

Course on Momentum Transfer in Process Engineering
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Lecture 25

Module 5

Problems and solution for flow through flat plates or slits

So we let with homogenization and we said that we will derive how the velocity profile and other profiles are getting changes or how can we find out the average velocity or (\bar{v}) (0:38) velocity or maximum velocity things like that, okay. So let us look into that so it will be like this just a minute one more yeah, so here it is we have done this, right? And then we have found out this velocity profile average velocity and we also have found out what is the relation between fanning friction factor and delta P, right? This also we have found out, right?

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The whiteboard contains the following derivations:

$$\Delta P = f \rho \frac{L}{2\delta} v_{av}^2 = 2 f \rho \left(\frac{L}{2\delta}\right) \left(\frac{D_{sl}}{2}\right) \quad \text{slit flow}$$

$$= 4 f \rho \left(\frac{L}{D}\right) v_{av}^2 \quad \text{pipe flow.} \quad f = \frac{16}{N_{Re}} \rightarrow \text{pipe}$$

$$f = \frac{24\mu}{L \rho v_{av}} = \frac{2 \times 3 \mu L v_{av}}{\rho L \rho v_{av}^2} = \frac{6\mu}{\rho v_{av}} = \frac{24}{4 \rho v_{av} \mu} = \frac{24}{N_{Re}}$$

$$N_{Re} = \frac{D v_{av} \rho}{\mu} \rightarrow \text{pipe.} \quad N_{Re} = \frac{4 \delta \rho v_{av}}{\mu} \rightarrow \text{slit.}$$

$$f = \frac{16}{N_{Re}} \quad f = \frac{24}{N_{Re}}$$

Then we can say now up to this we had finished delta P was $4 f$ instead of $4 f$ we had said let us for pipe flow delta P was $f \rho L$ by 2δ into v average, right? L by 2δ into v average, so if that was there then then what we do we take this v average square this $2 f \rho$, right? L by 2δ into v average square by 2 this way we can write that $2 f \rho L$ by 2δ and v average square by 2 , right? Similar to the pipe flow that was $4 f \rho$ then L by D then v average square by 2 , right? This was there for for delta P versus f relation, right?

Now if we substitute this for v average, then we can write f is equals to $2 \Delta P$ del by this was for pipe flow this is this is for slit, okay. So if we substitute them $2 \Delta P$ del by $L \rho v$ average square, right? Is equals to 2 into $3 \mu L v$ average del over del square $L \rho v$ average square, right? This is equals to 6μ by del rho v average which can be written equal to 24 by 4 del rho v average divided by μ , now this is equals to 24 by NRe , right?

So then for slit we see that it is 24 by NRe f is 24 by NRe for pipe we have seen it was 16 by NRe , right? So this was for pipe, so this f is related for for your slit flow this is 24 by NRe , right? So the difference is that in the pipe flow we had NRe equals to Dv average rho by μ this was for pipe flow, whereas for slit flow NRe is equals to 4 del rho v average divided by μ , right? So this is the difference this is for slit flow this is the basic difference, right? And corresponding f tend to be 16 by NRe and corresponding f tend to be 24 by NRe , right?

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Problem:- A plate heat exchanger is used to sterilize apple juice. The gap between the plates is 10 mm and of 3 m long. Assume density and viscosity of apple juice to be 1060 kg m^{-3} and 1×10^{-4} respectively. What is the average velocity and pressure drop if the Reynolds Number is 1200?

Sol.:- from the relation $N_{Re} = \frac{4\delta v_{av} \rho}{\mu}$

$$\therefore v_{av} = \frac{N_{Re} \mu}{4\delta \rho} \quad \text{here, } 2\delta = 10 \text{ mm}$$

$$v_{av} = \frac{1200 \times 10^{-4}}{2 \times 10 \times 10^{-3} \times 1060} = 0.056 \text{ m s}^{-1}$$

and, $\Delta p = \frac{3v_{av} \mu L}{\delta^2} = \frac{3 \times 0.056 \times 1 \times 10^{-3} \times 3}{(5 \times 10^{-3})^2} = 20.16 \text{ Pa}$

