta$, right? And this was also under root $3 \mu Q / \rho g w \cos \beta$.

So here it is said unit width so w will go out and this was also given as if you remember $Q / \sqrt{3 \mu m \dot{m} / \rho}$ this was this was ρ cube, right? This was ρ cube root square $g \cos \beta$, right? So this was for Reynolds number given, right? And if you remember that this Q was also given as $\rho g w \delta^2 \cos \beta / 3 \mu$, right?

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A hand holding a black marker points to a whiteboard. The whiteboard has a grid pattern and a small logo in the top right corner that reads "© CET I.I.T. KGP". The handwritten equation on the whiteboard is:

$$v_{av} = \frac{\rho g \cos \beta \delta^2}{3 \mu}$$

So that was like that and we also gave that this to be average velocity that we had also given that v average this average velocity we also had given as $\rho g (\cos \beta) \delta^2$ square by 3μ .

So all these given and we could find out the corresponding values, right? So this is what we want to make that when you have been given a given problem so depending on the requirement you find out the relation and then from that relation you can give whatever has been asked for, right? So for example here we had given a all the all the necessary things, but if normally normally the density of density of this thing oil is very very important to know in many cases, right?

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$$v_{av} = \frac{\rho g h^2}{3 \mu}$$

$S = 1.7 \times 10^{-3} \text{ m}; \mu = 0.2 \text{ Pa.s. } Nu = 16 \text{ average velocity } 0.03 \text{ m/s.}$

$\rho = ?$

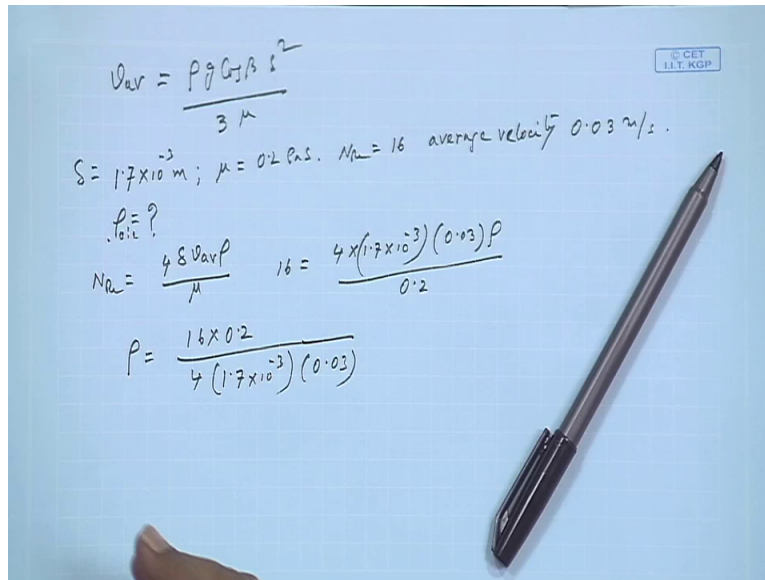
$$Nu = \frac{4 \rho v_{av} h}{\mu} \quad 16 = \frac{4 \times (1.7 \times 10^{-3}) (0.03) \rho}{0.2}$$
$$\rho = \frac{16 \times 0.2}{4 (1.7 \times 10^{-3}) (0.03)}$$

Suppose if we have the other values so we can say like that if the oil has the same problem 1.7 into 10 to the power minus 3 meter thickness of the film, right? And if we say the viscosity of the oil was say 0.2 Pascal second and if you say that if you say that the Reynolds number for the flow was say 16, right? Reynolds number for the flow was say 16, then how can we find out how can we find out v average it has say average velocity, yeah average velocity say say around 0.03 meter per second, right?

Then find out that what is the density of the oil, right? So in that case first from the from the Reynolds number value you can find out $4 \Delta v$ average, right? ρ divided by μ , right? So from this we can find out what is the value of density. So Reynolds number given is 16 $4 \Delta v$ given is 1.7 into 10 to the power minus 3 meter, right? Average velocity given 0.03 meter per second, right? So ρ we found out and viscosity 0.2 Pascal per Pascal second.

