Course on Momentum Transfer in Process Engineering

By Professor Tridib Kumar Goswami
Department of Agricultural & Food Engineering
Indian Institute of Technology, Kharagpur

Lecture 23
Module 5
Stoke's law

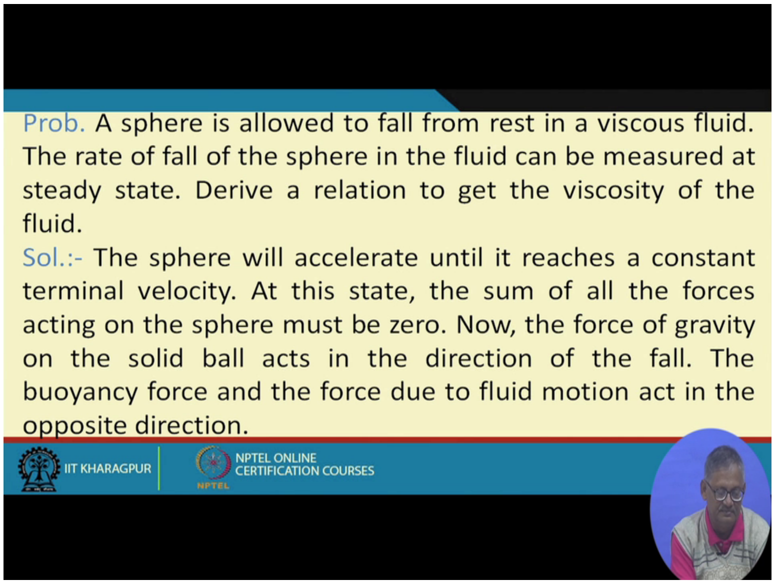
Okay, now you remember in the previous class we had developed the flow both flow and velocity as well as the shear stress profile. Now we also said that in cases we will do some problems and the moment you do some problems that will give you some confidence that yes I can solve some of the problems, right? So that if we can see say this we also said that viscosity we had developed we had determine theoretically of course in some cases that was through capillary in earlier class, right?

Now another way of finding out viscosity is that you you have seen that (())(1:22) viscometer, right? The in the very very earlier classes you have seen that we said that one one fixed container and one rotating container inner inner container was fixed and outer container was rotating and by that way the viscosity can be determine, right? And we (ex) we found some expressions for that shear stress also, okay as well as the velocity profile. And in that case we would said that yes you can develop the the theoretical relations for finding out the viscosity and same is used for flow film or we also showed through capillary tube, right?

Now another one we would like to also highlight a rest of this that is when you have a small tube through which a ball is dropped and if the ball is dropped then depending on the viscosity of the fluid through which it is flowing through you use the ball is dropping it will take the time. So if you know the time required from one height to another height how much it is taking time then from there also you can determine the viscosity this is another way of determining the viscosity, right?

So let us do one such problem by which we can determine the viscosity theoretically, this is of course based on definitely on the (buoyancy) (())(3:17) forces acting on it and the the the relation is also known in most of the cases, but still we had to find out we let us do that.

(Refer Slide Time: 3:36)

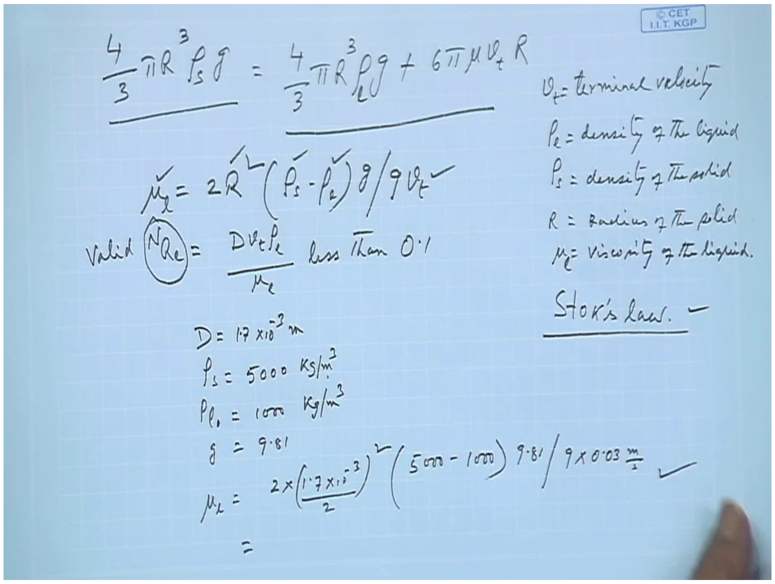


The problem is like this, a sphere is allowed to fall from rest that is when it is not moving, it is at rest is fall from a rest in a viscous fluid. The rate of fall of the sphere the rate of fall the sphere in the fluid can be measured at steady state. Derive a relation to get the viscosity of the fluid.