So once we know this, then we can do a problem like this, a plate heat exchanger is used to sterilize apple juice. The gap between the plates is 10 millimeter and of 3 meter long. Assume density and viscosity of apple juice to be $1060 \text{ kg per meter cube}$ and 1 into 10 to the power minus 4 Pascal second respectively. What is the average velocity and pressure drop if the Reynolds number is 1200 , right? So I repeat the problem this problem says that a plate heat exchanger is used to sterilize apple juice. The gap between the plates is 10 millimeter and of 3 meter long. Assume density and viscosity of apple juice to be $1060 \text{ kg per meter cube}$ and 1 into

10 to the power minus 4 Pascal second respectively, then what is the average velocity and pressure drop if the Reynolds number is 1200, right?

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$$\Delta P = f P \frac{L}{2\delta} v_{av} = 2 f P \left(\frac{L}{2\delta}\right) v_{av}$$

$$= (4 f P) \left(\frac{L}{D}\right) v_{av}$$

$$f = \frac{24 P \delta}{L \rho v_{av}^2} = \frac{2 \times 3 \mu L \rho v_{av} \delta}{\delta^2 L \rho v_{av}^2} = \frac{6 \mu}{\rho v_{av} \delta} = \frac{24}{\frac{4 \delta \rho v_{av} \delta}{\mu}} = \frac{24}{Re}$$

$$N_{Re} = \frac{D v_{av} \rho}{\mu} \rightarrow \text{Pipe} \quad N_{Re} = \frac{4 \delta \rho v_{av} \delta}{\mu} \rightarrow \text{slit}$$

$$f = \frac{16}{N_{Re}} \quad f = \frac{24}{N_{Re}}$$

$$N_{Re} = \frac{4 \delta \rho v_{av} \delta}{\mu} \quad \therefore v_{av} = \frac{N_{Re} \mu}{4 \delta \rho}$$

$$\therefore v_{av} = \frac{1200 \times 1 \times 10^{-4}}{4 \times 5 \times 1040}$$

$$2\delta = 10 \text{ mm}; \delta = 5 \text{ mm}$$

$$\mu = 1 \times 10^{-4} \text{ Pa}\cdot\text{s}$$

$$\rho = 1060 \text{ kg/m}^3$$

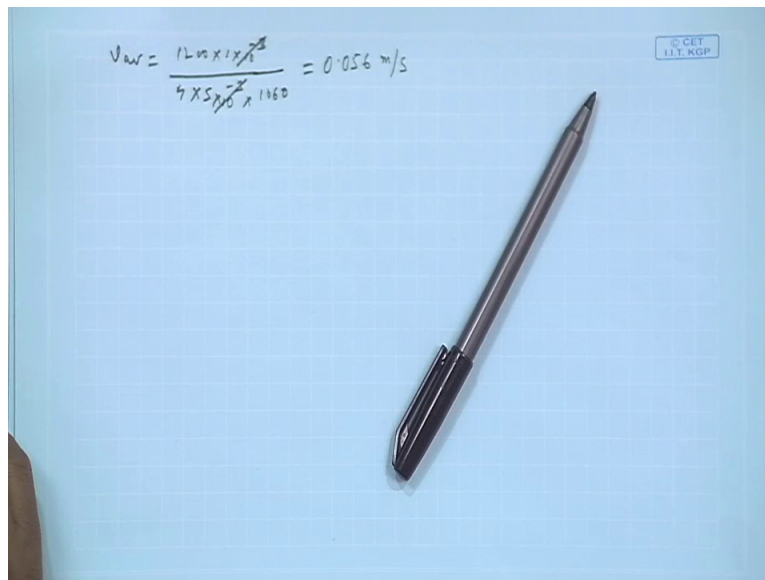
$$N_{Re} = 1200$$

Now from here we see that the Reynolds number is given solution of the problem that Reynolds number is given equals to 4 del v average into rho by mu, right? Therefore, we can write v average is equals to NRe into mu divided by 4 del sorry 4 del into rho, right? So what is given given here, 2 del is equals to 10 millimeter, right? therefore, we can write v average 2 del del is equals to 10 millimeter other property values what we have seen they are your mu is equals to 1 into 10 to the power minus 4, right? 1 into 10 to the power minus 4 Pascal seconds and rho density is given 1060 kg per meter cube, right?

