Course on Momentum Transfer in Process Engineering
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Lecture 28
Module 6
Flow through nozzle-2

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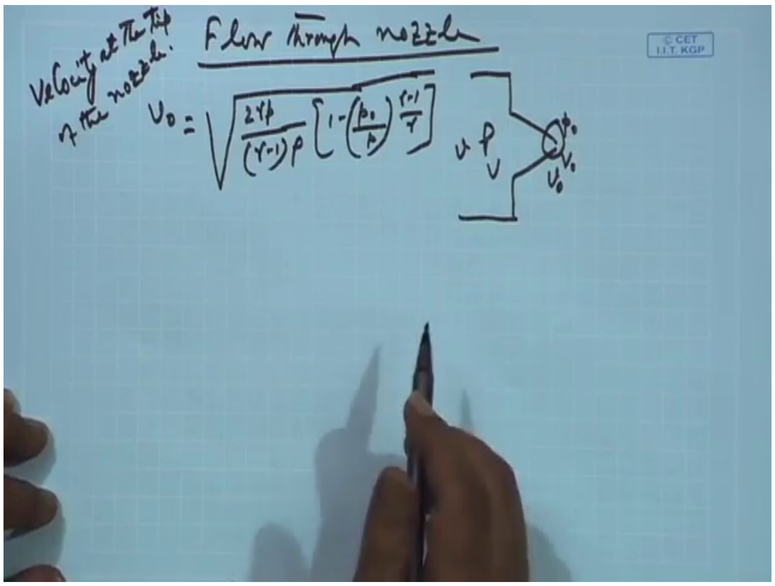

So we have seen that velocity at the tip this is flow through nozzle continuation flow through nozzle continuation that we have found out velocity at the tip we said our system was like this, right? So pressure was this volume was this specific volume was this here pressure is p0 at the tip specific volume is v0 at the tip, right? And velocity is here at v velocity here at the tip is v0. So velocity at the tip we found out to be equals to under root 2 gamma p over gamma minus 1 into rho times 1 minus p0 over p to the power gamma minus 1 by gamma, right? This was our thing.

And how did we come to know this was in terms of v by v0 so from v by v0 pV gamma is constant from there we found out what is the value of v0 by v to the power 1 minus gamma, right? That was 1 minus gamma from there we found out what is the value of p0 over p and that came to be equal to after all making after all interchanging and simplifying we got 2 gamma p by gamma minus 1 into rho in times 1 minus p0 by p to the power gamma minus 1 by gamma, right?

So this was velocity at the tip, right? And the tip of the nozzle, right? Now if we know the velocity at the tip, then we should be able to find out what is the discharge, discharge means how much how much how much fluid is flowing through the nozzle that we must be able to find out how much fluid is flowing through that nozzle, right?

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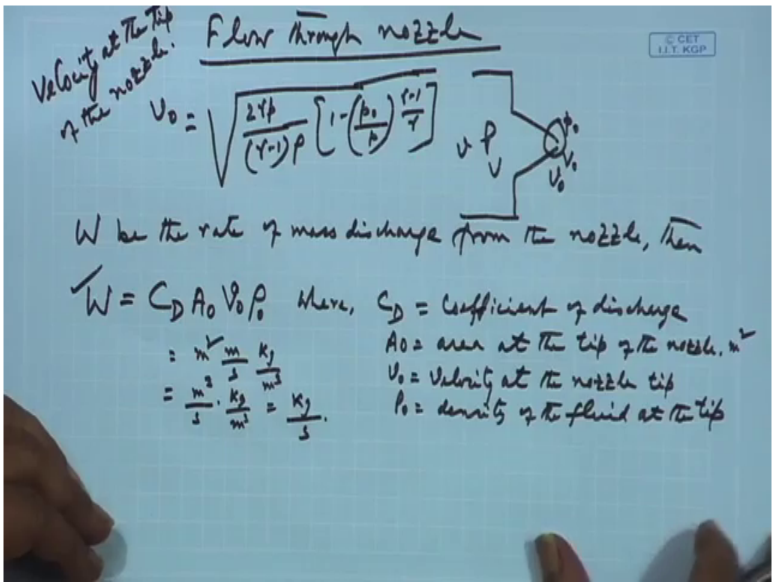

And to know that we must be find out that what is the discharge rate, right? Now if w be the rate of mass discharge if w be the rate of mass discharge from the nozzle, then we can write that w is equals to Cd into A0 into v0 into rho 0 this v is the velocity, right? Where Cd is the coefficient of discharge and this is given by the company who is making the nozzle, right? This Cd is value is given along with the nozzle who is manufacturing that manufacturer gives the value of Cd so how much you can take the value of Cd because this is coefficient of discharge that means your discharge cannot be 100 percent, right?