I repeat, a sphere is allowed to fall from rest in a viscous fluid. The rate of fall of the sphere in the fluid can be measured at steady state. Derive a relation to get the viscosity of the fluid, right? So obviously the sphere when it is falling it will accelerate until it reaches a constant velocity which is known as the terminal velocity, right? So at this state when the terminal velocity is attain some of the forces acting on the sphere must be equal to the some of the forces when acting on this sphere is equals to 0 because that time it will attain the terminal velocity, right?

And these forces are force of gravity on the solid ball acting in the direction of the fall and the buoyancy force and the force due to flow of the motion of the wall acting in the opposite direction. So these two forces when you are countering when you are accounting for, then that will give a relationship from there you can determine the viscosity, right? So let us look into the sphere will accelerate until it reaches a constant terminal velocity. At this state, the sum of all the forces acting on the sphere must be 0 and now the force of gravity on the solid ball acts in the direction of the fall and the buoyancy force and the force due to fluid motion act on the opposite direction, right?

(Refer Slide Time: 6:31)



So this we can say then then the volume of the sphere is four third pi R cube, right? And density of this sphere if it is rho s sphere and the gravity is that, so this is four third pi R cube rho s into g which is the gravity force acting on the ball and the buoyancy force which is acting on the ball will also the volume of the sphere four third pi R cube into the density of the liquid rho into g plus the 6 pi mu v turbulent velocity into R.

So the we we said the gravity force and the some of the forces acting on the it is equals to 0 so if we take all of them in one side, then becomes this minus this minus this equal to 0, right? So here that rho is the density of the solid and this rho is the density liquid of the liquid or say rho L if we write in a better way. So this is the density of the liquid, right? And vt is the terminal velocity terminal velocity, right? rho L is the density of the liquid, then rho s is the density of the sphere solid, right?

And R is the radius of the solid sphere, right? g is the acceleration due to gravity that we know, right? And mu is the viscosity of the liquid, right? So if this be true, then from this relation we can write that mu is equals to 2 R square into rho s minus rho liquid (ove) into g over 9 v terminal, right? So this mu is equals to 2 r square rho s minus rho liquid into g over 9 v terminal, right? I hope you have seen this when you have done Stoke’s law, right?

So this is nothing but Stoke’s law and from there if we know the terminal velocity if we know the density of the liquid, if we know density of the solid, if we know the diameter of the sphere, then we can find out the viscosity of the liquid, right? So this mu is rather mu L that should be said of course all it that does not have any viscosity. So this is again valid when Re or NRe Reynolds number is Dv terminal rho of the liquid by mu is less than 0.1 this is valid when the Reynolds number is less than 0.1 defined as Dv terminal rho of the liquid by mu of the liquid, so if it is less than 0.1 then we can determine that viscosity, right?

So let us can we do one just frame out one problem and do it for that you remember earlier we had given diameter of this say solid is 1.7 into 10 to the power minus 3 meter, right? And the rho of the solid depending on that the viscosity will be of course determine we had taking arbitrary. So rho of the solid that if we take to be equal to say around 50 or okay 5000 kg per meter cube arbitrary of course I am assuming rho of the liquid say water 1000 kg per meter cube this density might be too low, however rho of the liquid is that and g we know to be 9.81, right?