So we put them and and also given NRe is equals to 1200, right? So we put NRe is 1200 into into 1 into 10 to the power minus 4 is the mu divided by 4, right? 2 del is 10 millimeter so del is equals to 5 millimeter. So 4 into 5 into rho that is 1060 so how much it comes let us look into how much it comes that is 1200 into 10 to the power minus 4 divided by 20 4 into 5 is 20 divided by 1060, right? Is this much and that divided by that divided by 10 to the power 4 so it is 5.66 5.66 10 to the power minus 6 meter per second, right?

Let us put it down, so it is okay okay if we if we change a little change a little in the sense say in the value we change this to perhaps that was a we change this value to 3, right? Then, how it is coming?

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A photograph of a blue grid background with a silver pen lying diagonally. Handwritten in black ink is the equation:
$$v_{av} = \frac{1200 \times 10^{-3}}{4 \times 5 \times 10^3 \times 1060} = 0.056 \text{ m/s}$$

How it is coming? It is coming NRe v average is equals to 1200 into 1 into 10 to the power minus 3 divided by 4 into 4 into 5, right? 5 millimeter, right?

So that we forgot 5 into 10 to the power minus 3, right? Into into 1060, right? So 10 to the power minus 3 this 10 to the power minus 3 goes out, right? These 5 fours are 20 okay let us put directly there 1200 divided by 20 divided by 1060 so it is 0.056 yeah 0.056 meter per second, right? So that is what here also we got but in the problem it was been shown 10 to the power minus 4 that that why it was coming between erroneous, okay.

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The image shows handwritten calculations on a blue grid background. The first equation is $v_{av} = \frac{120 \times 10^{-3}}{4 \times 5 \times 10^{-3} \times 1160} = 0.056 \text{ m/s}$ with a checkmark. The second equation is $\Delta P = \frac{3 v_{av} \mu L}{b} = \frac{3 \times 0.056 \times 1 \times 10^{-3} \times 3}{(5 \times 10^{-3})^2} = 20.16 \text{ Pa}$. A silver pen is visible on the right side of the grid.

Now this is average velocity, so what about delta P? delta P is $3 v$ average μL divided by del square, right? So this is 3 into 0.056 , μ is 1 into 10 to the power minus 3 and L is given 3 meter long into 3 divided by del square del is 5 into 10 to the power minus 3 square, right? So if we do this we can see that it is coming if we do this we can see that what it is coming 3 into 0.056 0.056 into 10 to the power minus 3 into 10 to the power 3 plus minus sorry sorry, I made mistake.

So 3 into 0.056 0.056 into 10 to the power minus 3 10 raise to the power 3 plus minus, so it is that into 3 into 3 this is equals to that divided by 5 into 10 to the power minus 3 10 to the power minus 3 this square, right? So that means this is 20.16 is equals to 20.16 delta P so much Pascal, right? So that is what we are also getting here, right? Though this means if we do some problems if we do some solutions, then obviously we come across with the formulae more in in more in close way and you can solve the problems and figure out what is the value this values also we should know here delta P is around 20.16 Pascal, right?

So normal pressure is 100 1 , right? 101.325 kilo Pascal, so in that case this is so much know that delta P pressure drop is not very very high, okay.



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Prob2.:- Whole milk is heated in a tubular heat exchanger having a diameter of 10 mm. The Reynolds number is 5000. The density and viscosity of whole milk is 1030 kg m^{-3} and $2.12 \times 10^{-3} \text{ Pa}\cdot\text{s}$ respectively. Calculate the average velocity of milk and pressure drop in the heat exchanger of 3 m long.

Sol.:- For pipe flow
$$v_{av} = \frac{N_{Re}\mu}{D\rho} = \frac{5000 \times 2.12 \times 10^{-3}}{10 \times 10^{-3} \times 1030} = 1.029 \text{ m s}^{-1}$$

from *Moody's diagram*, corresponding to $N_{Re} = 5000$,
friction factor is 0.011.

$$\therefore \Delta p = 4f\rho \frac{L}{D} \frac{v_{av}^2}{2} = 4 \times 0.011 \times 1030 \times \frac{3}{10 \times 10^{-3}} \times \frac{(1.029)^2}{2}$$
$$= 6.543 \text{ kPa}$$

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So let us look into another problem this problem says, whole milk is heated in a tubular heat exchanger having a diameter of 10 millimeter. The Reynolds number is 5000. The density and viscosity of whole milk is $1030 \text{ kg per meter cube}$ and $2.12 \text{ into } 10 \text{ to the power minus } 3 \text{ Pascal second}$ respectively. Calculate the average velocity of milk and pressure drop in the heat exchanger of 3 meter long.