So how much it will be discharging depending on the manufacturer who is manufacturing he will (())(4:54) that this much of flow rate is there mass flow rate or volumetric flow rate whatever if one is given the other can also be found out. So mass flow rate or volumetric flow rate for this the discharge coefficient Cd is so much that is given by the manufacturer. So if Cd is the coefficient of discharge, A0 is the area at the tip of the nozzle at the tip of the nozzle if this is area and if we if you see that this is of course in area means meter square not meter cube by mistake maybe in the slide it was in meter cube, okay.

So area of the nozzle and Cd is that v0 is the velocity at the nozzle nozzle tip and rho 0 is the density of the fluid at the tip, right? So if we know this, then we can find out what is the discharge in Cd, right? Coefficient of discharge is a fraction so A0 is in meter square, v0 is in meter per second and rho 0 is in kg per meter cube. So this becomes equals to meter cube per second kg per meter cube so kg per second so that is the mass flow rate or w, right?

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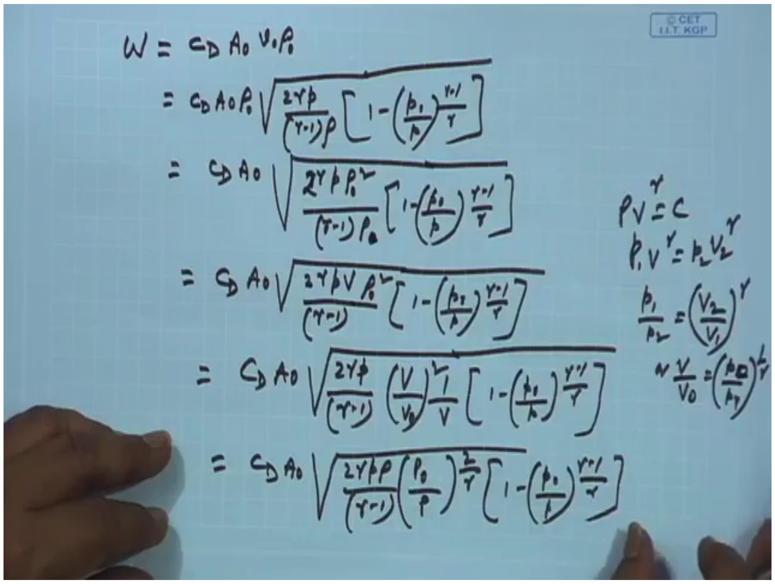

Now if w with that, then we have already found out what is the value of v0 so then w can be written as Cd A0 v0 rho 0, right? Now if we substitute v0 value of v0 then we write Cd A0, right? Into rho 0, right? Into v0 was 2 gamma p by gamma minus 1 into rho into 1 minus p0 over p to the power gamma minus 1 by gamma, right? If it was like that, then we can write now this p0 we can take this is a square so when it goes into the inside, then there is this rho 0 rather rho 0 can be also taken inside and for that this is square so it is rho 0 square, right?

So assuming this rho 0 and this is same so we can write Cd A0, right? 2 gamma 2 gamma p this is rho 0 square over gamma minus 1 into rho 0 is 1 minus p0 by p to the power gamma minus 1 by gamma, right? So this on simplification we can write Cd A0 under root 2 gamma p, right? 2 gamma p okay if we do not assume this to be rho 0, then 2 gamma p and this will goes top so gamma pV, right? By gamma minus 1 that is true and we can write this was rho 0 square, right? Into 1 minus p0 over p to the power gamma minus 1 by gamma.

So that we can simplify as Cd A0, right? Under root so this we can write as as rho, right? Means 1 by v that is 2 gamma p by gamma minus 1 so if it is rho 0, right? So rho 0 means 1 by v0, right? So it is then v by v0 square and since we have introduced one more v here so we need to write 1 by v here. So into 1 minus p0 by p this to the power gamma minus 1 by gamma, right? This is under root, right? So this we can simplify and say to be equals to Cd A0, right? 2 gamma so this v again we can bring it back to rho as 2 gamma p rho by gamma minus 1, right? And this is v by v0 so that means p0 by p, right? This is p0 by p and this is by 2 by gamma because pV gamma is equals to constant.