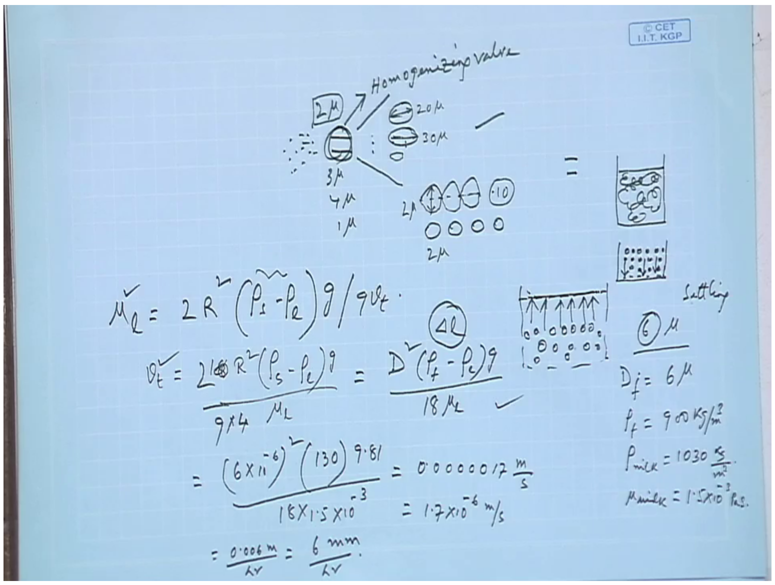
Then the viscosity that can be written as mu liquid this is 2 into 1.7 10 to the power minus 3, right? Whole square divided by 2, right? Into this is 5000 minus 1000 into g is 9.81, right? And terminal velocity let us assume to be say this is 9 into as we have seen earlier say it is 0.03 meter per second. Then if we look at this the value find out the value and then that will give you the viscosity of the liquid, right? So this way we can find out the viscosity of the liquid and obviously this is an application of the Stoke's law which we know, right?

Now let us go into another into another situation or fresh situation where we can have you you know that for homogenization that you get milk every now and then, so that milk is normally homogenized you know. So for homogenization what is done you know high pressure liquid is or milk is passed through a very small orifice that orifice determines what will be the diameter of the (())(14:50). Now first of all homogenization means that you are homogenizing fat particles, in milk if you take you (hav) you you might have seen that the fat particles comes out and and floats on the surface with time.

You you buy from the market not the homogenized milk, normal normal wherever is available fresh milk if you buy and then keep it just like that for couple of hours you will see that fat particles are coming up on the surface and they because the their sizes are big and and this from this what Stoke's law we have seen there also you find out what is the time required for the fat particles to come up, right? The same thing, okay.

So let us look into that then perhaps with the next thing would be much more easier to visualize, right? But before that let me tell the situation what happens we will do a again one problem here (als) only we will we will constitute that problem we will frame out that problem and solve it with the help of that Stoke's law which we have just defined, right? And say now what is happening in homogenization?

(Refer Slide Time: 16:32)



That a high pressure say say this is this is a slit through which a high pressure liquid is coming, right? here that milk is coming and going through this enlarge view, this size could be 2 micron or 3 micron or 4 micron or 1 micron depending on what you need at the exit of the homogenizer.

If this is the homogenizer, right? This is the point where through which homogenization is taking place called homogenizing genizing valve, right? This is called homogenizing valve. So this is another application unless we know the applications where you are using because then it becomes only only mathematics and little physics not engineering. So unless we know where you are applying for what you are developing so no I think the concept becomes not clear or the applications are not known.

So that is why in every stage I am trying to give some or other application where you can apply the things which we are being teaching, right? So when you are doing this that are the site pressure through when it is coming through this this molecules rather rather the fat globules this is called not called molecules each fat globules of different size maybe say this is say 20 micron say this is say maybe 30 micron say this is 10 micron they do path go together and come out of this slit, right? This slit diameter we assume to be said to micron, right?

So in that case when it is going through, then what happens there are both normal and shear force act on that and the molecular this fat globule that appears like this, right? And the forces acting on that then that this integrates it into depending on the size if it is 20 micron so 10 such 10 such things will be coming and each will around say 2 micron in size. So you will get membrane sorry a fat molecule like this which will come out from here of 2 micron.

So this is the application which we will do in the if possible in start with this class or maybe we will be doing in the next class depending on the time available, right? So I hope is very (()) (19:49) to to go into this before that let us also find out that the previously we had done that Stoke's law that equation if you remember that was mu, right? That was mu liquid is equals to 2 R square and rho s minus rho liquid, right? Into g divided by 9 vt, right? v terminal then becomes equals to 18 18, right? R square into rho in this case this will be whichever is higher rho liquid rho this, right? rho s minus okay rho liquid into g by mu liquid, right?