I repeat, whole milk is heated in a tubular heat exchanger having a diameter of 10 millimeter. The Reynolds number is 5000. The density and viscosity of whole milk is $1030 \text{ kg per meter cube}$ and $2.12 \text{ into } 10 \text{ to the power minus } 3 \text{ Pascal second}$ respectively. Calculate the average velocity of milk and pressure drop in the heat exchanger of 3 meter long, right?

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$$V_{av} = \frac{120 \times 10^{-3}}{4 \times 5 \times 10^{-3} \times 1160} = 0.056 \text{ m/s} \checkmark$$

$$\Delta P = \frac{3 V_{av} \mu L}{b^2} = \frac{3 \times 0.056 \times 10^{-3} \times 3}{(5 \times 10^{-3})^2} = 20.16 \text{ Pa}$$

$$V_{av} = \frac{N_{Re} \mu}{D \rho} = \frac{5000 \times 2.12 \times 10^{-3}}{10 \times 10^{-3} \times 1030} = 1.03 \text{ m/s}$$

given,
 $D = 10 \text{ mm}$
 $N_{Re} = 5000$
 $\rho = 1030 \text{ kg/m}^3$
 $\mu = 2.12 \times 10^{-3} \text{ Pa.s.}$
 $L = 3 \text{ m}$

So the solution then this can be that average velocity this is equals to $N_{Re} \mu$ over D into ρ , right? So it had been given having a diameter of 10 millimeter, so 10 millimeter diameter is already given like the previous one where it was 2 del equal to 10 millimeter, right?

So this can be and here given are diameter D is equals to 10 millimeter, right? And Reynolds number N_{Re} is equals to 5000, right? And density equals to 1030 kg per meter cube and viscosity μ is given as 2.12 into 10 to the power minus 3 Pascal second, right? As well the heat exchanger has a length L is equals to 3 meter, right? So then from here what we can write, we can write the Reynolds number to be 5000 μ to be 2.12 into 10 to power minus 3, then D given 10 millimeter, so 10 into 10 to power minus 3 this (18:54) will give if we just tend if we did not do this change of millimeter to meter so this are be careful when being done and this is density 1030, right?

So from this we can calculate and say that how much unless you do with calculations on your you do not find out the mistakes normally we we do, right? 5000 into 2.12 2.12 into 10 to the power minus 3 10 to the power minus 3 so it is so much divided by 10 divided by 1030 divided by 10 to the power minus 3.

So this is equals to 1.03 say two lines means 3 1.03 meter per second, right? Hopefully we also got there similar. So 1.029 give a 3 meter per second, right? So now what we need to know that

corresponding this to this because here you see from the problem given what we understand that whole milk is heated with a tubular heat exchanger.

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Handwritten calculations and diagrams on a blue grid background:

- Velocity calculation: $v_{av} = \frac{1.2 \times 10^{-3}}{4 \times 5 \times 10^{-3} \times 1160} = 0.056 \text{ m/s}$ ✓
- Pressure drop calculation: $\Delta P = \frac{3 v_{av} \mu L}{b} = \frac{3 \times 0.056 \times 1 \times 10^{-3} \times 3}{(5 \times 10^{-3})} = 20.16 \text{ Pa}$
- Average velocity calculation: $v_{av} = \frac{N_{Re} \mu}{D \rho} = \frac{5000 \times 2.12 \times 10^{-3}}{10 \times 10^{-3} \times 1030} = 1.03 \text{ m/s}$
- Given parameters: $D = 10 \text{ mm}$, $N_{Re} = 5000$, $\rho = 1030 \text{ kg/m}^3$, $\mu = 2.12 \times 10^{-3} \text{ Pa}\cdot\text{s}$, $L = 3 \text{ m}$
- Pressure drop calculation using friction factor: $\Delta P = 4 f \rho \frac{L}{D} \frac{v_{av}^2}{2} = 4 \times 0.011 \times \frac{3 \times 1030}{10 \times 10^{-3}} \times \frac{(1.03)^2}{2} = 7.21 \text{ Pa}$ ✓
- Diagram: A tube of diameter D and length L with flow direction indicated. A Moody chart is shown with $f = 0.011$ and $N_{Re} = 5000$.