So p1 v1 gamma is equals to p2 v2 gamma, right? So p1 by p2 is equals to v2 by v1 to the power gamma, right? So we can write this at v by v0 is equals to or v by v0 this is equals to p1 by p2 or p2 by p1 p2 by p1 or in this case p0 by p to the power 1 by gamma, right? So it comes p0 by p to the power 2 by gamma times 1 minus p0 by p to the power gamma minus 1 by gamma, right?

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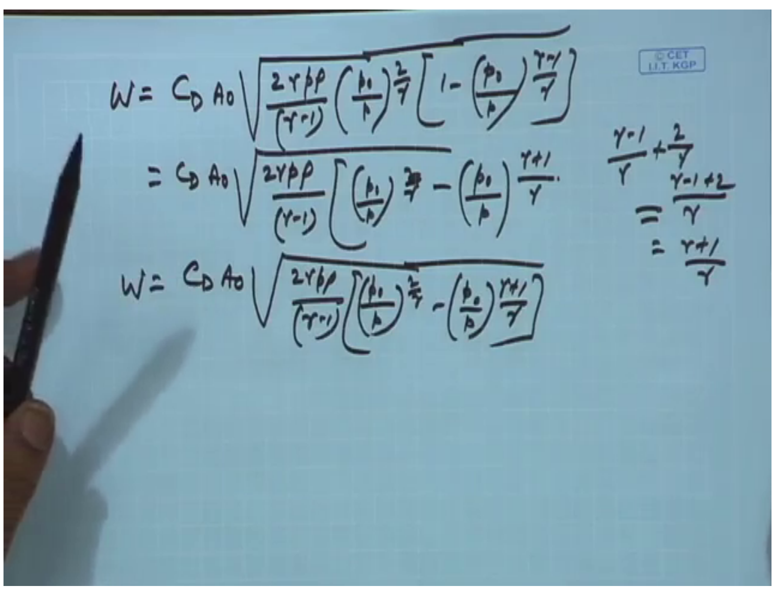

So this we can write and then we can say that this is nothing but equal to this is nothing but equal to let us keep it here so that we can see, so it is w is equals to Cd A0 into 2 what it was I am writing 2 gamma p rho by gamma minus 1, right? Into p0 by p to the power 2 by gamma into 1 minus p0 by p to the power gamma minus 1 by gamma, right? Now if we take this thing inside, then we can write Cd A0 under root 2 gamma p rho by gamma minus 1, right? And if we take this inside then it is p0 by p to the power 2 by gamma p0 by p to the power 2 by gamma minus p0 by p, right?

So we add this 2 by gamma, then it becomes p0 by p to the power gamma plus 1 by gamma. So in this case minus gamma minus 1 by gamma plus 2 by gamma, right? So it becomes equals to gamma the denominator gamma minus 1 plus 2 is equals to gamma plus 1 by gamma that is what it has come here, right? So the discharge we can we have written W as Cd A0 2 gamma p rho over gamma minus 1 into p0 by p to the power 2 by gamma minus p0 by p to the power gamma plus 1 by gamma, right? This is the discharge, right?

Now, this is normal discharge, now how could be know what is the discharge which will be maximum, right? Whatever Cd value has been given depending on that Cd value there will be a maximum discharge, so what will be that maximum discharge how can we find out, right?

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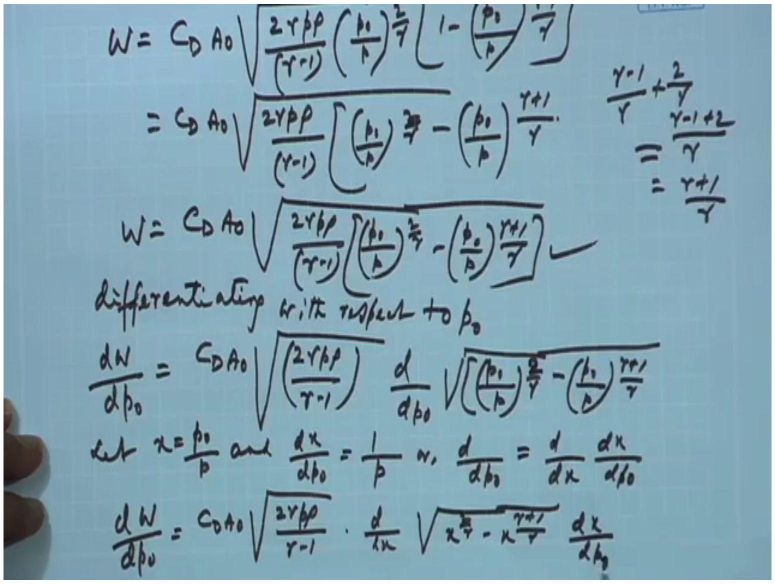