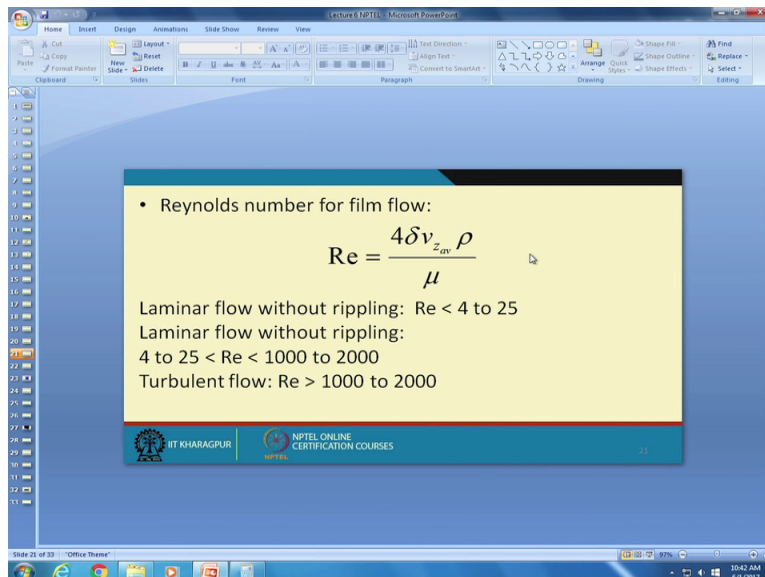
So in that case if we say then the ρ becomes equals to 16 into 0.2 over 4 into 1.7 into 10 to the power minus 3 into 0.03 so much so much your density. So let us let us just use this is a one which we need a calculator, right? So we have 16 into 0.2 divided by 4 into 1.7 into 0.03 , right? So this becomes this and so this is this times 100 this time 100 no this time 1000 , right? This times 1000 , so so much kg per meter cube then somewhere something wrong might have been done, right?

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So what we have done wrong you let us look into 4 into 1.7 into 10 to the power minus 3, right? 4 into 1.7 into 10 to the power minus 3 average velocity is given 0.03 that is what we saw meter per second density is so much, right? And and and and this viscosity is yes 0.4, so so it becomes it becomes 16 into into 0.23 let us look into that into 0.23, right?

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$$v_{av} = \frac{\rho g G \beta S^2}{3 \mu}$$
$$S = 1.7 \times 10^{-3} \text{ m}; \mu = 0.2 \text{ Pa.s. } N_{Re} = 16 \text{ average velocity } 0.03 \text{ m/s.}$$
$$\rho = ?$$
$$N_{Re} = \frac{4 v_{av} \rho}{\mu} \quad 16 = \frac{4 \times (1.7 \times 10^{-3}) (0.03) \rho}{0.2}$$
$$\rho = \frac{16 \times 0.2}{4 (1.7 \times 10^{-3}) (0.03)}$$

4 Δv average ρ by μ , so that is 16 given 4 1.7 into 10 to the power minus 3 0.03 ρ by 0.2.

So 16 into 0.2 by 4 into 1.7 into 10 to the power minus 3 into 0.03 we have not done anything wrong, so let us check the calculations once again that the where it has done wrong whether at all so it was 16 into 0.2, right? This is so much divided by it was 4 into 1.7 into 0.03, right? Into 0.03 so it becomes this much that is coming 15 and 10 to the power 3 that goes up then it is 15000 though, then where did we do anything wrong. So v average Reynolds number is so much, right? And v average we found out is given 0.3 0.03 N_{Re} is given that then where did we make the thing wrong, right?

So Reynolds number is 4 Δv average ρ by μ that is true, so if you do the other way round if you find out say Reynolds number to be found out like the previous one.