So one one milk tubular one one over other so like that annular which is shown in the morning so this is one heat exchanger, right? And that is another, right?

So one fluid is moving say this way in a a tubular heat exchanger it is not been said whether co current are counter current generally we take in the counter current spatial. So having a diameter of 10 millimeter so tubular heat exchanger diameter of 10 millimeter so in this it is being heated, now the Reynolds number is 5000, the density viscosity has this much, calculate the average pressure and the (velo) velocity. Now when we have taken diameter and length in the previous sections or previous classes we had said that whenever you have such a thing where N_{Re} is given you do not know the friction factor, then you find out it from the Moody's chart, right?

The Moody's chart was what like that there we log log graph, right? This f versus N_{Re} and corresponding to this epsilon by D has to be found out. So what is the epsilon by D that can be taken in this case epsilon separately not given but let us take that this epsilon by D corresponding to that and N_{Re} to be 5000 the corresponding f value you can achieve say around 0.011, right? So if assume 0.011 to be to be f value, then ΔP is equals to $4 f \rho L$ by $D v$ average square by 2, right? That is normal pipe flow, right?

And if that be true, then you can write this is 4 into 0.011 into L is given 3 and D is given 10, right? So 10 millimeter is 10 raise to minus 3 meter and v average which you have found out is 1.03 square by 2, right? So this comes equal to let us look into that calculator, so what is coming that 4 into 0.011 4 into 0.011, right? Into 3 into 3, right? Into so 4 f L okay into point 1.03 whole square into 1.03 square. So this is that divided by 10 divided by 2 divided by 10 to the power minus 3, right? 10 to the power minus 3.

So this becomes equals to 7.00194 that means 7.002 7.002, right? So much of Pascal let us look into this as 6.543 6.543, why 6.543? Because you must have done some mistake somewhere so 4 f was 0.011 into we have not taken density that is what is so 7.02 002 into 1030 if we make 7.002 into 1030 so this becomes equals to 7211 that means 7.2 kilo Pascal, right? This becomes 7.212 kilo Pascal, right? We hope there also 6.543 the reason being the difference is here we have taken 0.011, right? Whereas, in this case it has been taken 0.1.

So that is the fundamental difference why this 6.543 is coming and we have got 7.212, right? 1 point and here also we have taken 1.03 they have taken 1.029 square. So for which the value is obviously a little less, right? But the thing is like that when you have this values and when you have here you see though it was pipe flow though it was problem where it was not explicitly said that it is a pipe flow, right? It is said that whole milk is heated in a tubular heat exchanger and as I showed that tubular heat exchanger is like this, right? So in which one fluid is entering from the inner and another through the annular, so that is the annular heat exchanging is happening. So the way it is said it appears that the milk is appearing going through the inner one, right? So heating medium is not said that is a () (27:57) medium. Now when it is said it is also said that if the flow is such that N_{Re} is 5000, right? And also it is said that it is the fluid milk which has a density of 1030 and viscosity 2.12 length which it is passing is L that is 3 meter, right?

And and also viscosity 2.12 into 10 to the power minus 3 Pascal second. Now consequently it came to the mind that which shall we do since it is a simple pipe flow, so we have taken pipe flow the relation for that and in the pipe flow we could have also used Hagen–Poiseuille's equation but the moment you take a Hagen–Poiseuille equation you will see the difference between Hagen–Poiseuille and using the fanning friction there is a difference.

So 5, 10 percent difference what we will get what will (29:08) that. So we we have taken that fanning friction relation with the delta P and that fanning friction relation with the delta P the moment you put it so this becomes a simple pipe flow and the delta P with the fanning friction factor whatever is coming. So this way if you if you practice some more problems from difference sources and solve them then I hope more and more confidence level will gain and you can do and solve very easily, okay thank you.