Now if we differentiate this discharge, right? With respect to pressure, so differentiating so differentiating with respect to with respect to the pressure at the tip is 0, we can write dw dp0 this is equals to Cd A0 then 2 gamma p rho by gamma minus 1 this times d dp0 of under root p0 minus p po over p to the power 2 by gamma p0 by p to the power 2 by gamma minus p0 by p to the power gamma plus 1 by gamma, right?

This, now if we do let us assume that let x equals to p0 by p and therefore dx over dp0 is equals to nothing but 1 by p and the dp0 this is equals to d dx of dx dp0, right? So therefore we can rewrite that dw dp0 this is equals to Cd A0 under root 2 gamma p rho divided by gamma minus 1 times d dx of under root x to the power 2 by gamma minus x to the power gamma plus 1 by gamma, right? This x to the power 2 by gamma by (())(18:14) gamma, right? Into dx of dp0, right? dx dp0 then, right?

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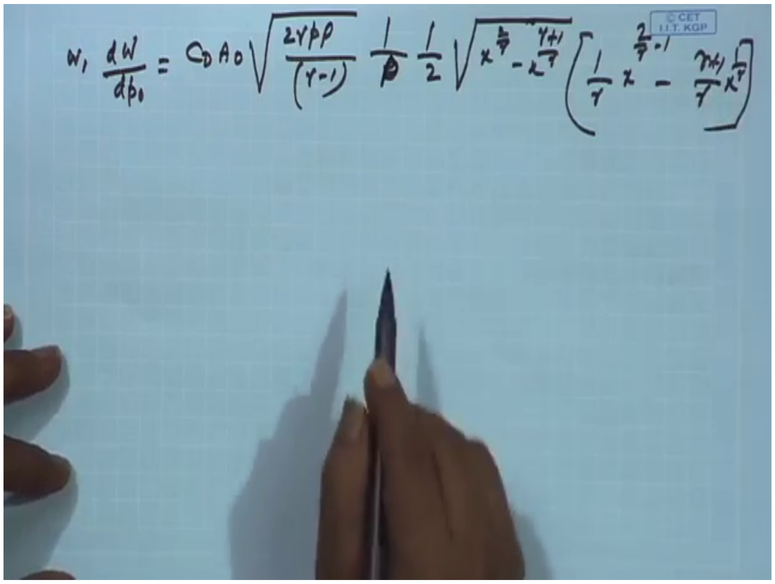

Let us keep it here, then this we can rewrite is equals to or dw dp0 this is equals to Cd A0, right? Over this under root 2 gamma p rho over gamma minus 1 into 1 by p into 1 by p into 1 by 2 into x to the power 2 by gamma 2 by gamma minus x to the power gamma plus 1 by gamma into 2 by gamma x to the power 2 by gamma minus 1 minus gamma plus 1 by gamma into x to the power 1 by gamma, right? So this is dw dp0 Cd A0 under root 2 gamma p rho by gamma minus 1 into 1 by p into 1 by 2 into x to the power 2 by gamma minus x to the power gamma plus 1 by gamma times 1 by gamma x to the power 2 by gamma minus 1 minus gamma plus 1 by gamma x to the power 1 by gamma, right? So then we write dw dp0 as this, now de the discharge will be maximum when dw dp0 becomes 0, right?