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$$\Delta_{var} = \frac{\rho g C_D \beta \delta^2}{3 \mu}$$

$\delta = 1.7 \times 10^{-3} \text{ m}; \mu = 0.2 \text{ Pa.s. } N_{Re} = 16 \text{ average velocity } 0.03 \text{ m/s.}$

$P_{oil} = ?$

$$N_{Re} = \frac{4 \delta \Delta_{var} \rho}{\mu} \quad 16 = \frac{4 \times (1.7 \times 10^{-3})^3 (0.03) \rho}{0.2}$$

$$P = \frac{16 \times 0.2}{4 (1.7 \times 10^{-3})^3 (0.03)} \quad R_{Re} = \frac{4 \times 1.7 \times 10^{-3} \times 820}{0.2}$$

$$= 27.88$$

So 4 into 1.7 into 10 to the power minus 3, right? Say here we had taken 820 and here we had taken 0.2, so under that situation it becomes equal to it becomes equal to let us look into that 4 into 1.7, right? 6.8 into 820, right? Divided by 0.2, right? So this divided by 1000 so 27.88 27.88.

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$$\Delta_{var} = \frac{\rho g C_D \beta \delta^2}{3 \mu}$$

$\delta = 1.7 \times 10^{-3} \text{ m}; \mu = 0.2 \text{ Pa.s. } N_{Re} = 16 \text{ average velocity } 0.03 \text{ m/s.}$

$P_{oil} = ?$

$$N_{Re} = \frac{4 \delta \Delta_{var} \rho}{\mu} \quad 16 = \frac{4 \times (1.7 \times 10^{-3})^3 (0.03) \rho}{0.2}$$

$$P = \frac{16 \times 0.2}{4 (1.7 \times 10^{-3})^3 (0.03)} \quad R_{Re} = \frac{4 \times 1.7 \times 10^{-3} \times 820 \times 0.03}{0.2}$$

$$= 27.88$$

$$27.88 = 4 \times 1.7 \times 10^{-5} \times$$

So that means there is some turbulence with this value, right? There is some turbulence with this value and because turbulence not turbulence there is some rippling like this as we said that some rippling like this may happen, right?

And after that when we if we take this that Reynolds number to be okay 27.88 27.88 and 4 this is 1.7 into 10 to the power minus 5 1.7 into 10 to the power minus 5, right? And average velocity we had taken how how come this we had missed one how come this we should have okay okay we had missed one, so here the viscosity velocity also should have been taken which we did not take, so that is why it was coming so so it was 0.03 so in 0.03 0.83, right?

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$$v_{av} = \frac{\rho g C \beta \delta^2}{3 \mu}$$

$$S = 1.7 \times 10^{-3} \text{ m}; \mu = 0.2 \text{ Pa.s. } Re = 16 \text{ average velocity } 0.03 \text{ m/s.}$$

$$Re = \frac{4 S v_{av} \rho}{\mu} \quad 16 = \frac{4 \times (1.7 \times 10^{-3}) (0.03) \rho}{0.2}$$

$$\rho = \frac{16 \times 0.2}{4 (1.7 \times 10^{-3}) (0.03)} \quad Re = \frac{4 \times 1.7 \times 10^{-3} \times 820 \times 0.03}{0.2}$$

$$27.88 = 4 \times 1.7 \times 10^{-5} \times$$

$$= \frac{27.88 \times 0.836}{0.836}$$

So this value is 0.83 and 4 into 1.7, right? Into 10 to the power minus 3 into 820 into 0.03, right? Divided by 0.2 so this was that divided by 1000, so values coming 0.8364 that is too low too low, however so what is needed is that generally generally for this kind of problem this density is a physical property so that to be measured somewhere with some other devices not with this kind of relations, with this kind of relations we can easily find out like (viscosity) like velocity like Reynolds number like thickness this things can be easily found out because other physical parameters they are normally determine with the devices which are known density like density as you have seen find out from the specific gravity what will.

So that value will be more accurate and from there this physical properties are normally know, similarly viscosity also can be determine as we have done in some class that how can you predict the viscosity with certain situation, right? Maybe in future also similar thing we will do and but these are based on the relations exactly, right? So try as much problems you can handle and let us stop it today here good thank you.