So this is that one which we know by the Stoke's law 9 into becomes no, so this not 18, so vt will be mu L comes down, right? And vt goes there. So this is 2 by 9 sorry not 18 2 by 9, right? And this is d square, right? So that is 2 2 4 R is equals to d by 2, right? d square by 4, right? So that means this becomes d to the power d square rho in this case let us write fat minus rho liquid into g over this goes out, so 18 mu L, right? So now if we say that how much time it will take what hour at what velocity this fat molecules or fat globules will come to the surface, right?

At what level this is going through the milk, right? So milk has a density so then it has also a viscosity and if this is the surface of the container if this is the container if it is the surface of the container, so at what velocity these fat globules will move we take the average fat diameter to be some value, say say 5 micron, 6 micron average, right? So 6 micron average we take and at what velocity this will come to the surface, right? If we can find out this one then this will give us some idea that why we need to go for homogenization of the milk, right?

Then we will go how the velocity profile etcetera is occurring into into this, right? So here we said that average size of the fat globules is 6 micron, density of liquid, okay then D fat is equals to 6 micron, density of liquid is liquid in this case milk rho fat is equals to say 900 kg per meter cube and density of milk is say 1030 kg per meter cube, right? And another thing is required mu milk, right? Is say earlier we had seen around it should be it should be around say 1.5 into 10 to the power minus 3 Pascal second, right?

In this case then the terminal velocity that should be D is 6 micron 6 into 10 to the power minus 6 square, right? Into rho (liq) fat minus rho liquid is take delta l this delta l means higher minus lower if rho fat is density is low, then rho liquid then it will go up and in case of if it is solid, right? Like like the same is applicable to another situation like where you have a container and you have some mud, right? This mud is there and if you allow it to keep for some time it settles down that is called settling, right? That settles down or called settling.

There also similar this mud or solids they do settle down and the rate at which it is settling is also can be predicted or can be determine with the help of this equation , right? And there you find out the terminal velocity at which it is it is dropping and and then then coming to the bottom layer, right? So here there the density of the solid is higher than the density of the liquid in this case density of the liquid fat is lighter than that of the milk.

So that is why fat globules will go up and here this will go down, so that is why delta tl delta l is taken, so depending if the turbulent velocity becomes negative, that means it is going up velocity cannot be negative, so it is going up and if it is positive that means it is going down, right? So 6 into 10 to the power minus 6 in this case if we take the delta or the mod value then it becomes 1030 minus 900 that means you have 900, 100, 1030.

So the difference is 130 and g is 9.81 and this 18 into 1.5 into 10 to the power minus 3. So this becomes equal to let us look into calculator that 6 into minus 6 6 into or 6 square rather, right? Into 130, right? Into 9.81, right? Divided by 130, right? So much what did I do, it is 6 square into 130 into 9.81 is equals to so much divided by 18 divided by 1.5, right? Divided by 10 to the power 6 and 3 9 10 to the power 6 9 that is 10 to the power 9 10 to the power 9 is equals to 0.000, 0.0000017 meter per second, right? It is so low that is 1.7 into 10 to the power 10 to the power 1.7 1, 2, 3, 4, 5, 6 10 to the power minus 6 meter per second, right?

So if we if we take in terms of meter per hour, then it will be 3600 into into 3600 so that means it will be 0.006 meter per hour 006 meter per hour that means if we take again meter that means if we take 100 or or 1000 6 millimeter per hour is the velocity 6 millimeter per hour that will travel if through this. So so you see 6 millimeter means roughly this, so distance it will take around 6 millimeter that is why you need to you need to keep it for some time so that the so that the fat globules are coming out, right?

So this is what is required, otherwise otherwise you will not be able to able to find out the meaning of the the requirement for your developing this kind of relations, right? So this this we have done, okay here this was if you would have taken this instead of 130 positive it would have been negative so that would have been the negative value of vt but velocity cannot be negative so that is why the negative sign would have applied the it is going towards the upward direction, right?

And if it would have been towards the bottom site, that is if it would have done with the help of denser denser particles then that would have come to the bottom of the container and then this 130 is would have been positive, right? And velocity indirectly would have come to the positive value, right? So this implies that why we need to know that how the homogenization is required or what is the usefulness of the thing which we are going to develop or which next we will be doing, right? That will be flow through the small orifice or flow the small fat plate between two fat plate which has which has very fine apart, right?

So there similar kind of things will happen, okay. So we will do in the next class, thank you.