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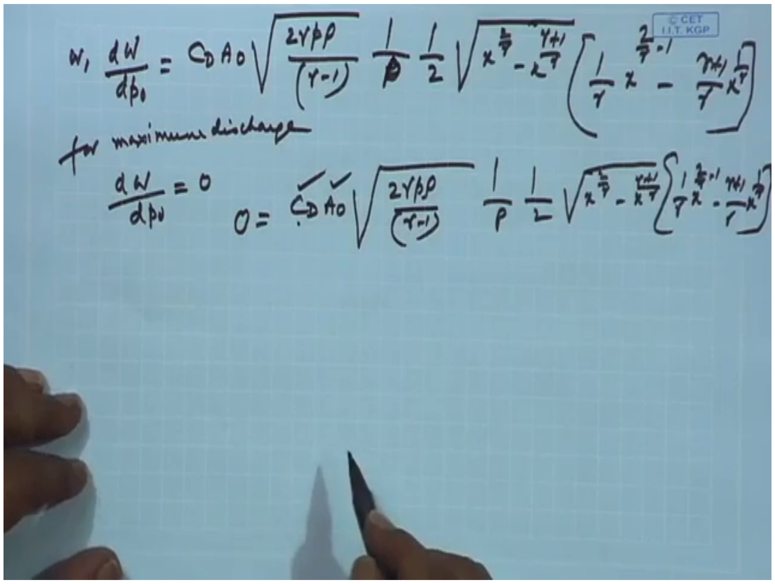

So discharge will be maximum for maximum for maximum discharge we can write for maximum discharge we can write that dw dp0 is equals to 0, right? So since dw dp0 is 0 for maximum discharge, then we can write 0 is equals to Cd A0 under root 2 gamma p rho by gamma minus 1 into 1 by p into 1 by 2 into x to the power 2 by gamma minus (ga) x to the power gamma plus 1 by gamma into 1 by gamma x to the power 2 by gamma minus 1 minus gamma plus 1 by gamma x to the power 1 by gamma, right? Now out of all these Cd cannot be 0, right? That is the coefficient of discharge, so this cannot be 0 it is already given by the company A0 area that is the area through which it is flowing that is also cannot be 0, right?

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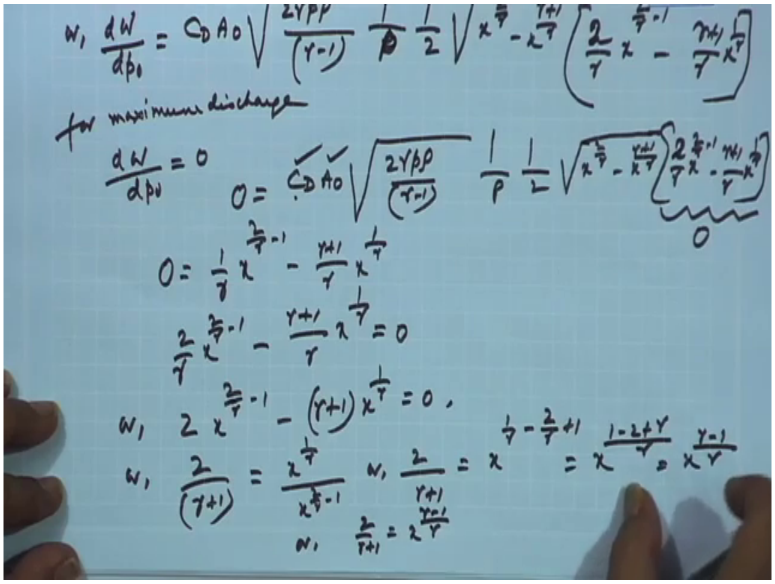

Now any one of these cannot be 0 because 2 is constant gamma is also constant there is a definite pressure there is a definite density gamma value is also fixed or specific for a given fluid. So this term cannot be 0, 1 by p since p is also having a definite value so 1 by p cannot be also 0. Similarly, 1 by 2 cannot be 0, right? Then x to the power 2 by gamma, right? x to the power 2 by gamma minus x to the power gamma plus 1 by gamma, right? If this can be 0 or 1 by gamma times x to the power 2 by gamma minus 1 minus gamma plus 1 by gamma into x to the power 1 by gamma, right?

This has to be 0 since this is under root if this is 0, then it becomes a un-imaginary so which is also not possible. So the only possibility is that this term becomes equals to 0, right? If this term becomes equal to 0, then we can write that 0 is equals to 1 by gamma x to the power 2 by gamma minus 1 minus gamma plus 1 by gamma x to the power 1 by gamma, right? So these we can rewrite this we can rewrite in terms of x and gamma as 2 by gamma x to the power 2 by gamma minus 1 minus minus gamma plus 1 by gamma x to the power 1 by gamma. So this is equals to 0, right?

So we might have missed one here, here we might have missed one this was 2 gamma d dx of this okay this was x to the power 2 by gamma dx dp0, okay so dx dp0 so when it was that, then we got this 1 by 2, right? This was not 1 by gamma this was 2 by gamma, right?

The mistake we carried over there 2 by gamma x to the power 2 by gamma minus 1 minus gamma plus 1 by gamma x to the power 1 by gamma. So it comes 2 by this cannot be 0 so this 2 by gamma this factor is equals to 0, so 2 by gamma x to the power 2 by gamma minus 1 is minus rather minus gamma minus gamma plus 1 by gamma into x to the power 1 by gamma equals to 0 this we can write or 2 x to the power 2 by gamma minus 1 minus gamma plus 1 x to the power 1 by gamma this we can rewrite rearrange, right?

So 2 by gamma x to the power 2 by gamma minus 1 minus gamma plus 1 by gamma x to the power 2 by gamma, right? This we can rewrite as this gamma this gamma goes out, right? 2 into x to the power 2 by gamma minus 1 minus gamma plus 1 into x to the power 1 by gamma this is equals to 0, right? Or we can write 2 over gamma plus 1 2 over gamma plus 1 is equals to x to the power 1 by gamma divided by x to the power 2 by gamma minus 1, right? Or we can write 2 by gamma plus 1 this is equals to x to the power 1 by gamma minus 2 by gamma plus 1, right? Is equal to x to the power 1 minus 2 plus gamma over gamma is equal to x to the power gamma minus 1 by gamma, right? Or we can write that 2 by gamma plus 1 is equals to x to the power gamma minus 1 by gamma, right?

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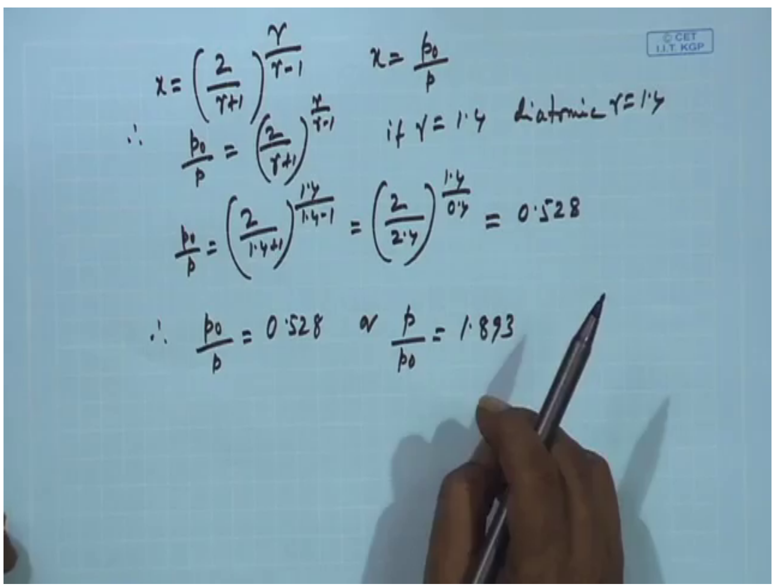

So if that be true, then we can write that x is equals to 2 by gamma plus 1 whole to the power gamma by gamma minus 1, right? This has come from here 2 by 2 gamma plus 1 is equals to x to the power gamma minus 1 by gamma, so x is equals to that it goes inverse gamma 2 by gamma plus 1 to the power gamma by gamma minus 1, right? So x is 2 by gamma plus 1 to the power gamma by gamma minus 1. Now we know x is equals to p0 by p, right? Therefore, we can write p0 by p is equals to 2 by gamma plus 1 to the power gamma by gamma minus 1, right? Now if the value of gamma given is say 1.4 for example for (diatomatic) diatomic gas like air gamma value is 1.4.

Therefore, p0 by p is equals to 2 by 1.4 plus 1 to the power 1.4 by 1.4 minus 1 this is equals to 2 by 2.4 to the power 1.4 by 0.4, right? This becomes equals to 0.528 this becomes equals to 0.528. Therefore, p0 by p is equals to 0.528 or p by p0 is equals to 1.893, right? So now we are running out of time, so let us stop here today next day next class we will begin from there, okay. Thank you.